



# Phenomenology of multi-hadron and jet production in heavy ion collisions at Large Hadron Collider

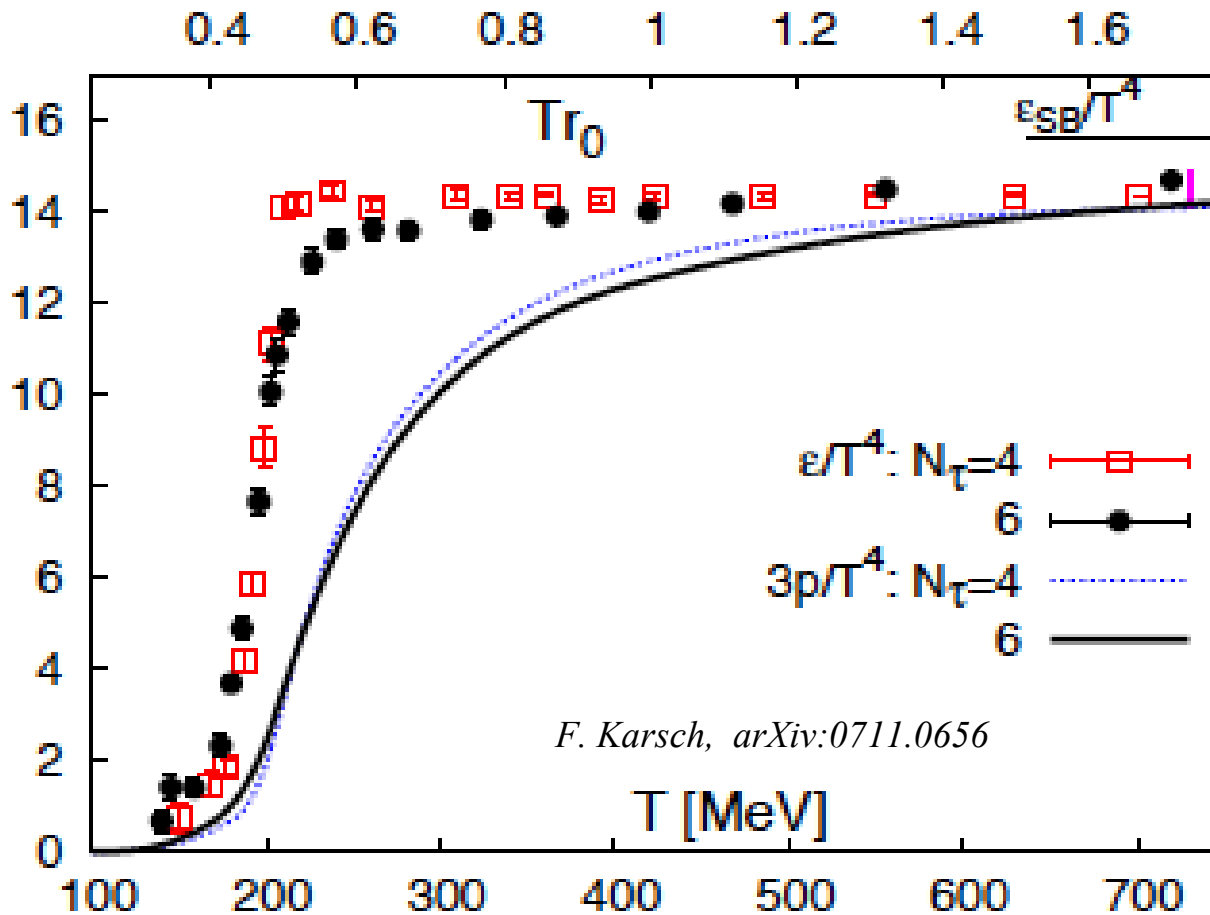


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June 24– July 1, 2015, Samara, Russia*

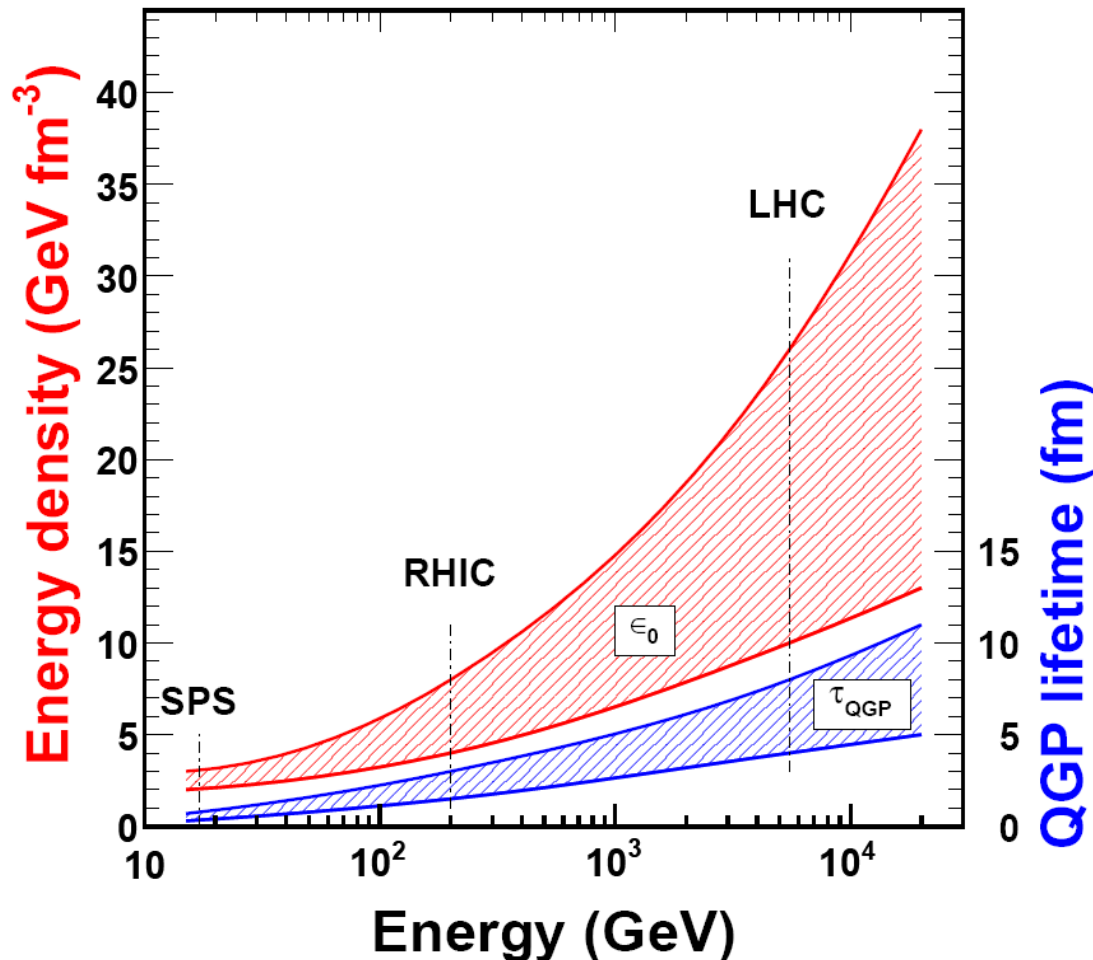
# Deconfinement of nuclear matter



*Deconfinement of nuclear matter and quark-gluon matter (QGM) formation – the prediction of Lattice Quantum Chromodynamics (QCD) for systems with high enough temperature and/or baryon density*

# Study of quark-gluon matter in relativistic heavy ion collisions

SPS (CERN) → RHIC (BNL) → LHC (CERN)



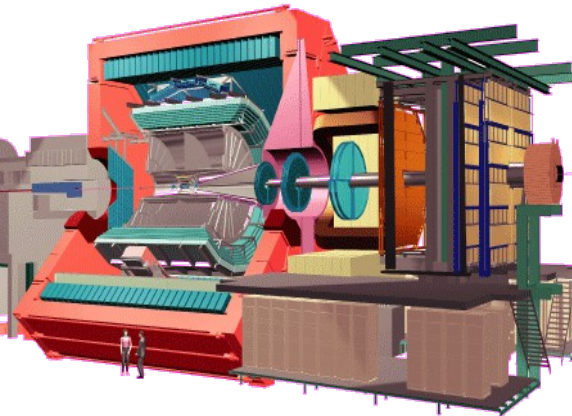
# Heavy ion physics at the LHC

2010, 2011: PbPb ( $\sqrt{s_{\text{NN}}} = 2.76 \text{ TeV}$ );

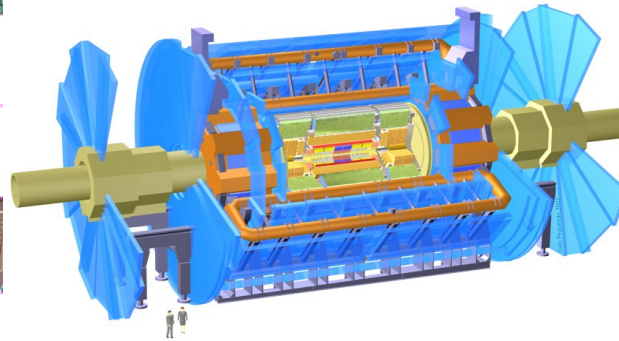
2012/2013: pPb ( $\sqrt{s_{\text{NN}}} = 5.02 \text{ TeV}$ );  $\geq 2015$ : PbPb ( $\sqrt{s_{\text{NN}}} = 5.1\text{-}5.5 \text{ TeV}$ );...

New regime of heavy ion physics with the important role of hard QCD-processes in hot and long-lived quark-gluon medium  
complementary measurements from ALICE & CMS/ATLAS

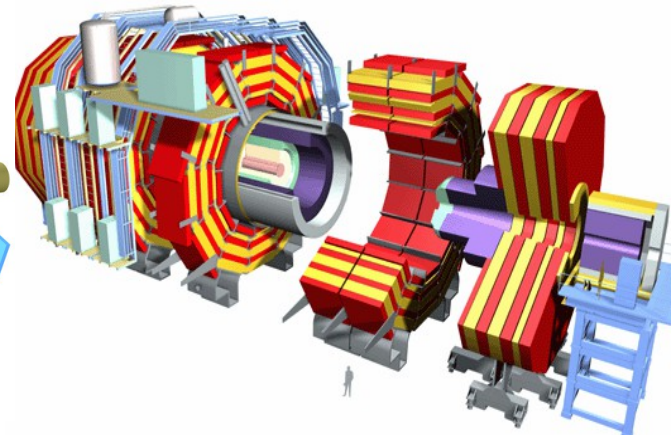
**ALICE**



**ATLAS**



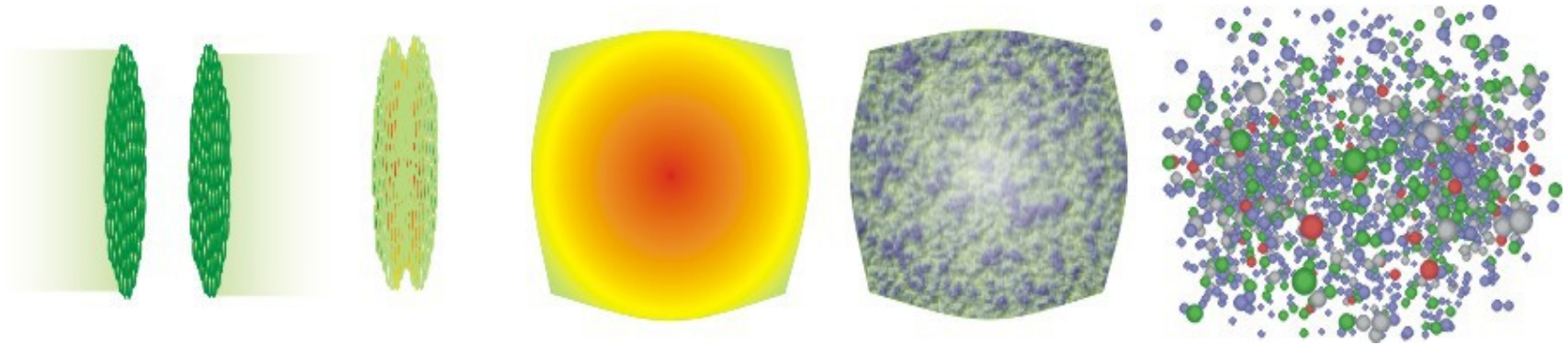
**CMS**



ALICE (low- $p_{\text{T}}$  charged particle tracking, hadron ID, central  $e$ , forward  $\mu$  ( $J/\psi$ ,  $\Upsilon$ ), soft  $\gamma$ ,...)   
soft probes + selected hard probes

