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INFLUENCE OF LONG-RANGE INTERACTIONS ON THE PHASE TRANSITION TEMPERATURE OF THE ISING MODEL

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ISING MODEL AND ITS RESEARCHING

Ising model has been studied since 1924 for the description of phase transitions in magnetic materials.

E. Ising, Beitrag zur Theorie des Ferro- und Paramagnetimus, Hamburg, 1924

L. Onsager, Crystalstatistics. A two-dimensional model with order-disorder transitions. // PhysRev, 1944;

B.M. McCoy, Tai Tsun Wu, The two-dimensional Ising model. // Cambridge, MA: Harvard Univ.Press, 1973

ISING MODEL AND ITS RESEARCHING

In recent the papers devoted to the Ising models with interaction between nearest and second nearest neighbors have appeared.

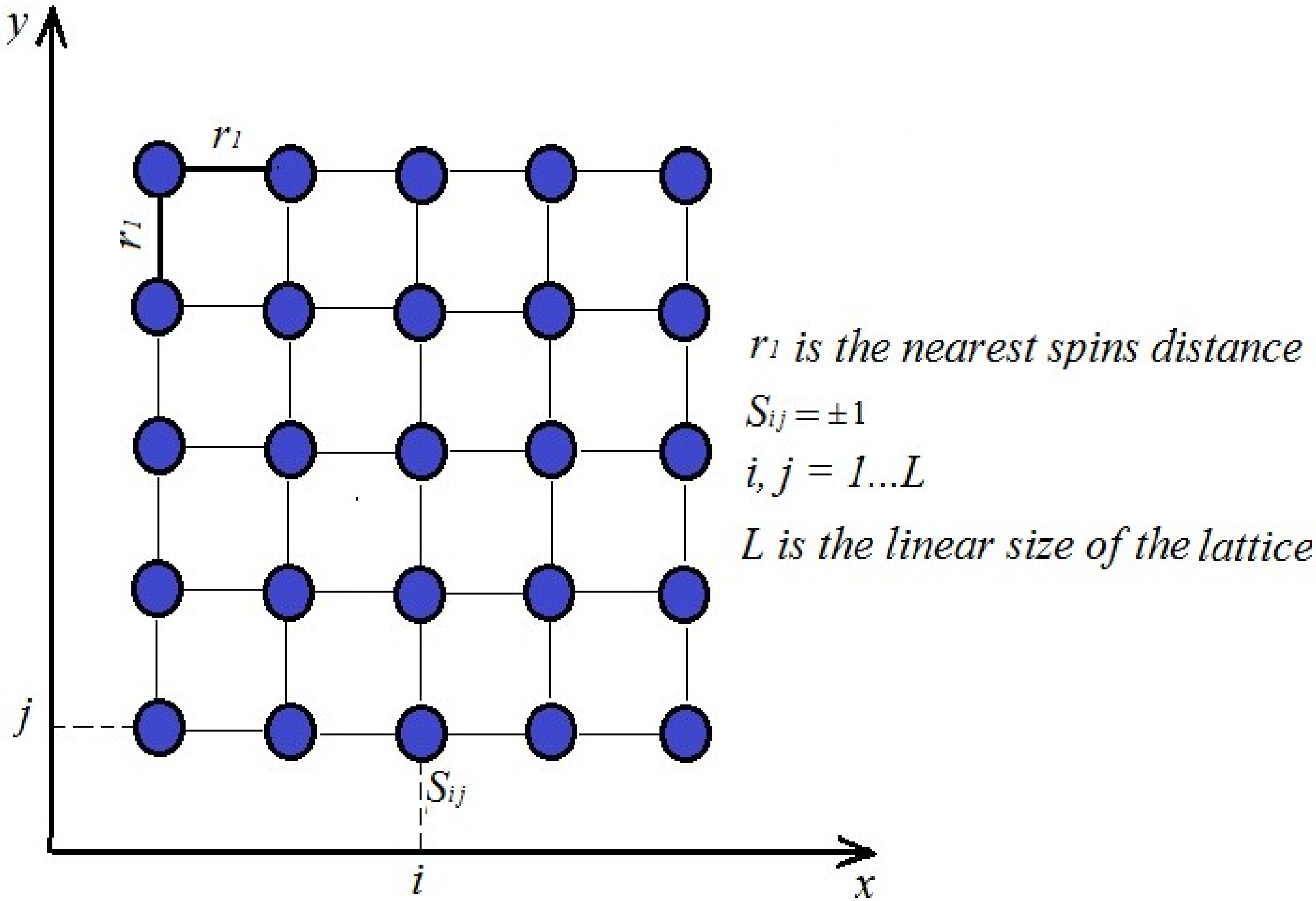
A.J. Ramirez-Pastor, F. Nieto, E.E. Vogel, Ising lattices with ±J second-nearest-neighbor interactions. // PhysRev, 1997, v. 55, № 21, p. 14323-14329;

