Two-Point Correlators of Fermionic Currents in External Magnetic Field

Ilya Karabanov & Alexander Parkhomenko

P. G. Demidov Yaroslavl State University

The XXIII International Workshop
"High Energy Physics and Quantum Field Theory"
Yaroslavl, Russia, 26 June – 3 July, 2017

Outline

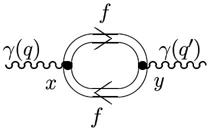
Introduction: Photon Polarization Operator

- Photon polarization operator is the typical example of the two-point correlation function
- Lagrangian density of fermion-photon interaction in QED

$$\mathcal{L}_{\rm QED}(x) = eQ_f \left[\bar{f}(x)\gamma_\mu f(x)\right] A^\mu(x)$$

- Matrix element of the $\gamma \to \gamma$ transition

$$iM_{\gamma\to\gamma} = \Pi^{\mu\nu}(q)\, \varepsilon_\mu(q)\, \varepsilon^*_\nu(q),$$



- Here, $\Pi^{\mu\nu}(q)$ is the two-point correlator of two vector currents
- In an external field, the field modification of the fermion propagator should be taken into account

Introduction: Axion Self-Energy

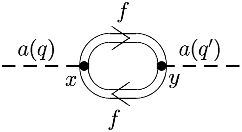
[Skobelev V. V., Phys. At. Nucl. 61 (1998); Borisov A. V. & Sizin P. E., JETP 86 (1999); Vassilevskaya L. A. et al., Phys. At. Nucl. 64 (2001)]

- Other example is the axion self-energy
- Lagrangian density of fermion-axion interaction

$$\mathcal{L}_{af}(x) = \frac{g_{af}}{2m_f}\left[\bar{f}(x)\gamma^{\mu}\gamma_5 f(x)\right]\partial_{\mu}\, a(x)$$

- $g_{af} = C_f m_f / f_a$ — dimensionless Yukawa constant
  C_f — dimensionless factor specifying the axion model
- Matrix element of $a \to a$ transition determines the electromagnetic correction to axion mass squared m_a^2

$$M_{a\to a} = -\delta m_a^2$$



- Here, δm_a^2 is the two-point correlator of two axial-vectors

Introduction: General Case of Two-Point Correlator

[Borovkov M. Yu. et al., Phys. At. Nucl. 62 (1999)]

- Lagrangian density of local fermion interaction

$$\mathcal{L}_{\rm int}(x) = \left[\bar{f}(x)\Gamma^{A} f(x)\right] J_{A}(x)$$

- J_A — generalized current (photon, neutrino current, etc.)
- Γ_A — any of γ-matrices from the set $\{1,\, \gamma_5,\, \gamma_\mu,\, \gamma_\mu\gamma_5, \sigma_{\mu\nu} = [\gamma_\mu, \gamma_\nu]/2\}$
- Two-point correlation function of general form

$$\Pi_{AB} = \int d^4 X\, \mathrm{e}^{-i(qX)}\, \mathrm{Sp}\left\{S_{\rm F}(-X)\, \Gamma_A\, S_{\rm F}(X)\, \Gamma_B\right\}$$

- $S_{\rm F}(X)$ — Lorentz-invariant part of the exact propagator
- Correlations of scalar, pseudoscalar, vector and axial-vector currents were studied
- Consider correlations of a tensor current with the other ones

Propagator in Constant Homogenious Magnetic Field

- Dirac equation in an external electromagnetic field

$$\left[i\,\hat{\partial} - e\,Q_f\,\hat{A}(\mathbf{r},t) - m_f\right]\Psi(\mathbf{r},t) = 0$$

- Q_f and m_f are the relative charge and mass of the fermion
  $\hat{\partial} = \partial_\mu\,\gamma^\mu, \qquad \hat{A} = A_\mu\,\gamma^\mu$
- Pure constant homogeneous magnetic field: $\mathbf{B} = (0, 0, B)$
- Four-potential (in Lorentz-covariant form):
  $A_\mu(x) = -F_{\mu\nu}x^\nu$
- $F_{\mu\nu}$ — strength tensor of external electromagnetic field
- Equation for fermion propagator in the magnetic field

$$\left[i\,\hat{\partial} - e\,Q_f\,\hat{A}(x) - m_f\right]G_{\mathrm{F}}(x,y) = \delta^{(4)}(x-y)$$

