Extremal properties of the extended Higgs sector

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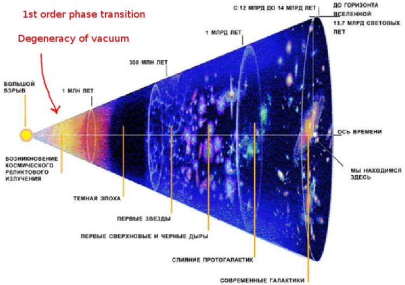
Samara, QFTHEP-2015

June 27 2015

Outline of report

* Introduction
1 Elements of catastrophe theory
2 Shape of Higgs MSSM potential
* Conditions for MSSM control parameters and Higgs masses
* Bifurcations sets in MSSM and catastrophe functions
3 Conditions for NMSSM control parameters
* Summary

Relevance of the study

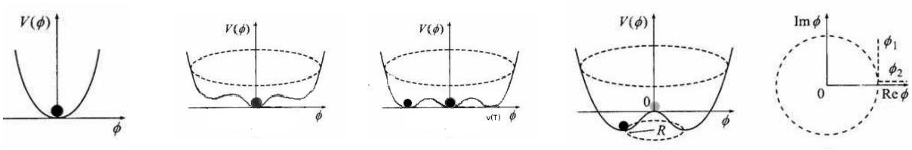


T $\sim$ 100 GeV, $SU(2)_W \times U(1)_Y$, $VEV = 0$

$\downarrow$ 1st order phase transition$\rightarrow$ SSB
$=$the interaction of nonlinearity and the SUSY potential

$T \sim 0$, $U(1)_{em}$, $VEV \neq 0$.

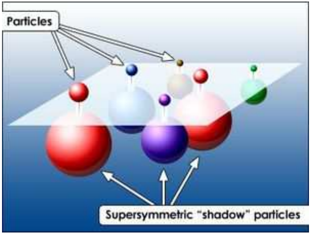
We will discribe a phase transition in term of catastrophe theory

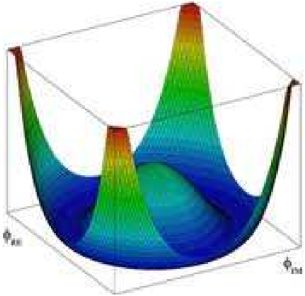


———————————— symmetry breaking dynamics ————————————

$T \gg T_c$ $\quad$ $T = T_c$ $\quad$ $T < T_c$

We study the evolution of Higgs potential shape in the framework of catastrophe theory for predicting conditions for the stable minimum existence, i.e. the true minimum, in which our Universe is expected now





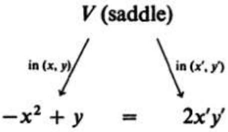
We take the effective 2HDM potential for MSSM and NMSSM with additional Higgs singlet, where the control parameters of Higgs potentials depend on the temperature.

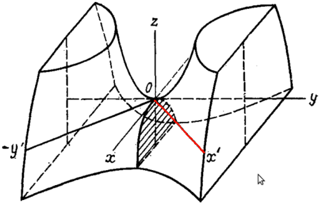
Elements of catastrophe theory

Two surfaces are **qualitatively similar** if we can find a smooth change of coordinates so that the functional form for V', expressed in terms of the new coordinates, is equal V in the original coordinate system:

$$V(x) = V'(x')$$

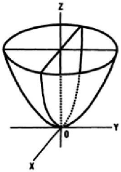
Example





We have canonical form after a smooth change of variables ($\doteq$)

Conditions	*Canonical form*	*Theorem/Lemma*
1. $\nabla V = 0$, но $det V_{ij} \neq 0$	$M_i^n = \sum_{i=1}^n \lambda_i y_i^2$	Morse Lemma
2. $\nabla V = 0$, $det V_{ij} = 0$	$f_{NM}(y_1, ..., y_l) + M_i^{n-l}$	Splitting Lemma
V "общая"	$CG(l) + M_i^{n-l}$	Thom Theorem
$k \leq 5$	$Cat(l,k) + M_i^{n-l}$	Thom Theorem



