# Azimuthal decorrelation of the hadronic dijets in the Regge limit of QCD

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# Outline

# Production of two high-$p_T$ jets

**Motivation**

- Multiple jet production is the dominant high transverse-momentum ($p_T$) process at LHC energies.
- Azimuthal decorrelations between the two central jets with the largest transverse momenta are sensitive to the dynamics of events with multiple jets.
- Particularly, the measurements of decorrelations in the azimuthal angle between the two most energetic jets, $\Delta\varphi$, as a function of number of produced jets, give the chance to separate directly leading order (LO) and next-to-leading orders (NLO) contributions in the strong coupling constant $\alpha_S$.
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- ATLAS Collaboration: $\sqrt{S} = 7$ TeV,$p_T > 30$ GeV, $|y_{jj}| < 1.1$
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- pQCD calculations, next-to-leading order (NLO) in three-parton production: Z. Nagy, Phys. Rev. D 68, 094002 (2003), Phys. Rev. Lett. 88, 122003 (2002).
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# Particle production at large $\sqrt{S}$

Higher-order perturbative corrections:

- Heavy final states (Higgs bosons, $t\bar{t}$, ...) produced by large-$x \sim 10^{-1}$ initial partons $\leftarrow$ soft and collinear gluons
- Light final states (small-$p_T$ quarkonia, single jets, prompt photons, ...) produced by small-$x \sim 10^{-3} \leftarrow$ additional hard jets $\leftarrow$ higher-order corrections in $\alpha_s \Rightarrow$ complicated task
- To obtain the agreement with experimental data one needs to perform the pQCD calculations in NLO order and higher $\Rightarrow$ much time and computational resources are involved.
- Even that not all the observables are successfully described in the framework of collinear parton model (CPM), such as azimuthal angle distributions in pair production of particles.

At small $x$ there are **2 dominant types of particle production kinematics: quasi-multi-Regge (QMRK) and multi-Regge (MRK)**. At such conditions imposed, the logarithms $\ln^n(1/x)$ can be resummed in all orders $n$ by BFKL-approach (see talk of V. A. Saleev) based on the property of gluon Reggeization.

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## TMD factorization

In the region of small $x \sim \mu/\sqrt{S}$ and the case whether the initial state radiation is highly separated in rapidity from the central region and can be factorized. In the small-$x$ regime, initial state partons carry the substantial transverse momentum (virtuality) $|\mathbf{q}_T| \sim x\sqrt{S}$, in contrast with the standard CPM where $|\mathbf{q}_T| \ll x\sqrt{S}$, and can be neglected.

TMD factorization theorems were formulated for a number of semi-inclusive processes including Drell-Yan processes and $e^+e^-$ annihilation, where the sensitivity to the partonic transverse momentum become important [Collins, Soper, Nadolsky, Yuan].

For particular properties in hadronic collisions, like heavy flavour or heavy boson (including Higgs) production, TMD factorization was formulated in the high-energy (small-$x$) limit. In this case the functions encoding the hadronic structure are called unintegrated parton distribution functions (uPDFs).

The large variety of different TMD PDF and uPDF parametrizations were recently collected into a single library, TMDlib [F. Hautmann *et al.*, Eur. Phys. J. C74, 3220 (2014)].

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## $k_T$-factorization approach

This is the standard setup of the $k_T$*-factorization* [L. V. Gribov *et. al.* 1983; J. C. Collins *et. al.* 1991; S. Catani *et. al.* 1991].
The standard $k_T$-factorization prescription for gluons states the polarization vector of initial-state gluon with 4-momentum $k = (k_0, \mathbf{k_T}, k_z)$: $\epsilon^\mu(k) = \frac{k_T^\mu}{|\mathbf{k_T}|}$.

The $k_T$-factorization with *Reggeized* initial-state partons which interactions are described by Fadin-Kuraev-Lipatov effective vertices we call Parton Reggeization Approach (PRA).

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# The advantages of the Parton Reggeization Approach

1. The number of matrix elements obtained using the standard prescription of the $k_t$-factorization approach for off-shell gluon polarization vectors has the incorrect parton model limits $q_{1T}, q_{2T} \to 0$, unlike the ones calculated in the PRA.

$$C_{RR}^{q\bar{q}} = \epsilon^{\alpha}(q_{1T})\epsilon^{\beta}(q_{2T})M_{\alpha\beta}(gg \to q\bar{q})$$
$$C_{RR}^{gg,\mu\nu} \neq \epsilon^{\alpha}(q_{1T})\epsilon^{\beta}(q_{2T})M_{\alpha\beta}^{\mu\nu}(gg \to gg),\ C_{RQ}^{gq,\mu} \neq \epsilon^{\alpha}(q_{1T})M_{\alpha}^{\mu}(gq \to gq)$$

2. The gauge invariance of the initial off-shell quarks is held only in the PRA.

$$C_{QR,\ a}^{qg,\ b,\ \mu}(q_1, q_2, k_1, k_2) = \frac{1}{2} g_s^2 \frac{q_2^-}{2\sqrt{t_2}} \bar{U}(k_1)\Big[\gamma_\sigma^{(-)}(q_1, k_1 - q_1)t^{-1} \times$$
$$\times\big(\gamma_{\mu\nu\sigma}(k_2, -q_2)n_\nu^+ + t_2 \frac{n_\mu^+ n_\sigma^+}{k_2^+}\big)\left[T^a, T^b\right] - \gamma^+(\hat{q}_1 - \hat{k}_2)^{-1}\gamma_\mu^{(-)}(q_1, -k_2)T^a T^b -$$
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# Unintegrated parton distribution functions

Normalization condition should be satisfied:

$$xF_g^{p,\bar{p}}(x,\mu^2) = \int^{\mu^2} \Phi_g^{p,\bar{p}}(x,|\mathbf{q_T}|^2,\mu^2)d|\mathbf{q_T}|^2,$$

where $F(x,\mu^2)$ - collinear PDF.
There are different sets of transverse-momentum-dependent PDFs:

- Balitsky-Fadin-Kuraev-Lipatov (BFKL) approach [J. Blümlein, preprint DESY 95–121 (1995) , arXiv:hep-ph/9506403]
- Kimber-Martin-Ryskin (KMR) approach [M. A. Kimber, A. D. Martin, M. G. Ryskin, G. Watt, 2001–2004].
- Ciafaloni-Catani-Fiorani-Marchesini (CCFM) approach [M. Ciafaloni, S. Catani, F. Fiorani, G. Marchesini (1988-1990)]

KMR PDFs are based on Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equations with contribution of the large logarithms $\log(\frac{\mu^2}{\Lambda^2_{QCD}})$ and include additionally (model dependent) BFKL corrections due to large logarithms $\log(\frac{S}{\mu^2}) \simeq \log(\frac{1}{x})$. The KMR procedure (realized as the open code in C++) is constructed in the way which takes into account the gluon Reggeization and so far it is clear to use it together with Reggeon effective vertices.