Rosana A. dos Anjos, J. Roberto Viana, J. Ricardo de Sousa, J. A. Plascak. Three-dimensional Ising model with nearest- and next-nearest-neighbor interactions // PhysRev E. Vol. 76(022103), 2007;

M. Picco, Critical behavior of the Ising model with long-range interactions. // arXiv:1207.1018v1, 2012;

M. Ch. Angelini, G. Parisi, F. Ricci-Tersenghi. Relations between short-range and long-range Ising models // PhysRev E. Vol. 89(062120), 2014.

ISING MODEL DESCRIPTION



Ising model may be described by the lattice with spins in the nodes. The lattice supposed with periodic boundary conditions. Lattice sides directed along the coordinate axes parallel to them. Coordinates of the spin are integer values *(i, j)*.

ISING MODEL

Ising model hamiltonian has a structure:

$$H(S_{ij}) = \sum_{lm} J S_{ij} S_{lm},$$

$$l = \overline{i-1, i+1}, \quad m = \overline{j-1, j+1}$$

with *J* – interaction value between spins S_i and S_j.

$$H = \sum_{i,j=0}^{N} \sum_{l,m} J S_{ij} S_{lm}$$

ISING MODEL WITH LONG-RANGE INTERACTIONS

Interaction value between spins defined as

$$J(r) = J_0 r^{-d-\sigma}$$

J_0 Is the nearest spin interaction value

d Is dimensional of the lattice

σ Is parameter

r Is the distance between spins

TWO-DIMENSIONAL ISING MODEL WITH LONG-RANGE INTERACTIONS

$$H(S_{ij}) = \sum_{l=i-N}^{i+N} \sum_{m=j-N}^{j+N} \frac{J_0}{r_{lm}^{2+\sigma}} S_{ij} S_{lm},$$

$r_{lm} = \sqrt{(i-l)^2 + (j-m)^2}$ is the distance between spins S_{ij} and S_{lm}

$S_{ij} = \pm 1$ for all i, j.

J_0 is the interaction constant between spins

Magnetization value: $$\langle M \rangle = \frac{1}{L^2} \sum_{i,j=1}^{L} S_{ij} Z^{-1} \exp\left(-\frac{1}{kT} H(S_{ij})\right)$$

Normalization value: $$Z = \exp\left(-\frac{1}{kT} H(+1)\right) + \exp\left(-\frac{1}{kT} H(-1)\right)$$