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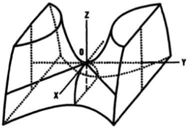

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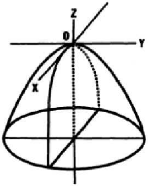


c

$$V \doteq CG(l) + \sum_{j=l+1}^{n} \lambda_j y_j^2$$

Elementary Catastrophes of Thom

Name	k	Germ	Perturbation
A_2	1	x^3	$a_1 x$
$A_{\pm 3}$	2	$\pm x^4$	$a_1 x + a_2 x^2$
A_4	3	x^5	$a_1 x + a_2 x^2 + a_3 x^3$
$A_{\pm 5}$	4	$\pm x^6$	$a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4$
A_6	5	x^7	$a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5$
D_{-4}	3	$x^2 y - y^3$	$a_1 x + a_2 y + a_3 y^2$
D_{+4}	3	$x^2 y + y^3$	$a_1 x + a_2 y + a_3 y^2$
D_5	4	$x^2 y + y^4$	$a_1 x + a_2 y + a_3 x^2 + a_4 y^2$
D_{-6}	5	$x^2 y - y^5$	$a_1 x + a_2 y + a_3 x^2 + a_4 y^2 + a_5 y^3$
D_{+6}	5	$x^2 y + y^5$	$a_1 x + a_2 y + a_3 x^2 + a_4 y^2 + a_5 y^3$
$E_{\pm 6}$	5	$x^3 \pm y^4$	$a_1 x + a_2 y + a_3 xy + a_4 y^2 + a_5 xy^2$

The two-doublet Higgs Potential for MSSM

$$U_{eff}(\phi_1,\phi_2) = -\frac{1}{2}\mu_1^2(\phi_1^\dagger\phi_1) - \frac{1}{2}\mu_2^2(\phi_2^\dagger\phi_2) - {\mu_{12}}^2(\phi_1^\dagger\phi_2) - ({\mu_{12}}^2)^*(\phi_2^\dagger\phi_1)$$

$$+\lambda_1(\phi_1^\dagger\phi_1)^2 + \lambda_2(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1)$$

$$+\frac{\lambda_5}{2}(\phi_1^\dagger\phi_2)^2 + \frac{{\lambda_5}^*}{2}(\phi_2^\dagger\phi_1)^2 + \lambda_6(\phi_1^\dagger\phi_1)(\phi_1^\dagger\phi_2) + {\lambda_6}^*(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_1) + \lambda_7(\phi_2^\dagger\phi_2)(\phi_1^\dagger\phi_2) + {\lambda_7}^*(\phi_2^\dagger\phi_2)(\phi_2^\dagger\phi_1)$$

Georgi H., Hadr. J. Phys. 1978

Lee T. D., Phys. Rev. D. 1973

Nilendra G. Deshpande, Ernest Ma, Phys.Rev. 1978

Dubinin M., Semenov A.,2004; Akhmetzyanova E.,Dolgopolov M., Dubinin M.,2005

where the vacuum expectation values

$$\langle\phi_i\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_i, \end{pmatrix}, \qquad (i = 1, 2).$$

$$v^2 = v_1^2 + v_2^2 = 246^2 \text{ GeV}^2, \qquad \tan\beta = \frac{v_2}{v_1}$$

$$\mu_i^2(T),\ \lambda_i(T),\ v_{1,2}(T)$$

Boundary conditions

On the scale of the superpartners M_{SUSY}

$$
\text{————————} m_t \text{———} \Leftarrow \text{————————————} M_{SUSY} \text{————————}
$$

The effective potential method,
the method of Feynman diagrams
& finite-temperature corrections

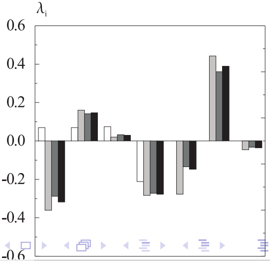
$$
\lambda_1^{SUSY} = \lambda_2^{SUSY} = \frac{g_1^2 + g_2^2}{8}, \qquad \lambda_3^{SUSY} = \frac{g_2^2 - g_1^2}{4},
$$

$$
\lambda_4^{SUSY} = -\frac{g_2^2}{2}, \qquad \lambda_5^{SUSY} = \lambda_6^{SUSY} = \lambda_7^{SUSY} = 0.
$$