CMS/ATLAS (high- $p_{\text{T}}$  charged particle tracking, central  $\mu$  ( $J/\psi$ ,  $\Upsilon$ ,  $Z$ ,  $W$ ), hard  $\gamma$ , calorimetric jets...)   
hard probes + selected soft probes

# Basic probes of hot and dense quark-gluon matter formation in PbPb collisions at Large Hadron Collider at $\sqrt{s_{NN}}=2.76$ TeV



## Hydrodynamical (collective) properties of multi-particle system

- Anisotropic flow
- Two-particle azimuthal correlations (“ridge”)

## Medium-induced energy loss of hard quarks and gluons (“jet quenching”)

- Transverse momentum imbalance in *jet+jet*,  $\gamma$ +*jet*, *Z+jet* production
- Suppression of hard hadron and jet yields
- Modification of internal jet structure

## Debye screening of colour charge and thermal charmonium production

- Specific pattern of quarkonium suppression ( $J/\psi$ ,  $Y$ )
- Regeneration and anisotropic flow of  $J/\psi$  mesons

# HYDJET and HYDJET++

## relativistic heavy ion event generators

**HYDJET (HYDroynamics + JETs)** - event generator to simulate heavy ion event as merging of two independent components (soft hydro-type part + hard multi-partonic state, the latter is based on **PYQUEN - PYthia QUENched**).

<http://cern.ch/lokhtin/hydro/hydjet.html>

*(latest version 1.9)*

Original paper: I.Lokhtin, A.Snigirev, Eur. Phys. J. C 46 (2006) 2011

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**HYDJET++ (HYDJET v.2.\*)** – continuation of HYDJET (identical hard component + improved soft component including full set of thermal resonance production).

<http://cern.ch/lokhtin/hydjet++>

*(latest version 2.2)*

Original paper: I.Lokhtin, L.Malinina, S.Petrushanko, A.Snigirev, I.Arsene, K.Tywoniuk, Comp. Phys. Comm. 180 (2009) 779

# HYDJET++ (soft component): physics frames

Soft (hydro) part of HYDJET++ is based on the adapted FAST MC model:

**Part I:** *N.S.Amelin, R.Lednisky, T.A.Pocheptsov, I.P.Lokhtin, L.V.Malinina, A.M.Snigirev, Yu.A.Karpenko, Yu.M.Sinyukov, Phys. Rev. C 74 (2006) 064901*

**Part II:** *N.S.Amelin, R.Lednisky, I.P.Lokhtin, L.V.Malinina, A.M.Snigirev, Yu.A.Karpenko, Yu.M.Sinyukov, I.C.Arsene, L.Bravina, Phys. Rev. C 77 (2008) 014903*

- ✓ **fast** HYDJET-inspired MC procedure for soft hadron generation
- ✓ multiplicities are determined assuming **thermal equilibrium**
- ✓ hadrons are produced on the hypersurface represented by a **parameterization**
  - of relativistic hydrodynamics with given **freeze-out conditions**
- ✓ **chemical and kinetic freeze-outs** are separated
- ✓ decays of **hadronic resonances** are taken into account (360 particles from SHARE data table) with “home-made” decayer
- ✓ written within **ROOT** framework (C++)
- ✓ contains 16 **free parameters** (but this number may be reduced to 9)

# HYDJET++ (hard component): PYQUEN (PYthia QUENched)

Initial parton configuration

PYTHIA6.4 w/o hadronization: `mstp(111)=0`



Parton rescattering & energy loss (collisional, radiative) + emitted g

PYQUEN rearranges partons to update ns strings



Parton hadronization and final particle formation

PYTHIA6.4 with hadronization: call PYEXEC

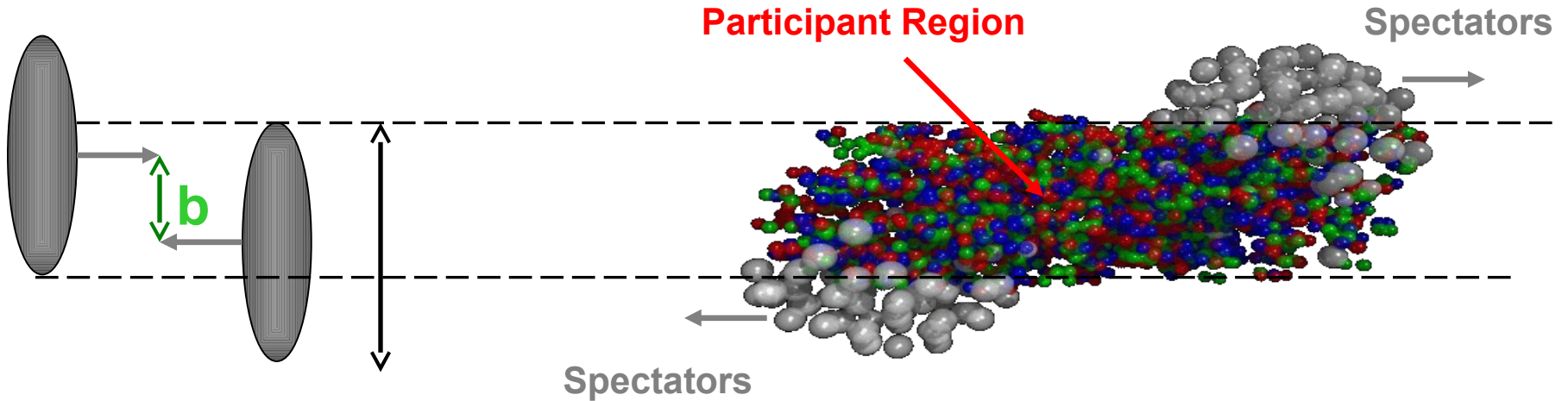
Three model parameters: initial maximal QGP temperature  $T_0$ , QGP formation time  $\tau_0$   
and number of active quark flavors in QGP  $N_f$

(+ minimal  $p_T$  of hard process `Ptmin` to specify the number of hard NN collisions)

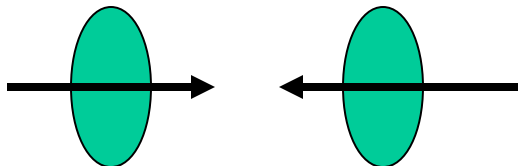
*I.P.Lokhtin, A.M.Snigirev, Eur. Phys. J. 45 (2006) 211 (latest version 1.5.1)*



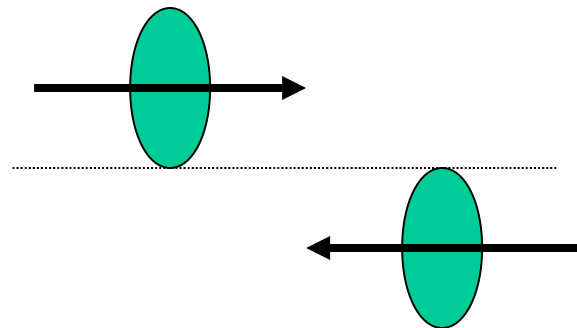
# Centrality of nucleus-nucleus interactions



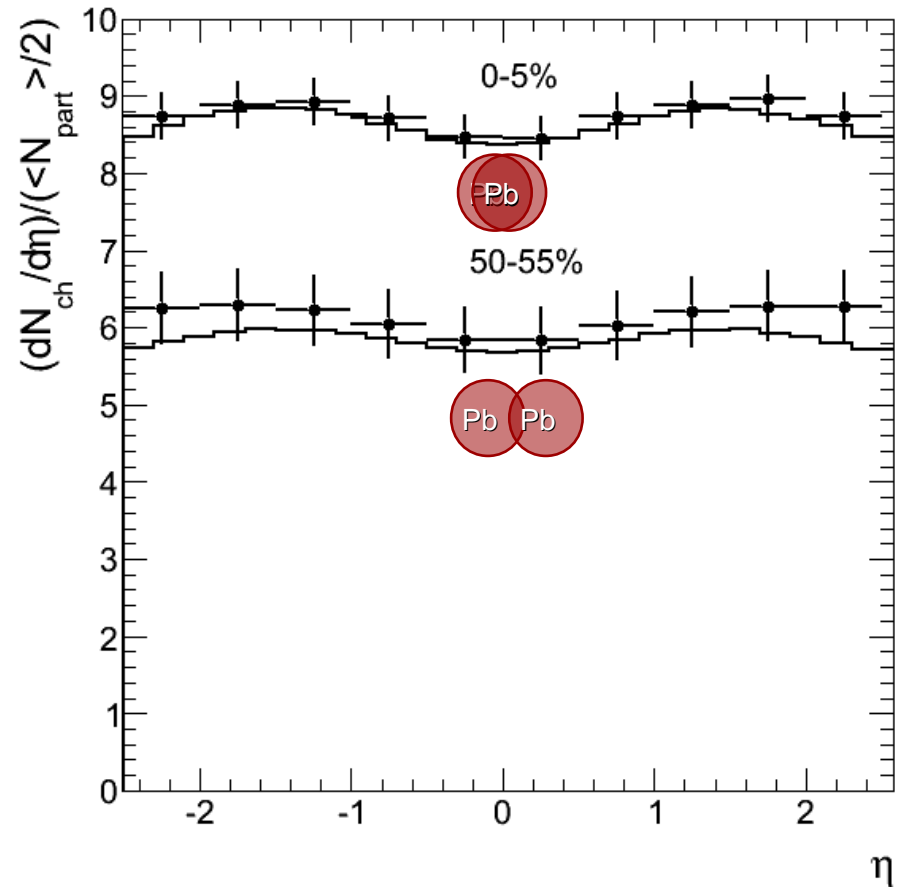
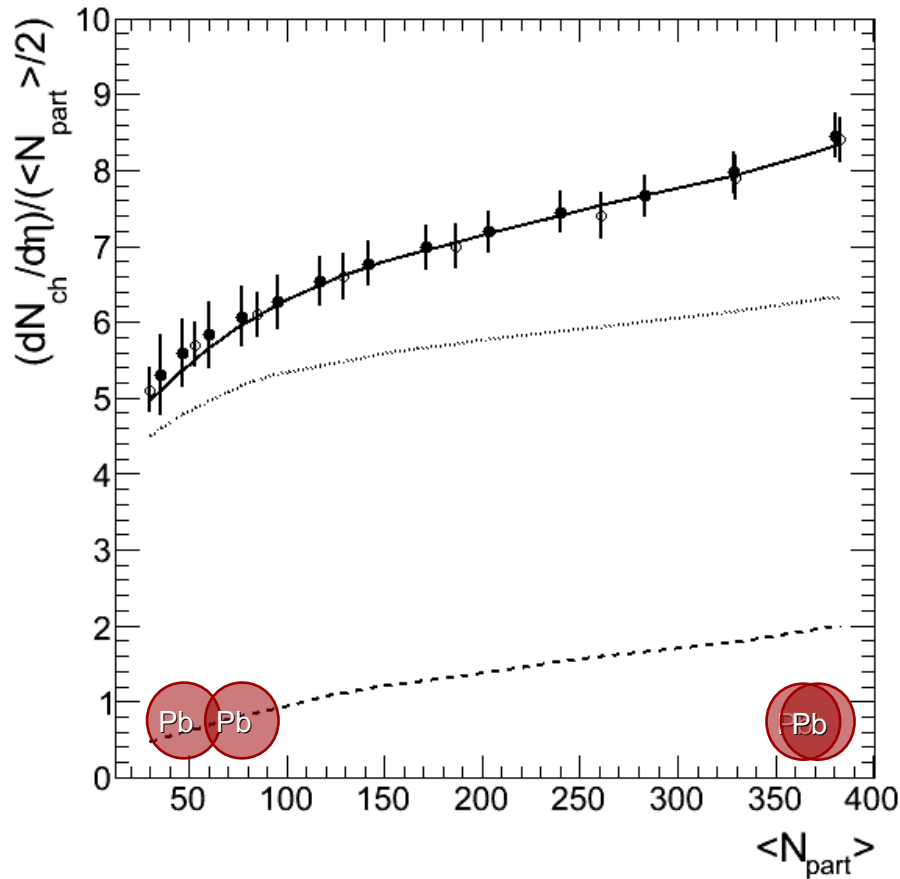
central collision