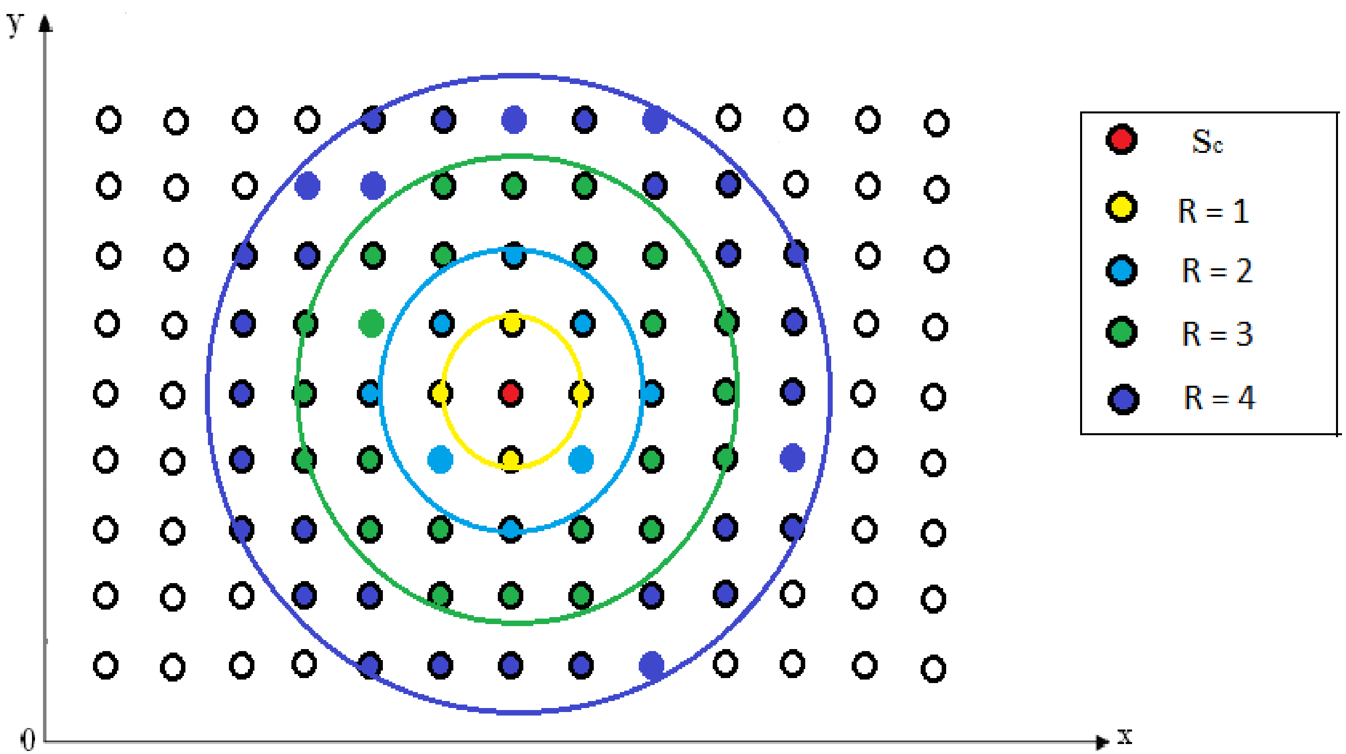
k is the Boltzmann constant

T is the temperature

L is the linear size of the lattice

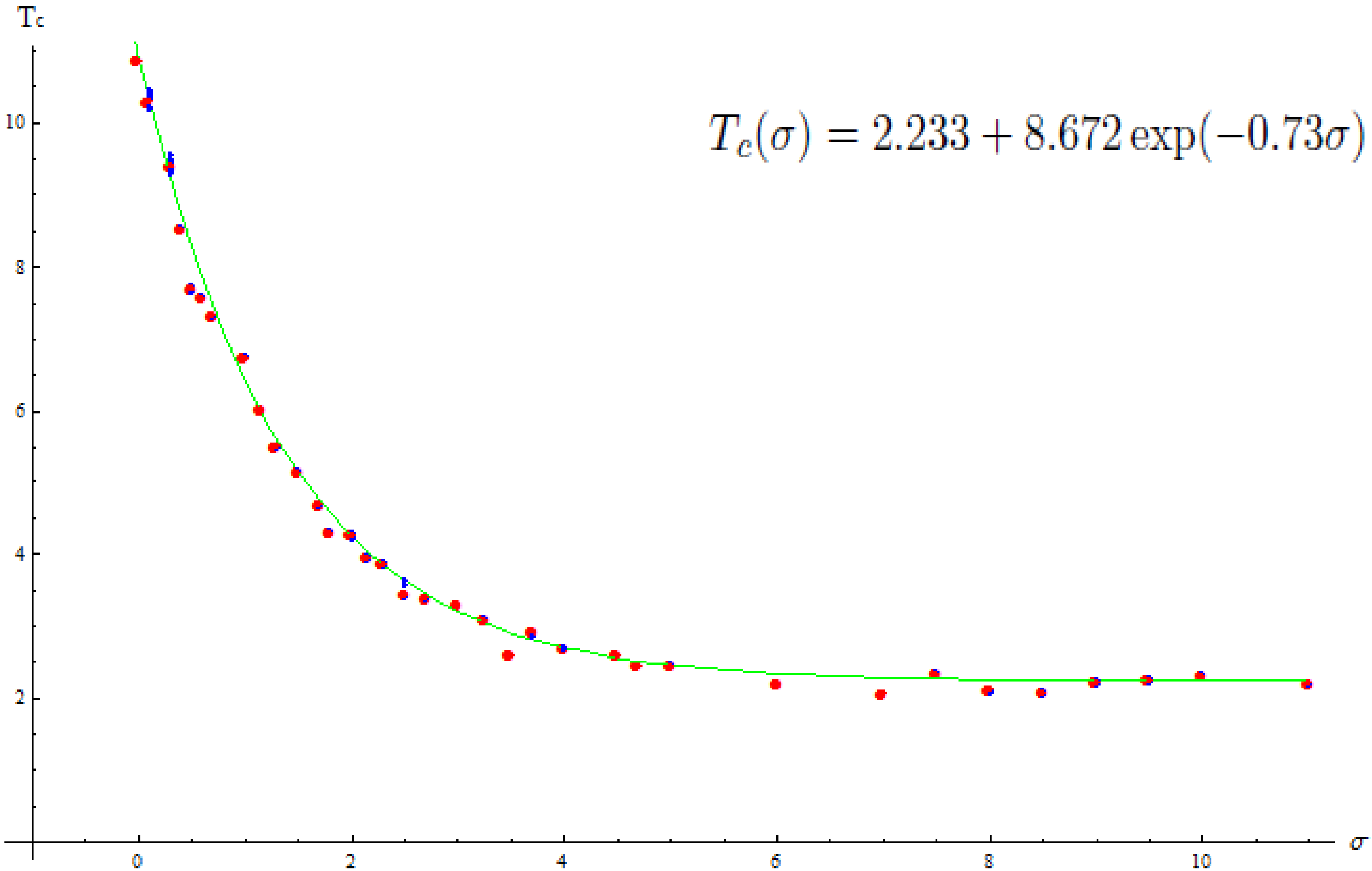
The summing is over points which are situated in the circle of the radius R and the center (i, j):

$$\sqrt{(l-i)^2 + (m-j)^2} \leqslant R = N.$$



Periodic boundary conditions are taken into account for the summing

PHASE TRANSITION TEMPERATURE OF THE TWO-DIMENSIONAL ISING MODEL



THREE-DIMENSIONAL ISING MODEL WITH LONG-RANGE INTERACTIONS

$$H(S_{ijs}) = \sum_{l=i-N}^{i+N} \sum_{m=j-N}^{j+N} \sum_{n=s-N}^{s+N} \frac{J_0}{r_{lmn}^{3+\sigma}} S_{ijs} S_{lmn},$$

$r_{lmn} = \sqrt{(i-l)^2 + (j-m)^2 + (s-n)^2}$ is the distance between spins S_{ijs} and S_{lmn}