The deviation from the parameters

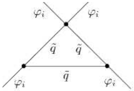
$$
\lambda_i = \lambda_i^{SUSY} - \triangle\lambda_i
$$

Dolgopolov M., Dubinin M., Rykova E. Threshold corrections to the MSSM finite-temperature Higgs potential. Journal of Modern Physics. 2011. PP.301-322



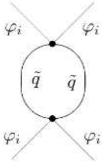
Threshold corrections (example for λ_1)

$$\lambda_i = \lambda_i^{SUSY} - \triangle\lambda_i^{th}$$



$$\triangle\lambda_1^{thr} = 3h_t^4|\mu|^4 I_2[m_Q, m_U] + 3h_b^4|A_b|^4 I_2[m_Q, m_D] + h_t^2|\mu|^2(-\frac{g_1^2 - 3g_2^2}{2} I_1[m_Q, m_U]$$

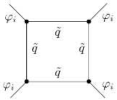
$$2g_1^2 I_1[m_U, m_Q]) + h_b^2|A_b|^2(\frac{12h_b^2 - g_1^2 - 3g_2^2}{2} I_1[m_Q, m_D] + (6h_b^2 - g_1) I_1[m_d, m|$$



$$-\triangle\lambda_1^{f} = (h_b^2 - \frac{g_1^2}{6})^2 (I(m_Q) + I(m_D)) + \frac{g_1^4}{9} I(m_U)$$

$$\triangle\lambda_1^{log} = -\frac{1}{384\pi^2}(11g_1^4 - 36h_b^2 g_1^2 + 9(g_2^4 - 4h_b^2 g_2^2 + 16h_b^4)) ln(\frac{m_Q m_U}{m_t^2})$$

where



$$I_0[m_1, m_2] = \sum_{n=-\infty, n\neq 0}^{\infty} \frac{1}{4\pi(\sqrt{4\pi^2 n^2 T^2 + m_1^2} + \sqrt{4\pi^2 n^2 T^2 + m_2^2})}$$

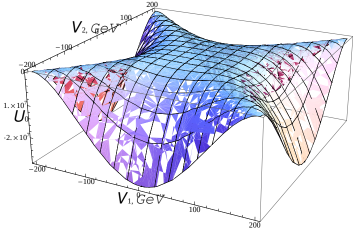
$$I_1[M_1, M_2] = -\frac{1}{64\pi^4 T^2} \sum_{n=-\infty, \neq 0}^{\infty} \frac{1}{\sqrt{M_1^2 + n^2}(\sqrt{M_1^2 + n^2} + \sqrt{M_2^2 + n^2})^2}$$

$$I_2[M_1, M_2] = \frac{1}{256\pi^5 T^4} \sum_{n=-\infty, n\neq 0}^{\infty} \frac{1}{\sqrt{(M_1^2 + n^2)(M_2^2 + n^2)}(\sqrt{M_1^2 + n^2} + \sqrt{M_2^2 + n^2})^3}$$

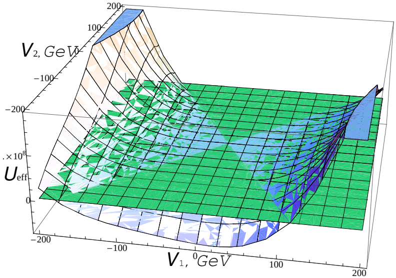
Higgs potential minima surfaces at $T = 0$

$$U_0(v_1, v_2) = -\frac{(g_1^2 + g_2^2)(v_1^2 - v_2^2)^2}{32} \quad \longrightarrow$$