peripheral collision



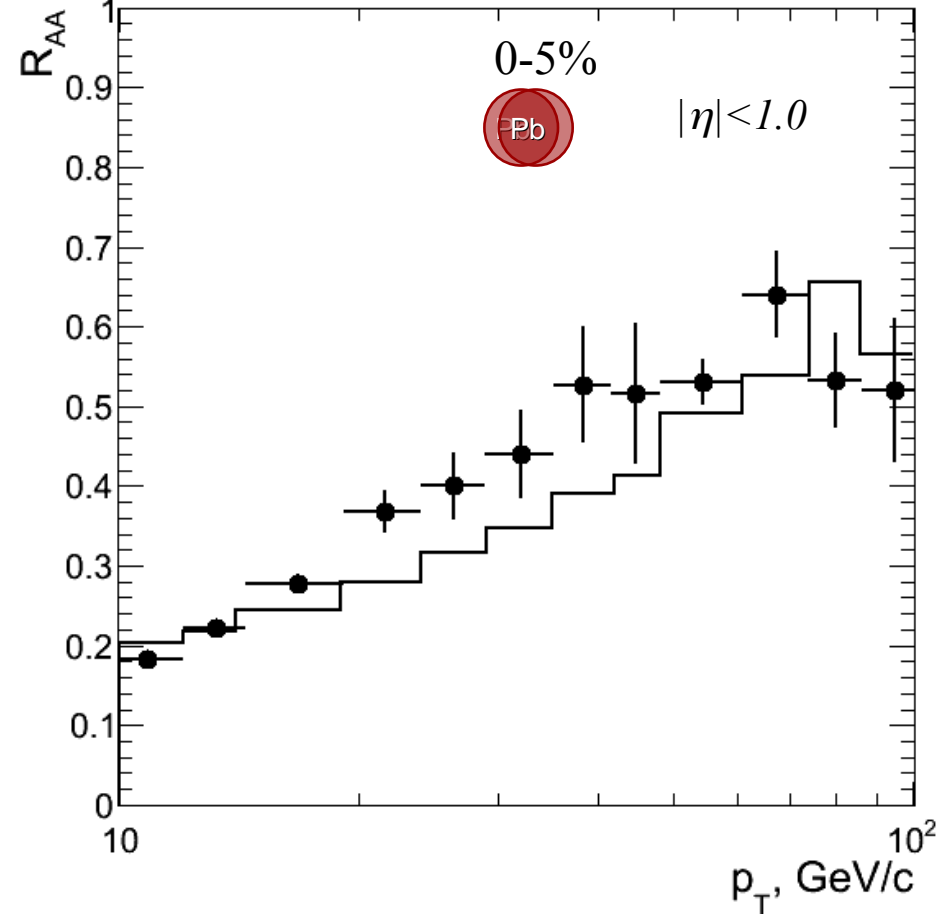
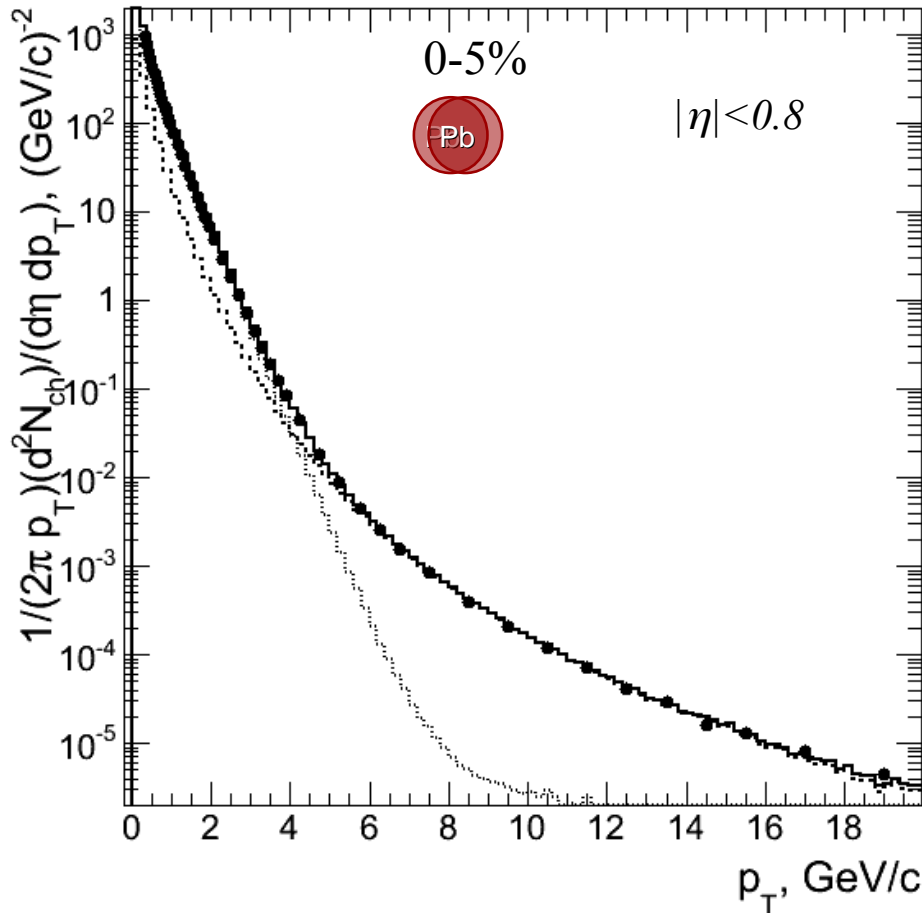
# Charged multiplicity vs. centrality and pseudorapidity



Open points: ALICE data (*PRL 106 (2011) 032301*), closed points: CMS data (*JHEP 1108 (2011) 141*);  
 histograms: HYDJET++

Tuned HYDJET++ reproduces multiplicity vs. event centrality (down to very peripheral events) with contribution of hard component to multiplicity in mid-rapidity for central PbPb  $\sim 30\%$ , as well as approximately flat pseudorapidity distribution.

# $P_T$ -spectrum and nuclear modification factor $R_{AA}$ for inclusive charged hadrons



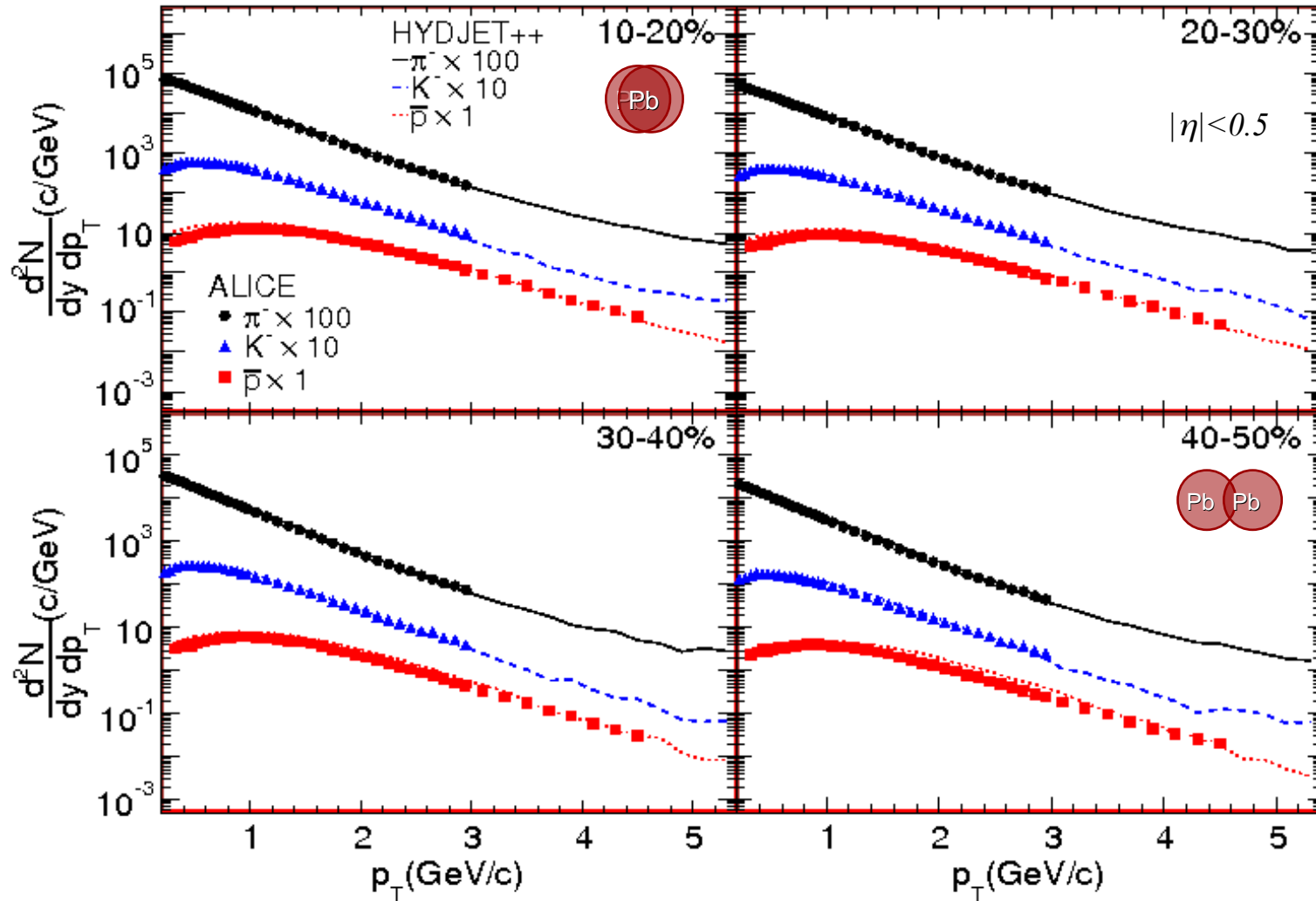
$$R_{AA} = \frac{\sigma_{pp}^{inel}}{\langle N_{coll} \rangle} \frac{d^2 N_{AA} / dp_T d\eta}{d^2 \sigma_{pp} / dp_T d\eta} \sim \begin{cases} \text{“QCD Medium”} \\ \text{“QCD Vacuum”} \end{cases} \left\{ \begin{array}{l} R_{AA} > 1: \text{enhancement} \\ R_{AA} = 1: \text{no medium effect} \\ R_{AA} < 1: \text{suppression} \end{array} \right.$$