## UPDF sets

- The KMR prescription to obtain unintegrated PDF from collinear one is based on the mechanism of last step parton $k_T-$dependent radiation and the assumption of strong angular ordering:

$$\Phi_g(x, k_T^2, \mu^2) = T_g(k_T, \mu) \frac{\alpha_s(k_T^2)}{2\pi} \int_x^{1-\Delta} dz \int^{k_T^2} \frac{dq_T^2}{q_T^2} \times$$
$$\times \left[ P_{gg}(z) f_g\left(\frac{x}{z}, q_T^2\right) + P_{gq}(z) f_q\left(\frac{x}{z}, q_T^2\right)\right].$$

Where $P_{gg}(z)$, $P_{gq}(z)$- DGLAP splitting functions, $T_g(k_T, \mu)$- Sudakov formfactor.

- Blümlein distribution: the $q_T$-dependent gluon distribution is calculated numerically accounting for the resummation of small x effects due to the BFKL equation. It is represented by a convolution of a gluon density in the collinear limit $g(x, \mu^2)$ and a universal function $\mathcal{G}(x, q_T^2, \mu^2)$ for which an analytic expression is derived:

$$\Phi(x, q_T^2, \mu^2) = \mathcal{G}(x, q_T^2, \mu^2) \otimes g(x, \mu^2)$$

- The TMDlib [F. Hautmann *et al.*, Eur. Phys. J. C74, 3220 (2014)] includes about 20 parametrizations, mostly based on CCFM evolution equations, both for quarks and gluons. They depend on two light-cone momentum fractions $x^+$ and $x^-$ carried by the parton.

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# Hard scattering matrix elements

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polarization vector of initial-state gluon with 4-momentum $k = (k_0, \mathbf{k_T}, k_z)$:
$\epsilon^\mu(k) = \frac{k_T^\mu}{|\mathbf{k_T}|} \Rightarrow$ no gauge-invariance in the case of gluons in final state; no generally accepted prescription for the treatment of off-shell initial-state quarks.

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## Parton Reggeization Approach

**The Regge limit of QCD: the center-of-mass energy is large $\sqrt{S} \to \infty$ and the momentum transfer $\sqrt{-t}$ is fixed**

We propose the c.m. energy of LHC $\sqrt{S} = 7$ TeV to be large enough, and the finiteness of $t$ is controlled by fixed $p_T$ of final jets.

The most appropriate approach for the description of scattering amplitudes is given by the theory of complex angular momenta (Gribov-Regge theory)

The Regge kinematics is a particular case of **multi-Regge kinematics (MRK)**. MRK is the kinematics where all particles have limited (not growing with s) transverse momenta and are combined into jets with limited invariant mass of each jet and large (growing with $s$) invariant masses of any pair of the jets. The MRK gives dominant contributions to cross sections of QCD processes at high energy.

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**The Regge limit of QCD: the center-of-mass energy is large $\sqrt{S} \to \infty$ and the momentum transfer $\sqrt{-t}$ is fixed**
We propose the c.m. energy of LHC $\sqrt{S} = 7$ TeV to be large enough, and the finiteness of $t$ is controlled by fixed $p_T$ of final jets.

The most appropriate approach for the description of scattering amplitudes is given by the theory of complex angular momenta (Gribov-Regge theory)

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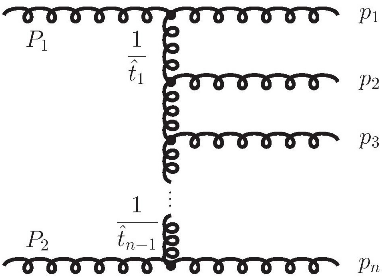
# Parton Reggeization Approach: multi-Regge kinematics



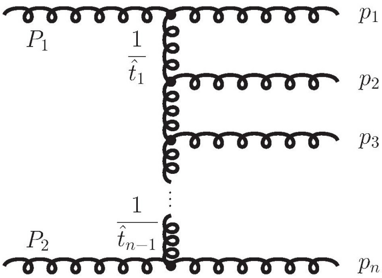

$$
\begin{aligned}
p_i &= \beta_i P_1 + \alpha_i P_2 + p_{iT} \\
S &= (P_1 + P_2)^2 \\
S\alpha_i\beta_i &= p_i^2 - p_{iT}^2 \\
1/S \sim \beta_{n+1} &\ll \beta_n \ll \ldots \ll \beta_0 \sim 1 \\
1/S \sim \alpha_0 &\ll \alpha_1 \ll \ldots \ll \alpha_{n+1} \sim 1 \\
S_i = (p_{i-1} + p_i)^2 &= S\beta_{i-1}\alpha_i \\
S_i \gg |p_{iT}^2| &\sim |t_i| = |q_i^2|
\end{aligned}
$$

Despite of a great number of contributing Feynman diagrams it turns out that at the Born level in the MRK amplitudes acquire a simple factorized form. In the leading logarithmic approximation (LLA) the n-gluon production amplitude in this kinematics has the multi-Regge form

$$A_{2+n}^{LLA} = A_{2+n}^{tree} \prod_{i=1}^{n+1} s_i^{\omega(t_i)}$$

Radiative corrections to these amplitudes do not destroy this form, and their energy dependence is given by Regge factors $s_i^{\omega(t_i)}$. This phenomenon is called **gluon Reggeization**.

