2HDM in terms of observable quantities and problems of renormalization

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Two Higgs doublet model – 2HDM

The 2HDM describes a system of two scalar isospinor fields ϕ_1, ϕ_2 with hypercharge $Y = 1$. The most general form of the 2HDM potential is

$$V = \frac{\lambda_1}{2}(\phi_1^\dagger\phi_1)^2 + \frac{\lambda_2}{2}(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1)$$
$$+ \left[\frac{\lambda_5}{2}(\phi_1^\dagger\phi_2)^2 + \lambda_6(\phi_1^\dagger\phi_1)(\phi_1^\dagger\phi_2) + \lambda_7(\phi_2^\dagger\phi_2)(\phi_1^\dagger\phi_2) + \text{h.c.}\right]$$
$$- \frac{m_{11}^2}{2}(\phi_1^\dagger\phi_1) - \frac{m_{22}^2}{2}(\phi_2^\dagger\phi_2) - \left[\frac{m_{12}^2}{2}(\phi_1^\dagger\phi_2) + \text{h.c.}\right]$$

Its coefficients (brawn are real, blue are complex) are restricted by the requirement that the potential be positive at large quasiclassical values of ϕ_i (*positivity constraints*). This potential is described by 14 real parameters

The model contains two doublets of scalar fields with identical quantum numbers $\Rightarrow$ it can be described either in terms of the original fields ϕ_1, ϕ_2 or in terms of fields ϕ'_1, ϕ'_2, which are obtained from ϕ_k by a global unitary reparameterization transformation $\hat{\mathcal{F}}$

$$\begin{pmatrix}\phi'_1\\ \phi'_2\end{pmatrix} = \hat{\mathcal{F}}_{gen}(\theta,\tau,\rho)\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix},$$

$$\hat{\mathcal{F}}_{gen} = e^{-i\rho_0}\begin{pmatrix}\cos\theta\, e^{i\rho/2} & \sin\theta\, e^{i(\tau-\rho/2)}\\ -\sin\theta\, e^{-i(\tau-\rho/2)} & \cos\theta\, e^{-i\rho/2}\end{pmatrix}.$$

with corresponding transformation of the parameters $\lambda_i \to \lambda'_i$. We refer to these different choices as different RPa bases.

This transformation is described by 3 angles θ, ρ, τ and ρ_0, the parameter ρ_0 don't influence for coefficient

$\Longrightarrow$ Model is described by 11 relevant parameters.

We develop a method for finding the minimal and a comprehensive set of directly measurable quantities defining the 2HDM and have built simple example of such set. We call these quantities observables and call the chosen complete set the basic set of observables. This basic set is subdivided naturally into two subsets, defined below.
We have found simple explicit expressions for the parameters of potential of the model via these observables (and non-physical parameters, fixing RPa basis).
Fortunately, the obtained description appeared to be simple enough.

Extrema of the potential satisfy the stationarity equations $\partial V/\partial\phi_i|_{\phi_1=\langle\phi_1\rangle,\phi_2=\langle$
0 ($i = 1, 2$). The most general solution that describes the $SU(2)\times U(1)_Y \to U(1)_{EM}$ symmetry breaking is expressed via two positive numbers v_i and the relative phase factor $e^{i\xi}$ as:

$$\langle\phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle\phi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_2 e^{i\xi} \end{pmatrix},$$
$$v_1 = v\cos\beta, \quad v_2 = v\sin\beta, \quad v = \sqrt{v_1^2 + v_2^2}\,.$$

The ground state of potential (the vacuum) is the extremum with the lowest energy, and its vacuum expectation value (v.e.v.) is $v =$ 246 GeV.
The fields ϕ_i are then decomposed into their v.e.v.'s and the quantized component fields, their linear combinations describe Goldstone modes $G^\pm$, G^0, charged Higgses $H^\pm$ with mass $M_\pm$ and neutral Higgses $h_{1,2,3}$ with masses $M_{1,2,3}$