Points: ALICE (left) (*PLB* 696 (2011) 30) & CMS (right) (*EPJ C* 72 (2012) 1945) data;

histograms: HYDJET++

HYDJET++ reproduces  $p_T$ -spectrum and  $R_{AA}$  for central PbPb in mid-rapidity up to  $p_T \sim 100$  GeV/c

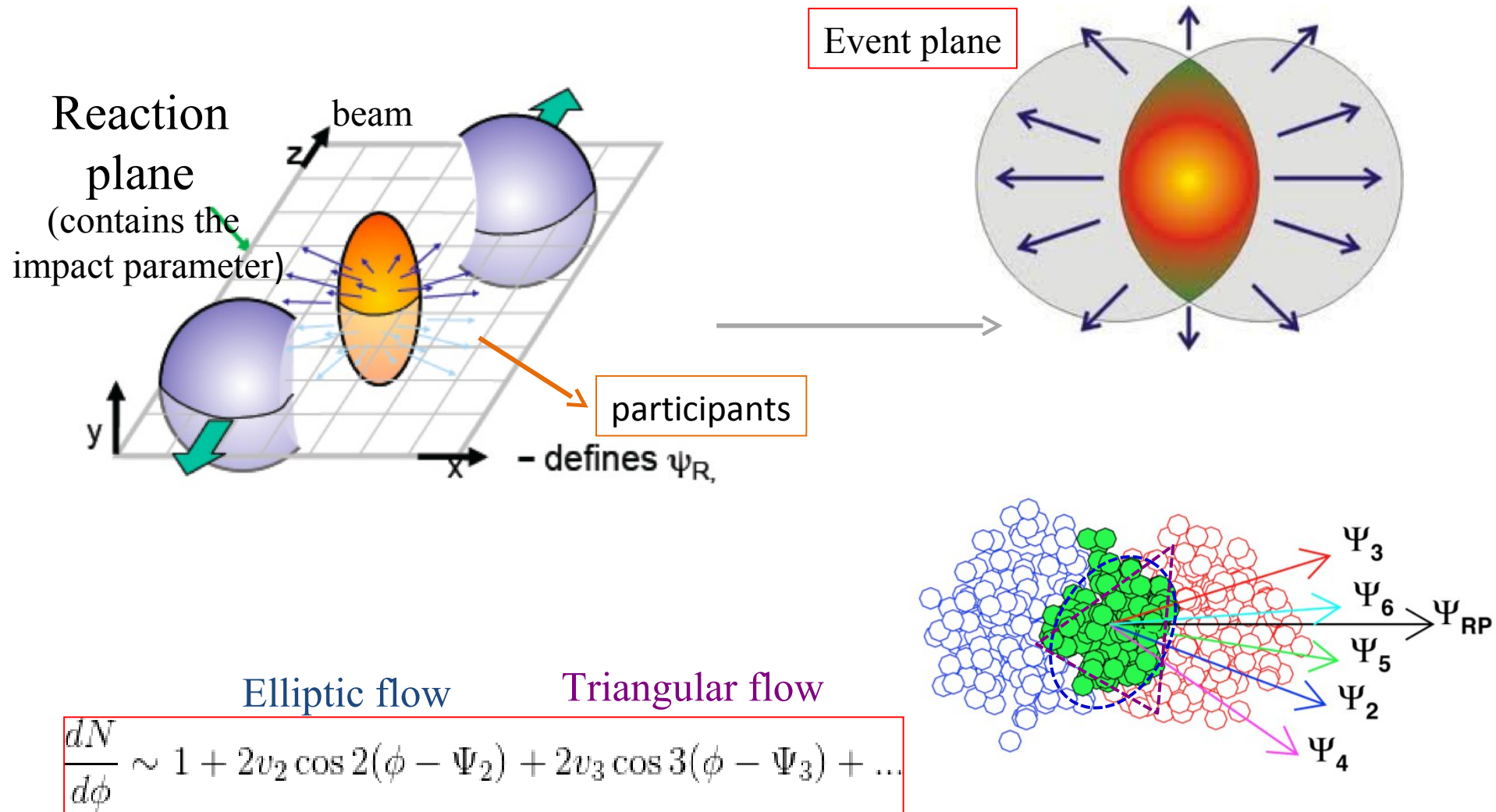
# $P_T$ -spectra of identified hadrons



Points: ALICE data (*APP B 43 (2012) 555*); histograms: HYDJET++

HYDJET++ reproduces  $p_T$ -spectrum of pions, kaons and (anti-)protons as well

# Azimuthal correlations and flow



# Anisotropic flow generation in HYDJET++ (soft component)

## Elliptic flow $v_2$

$$v_2 \propto \frac{2(\delta - \epsilon)}{(1 - \delta^2)(1 - \epsilon^2)}$$

- spatial modulation of freeze-out surface
- fluid velocity modulation

### Spatial anisotropy

$$\epsilon(b) = \frac{R_y^2 - R_x^2}{R_y^2 + R_x^2},$$

$R(b)$  – surface radius

### Momentum anisotropy

$$\tan \varphi_u = \sqrt{\frac{1 - \delta(b)}{1 + \delta(b)}} \tan \varphi.$$

$\varphi_u$  : azimuthal angle of fluid velocity

$\varphi$  : spatial azimuthal angle

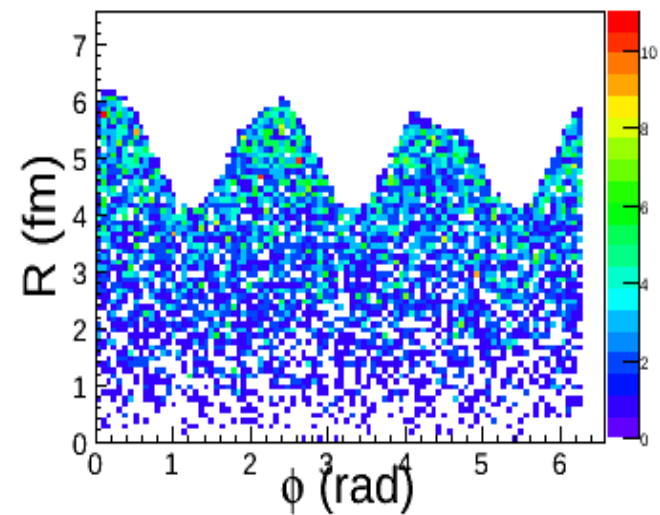
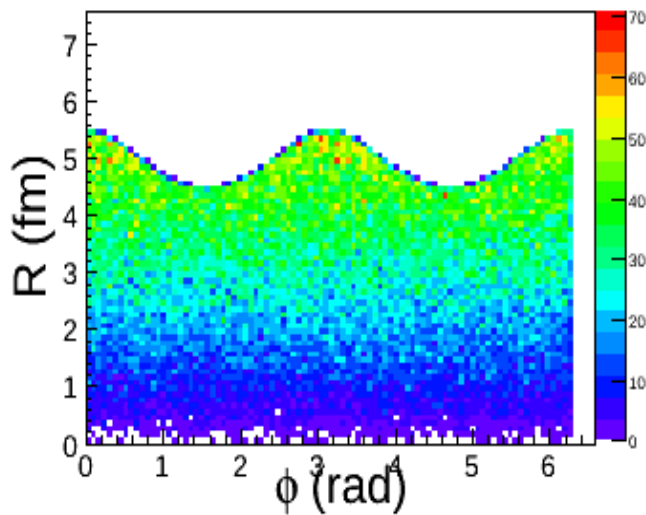
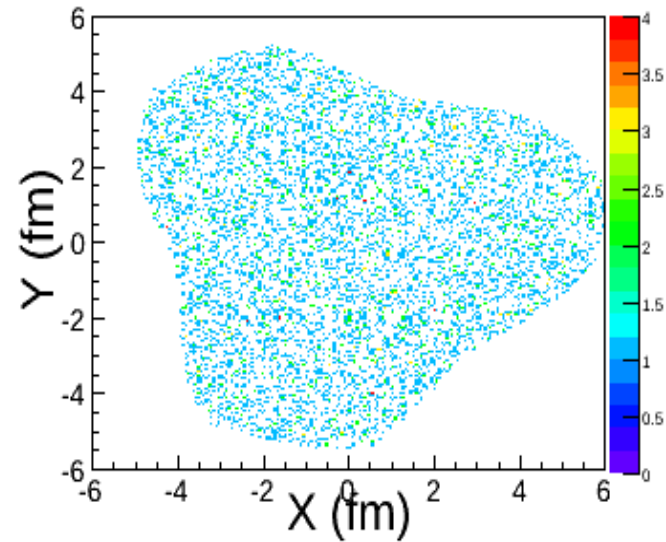
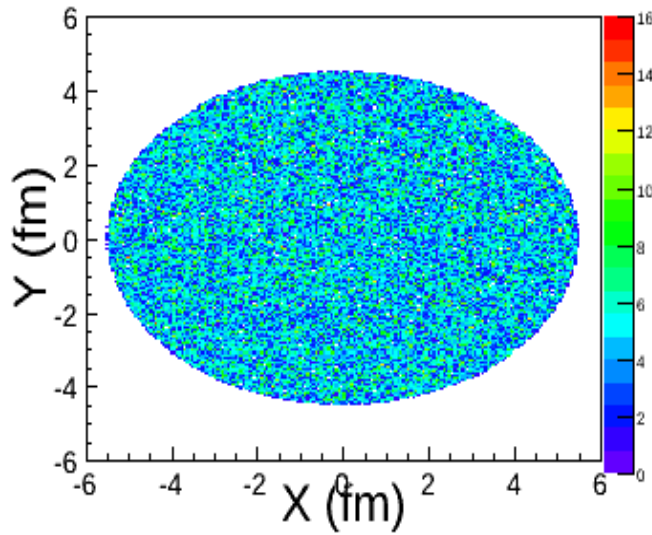
## Triangular flow $v_3$

*Spatial modulation of freeze-out surface as  $\cos(3\varphi)$  with independent phase  $\Psi_3$  and parameter  $\epsilon_3$*