Threshold corrections



$$v_1 = \pm v_2$$



Saddle

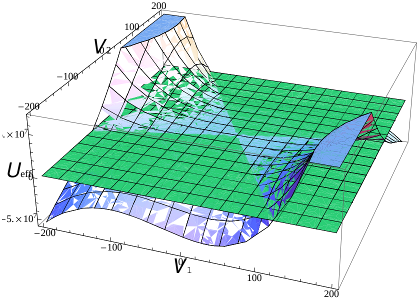
Higgs potential minima surfaces at $T \neq 0$

$$U(\psi, T) = \frac{1}{2}\Lambda_2(T)\psi^2 - ET\psi^3 + \frac{1}{4}\Lambda_4(T)\psi^4$$

Shaposhnikov criteria

$$\frac{v_c}{T_c} = \frac{2E}{\Lambda_4(T_c)} > 1$$

M.E. Shaposhnikov, JETP Lett. 44 (1986) 465



Superpartners mass parameters $m_Q = 500$ GeV, $m_U = 200$ GeV, $m_D = 800$ GeV, $T = 200$ GeV, $\mu = 500$ GeV, $A = A_t = A_b = 1200$ GeV, $\tan\beta = 5$

Local minimum conditions

In the space of (v_1, v_2) and μ_1, μ_2, μ_{12}, $\lambda_1(T), .., \lambda_7(T)$

$$U_{eff}(v_1, v_2) = -\frac{1}{2}\mu_1^2 v_1^2 - \frac{1}{2}\mu_2^2 v_2^2 - Re\mu_{12}^2 v_1 v_2 + \frac{1}{4}\lambda_1 v_1^4 + \frac{1}{4}\lambda_2 v_2^4$$

$$+\frac{1}{4}\lambda_{345} v_1^2 v_2^2 + \frac{1}{2}Re\lambda_6 v_1^3 v_2 + \frac{1}{2}Re\lambda_7 v_1 v_2^3,$$

where $\lambda_{345} = \lambda_3 + \lambda_4 + Re\lambda_5$.

1. $\mu_1^2 = -Re\mu_{12}^2 tg\beta + \lambda_1 v^2 c_\beta^2 + \frac{\lambda_{345}}{2} v^2 s_\beta^2 + \frac{3}{4} Re\lambda_6 v^2 s_{2\beta} + \frac{Re\lambda_7}{2} v^2 tg\beta s_\beta^2$

   $\mu_2^2 = -Re\mu_{12}^2 ctg\beta + \lambda_2 v^2 s_\beta^2 + \frac{\lambda_{345}}{2} v^2 c_\beta^2 + \frac{Re\lambda_6}{2} v^2 ctg\beta c_\beta^2 + \frac{3}{4} Re\lambda_7 v^2 s_{2\beta}$

   ——————————————————————————————————————

2. $Re\mu_{12}^2 = s_\beta c_\beta \left(m_A^2 - \frac{v^2}{2}(2Re\lambda_5 + Re\lambda_6 ctg\beta + Re\lambda_7 tg\beta) \right)$

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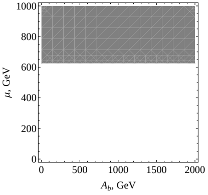
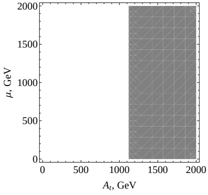
3. $DetH \geq 0$, $TrH > 0$, where $H = \partial^2 U / \partial v_i \partial v_j$:

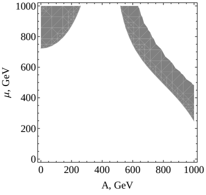
   $\lambda_1 > 0, \qquad \lambda_2 > 0, \qquad |\lambda_3 + \lambda_4 - |\lambda_5|| \leq 2\sqrt{\lambda_1 \lambda_2}$