$S_{ijs} = \pm 1$ for all $i, j\ s$

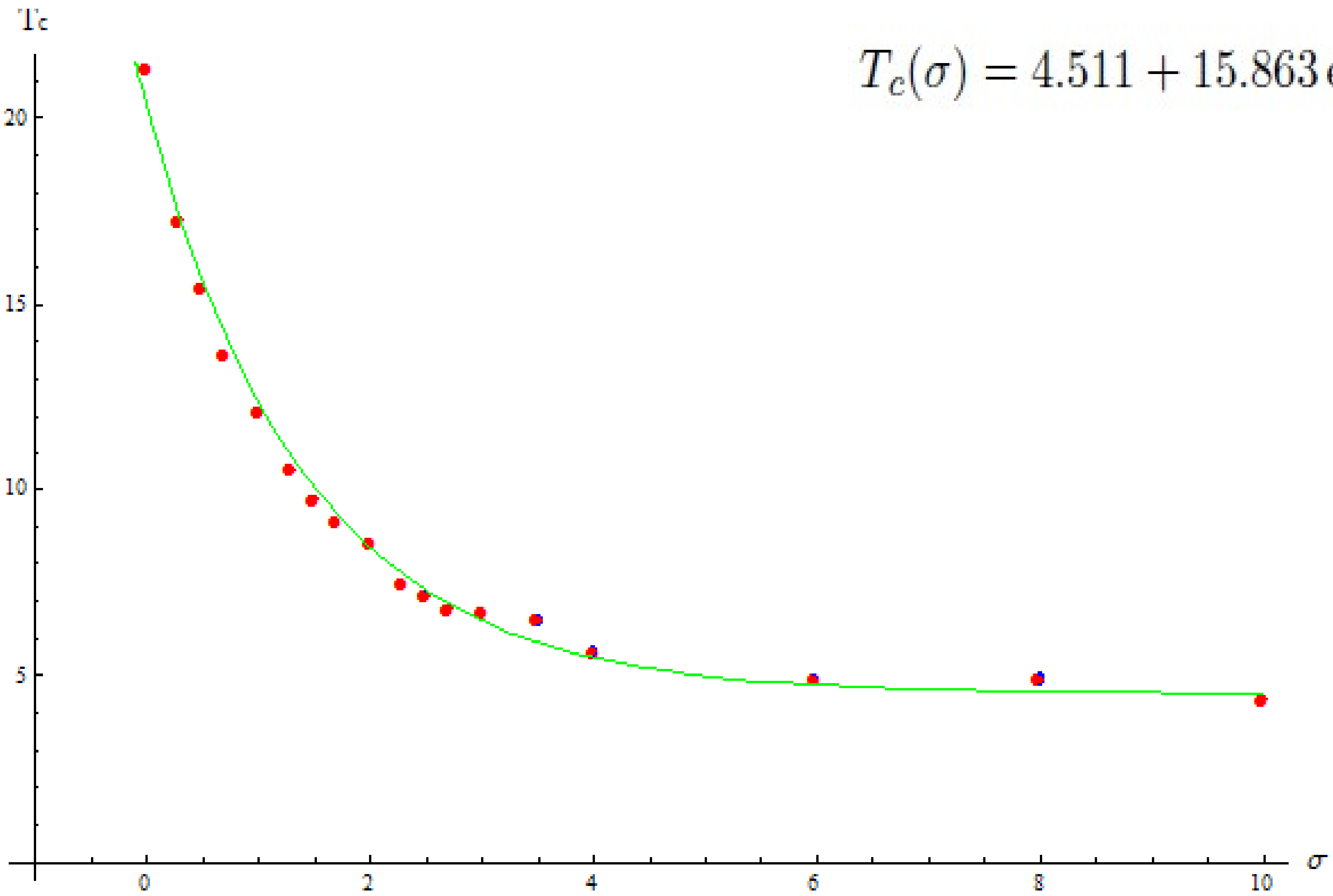
J_0 is the interaction constant between spins

Magnetization value: $\langle M \rangle = \frac{1}{L^3} \sum_{i,j,s=1}^{L} S_{ijs} Z^{-1} \exp\left(-\frac{1}{kT} H(S_{ijs})\right),$

Normalization value: $Z = \exp\left(-\frac{1}{kT} H(+1)\right) + \exp\left(-\frac{1}{kT} H(-1)\right)$

$$\sqrt{(l-i)^2 + (m-j)^2 + (n-s)^2} \leqslant R = N,$$

PHASE TRANSITION TEMPERATURE OF THE THREE-DIMENSIONAL ISING MODEL



For $\sigma \to \infty$ $T_c(\sigma) = 4.511$ performed in K. Binder's work for the nearest-interaction three-dimensional model.

K. Binder, E. Luijten. Monte Carlo test of renormalization group predictions for critical phenomena in Ising models. // Phys Rep, 344, 2001. pp.179 — 253.

PHASE TRANSITION TEMPERATURE

$$T_c(\sigma) = A + B\exp(-c\sigma),$$

$A = 2.233,\ B = 8.672,\ c = 0.73$ for two-dimensional model

$A = 4.511,\ B = 15.863,\ c = 0.7$ for three-dimensional model

The relative intensity interaction J (R) between the spins located at a distance R,

and spins at a distance R = 1 represented by the expression

$$\delta = \frac{J(R)}{J_0} = \frac{1}{R^{(d+\sigma)}},$$ then

$$\sigma = -\left(d + \frac{\ln\delta}{\ln R}\right).$$

PHASE TRANSITION TEMPERATURE DEPENDENCE FROM THE INTERACTION AREA RADIUS

We'll presume $\delta = 10^{-d}$, the intensity of the interaction between the spins is taken into account up to a distance R_k: i.e.