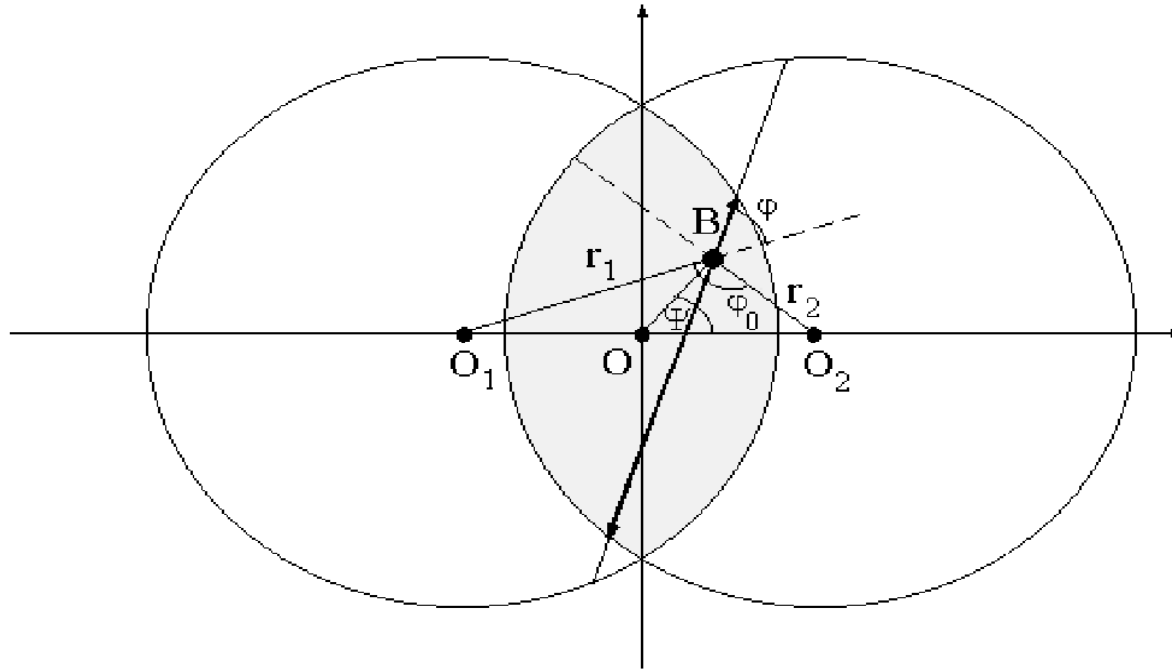
$$R(b, \phi) = R_f(b) \frac{\sqrt{1 - \epsilon^2(b)}}{\sqrt{1 + \epsilon(b) \cos 2\phi}} [1 + \epsilon_3(b) \cos 3(\phi + \Psi_3^{\text{RP}})]$$

Three parameters  $\epsilon(b_0)$ ,  $\epsilon_3(b_0)$  и  $\delta(b_0)$  is tuned to fit the data

# Anisotropic flow generation in HYDJET++ (soft component)



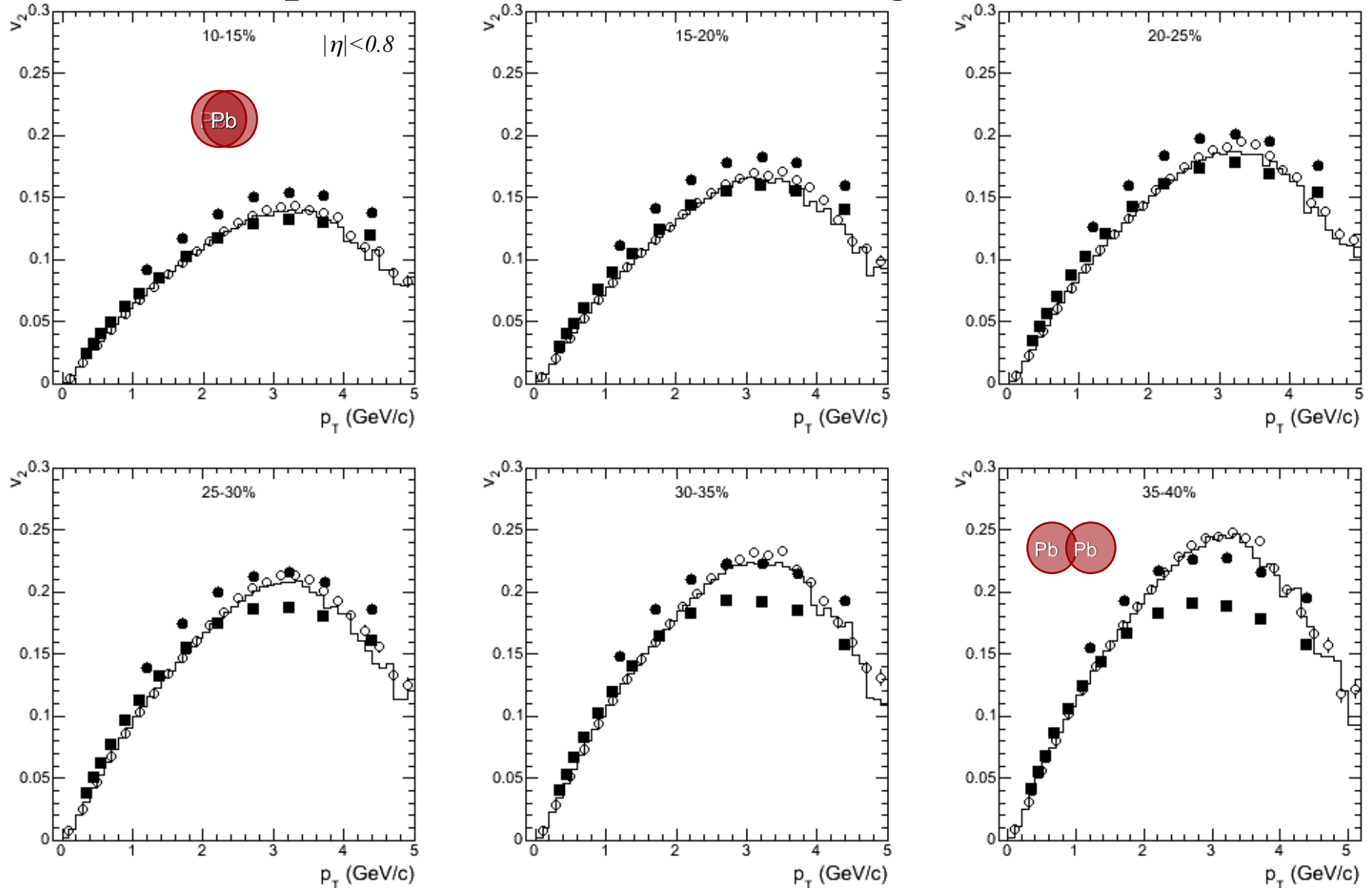
# Anisotropic flow generation in HYDJET++ (hard component)



Some anisotropic flow for hard component ( $v_2$  and higher even harmonics at high transverse momenta) is generated due to partonic rescattering and energy loss in azimuthally-asymmetric volume of the medium



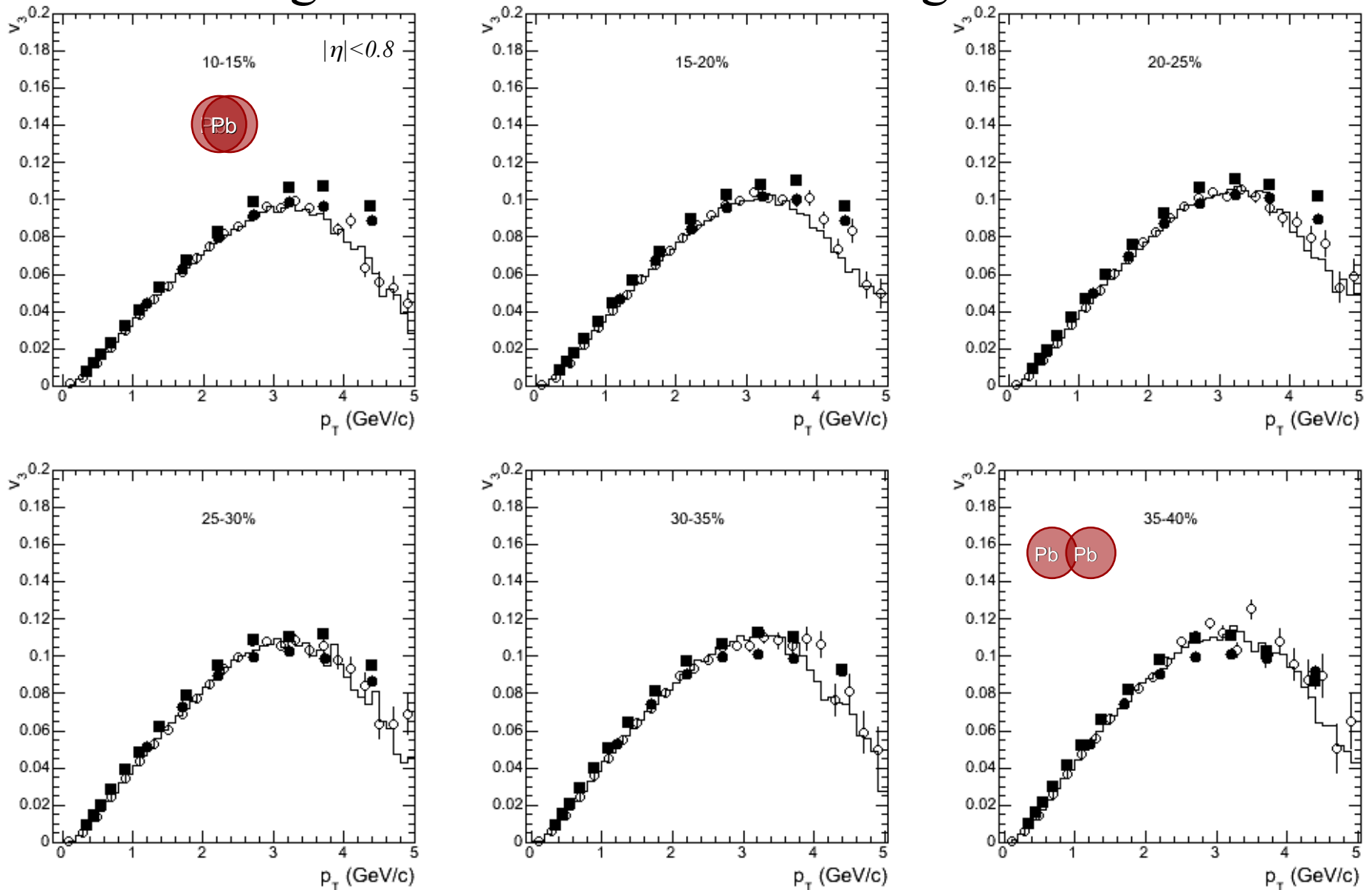
# Elliptic flow of inclusive charged hadrons



Closed circles and squares: CMS data  $v_2\{2\}$  &  $v_2\{\text{LYZ}\}$  (*PRC* 87 (2013) 014902);

histograms and open circles: HYDJET++ ("true"  $v_2(\psi_2)$  &  $v_2\{\text{EP}\}$ )