# Parton Reggeization Approach: multi-Regge kinematics




$$p_i = \beta_i P_1 + \alpha_i P_2 + p_{iT}$$
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$$S\alpha_i\beta_i = p_i^2 - p_{iT}^2$$
$$1/S \sim \beta_{n+1} \ll \beta_n \ll \ldots \ll \beta_0 \sim 1$$
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# Particle Reggeization

The gluons in each crossing channel $t_i$ are Reggeized if one takes into account the radiative corrections to the Born production amplitude $A^{tree}_{2+n}$.

The production amplitude in the tree approximation has the factorised form:

$$A^{tree}_{2+n} = 2Sg_s T^{c_1}_{AA'} \Gamma_{A'A} \frac{1}{t_1} g_s T^{d_1}_{c_2 c_1} \Gamma^1_{21} \frac{1}{t_2} \ldots g_s T^{d_n}_{c_{n+1} c_n} \Gamma^n_{n+1,n} \frac{1}{t_{n+1}} g_s T^{c_{n+1}}_{B'B} \Gamma_{B'B},$$

$\Gamma^r_{r+1,r}$ – Reggeon-Reggeon-particle (RRP) vertex,
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**The effect of particle Reggeization was discovered in QED in 1964**: *M. Gell-Mann, M. L. Goldberger, F. E. Low, E. Marx, and F. Zachariasen, Phys.Rev.* ***133***, *B161-B174 (1964).*

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## Parton Reggeization Approach: effective vertices

There are **two ways to derive effective vertices**:

- From the analyticity and unitarity constraints for multiparticle production amplitudes. These methods were developed in the works of Lipatov, Fadin, Kuraev and co-authors.
- From the Lagrangian of non-Abelian gauge invariant effective theory, which includes fields of Reggeized particles, firstly written down in *L. N. Lipatov, Nucl. Phys.* ***B452****, 369 (1995).*

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# Parton Reggeization Approach: the factorization hypothesis

In the leading order of PRA the hypothesis of factorization of the effects of long and short distances is proved:

$$
\begin{aligned}
& d\sigma(p+p\to\mathcal{H}+X,S)=\int\frac{dx_1}{x_1}\int d|\mathbf{q}_{1T}|^2\int\frac{d\varphi_1}{2\pi}\Phi(x_1,|\mathbf{q}_{1T}|^2,\mu^2)\\
& \times\int\frac{dx_2}{x_2}\int d|\mathbf{q}_{2T}|^2\int\frac{d\varphi_2}{2\pi}\Phi(x_2,|\mathbf{q}_{2T}|^2,\mu^2)\\
& \times d\hat{\sigma}(R+R\to\mathcal{H}+X,\mathbf{q}_{1T},\mathbf{q}_{2T},\hat{s}),
\end{aligned}
$$

# Dijet production in QMRK

The production of gluon pairs with close rapidities in the central region whereas the protons remnants have large modula of rapidities satisfies the conditions of quasi-multi-Regge kinematics (QMRK). MRK is a particular case of QMRK.

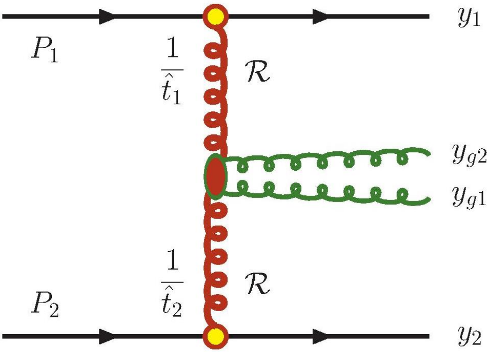


Figure 1 : QMRK: $y_1 \ll y_{g1} \simeq y_{g2} \ll y_2$

# Dijet production: the hard subprocesses

The full number of hard subprocesses in QMRK contributing to dijet production:

$$\begin{aligned} R+R &\to g+g, \\ R+R &\to q+\bar{q}, \\ Q+R &\to q+g, \\ Q+Q &\to q+q, \\ Q+Q' &\to q+q', \\ Q+\bar{Q} &\to q+\bar{q}, \\ Q+\bar{Q} &\to q'+\bar{q}', \\ Q+\bar{Q} &\to g+g. \end{aligned}$$

At the LHC the dominant partonic subprocess is:

$$\mathcal{R}(q_1)+\mathcal{R}(q_2) \to g(k_1)+g(k_2)$$

$q_i^\mu = x_i P_i^\mu + q_{iT}^\mu$ $(i=1,2)$ – four-momenta of the Reggeized gluons;
$P_{1,2}^\mu = (\sqrt{S}/2)(1,0,0,\pm 1)$ – four-momenta of the incoming protons;
$q_{iT}^\mu = (0, \mathbf{q}_{iT}, 0)$, $t_i = -q_{iT}^2 = \mathbf{q}_{iT}^2$.
$k_{1,2}$ – four-momenta of the final gluons, $k_1^2 = k_2^2 = 0$.

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Q + Q &\to q + q, \\
Q + Q' &\to q + q', \\
Q + \bar{Q} &\to q + \bar{q}, \\
Q + \bar{Q} &\to q' + \bar{q}', \\
Q + \bar{Q} &\to g + g.
\end{aligned}
$$

At the LHC the dominant partonic subprocess is:

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# Dijet production: the hard subprocesses

**The effective $\mathcal{R}\mathcal{R}gg$ vertex**

$$C^{gg,\ cd,\ \mu\nu}_{RR,ab}(q_1,q_2,k_1,k_2) = g_s^2 \frac{q_1^+ q_2^-}{4\sqrt{t_1 t_2}} \times$$

$$\times\Big( T_1 s^{-1} \Gamma^{(+-)\sigma}(q_1,q_2)\gamma_{\mu\nu\sigma}(-k_1,-k_2) +$$

$$+T_3 t^{-1}\Gamma^{\sigma\mu-}(q_1,k_1-q_1)\Gamma^{\sigma\nu+}(k_2-q_2,q_2) -$$

$$-T_2 u^{-1}\Gamma^{\sigma\nu-}(q_1,k_2-q_1)\Gamma^{\sigma\mu+}(k_1-q_2,q_2) -$$

$$-T_1(n_\mu^- n_\nu^+ - n_\nu^- n_\mu^+) - T_2(2g_{\mu\nu} - n_\mu^- n_\nu^+) - T_3(-2g_{\mu\nu} + n_\nu^- n_\mu^+) +$$

$$+\Delta^{\mu\nu+}(q_1,q_2,k_1,k_2) + \Delta^{\mu\nu-}(q_1,q_2,k_1,k_2)\Big)$$

$$T_1 = f_{cdr}f_{abr}, \quad T_2 = f_{dar}f_{cbr}, \quad T_3 = f_{acr}f_{dbr}, \quad T_1+T_2+T_3=0$$

$$\Delta^{\mu\nu+}(q_1,q_2,k_1,k_2) = 2t_2 n_\mu^+ n_\nu^+ \left(\frac{T_3}{k_2^+ q_1^+} - \frac{T_2}{k_1^+ q_1^+}\right),$$