- Use the Fock-Schwinger method for its solution

Basic Tensors in Presence of Magnetic Field

- Minkowski space filled with external magnetic field is divided into two subspace:
  - Euclidean with metric tensor $\Lambda_{\mu\nu} = (\varphi\varphi)_{\mu\nu}$ orthogonal plane to the field direction
  - Pseudo-Euclidean with metric tensor $\tilde{\Lambda}_{\mu\nu} = (\tilde{\varphi}\tilde{\varphi})_{\mu\nu}$
  - Metric tensor of Minkowski space $g_{\mu\nu} = \tilde{\Lambda}_{\mu\nu} - \Lambda_{\mu\nu}$
- Dimensionless tensor of the external magnetic field and its dual

$$\varphi_{\alpha\beta} = \frac{F_{\alpha\beta}}{B}, \qquad \tilde{\varphi}_{\alpha\beta} = \frac{1}{2}\varepsilon_{\alpha\beta\rho\sigma}\varphi^{\rho\sigma}$$

- Arbitrary four-vector $a^\mu = (a_0, a_1, a_2, a_3)$ can be decomposed into the two orthogonal components

$$a_\mu = \tilde{\Lambda}_{\mu\nu}a^\nu - \Lambda_{\mu\nu}a^\nu = a_{\|\mu} - a_{\perp\mu}$$

- For the scalar product of two four-vectors one has

$$(ab) = (ab)_\| - (ab)_\perp$$

$$(ab)_\| = (a\tilde{\Lambda}b) = a^\mu\tilde{\Lambda}_{\mu\nu}b^\nu, \quad (ab)_\perp = (a\Lambda b) = a^\mu\Lambda_{\mu\nu}b^\nu$$

Propagator in the Fock-Schwinger Representation

- General representation of the propagator [Itzikson & Zuber]

$$G_{\rm F}(x,y) = {\rm e}^{i\Omega(x,y)}\, S_{\rm F}(x-y)$$

- Lorentz non-invariant phase factor

$$\Omega(x,y) = -eQ_f \int_y^x d\xi^\mu \left[A_\mu(\xi) + \frac{1}{2}F_{\mu\nu}(\xi-y)^\nu\right]$$

- In two-point correlation function phase factors canceled

$$\Omega(x,y) + \Omega(y,x) = 0$$

- Lorentz-invariant part of the fermion propagator ($\beta = eB|Q_f|$)

$$\begin{aligned} S_{\rm F}(X) \;&=\; -\frac{i\beta}{2(4\pi)^2}\int\limits_0^\infty \frac{ds}{s^2}\left\{(X\widetilde{\Lambda}\gamma)\cot(\beta s) - i(X\widetilde{\varphi}\gamma)\gamma_5 - \right.\\ &\quad \left. -\; \frac{\beta s}{\sin^2(\beta s)}(X\Lambda\gamma) + m_f s\,[2\cot(\beta s) + (\gamma\varphi\gamma)]\right\} \times \\ &\quad \times\; \exp\left(-i\left[m_f^2 s + \frac{1}{4s}(X\widetilde{\Lambda}X) - \frac{\beta\cot(\beta s)}{4}(X\Lambda X)\right]\right),\end{aligned}$$

Orthogonal Basis Motivated by Magnetic Field

- Correlators having rank non-equal to zero, should be decomposed in some orthogonal set of vectors
- In magnetic field, such a basis naturally exists

$$b^{(1)}_\mu = (q\varphi)_\mu, \qquad b^{(2)}_\mu = (q\tilde{\varphi})_\mu$$
$$b^{(3)}_\mu = q^2\,(\Lambda q)_\mu - (q\Lambda q)\,q_\mu, \quad b^{(4)}_\mu = q_\mu$$

- Arbitrary vector a_μ can be presented as

$$a_\mu = \sum_{i=1}^{4} a_i\,\frac{b^{(i)}_\mu}{\left(b^{(i)}b^{(i)}\right)}, \qquad a_i = a^\mu b^{(i)}_\mu$$