μ, GeV
A_b, GeV

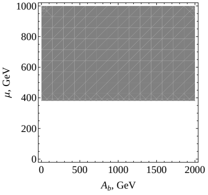
a


μ, GeV
A_t, GeV

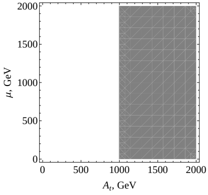
b


μ, GeV
A, GeV

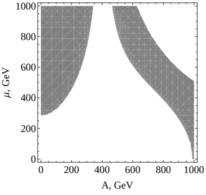
c


μ, GeV
A_b, GeV

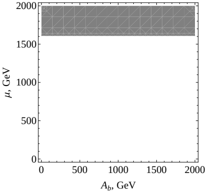
d


μ, GeV
A_t, GeV

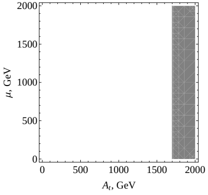
e


μ, GeV
A, GeV

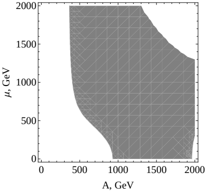
f


μ, GeV
A_b, GeV

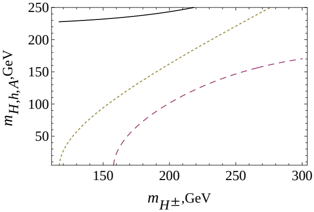
h


μ, GeV
A_t, GeV

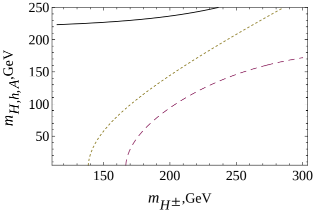
i


μ, GeV
A, GeV

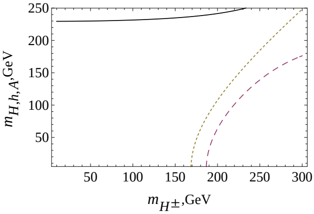
j



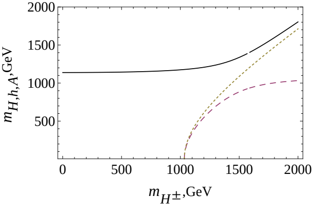
$(T = 100 \text{ GeV})$



$(T = 120 \text{ GeV})$



$(T = 150 \text{ GeV})$



$(T = 1000 \text{ GeV})$

Nonlinear transformations (Peccei–Quinn symmetry)

in first coordinate system

$$\lambda_1 v_1^3 + \frac{\lambda_{345}}{2} v_1 v_2^2 - \mu_1^2 v_1 - \mu_{12}^2 v_2 = 0$$
$$\lambda_2 v_2^3 + \frac{\lambda_{345}}{2} v_1^2 v_2 - \mu_2^2 v_2 - \mu_{12}^2 v_1 = 0$$

in new coordinate system

1. $U_{\overline{v}_1,\overline{v}_2} = \overline{\mu}_1^2 \overline{v}_1^2 + \overline{\mu}_1^2 \overline{v}_2^2$ (Morse lemma)

$$\overline{v}_1 (\lambda_1 \overline{v}_1^2 + \frac{\lambda_{345}}{2} \overline{v}_2^2 - \overline{\mu}_1^2) = 0$$
$$\overline{v}_2 (\lambda_2 \overline{v}_2^2 + \frac{\lambda_{345}}{2} \overline{v}_1^2 - \overline{\mu}_2^2) = 0$$

$$\overline{\mu}_{1,2}^2 = \frac{1}{2}\left(\mu_1^2 + \mu_2^2 \pm \sqrt{(\mu_1^2 - \mu_2^2)^2 + 4Re\mu_{12}^4}\right), \qquad \cos^2\theta = \frac{1}{2} - \frac{1}{2}\frac{|\mu_1^2 - \mu_2^2|}{\sqrt{(\mu_1^2 - \mu_2^2)^2 + 4Re\mu_{12}^4}}$$

2. $U = U_{NM}(\overline{v}_1, \overline{v}_2) + (\overline{\mu}_1^2 \overline{v}_1^2 + \overline{\mu}_2^2 \overline{v}_2^2)$ (Thom theorem)