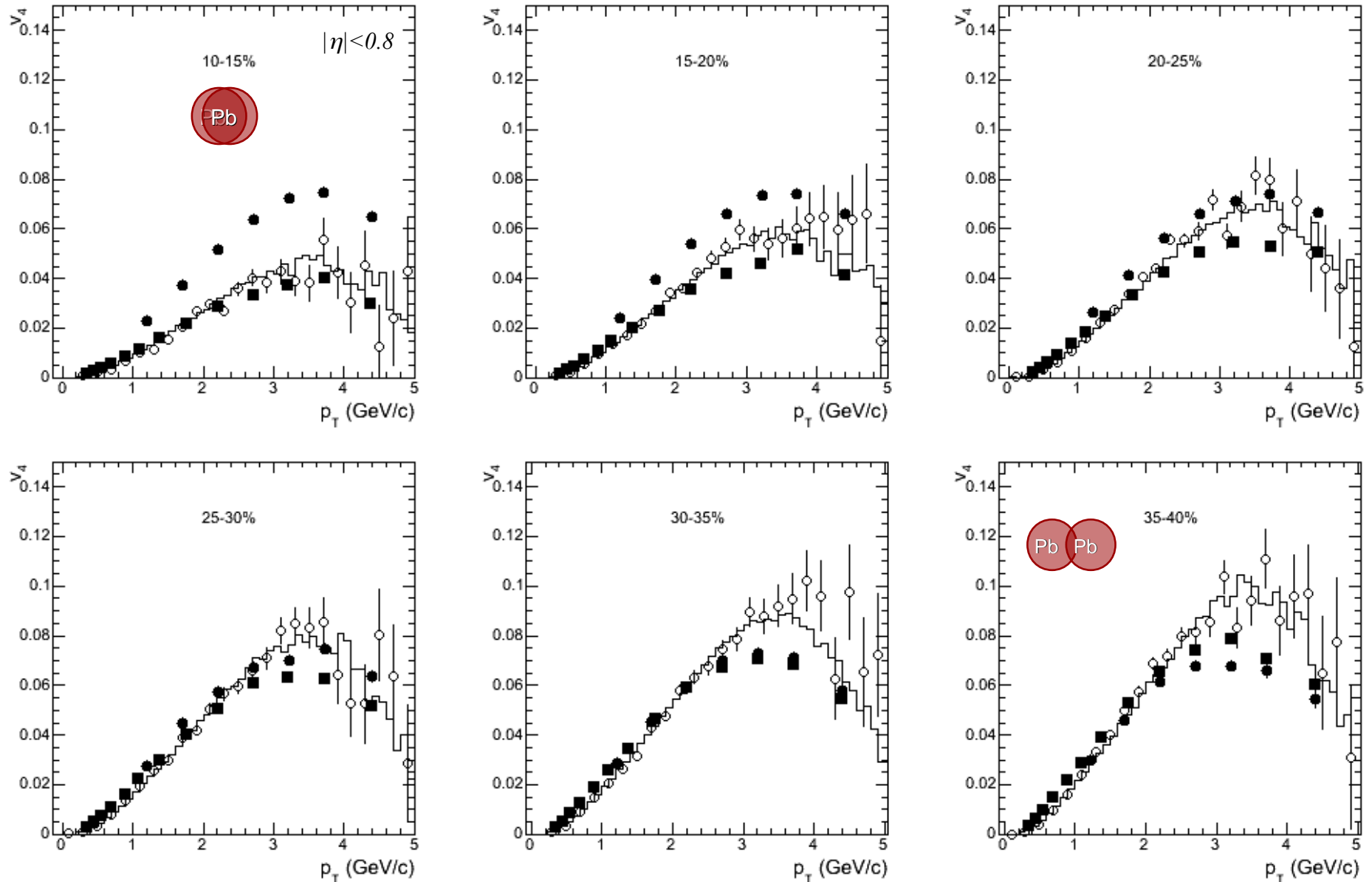
# Triangular flow of inclusive charged hadrons



Closed circles and squares: CMS data  $v_3\{2\}$  &  $v_3\{EP\}$  (*PRC 89 (2014) 044906*);

histograms and open circles: HYDJET++ (“true”  $v_3(\psi)$  &  $v_3\{EP\}$ )

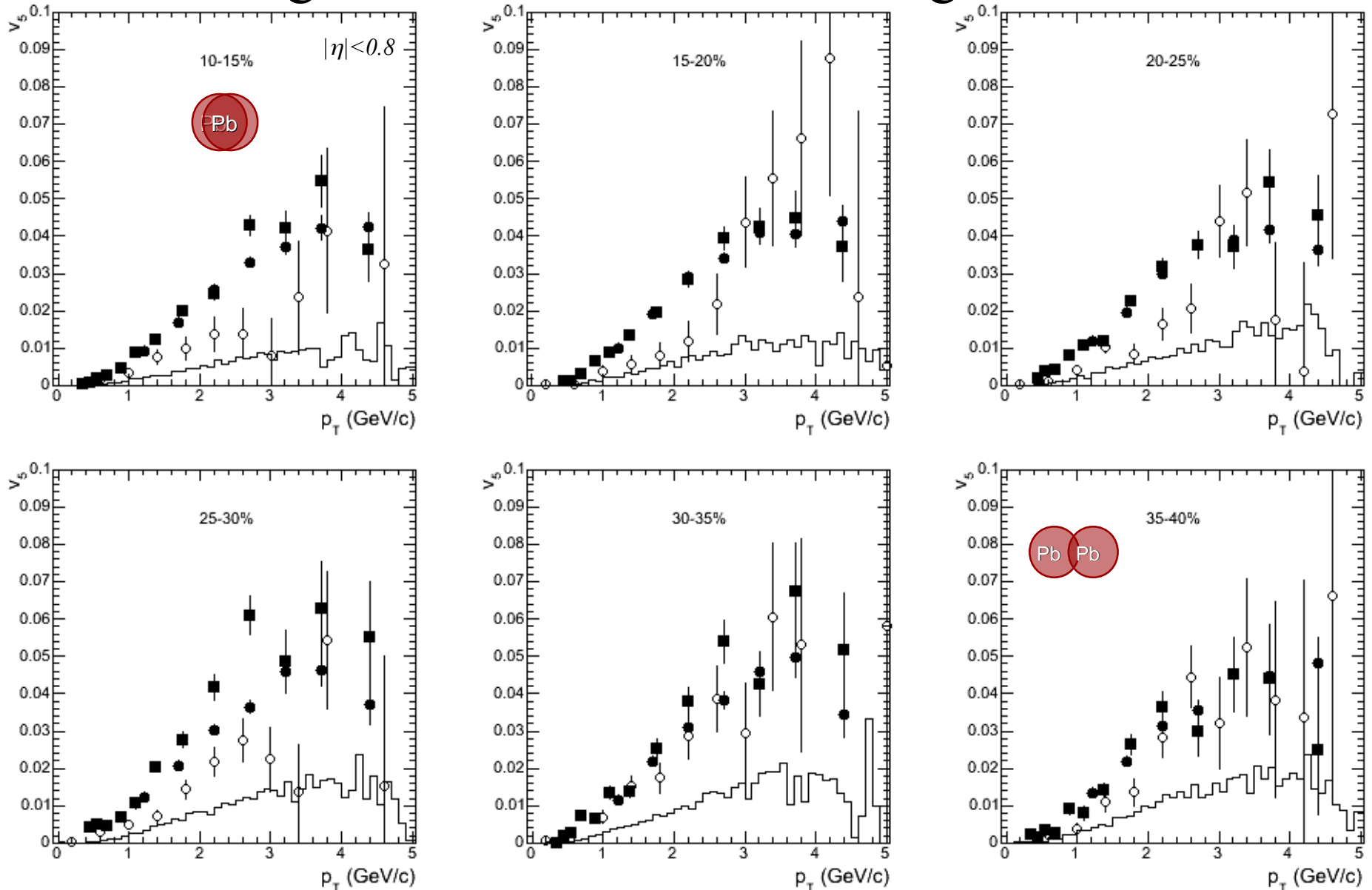
# Quadrangular flow of inclusive charged hadrons



Closed circles and squares: CMS data  $v_4\{2\}$  &  $v_4\{LYZ\}$  (*PRC* 89 (2014) 044906);

histograms and open circles: HYDJET++ ("true"  $v_4(\psi_2)$  &  $v_4\{EP\}$ )

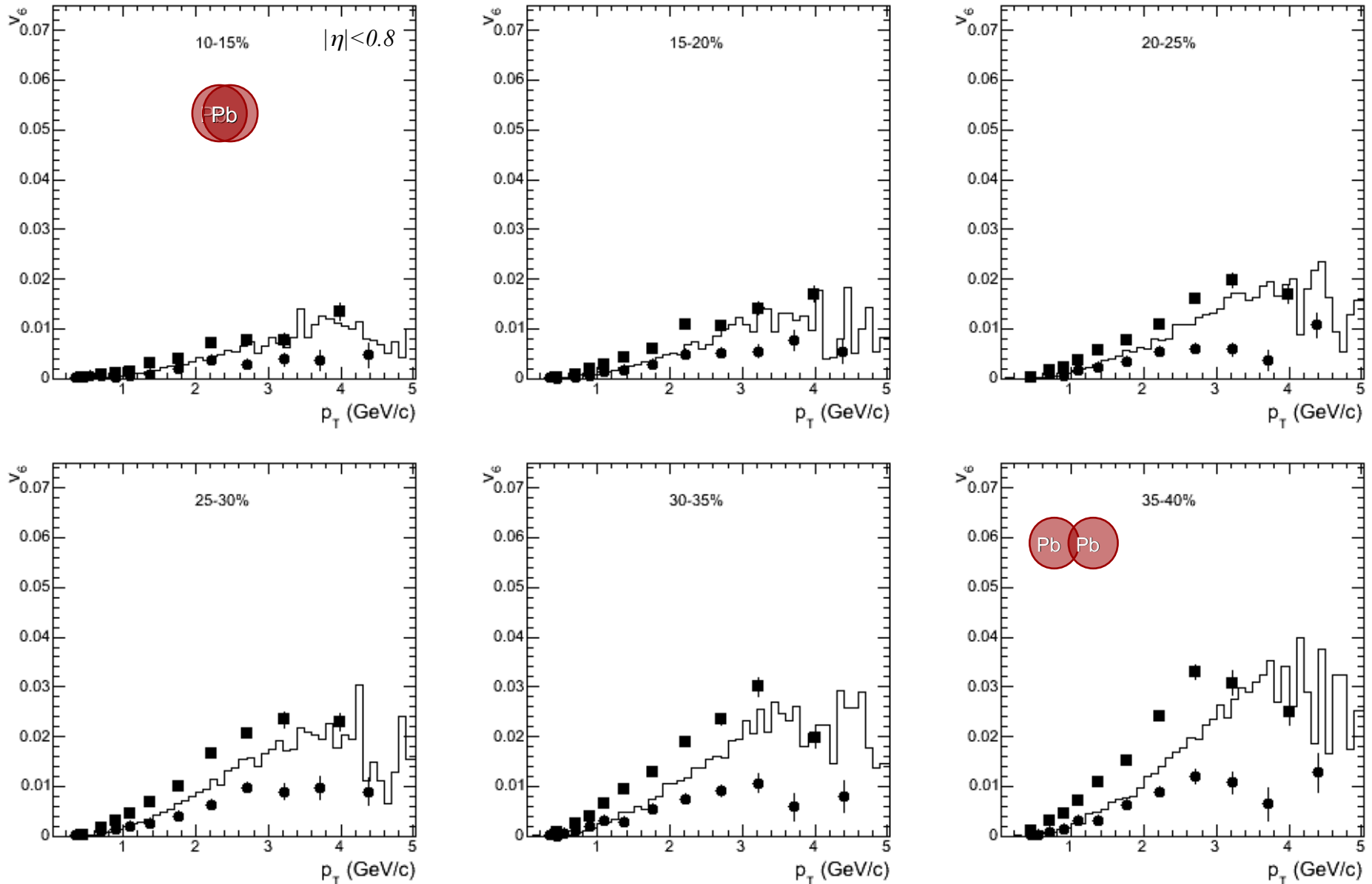
# Pentagonal flow of inclusive charged hadrons