Relative couplings. We consider couplings of each neutral Higgs boson to a fundamental particle P by g_a^P ($P = \{V(W, Z)$, $f = q(t, b, c, ...)$, $\ell(\tau, \mu$ and similar couplings of the standard Higgs boson of SM g_{SM}^P. We use the relative couplings

$$\chi_a^P = g_a^P / g_{\mathsf{SM}}^P \,.$$

Besides, we introduce dimensionless relative couplings:

$$\chi_a^{H^+W^-} = \frac{g(H^+W^-h_a)}{M_W/v} \equiv \left(\chi_a^{H^-W^+}\right)^* \,; \quad \chi_a^{\pm} = g(H^+H^-h_a)/(2M_{\pm}^2/v) \,.$$

The neutrals h_a generally have no definite CP parity. Couplings χ_a^V and $\chi_a^{\pm}$ are real due to Hermiticity of Lagrangian, while other couplings are generally complex.

Higgs basis

We analyze the model with known vacuum using the special RPA transformed basis with $v_2 = 0$ the Higgs (or Georgi) basis. It is obtained from mentioned basis with known v.e.v.'s by RPa transformation:

$$\begin{pmatrix}\Phi_1\\ \Phi_2\end{pmatrix} = \hat{\mathcal{F}}_{HB}\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix}, \quad \hat{\mathcal{F}}_{HB} = e^{-i\rho_0}\begin{pmatrix}\cos\beta\, e^{i\rho/2} & \sin\beta\, e^{i(\rho/2-\xi)}\\ -\sin\beta\, e^{-i(\rho/2-\xi)} & \cos\beta\, e^{-i\rho/2}\end{pmatrix}$$

(Here $\hat{\mathcal{F}}_{HB}$ is obtained from $\hat{\mathcal{F}}_{gen}$ by substitution $\theta = \beta, \tau = \rho - \xi$. The phase factor $e^{\pm i\rho/2}$ represents the remaining RPh freedom in the choice of the Higgs basis that is, independence of the physical picture from the choice of relative phase ϕ_i, the RPh phase.

Vise versa, any form of the potential can be obtained from the Higgs basis form with the transformation, $\hat{\mathcal{F}}_{HB}^{-1} = \hat{\mathcal{F}}_{gen}(\theta = -\beta, \tau = \rho + \xi)$ with $\rho \to -\rho$, $\rho_0 \to -\rho_0$. We do not fix in this definition the RPh phase ρ and the irrelevant parameter ρ_0.

The potential obtained can be rewritten in the form (capital letters for fields and parameters in Higgs basis)

$$V_{HB} = M_\pm^2 (\tilde\phi_2^\dagger \tilde\phi_2) + \frac{\Lambda_1}{2}\left(\Phi_1^\dagger \Phi_1 - \frac{v^2}{2}\right)^2 + \frac{\Lambda_2}{2}(\tilde\phi_2^\dagger \tilde\phi_2)^2 + \Lambda_3 \left(\Phi_1^\dagger \Phi_1 - \frac{v^2}{2}\right)(\tilde\phi_2^\dagger \tilde\phi_2)$$
$$+\Lambda_4 (\tilde\phi_1^\dagger \tilde\phi_2)(\tilde\phi_2^\dagger \tilde\phi_1) + \left[\frac{\Lambda_5}{2}(\tilde\phi_1^\dagger \tilde\phi_2)^2 + \Lambda_6 \left(\Phi_1^\dagger \Phi_1 - \frac{v^2}{2}\right)(\tilde\phi_1^\dagger \tilde\phi_2)\right.$$
$$\left. +\Lambda_7 (\tilde\phi_2^\dagger \tilde\phi_2)(\tilde\phi_1^\dagger \tilde\phi_2) + \text{h.c.}\right] + \mathcal{E}_{vac}, \quad \mathcal{E}_{vac} = -\frac{\Lambda_1}{8} v^4 .$$

In the Higgs basis, the decomposition of fields v.e.v. has simple form

$$\Phi_1 = \begin{pmatrix} G^+ \\ \dfrac{v + \eta_1 + iG^0}{\sqrt{2}} \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} H^+ \\ \dfrac{\eta_2 + i\eta_3}{\sqrt{2}} \end{pmatrix} .$$

To arrive to the description in terms of physically observable fields, one should start by substituting these expressions into the potential. By choosing the unitarity gauge for the gauge fields, we omit the Goldstone modes G^a from now on.