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# The matrix element of subprocess $\mathcal{RR} \to gg$

The general form of the squared amplitudes for all subprocesses

$$\overline{|\mathcal{M}|^2} = \pi^2 \alpha_S^2 A \sum_{n=0}^{4} W_n S^n,$$

For the subprocess $\mathcal{RR} \to gg$:

$$\begin{aligned}
A &= \frac{18}{a_1 a_2 b_1 b_2 s^2 t^2 u^2 t_1 t_2}, \\
W_0 &= x_1 x_2 s^2 t u t_1 t_2 \left(x_1 x_2 (tu + t_1 t_2) + (a_1 b_2 + a_2 b_1) tu\right), \\
W_1 &= x_1 x_2 s t_1 t_2 \Bigg[ t^2 u \Bigg( a_1 b_2 (a_2 b_2 + a_1 x_2)(t_1 + t_2) - a_2 b_1 (a_1 b_1 t_1 + a_2 b_2 t_2) + \\
&+ \left(x_2 (a_1^2 b_2 + a_2^2 b_1) + a_1 a_2 (b_1 - b_2)^2\right) u + x_1 x_2 a_1 b_2 t \Bigg) \Bigg] + \\
&+ \Big[ a_1 \leftrightarrow a_2, b_1 \leftrightarrow b_2, t \leftrightarrow u \Big], \\
W_2 &= a_1 a_2 b_1 b_2 t u \Bigg( x_1^2 x_2^2 \left[ 2(t_1 + t_2)(t^2 u + t_1 t_2 (s + u - t)) + \right. \\
&+ \left. tu((t_1 - t_2)^2 + t(u + 2t)) \right] + \\
&+ x_1 x_2 t t_1 t_2 \left(4(x_1 b_1 + x_2 a_2)(s + u) - (a_1 b_1 + a_2 b_2) u\right) +
\end{aligned}$$

# The matrix element of subprocess $\mathcal{R}\mathcal{R} \to gg$

$$
\begin{aligned}
W_3 &= x_1 x_2 a_1 a_2 b_1 b_2 \Big[ t^2 u \Big( 2a_1 b_2 (x_1 x_2 (t_1 + t_2)(2t - u - s) - (x_1 b_2 t_1 + x_2 a_1 t_2)(u + s)) + \\
&+ [x_1 t_1 (2(a_1 b_2^2 + a_2 b_1^2) + 3x_1 b_1 b_2) + x_2 t_2 (2(a_1^2 b_2 + a_2^2 b_1) + 3a_1 a_2 x_2)] u + \\
&+ 4x_1 x_2 t((a_1 b_2 + a_2 b_1)u + a_1 b_2 t) \Big) \Big] + \Big[ a_1 \leftrightarrow a_2, b_1 \leftrightarrow b_2, t \leftrightarrow u \Big], \\
W_4 &= x_1^2 x_2^2 a_1 a_2 b_1 b_2 \Big[ t \Big( a_1 a_2 b_1 b_2 u (t_1 + t_2)(t - u - s) + (a_1 b_2 + a_2 b_1)^2 t u^2 - \\
&- 2a_1 b_2 t (s + u)(2a_2 b_1 u - a_1 b_2 s) \Big) \Big] + \Big[ a_1 \leftrightarrow a_2, b_1 \leftrightarrow b_2, t \leftrightarrow u \Big].
\end{aligned}
$$

The invariant variables: $s = (q_1 + q_2)^2$, $t = (q_1 - k_1)^2$, $u = (q_1 - k_2)^2$,
$a_1 = 2k_1 \cdot P_2/S$, $a_2 = 2k_2 \cdot P_2/S$, $b_1 = 2k_1 \cdot P_1/S$, $b_2 = 2k_2 \cdot P_1/S$.

The amplitudes and squared matrix elements for the full set of $2 \to 2$ subprocesses with Reggeons in the initial state which give contribution to dijet production are presented in the work *M.A. Nefedov, V.A. Saleev, A. V Shipilova.* Dijet azimuthal decorrelations at the LHC in the parton Reggeization approach. Phys. Rev. D**87** (2013) 094030. The all squared matrix elements are checked to give the correct expressions in the Parton Model limit.

# The matrix element of subprocess $\mathcal{RR} \to gg$

$$
\begin{aligned}
W_3 &= x_1 x_2 a_1 a_2 b_1 b_2 \Big[ t^2 u \Big( 2a_1 b_2 \big(x_1 x_2 (t_1 + t_2)(2t - u - s) - (x_1 b_2 t_1 + x_2 a_1 t_2)(u + s)\big) + \\
&+ \big[x_1 t_1 \big(2(a_1 b_2^2 + a_2 b_1^2) + 3x_1 b_1 b_2\big) + x_2 t_2 \big(2(a_1^2 b_2 + a_2^2 b_1) + 3a_1 a_2 x_2\big)\big] u + \\
&+ 4x_1 x_2 t \big((a_1 b_2 + a_2 b_1) u + a_1 b_2 t\big) \Big) \Big] + \Big[ a_1 \leftrightarrow a_2, b_1 \leftrightarrow b_2, t \leftrightarrow u \Big], \\
W_4 &= x_1^2 x_2^2 a_1 a_2 b_1 b_2 \Big[ t \Big( a_1 a_2 b_1 b_2 u (t_1 + t_2)(t - u - s) + (a_1 b_2 + a_2 b_1)^2 t u^2 - \\
&- 2a_1 b_2 t (s + u)(2a_2 b_1 u - a_1 b_2 s) \Big) \Big] + \Big[ a_1 \leftrightarrow a_2, b_1 \leftrightarrow b_2, t \leftrightarrow u \Big].
\end{aligned}
$$

The invariant variables: $s = (q_1 + q_2)^2$, $t = (q_1 - k_1)^2$, $u = (q_1 - k_2)^2$,
$a_1 = 2k_1 \cdot P_2/S$, $a_2 = 2k_2 \cdot P_2/S$, $b_1 = 2k_1 \cdot P_1/S$, $b_2 = 2k_2 \cdot P_1/S$.