Closed circles and squares: CMS data  $v_5\{2\}$  &  $v_5\{EP\}$  (*PRC 89 (2014) 044906*);

histograms and open circles: HYDJET++ ("true"  $v_5(\psi_3)$  &  $v_5\{EP\}$ )

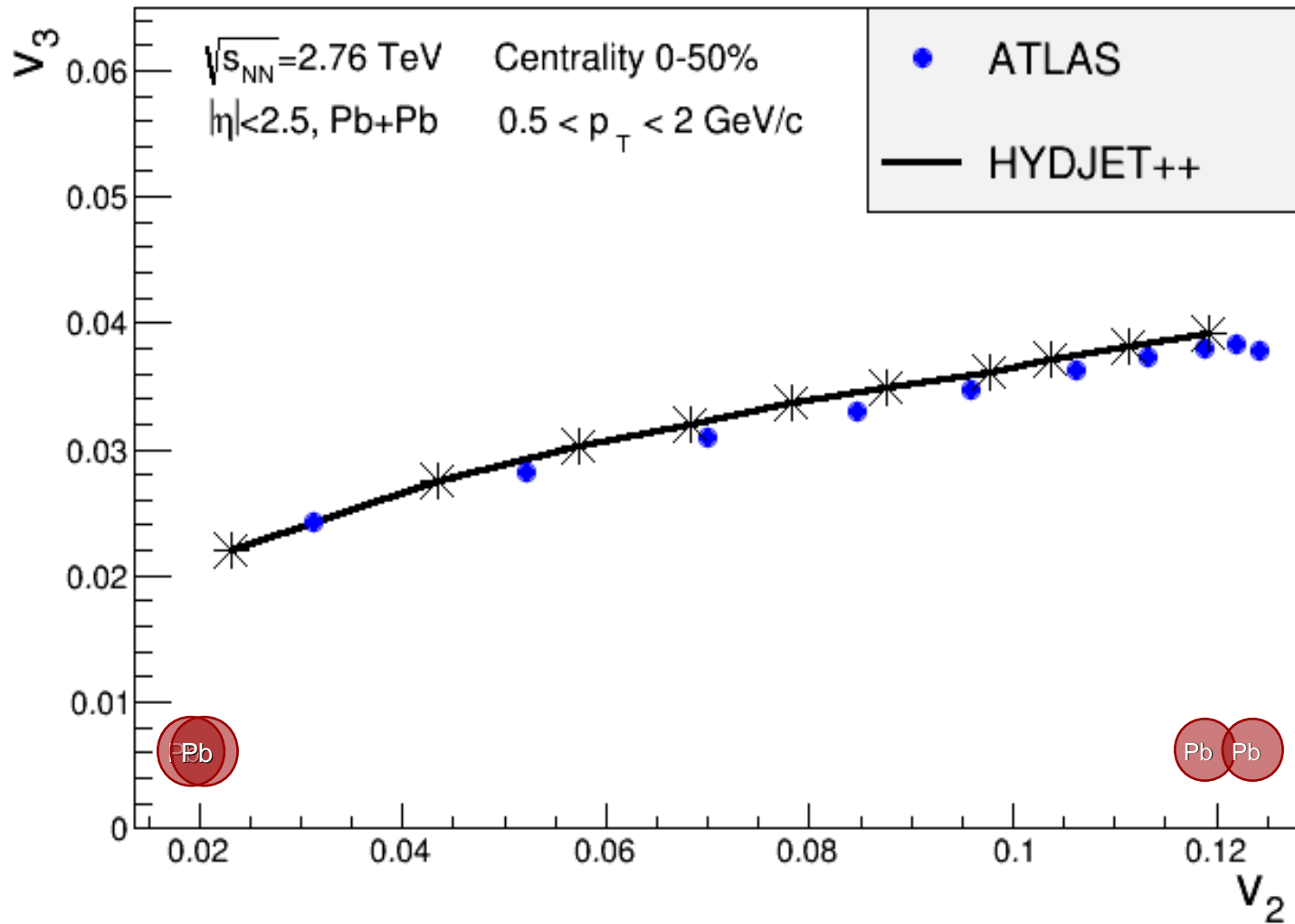
# Hexagonal flow of inclusive charged hadrons



Closed circles and squares: CMS data  $v_6\{\text{EP}/\psi_2\}$  &  $v_6\{\text{LYZ}\}$  (*PRC 89 (2014) 044906*); 21

histograms: HYDJET++ (“true”  $v_6(\psi_2)$ )

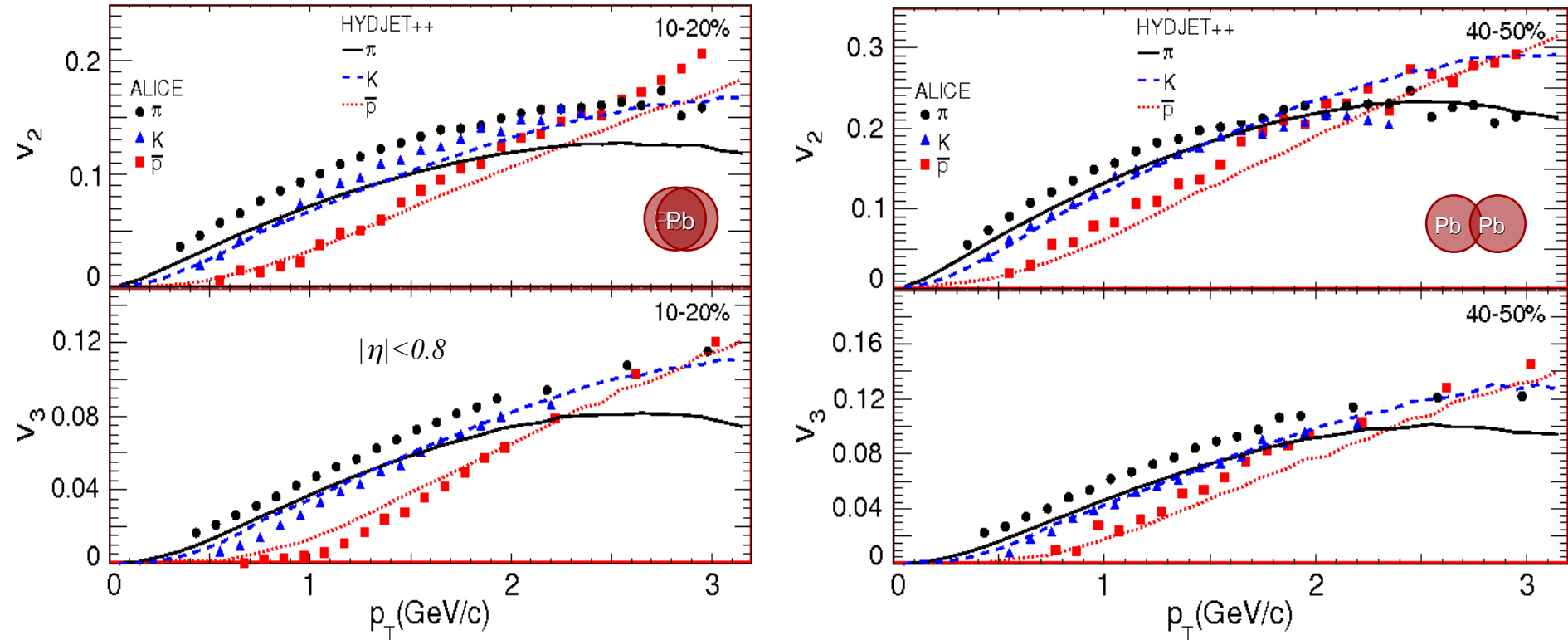
# Correlations between elliptic and triangular flows



Points: ATLAS data ([arXiv:1504.01289](https://arxiv.org/abs/1504.01289)); histogram: HYDJET++

HYDJET++ reproduces the correlation between elliptic and triangular flows 22

# Elliptic and triangular flows of identified hadrons



Points: ALICE data (*JPG 38 (2011) 124047*); histograms: HYDJET++

HYDJET++ reproduces  $v_2$  and  $v_3$  for kaons and (anti-)protons, but rather underestimates the data for pions (stronger non-flow correlations in the data than in the model?)<sup>23</sup>

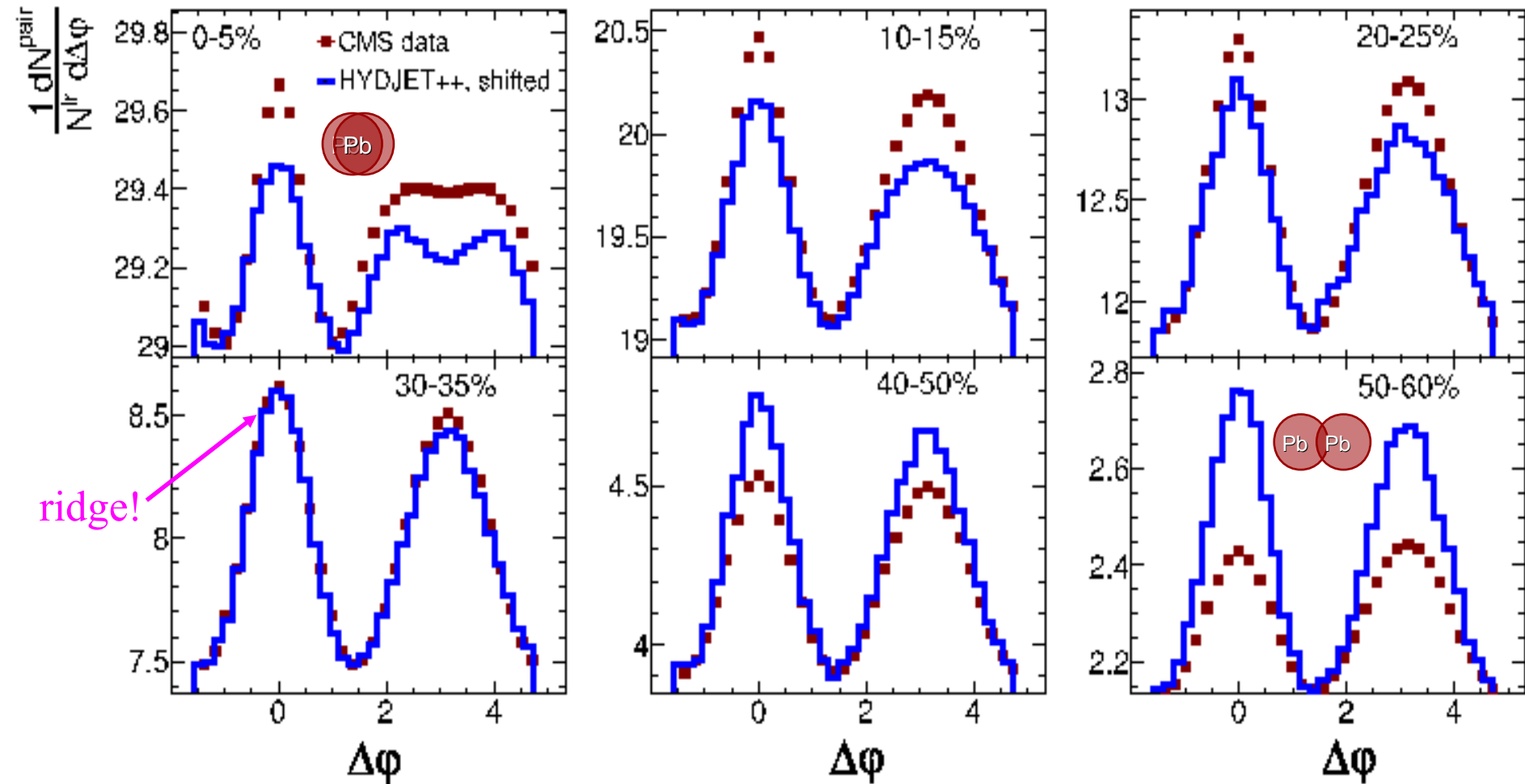
# Dihadron angular correlations

PbPb  $\sqrt{s_{NN}}=2.76$  TeV

$1 < p_T^a < 1.5$  GeV/c

$3 < p_T^r < 3.5$  GeV/c

$2 < |\Delta\eta| < 4$



Points: CMS data (EPJC 72 (2012) 2012); histograms: HYDJET++

Interplay of elliptic and triangular flows in HYDJET++ yields long-range 2-particle azimuthal correlations (ridge effect), but centrality dependence of the correlation strength seems to be strong