Now the potential takes the form:

$$V = M_{\pm}^2 H^+ H^- + \frac{M_{ij}}{2} \eta_i \eta_j + v T_i H^+ H^- \eta_i + v T_{ijk} \eta_i \eta_j \eta_k$$
$$+ C H^+ H^- H^+ H^- + \frac{B_{ij}}{2} H^+ H^- \eta_i \eta_j + Q_{ijkl} \eta_i \eta_j \eta_k \eta_l .$$

The coefficients M_{ij} form the neutral scalar *mass matrix*

$$M_{ij}=v^2\begin{pmatrix} \Lambda_1 & Re\,\Lambda_6 & -Im\,\Lambda_6 \\ Re\,\Lambda_6 & \dfrac{N+Re\Lambda_5}{2} & -Im\,\Lambda_5/2 \\ -Im\,\Lambda_6 & -Im\,\Lambda_5/2 & \dfrac{N-Re\Lambda_5}{2} \end{pmatrix}; \quad N=M_\pm^2/v^2+\Lambda_4$$

The physical neutral Higgs states h_a are such superpositions of fields η_i that diagonalize this mass matrix:

$$h_a=R_a^i\eta_i\,, \quad \eta_i=R_i^a h_a \Rightarrow M_{ij}\frac{\eta_i\eta_j}{2}=\sum_a\frac{M_a^2h_a^2}{2}, \quad M_{ij}=R_i^aR_j^aM_a^2.$$

The mixing matrix R_i^a is a real-valued orthogonal matrix determined by the parameters of the mass matrix. It can be parameterized with three Euler angles. One of them is responsible for rephasing transformation of fields, i.e. it is irrelevant.

The trace of the mass matrix is invariant under orthogonal transformations. Therefore $v^2(\Lambda_1+\Lambda_4)=\sum_a M_a^2 - M_\pm^2$.

One of the advantages of the Higgs basis as compared to other RPa bases is the fact that **the elements of rotation matrix are directly related to the relative couplings**, which are, in principle, measurable:

$$\chi_a^V = R_1^a, \quad \chi_a^{H^+W^-} \equiv \left(\chi_a^{H^-W^+}\right)^* = R_2^a + iR_3^a.$$

The phases of quantities $\chi_a^{H^+W^-}$, i.e. the ratios R_3^a/R_2^a cannot be fixed because of the rephasing freedom of potential in the Higgs basis, but their relative phases for different h_a are determined unambiguously.

The orthogonality of the mixing matrix means that its elements obey a set of relations:

$$\sum_i R_i^a R_i^b = \delta_{ab}\,, \qquad \sum_a R_i^a R_j^a = \delta_{ij};$$

in particular: $$\sum_a (\chi_a^V)^2 = 1\,, \quad |\chi_a^V|^2 + |\chi_a^{H^+W^-}|^2 = 1\,.$$

$$R_a^i = \begin{pmatrix} \chi_1^V & \chi_2^V & \chi_3^V \\ \frac{-\chi_1^V \chi_2^V}{\sqrt{1-(\chi_2^V)^2}} & \sqrt{1-(\chi_2^V)^2} & \frac{-\chi_2^V \chi_3^V}{\sqrt{1-(\chi_2^V)^2}} \\ \frac{\chi_3^V}{\sqrt{1-(\chi_2^V)^2}} & 0 & \frac{-\chi_1^V}{\sqrt{1-(\chi_2^V)^2}} \end{pmatrix} T, \qquad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\rho & \sin\rho \\ 0 & -\sin\rho & \cos\rho \end{pmatrix}$$

Finally, one can read the mass matrix, written above, as expressions of some Λ's via elements of the mass matrix and then, withto express them via the masses of Higgs bosons and their couplings to gauge bosons:

$$v^2\Lambda_1 = \sum_a (\chi_a^V)^2 M_a^2\,; \quad v^2\Lambda_4 = \sum_a M_a^2 - M_\pm^2 - v^2\Lambda_1;$$
$$v^2\Lambda_5^* = \sum_a (\chi_a^{H^+W^-})^2 M_a^2; \quad v^2\Lambda_6^* = \sum_a \chi_a^V \chi_a^{H^+W^-} M_a^2.$$

The observables entering into this equation form a first subset of the basic set of observables. Couplings $\chi_a^{H^+W^-}$ are expressed via χ_a^V. The phase freedom in the definition of these couplings is reproduced as a similar freedom in phases of Λ_5, Λ_6.