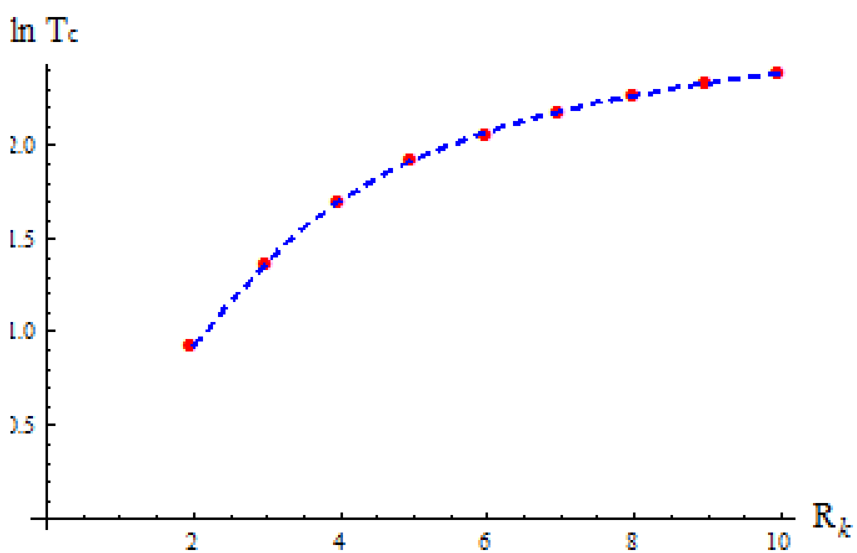
$$J(R_k) = 10^{-d} J_0;$$

$$J(R_k + 1) = 0.$$

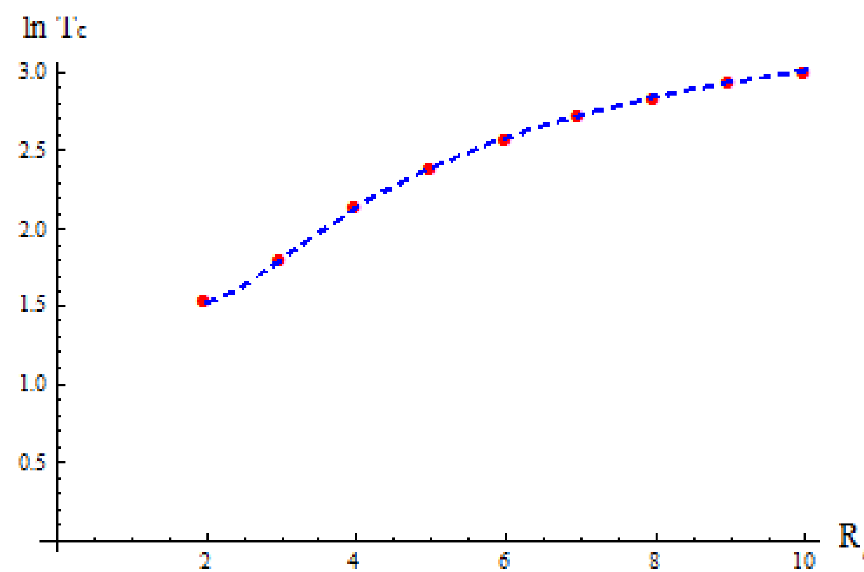
$$\sigma_k = -\left(d + \frac{-d \ln 10}{\ln R_k}\right), \quad k = 2, 3, ..., 10.$$

$$T_c(R_k) = A + B \exp\left[-cd\left(\frac{\ln 10}{\ln R_k} - 1\right)\right]$$

PHASE TRANSITION TEMPERATURE DEPENDENCE FROM THE INTERACTION AREA RADIUS



Two-dimensional model



Three-dimensional model

CONCLUSIONS

- Phase transition temperature essentially depends on the interaction radius area between spins.
- This dependence expressed analytically.

THANKS FOR YOUR ATTENTION!