The amplitudes and squared matrix elements for the full set of $2 \to 2$ subprocesses with Reggeons in the initial state which give contribution to dijet production are presented in the work *M.A. Nefedov, V.A. Saleev, A. V Shipilova.* Dijet azimuthal decorrelations at the LHC in the parton Reggeization approach. Phys. Rev. D**87** (2013) 094030. The all squared matrix elements are checked to give the correct expressions in the Parton Model limit.

# Dijet production: cross section.

Exploiting the hypothesis of high-energy factorization, we express the hadronic cross sections $d\sigma$ as convolutions of partonic cross sections $d\hat{\sigma}$ with unintegrated PDFs $\Phi_g^h$ of Reggeized gluons in the hadrons $h$.

$$\frac{d\sigma(pp \to ggX)}{dk_{1T}dy_1 dk_{2T}dy_2 d\Delta\varphi} = \frac{k_{1T}k_{2T}}{16\pi^3} \int dt_1 \int d\phi_1 \Phi_g^p(x_1, t_1, \mu^2)\Phi_g^p(x_2, t_2, \mu^2) \times \frac{\overline{|\mathcal{M}(RR \to gg)|^2}}{(x_1 x_2 S)^2},$$

where $k_{1,2T}$ and $y_{1,2}$ are final gluon transverse momenta and rapidities, respectively, and $\Delta\varphi$ is an azimuthal angle enclosed between the vectors $\vec{k}_{1T}$ and $\vec{k}_{2T}$,

$$x_1 = (k_1^0 + k_2^0 + k_1^z + k_2^z)/\sqrt{S}, \quad x_2 = (k_1^0 + k_2^0 - k_1^z - k_2^z)/\sqrt{S},$$

$$k_{1,2}^0 = k_{1,2T}\cosh(y_{1,2}), \quad k_{1,2}^z = k_{1,2T}\sinh(y_{1,2}).$$

# Dijet production at the LHC: comparison with experiment.

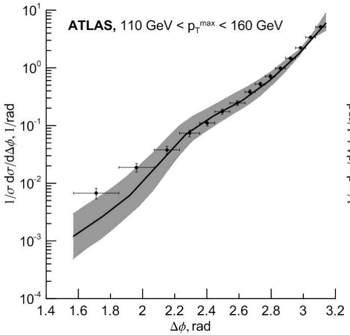


Figure 2 : The azimuthal dijet decorrelations at $\sqrt{S} = 7$ TeV, $|y_{jj}| < 1.1$

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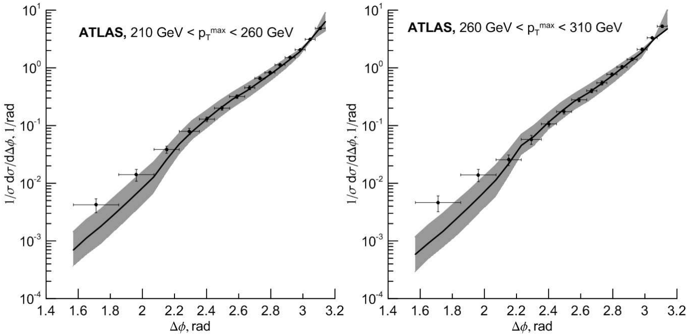


Figure 3 : The azimuthal dijet decorrelations at $\sqrt{S} = 7$ TeV, $|y_{jj}| < 1.1$

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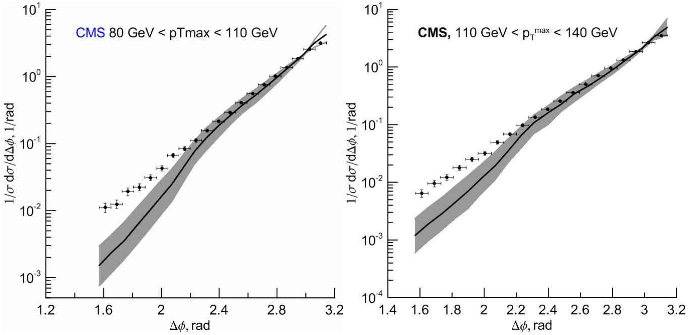


Figure 4 : The azimuthal dijet decorrelations at $\sqrt{S} = 7$ TeV, $|y_{jj}| < 1.1$

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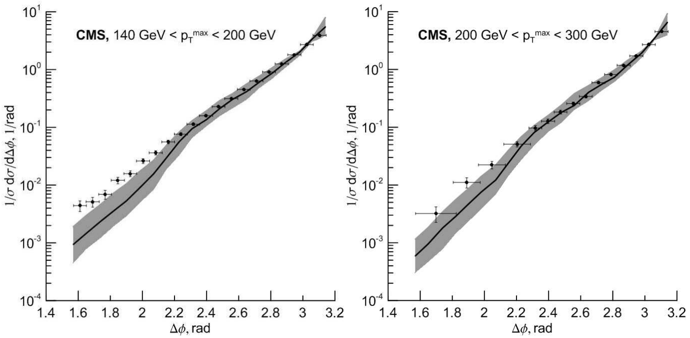


Figure 5 : The azimuthal dijet decorrelations at $\sqrt{S} = 7$ TeV, $|y_{jj}| < 1.1$

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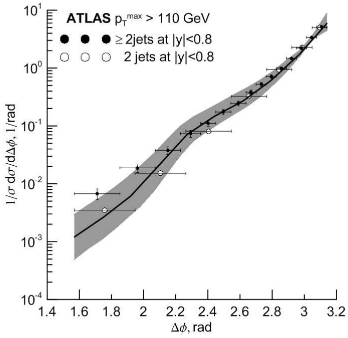



Figure 6 : The azimuthal dijet decorrelations at $\sqrt{S} = 7$ TeV, $|y_{jj}| < 1.1$

The next step of the analysis can be **an inclusion of a** $2 \to 3$ **process RR** $\to$ **ggg** to the calculations. That would lead to a more complete and precise description of the data which contain more than 2 jets in the final state.