U_{NM} – simple sprout of catastrophe A_4 or A_6

1 Bifurcation sets

№	Solutions	Hessian $H(\overline{v}_1, \overline{v}_2) =$	local minimum conditions
1	$\overline{v}_1 = 0, \quad \overline{v}_2 = 0$	$-\begin{pmatrix} \overline{\mu}_1^2 & 0 \\ 0 & \overline{\mu}_2^2 \end{pmatrix}$	$\overline{\mu}_1^2 + \overline{\mu}_2^2 < 0, \quad \overline{\mu}_1^2 \cdot \overline{\mu}_2^2 \geq 0$
2	$\overline{v}_1 = 0, \quad \lambda_2 \overline{v}_2^2 - \overline{\mu}_2^2 = 0$	$\begin{pmatrix} -\overline{\mu}_1^2 + \frac{\lambda_{345}}{2} \overline{v}_2^2 & 0 \\ 0 & 2\lambda_2 \overline{v}_2^2 \end{pmatrix}$	$-\overline{\mu}_1^2 + \overline{v}_2^2 (2\lambda_2 + \frac{1}{2}\lambda_{345}) > 0$ $(-\overline{\mu}_1^2 + \frac{1}{2}\lambda_{345}\overline{v}_2^2)\lambda_2 \overline{v}_2^2 \geq 0$
3	$\overline{v}_2 = 0, \quad \lambda_1 \overline{v}_2^2 - \overline{\mu}_1^2 = 0$	$\begin{pmatrix} 2\lambda_1 \overline{v}_1^2 & 0 \\ 0 & -\overline{\mu}_2^2 + \frac{\lambda_{345}}{2} \overline{v}_1^2 \end{pmatrix}$	$-\overline{\mu}_2^2 + \overline{v}_1^2 (2\lambda_1 + \frac{1}{2}\lambda_{345}) > 0$ $(-\overline{\mu}_2^2 + \frac{1}{2}\lambda_{345}\overline{v}_1^2)\lambda_1 \overline{v}_1^2 \geq 0$
4	$\lambda_1 \overline{v}_1^2 + \frac{\lambda_{435}}{2} \overline{v}_2^2 - \overline{\mu}_1^2 = 0,$ $\lambda_2 \overline{v}_2^2 + \frac{\lambda_{435}}{2} \overline{v}_1^2 - \overline{\mu}_2^2 = 0$	$\begin{pmatrix} 2\lambda_1 \overline{v}_1^2 & \lambda_{345} \overline{v}_1 \overline{v}_2 \\ \lambda_{345} \overline{v}_1 \overline{v}_2 & 2\lambda_2 \overline{v}_2^2 \end{pmatrix}$	$\lambda_1 \overline{v}_1^2 + \lambda_2 \overline{v}_2^2 > 0$ $\overline{v}_1^2 \overline{v}_2^2 (4\lambda_1\lambda_2 - \lambda_{345}^2) \geq 0$

2 Catastrophes

$$Cat(2;3) = \overline{v}_2^5 + a_1 \overline{v}_2 + a_2 \overline{v}_2^2 + a_3 \overline{v}_2^3$$
$$Cat(2;5) = \overline{v}_2^7 + a_1 \overline{v}_2 + a_2 \overline{v}_2^2 + a_3 \overline{v}_2^3 + \overline{v}_2^4 + \overline{v}_2^5$$

Higgs potential in NMSSM

$$U(\Phi_1, \Phi_2, S) = -\mu_1^2(\Phi_1^\dagger \Phi_1) - \mu_2^2(\Phi_2^\dagger \Phi_2) - \mu_3^2(S^\dagger S) +$$

$$+\frac{\lambda_1}{2}(\Phi_1^\dagger \Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger \Phi_2)^2 + \lambda_3(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4(\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) +$$

$$+k_1(\Phi_1^\dagger \Phi_1)(S^\dagger S) + k_2(\Phi_2^\dagger \Phi_2)(S^\dagger S) + k_3(\Phi_1^\dagger \Phi_2)(S^\dagger S^\dagger) + k_3(\Phi_2^\dagger \Phi_1)(SS) +$$

$$+k_4(S^\dagger S)^2 + k_5(\Phi_1^\dagger \Phi_2)S + k_5(\Phi_2^\dagger \Phi_1)S^\dagger + k_6 S^3 + k_6 (S^\dagger)^3$$

$$\langle \Phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \qquad \langle \Phi_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix}, \qquad \langle S \rangle = v_3.$$