- Arbitrary tensor $T_{\mu\nu}$ can be similarly decomposed

$$T_{\mu\nu} = \sum_{i,j=1}^{4} T_{ij}\,\frac{b^{(i)}_\mu b^{(j)}_\nu}{\left(b^{(i)}b^{(i)}\right)\left(b^{(j)}b^{(j)}\right)}, \qquad T_{ij} = T^{\mu\nu} b^{(i)}_\mu b^{(j)}_\nu$$

Examples of Correlators

- Axion self-energy

$$M_{a\to a}(q^2, q_\perp^2, \beta) = \sum_f \frac{g_{af}^2 \beta}{8\pi^2} \int_0^\infty \frac{dt}{\sin(\beta t)} \int_0^1 du \left[q_\parallel^2 \cos(\beta t) - q_\perp^2 \cos(\beta t u)\right] \times$$
$$\times \exp\left\{-i\left[m_f^2 t - \frac{q_\parallel^2}{4} t\,(1-u^2) + q_\perp^2 \frac{\cos(\beta t u) - \cos(\beta t)}{2\beta \sin(\beta t)}\right]\right\}$$

- Two proper times variables s_1 and s_2 replaced by $t = s_1 + s_2$ and $u = (s_1 - s_2)/t$
- Field-induced contribution to the $a \to a$ transition

$$\Delta M(q^2, q_\perp^2, \beta) = M_{a\to a}(q^2, q_\perp^2, \beta) - M_{a\to a}(q^2, 0, 0),$$

- This quantity is free from UV divergences

Correlator of Pseudoscalar and Tensor Currents

- Correlator of pseudoscalar and tensor currents is rank-2 tensor
- From six non-trivial coefficients in the basis decomposition, three ones only are independent

$$\Pi_{12}^{(PT)}(q^2, q_\perp^2, \beta) = \frac{\beta}{8\pi^2}\, q_\parallel^2\, q_\perp^2 \int\limits_0^\infty dt \int\limits_0^1 du\, \frac{\sin(\beta t u)}{\sin(\beta t)}\, [u \cot(\beta t u) - \cot(\beta t)] \times$$
$$\times \exp\left\{ -i \left[m_f^2\, t - \frac{q_\parallel^2}{4}\, t\,(1-u^2) + q_\perp^2\, \frac{\cos(\beta t u) - \cos(\beta t)}{2\beta \sin(\beta t)} \right] \right\}$$

- Coefficient $\Pi_{21}^{(PT)}$ differs by the sign from $\Pi_{12}^{(PT)}$ due to the tensor anti-symmetry
- Four other coefficients $\Pi_{23}^{(PT)} = -\Pi_{32}^{(PT)}$ and $\Pi_{24}^{(PT)} = -\Pi_{42}^{(PT)}$ are also calculated
- Correlators of other currents with the tensor one are also obtained

Applications of Correlators

- Polarization operator is related with correlator of two vector currents
- Models beyond the Standard Model can effectively produce the Pauli Lagrangian density

$$\mathcal{L}_{\rm AMM}(x) = -\frac{\mu_f}{4}\left[\bar{f}(x)\sigma_{\mu\nu}f(x)\right]F^{\mu\nu}(x)$$

- After combining with the QED Lagrangian, it contributes to the photon polarization operator
- Contribution linear in the fermion AMM is related with correlator of vector and tensor currents
- Its influence on photon requires detail discussion
- Strong-field limit is also important as expressions are simplified drastically
- Good check of correctness of correlators obtained within strong-magnetic-field formalism by Skobelev

Three-Point Correlators

- Technique we are developed can be extended for calculation of three-point correlators
- We have such an experience when calculated axion-two-photon vertex in crossed and magnetic field configurations
- The ones obtained later are differs from ours but a reason remains unclear
- Some other three-point vertecies are also of special importance

Conclusions

- Two-point correlators in presence of constant homogeneous external magnetic field are considered
- This analysis extended the previous one by inclusion of tensor currents into consideration
- With new correlators, modifications to photon polarization operator induced by Pauli Lagrangian can be studied
- Computer technique developed for two-point correlators is planned to be applied for three-point ones