# Dijet production at the LHC: comparison with experiment.

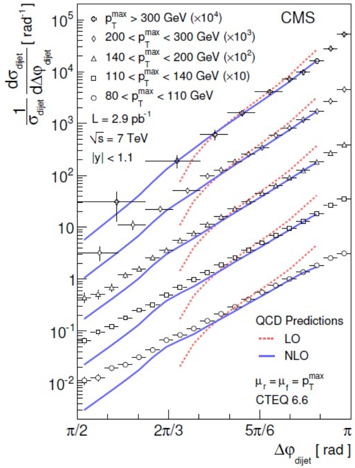
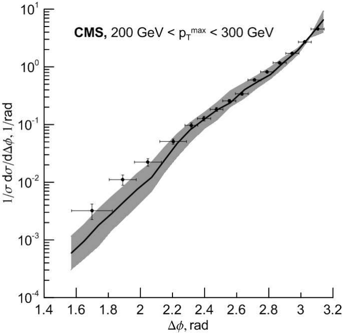



Figure 3: Normalized $\Delta\varphi_{\rm dijet}$ distributions in several $p_T^{\max}$ regions, scaled by the multiplicative factors given in the figure for easier presentation. The curves represent predictions from LO (dotted line) and NLO pQCD (solid line). Non-perturbative corrections have been applied to the predictions. The error bars on the data points include statistical and systematic uncertainties.



# $b\bar{b}$-pair production at the LHC: comparison with experiment.

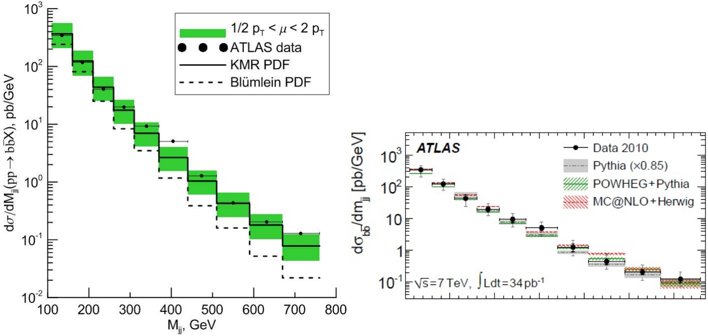



Figure 7 : The invariant mass spectra of $b\bar{b}$-pairs produced at $\sqrt{S} = 7$ TeV, $|y_{b,\bar{b}}| < 1.12$

# $b\bar{b}$-pair production at the LHC: comparison with experiment.

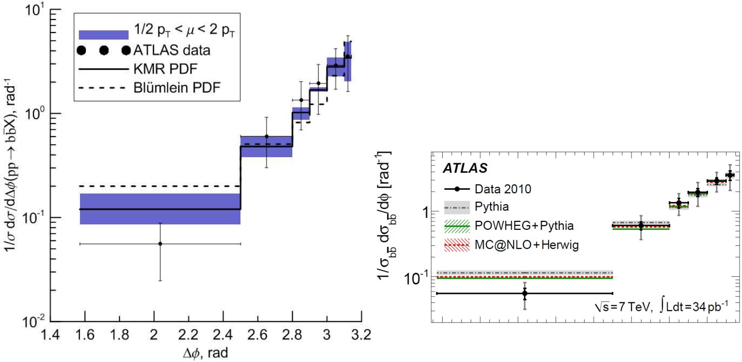



Figure 8 : The normalized azimuthal angle spectra of $b\bar{b}$-pairs produced at $\sqrt{S} = 7$ TeV, $|y_{b,\bar{b}}| < 1.12$

# $b\bar{b}$-pair production at the LHC: comparison with experiment.

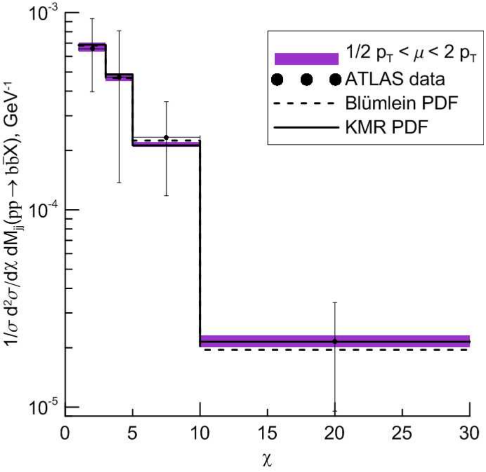



Figure 9 : The $b\bar{b}$-dijet cross-section as a function of $\chi$ for $b$-jets with $p_T > 40$ GeV, $|y| < 2.1$ and $|y_{boost}| = \frac{1}{2}|y_1 + y_2| < 1.1$, for dijet invariant mass range $110 < M_{jj} < 370$ GeV.

# $b\bar{b}$-pair production at the LHC: comparison with experiment.

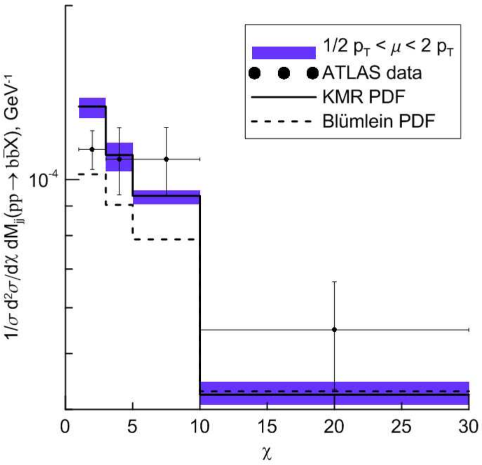



Figure 10 : The $b\bar{b}$-dijet cross-section as a function of $\chi$ for $b$-jets with $p_T > 40$ GeV, $|y| < 2.1$ and $|y_{boost}| = \frac{1}{2}|y_1 + y_2| < 1.1$, for dijet invariant mass range $370 < M_{jj} < 850$ GeV.