J.R. Ellis, J.F. Gunion, H.E. Haber, L. Roszhkowski and F. Zwirner, Phys. Rev. D39 – 1989

$$v^2 = v_1^2 + v_2^2 = 246^2 \text{ GeV}^2$$

$$v(T),\ \lambda_i(T),\ k(T)$$

Minimum conditions, U_{eff}

$$U(v_1, v_2, v_3) = -\frac{1}{2}\mu_1^2 v_1^2 - \frac{1}{2}\mu_2^2 v_2^2 - \mu_3^2 v_3^2 + v_1^4 + \frac{\lambda_2}{8} v_2^4 + \frac{1}{4}\lambda_3 v_1^2 v_2^2 + \frac{1}{4}\lambda_4 v_1^2 v_2^2 +$$

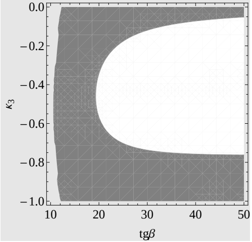
$$+\frac{1}{2}k_1 v_1^2 v_3^2 + \frac{1}{2}k_2 v_2^2 v_3^2 + k_3 v_1 v_2 v_3^2 + k_4 v_3^4 + k_5 v_1 v_2 v_3 + 2k_6 v_3^3.$$

Critical points μ_i

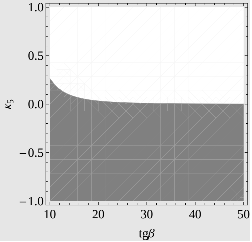
$$\mu_1^2 = \frac{v^2}{2}\,\left(\lambda_1 \cos^2\beta + (\lambda_3 + \lambda_4)\sin^2\beta\right) + k_1 v_3^2 + (k_3 v_3 + k_5)\, v_3\, \tan\beta,$$

$$\mu_2^2 = \frac{v^2}{2}\,\left(\lambda_2 \sin^2\beta + (\lambda_3 + \lambda_4)\cos^2\beta\right) + k_2 v_3^2 + (k_3 v_3 + k_5)\, v_3\, \cot\beta,$$

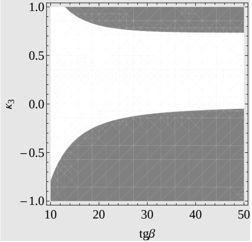
$$\mu_3^2 = \frac{v^2}{2}\,\left(k_1\, \cos^2\beta + k_2\, \sin^2\beta + k_3\, \sin 2\beta\right) + 2k_4\, v_3^2 + k_5\, \frac{v_1 v_2}{2v_3} + 3k_6 v_3.$$



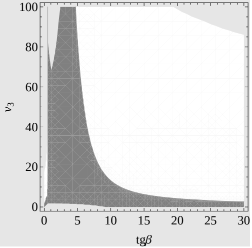
(a)



(b)



(c)



(d)

Рис.: Condition $DetH > 0$ (a, b, c, d) (Not painted areas are allowed) at $T = 100$ GeV, where a)$\forall k_{1,2,4,6}, k_5 = 1, v_3 = 1$; b) $\forall k_{1,2,3,4,6}, k_5 = -1, v_3 = 1$; c) $\forall k_{2,3,4,6}, k_1 = 1, v_3 = 1$; d) $\forall k_{2,3,4,6}, k_1 = 0, v_3 = 1$.

Summary

1 Bifurcation sets for Higgs potential at the case of Peccei–Quinn symmetry are obtain. These sets always describe system in a local minimum with a critical morse point.
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2 Constrains on MSSM and NMSSM allowed parameter space are evaluated at the presence of effective potential local minimum.
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3 Higgs prepotential as canonical morse form and non-morse term (catastrophe function at critical temperature) are reconstructed.
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Thank you for attention