The second subset of basic set form 3 triple Higgs couplings and one quartic coupling.

Each triple Higgs vertex depends on Λ_3, $Re\Lambda_7$, $Im\Lambda_7$, in addition to the parameters of the first subset. The cubic interaction of neutral and charged scalars can be written as $vT_i H^+H^-\eta_i$, with $T_i = (\Lambda_3, Re\Lambda_7, -Im\Lambda_7)_i$. After transformation to physical states $\eta_i = R_i^a h_a$, we obtain the corresponding couplings: $g(H^+H^-h_a) = vR_i^a T_i$. This expression is easy to solve for T_i by inverting the rotation matrix:

$$\Lambda_3 = (2M_\pm^2/v^2)\sum_a \chi_a^V \chi_a^\pm; \qquad \Lambda_7^* = (2M_\pm^2/v^2)\sum_a \chi_a^{H^-W^+}\chi_a^\pm\,.$$

The parameter Λ_2 can only be extracted from quartic couplings. (Each quartic Higgs vertex depends on parameter Λ_2 in addition to parameters determined from mass terms and triple Higgs couplings). We use for basic set **the vertex $H^+H^-H^+H^-$**. The $H^+H^-H^+H^-$ vertex enters Lagrangian in a very simple form $\frac{\Lambda_2}{2}H^+H^-H^+H^-$, and its observation offers the simplest way to measure Λ_2:

$$\Lambda_2 = 2g(H^+H^-H^+H^-)\,.$$

Possible strong interaction in the Higgs sector.

The fact that free parameters of the potential naturally fall into three very distinct categories, offers a new opportunity which was absent in the SM. Before the Higgs discovery, the large coupling constant λ was, in principle, possible within SM. In this case, the Higgs boson would be very heavy and wide, and it could not be seen as separate particle. Instead, its dynamics would be governed by the strong interaction in the Higgs sector, which would manifest itself in the form of resonances in the $W_L W_L$, $W_L Z_L$, $Z_L Z_L$ scattering in the 1-2 TeV energy range. In the SM this opportunity is closed by the discovery of the Higgs boson with $M \approx 125$ GeV.

Our analysis shows that, within 2HDM, the reasonably low values of all Higgs masses are well compatible with large Λ_3, $|\Lambda_7|$, Λ_2, i.e. with the strong interaction in the Higgs sector.

To-day I like to see for opportunity that "observable"(at 3σ level now) in WW, ZZ, hh production at $M{\sim}2$ TeV, observed by both CMS and ATLAS could be signal of such strong interaction.

For first summary

The observables of the basic set are measurable quantities, independent of each other. The models with arbitrary values of these observable parameters can in principle be realized. In some special variants of 2HDM, additional relations between these parameters may appear (for example, in the CP conserving case $\chi_3^V = \chi_3^{\pm} = 0$).
Our results open the door for the study of Higgs models in terms of measurable quantities alone. It allows to remove from the data analysis the widely spread intermediate stages with complex, often model-dependent, analysis of coefficients of Lagrangian.
The principal possibility to determine all parameters of 2HDM from the (future) data meet strong practical limitations (which can be hidden in other approaches). In the best case, it looks the problem for a very long time.

Renormalization scheme

The standard calculation of the radiative corrections (RC) in the model is based on the parameters of Lagrangian which are RPa dependent. This RPa ambiguity can be removed, for example, by using the renormalization procedure fixing parameters of the basic set. In the modern approach the calculation of any physical effect should be supplemented by calculation of renormalized values of masses and other parameters of basic set which should be taken into account in the data analysis. For example, in some particular variant of MSSM the value of triple Higgs coupling with RC looks essentially different from its tree form in the SM. However, within the same approximation the using of the renormalized mass M_1 makes the result close to the SM value.