# $b\bar{b}$-pair production at Tevatron: comparison with experiment.

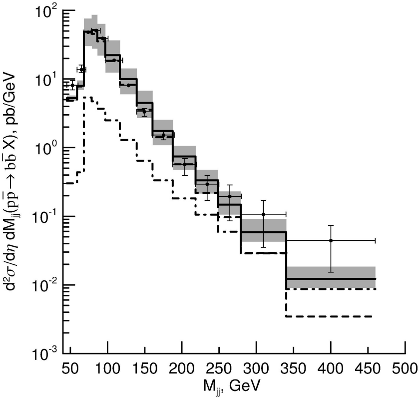



Figure 11 : The $b\bar{b}$-dijet cross-section as a function of the invariant mass of $b$-jets at $\sqrt{S} = 1.96$ TeV, $|\eta_{b,\bar{b}}| < 1.2$.

The contributions: $\mathcal{R} + \mathcal{R} \to b + \bar{b}$ (dash), $\mathcal{Q}_q + \bar{\mathcal{Q}}_q \to b + \bar{b}$ (dash-dot), sum of them both (solid).

# $b\bar{b}$-pair production at Tevatron: comparison with experiment.

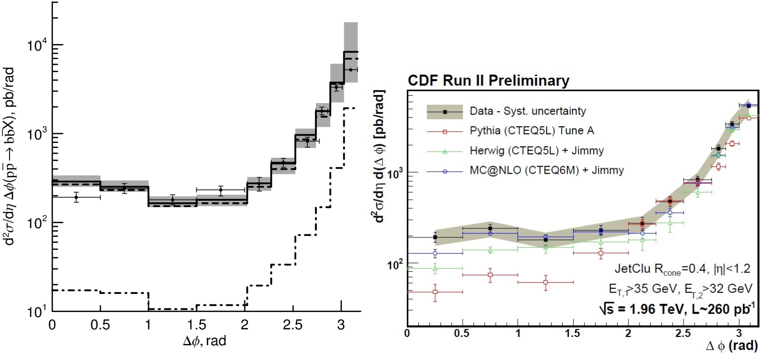



Figure 12 : The $b\bar{b}$-dijet cross-section as a function of the azimuthal angle between $b$-jets at $\sqrt{S} = 1.96$ TeV, $|\eta_{b,\bar{b}}| < 1.2$.

The contributions: $\mathcal{R} + \mathcal{R} \to b + \bar{b}$ (dash), $\mathcal{Q}_q + \bar{\mathcal{Q}}_q \to b + \bar{b}$ (dash-dot), sum of them both (solid).

# $b\bar{b}$-pair production at Tevatron: comparison with experiment.

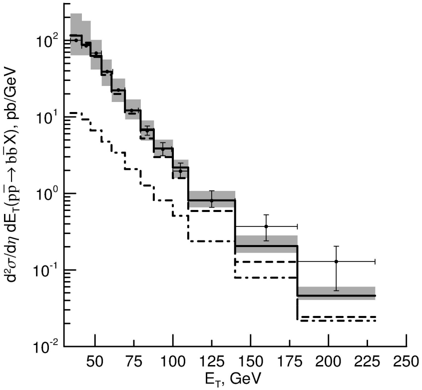



Figure 13 : The $b\bar{b}$-dijet cross-section as a function of the leading jet transverse energy for $b$-jets at $\sqrt{S} = 1.96$ TeV, $|\eta_{b,\bar{b}}| < 1.2$.

The contributions: $\mathcal{R} + \mathcal{R} \to b + \bar{b}$ (dash), $\mathcal{Q}_q + \bar{\mathcal{Q}}_q \to b + \bar{b}$ (dash-dot), sum of them both (solid).

# $D\bar{D}$ and $DD$ production

**Experimental data:** R. Aaij *et. al.*, LHCb Collaboration, JHEP 1206, 141.

- $DD$ production: $D^0 D^0$, $D^0 D^+$, $D^+ D^+$, $D^0 D_s$
- $D\bar{D}$ production: $D^0 \bar{D}^0$, $D^0 D^-$, $D^+ D^-$

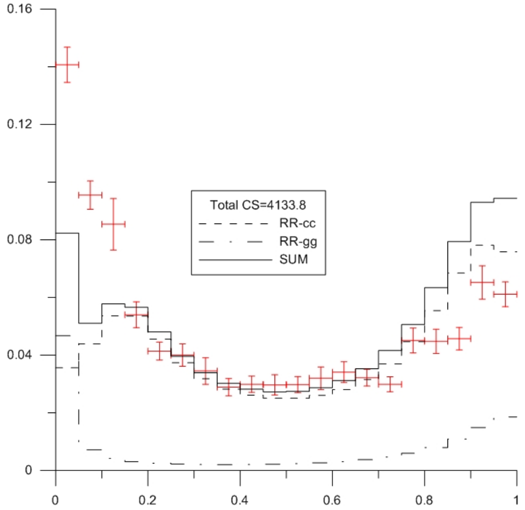
**Theoretical investigations:**

A. Szczurek *et. al.*, arXiv:1505.04067, 2015. Study in terms of double parton scattering

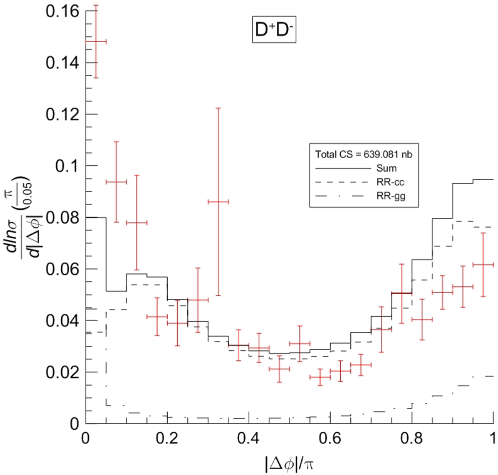
**Parton subprocesses in PRA:**

$$R + R \to c + \bar{c} \Rightarrow D\bar{D}\text{-pair production}$$
$$R + R \to g + g \Rightarrow \text{both } D\bar{D} \text{ and } DD\text{-pair production}$$

# $D\bar{D}$-pair production.



# $D\bar{D}$-pair production.



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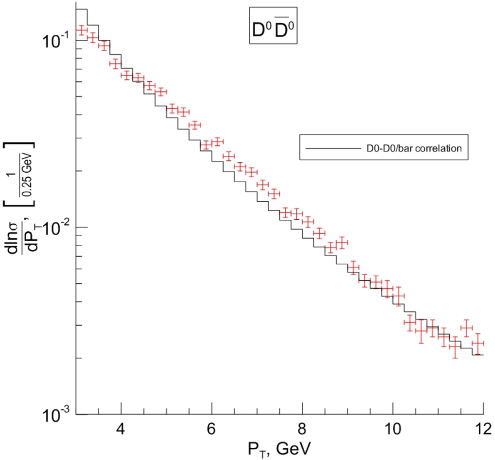

# Summary

- The good description of dijet azimuthal decorrelations is achieved just in the LO parton Reggeization approach, without any ad-hoc adjustments or input parameters, whereas in the collinear parton model, such a degree of agreement calls for NLO and NNLO corrections and complementary initial-state radiation effects and ad-hoc nonperturbative transverse momenta of partons.
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# The advantages of the Parton Reggeization Approach

1. The gauge invariance of the initial off-shell quarks is held only in the Parton Reggeization Approach.

$$C^{qg,\ b,\ \mu}_{QR,\ a}(q_1,q_2,k_1,k_2)=\frac{1}{2}g_s^2\frac{q_2^-}{2\sqrt{t_2}}\bar{U}(k_1)\Big[\gamma_\sigma^{(-)}(q_1,k_1-q_1)t^{-1}\times$$
$$\times\big(\gamma_{\mu\nu\sigma}(k_2,-q_2)n_\nu^+ + t_2\frac{n_\mu^+ n_\sigma^+}{k_2^+}\big)\left[T^a,T^b\right]-\gamma^+(\hat{q}_1-\hat{k}_2)^{-1}\gamma_\mu^{(-)}(q_1,-k_2)T^aT^b-$$
$$-\gamma_\mu(\hat{q}_1+\hat{q}_2)^{-1}\gamma_\sigma^{(-)}(q_1,q_2)n_\sigma^+T^bT^a+\frac{2\hat{q}_1 n_\mu^-}{k_1^-}\left(\frac{T^aT^b}{k_2^-}-\frac{T^bT^a}{q_2^-}\right)\Big],$$

2. The number of matrix elements obtained using the prescription of the $k_t$-factorization approach for off-shell gluon polarization vectors $\epsilon^\mu(q_T)=q_{T\mu}/\sqrt{\vec{q}_T^{\,2}}$ has the incorrect parton model limits $q_{1T},q_{2T}\to 0$, unlike the ones calculated in the Parton Reggeization Approach.

$$C_{RR}^{\mu,g}=\epsilon^\alpha(q_{1T})\epsilon^\beta(q_{2T})g_{\alpha\beta\mu}(q_1,q_2,q_1+q_2),\qquad C_{RR}^{q\bar{q}}=\epsilon^\alpha(q_{1T})\epsilon^\beta(q_{2T})M_{\alpha\beta}(gg\to q\bar{q})$$
$$C_{RR}^{gg,\mu\nu}\neq\epsilon^\alpha(q_{1T})\epsilon^\beta(q_{2T})M^{\mu\nu}_{\alpha\beta}(gg\to gg),\qquad C_{RQ}^{gq,\mu}\neq\epsilon^\alpha(q_{1T})M^\mu_\alpha(gq\to gq)$$

3. In the framework of Parton Reggeization Approach the NLO calculations can be

# The advantages of the Parton Reggeization Approach

1. The gauge invariance of the initial off-shell quarks is held only in the Parton Reggeization Approach.

$$C^{qg,\ b,\ \mu}_{QR,\ a}(q_1,q_2,k_1,k_2) = \frac{1}{2}g_s^2\frac{q_2^-}{2\sqrt{t_2}}\bar{U}(k_1)\Big[\gamma^{(-)}_\sigma(q_1,k_1-q_1)t^{-1}\times$$

$$\times\big(\gamma_{\mu\nu\sigma}(k_2,-q_2)n^+_\nu + t_2\frac{n^+_\mu n^+_\sigma}{k_2^+}\big)\left[T^a,T^b\right] - \gamma^+(\hat{q}_1-\hat{k}_2)^{-1}\gamma^{(-)}_\mu(q_1,-k_2)T^aT^b -$$

$$-\gamma_\mu(\hat{q}_1+\hat{q}_2)^{-1}\gamma^{(-)}_\sigma(q_1,q_2)n^+_\sigma T^bT^a + \frac{2\hat{q}_1 n^-_\mu}{k_1^-}\left(\frac{T^aT^b}{k_2^-}-\frac{T^bT^a}{q_2^-}\right)\Big],$$

2. The number of matrix elements obtained using the prescription of the $k_t$-factorization approach for off-shell gluon polarization vectors $\epsilon^\mu(q_T) = q_{T\mu}/\sqrt{\vec{q}_T^{\,2}}$ has the incorrect parton model limits $q_{1T}, q_{2T}\to 0$, unlike the ones calculated in the Parton Reggeization Approach.

$$C^{\mu,g}_{RR} = \epsilon^\alpha(q_{1T})\epsilon^\beta(q_{2T})g_{\alpha\beta\mu}(q_1,q_2,q_1+q_2), \qquad C^{q\bar{q}}_{RR} = \epsilon^\alpha(q_{1T})\epsilon^\beta(q_{2T})M_{\alpha\beta}(gg\to q\bar{q})$$

$$C^{gg,\mu\nu}_{RR} \neq \epsilon^\alpha(q_{1T})\epsilon^\beta(q_{2T})M^{\mu\nu}_{\alpha\beta}(gg\to gg), \qquad C^{gq,\mu}_{RQ} \neq \epsilon^\alpha(q_{1T})M^\mu_\alpha(gq\to gq)$$

3. In the framework of Parton Reggeization Approach the NLO calculations can be

Thank you for attention!