## 1) Thermal charm production in HYDJET++ (soft component)

Thermal charmed hadrons  $J/\psi$ ,  $D^0$ ,  $\bar{D}^0$ ,  $D^+$ ,  $D^-$ ,  $D_s^+$ ,  $D_s^-$ ,  $\Lambda_c^+$ ,  $\Lambda_c^-$  are generated within the statistical hadronization model

*(A.Andronic, P.Braun-Munzinger, K.Redlich, J.Stachel,*

*Phys.Lett. B 571 (2003) 36; Nucl. Phys. A 789 (2007) 334)*

$$N_D = \gamma_c N_D^{\text{th}} (I_1(\gamma_c N_D^{\text{th}}) / I_0(\gamma_c N_D^{\text{th}})), \quad N_{J/\psi} = \gamma_c^2 N_{J/\psi}^{\text{th}}$$

$\gamma_c$  - charm enhancement factor is obtained from the equation:

$$N_{cc} = 0.5 \gamma_c N_D^{\text{th}} (I_1(\gamma_c N_D^{\text{th}}) / I_0(\gamma_c N_D^{\text{th}})) + \gamma_c^2 N_{J/\psi}^{\text{th}}$$

where number of c-quark pairs  $N_{cc}$  is calculated with PYTHIA

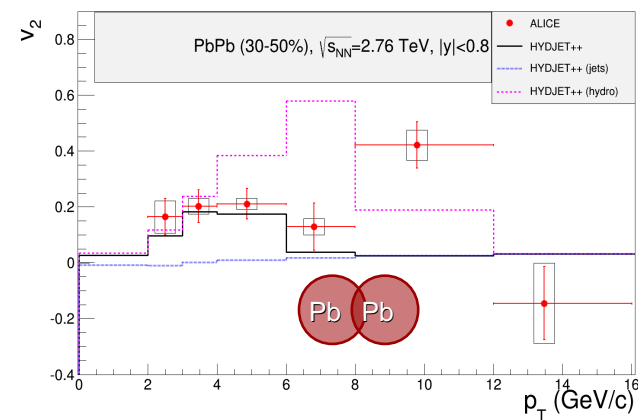
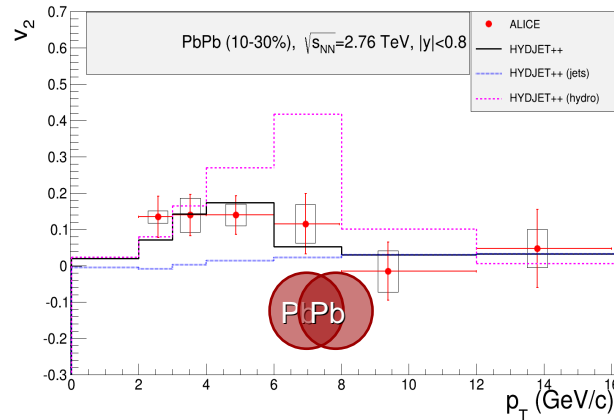
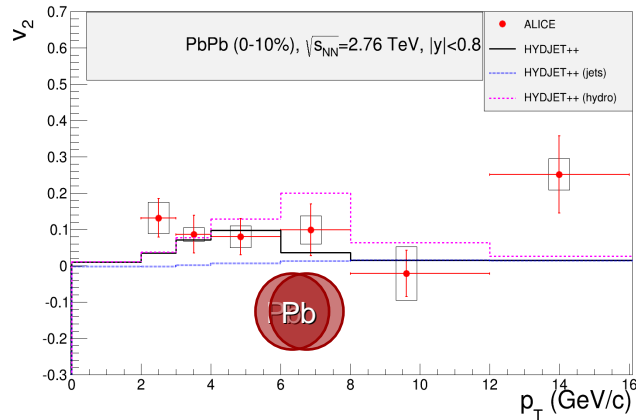
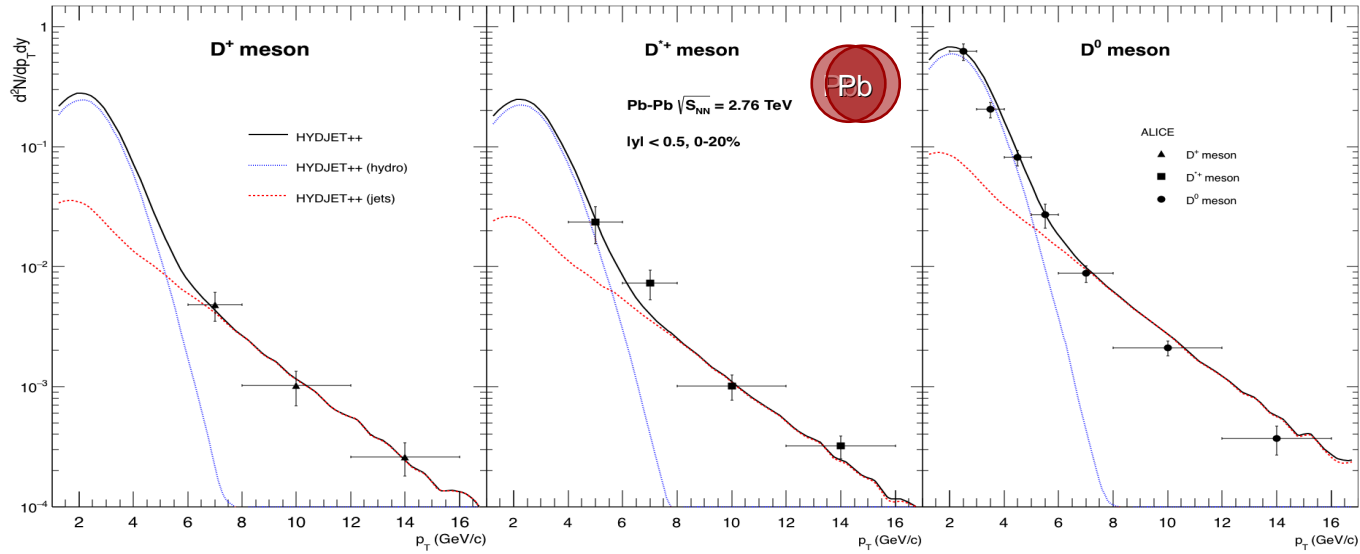
(the factor  $K \sim 2$  is applied to take into account NLO pQCD corrections)

and multiplied by the number of NN sub-collisions for given centrality

## 2) Non-thermal charm production in HYDJET++ (hard component)

Non-thermal charmed hadrons are generated within PYTHIA/PYQUEN taking into account medium-induced rescattering and energy loss of heavy quarks (b, c)

# $P_T$ -spectra and elliptic flow of $D^0$ -mesons

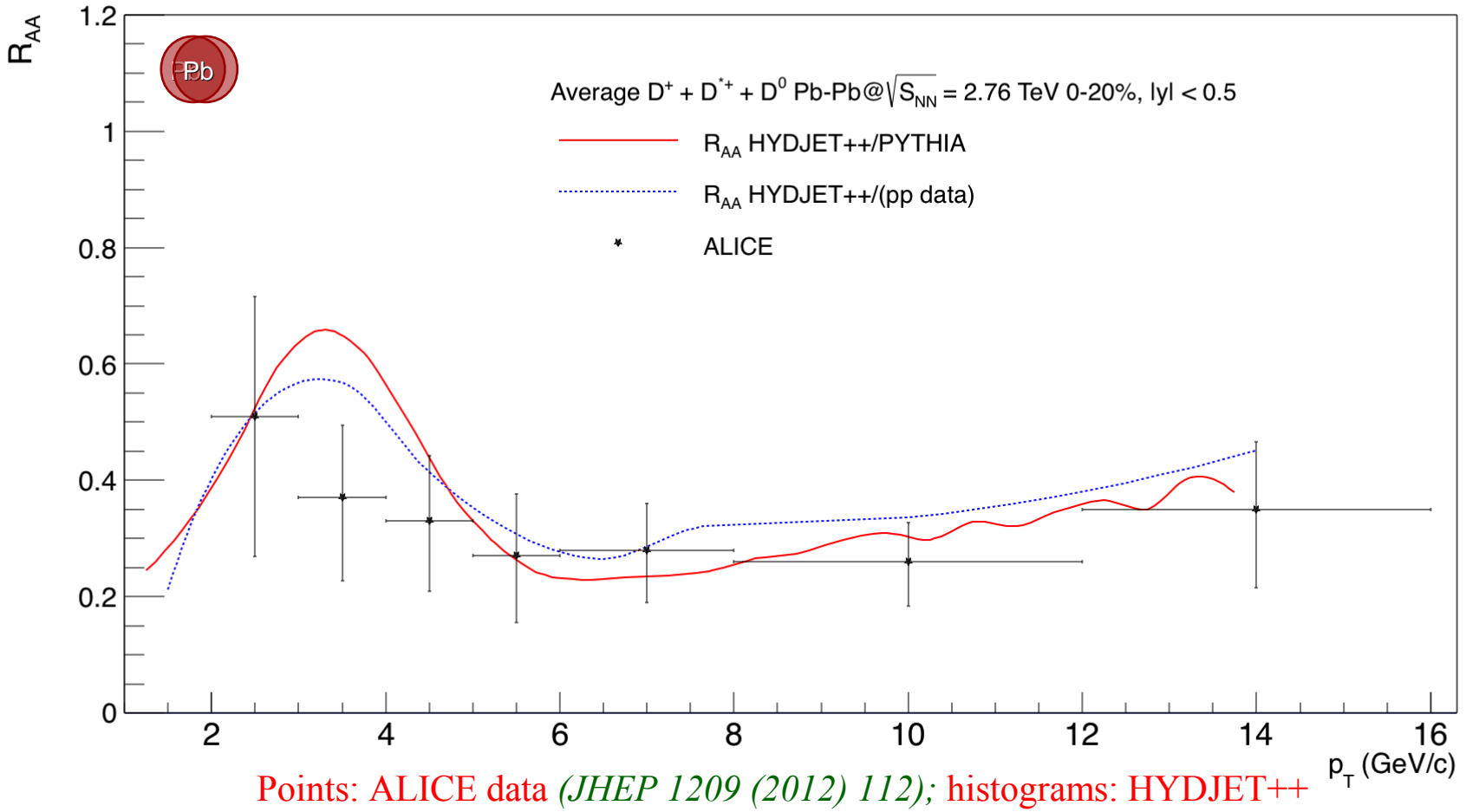


Points: ALICE data (*JHEP* 1209 (2012) 112; *PRC* 90 (2014) 034904); histograms: HYDJET++

HYDJET++ reproduces  $p_T$ -spectrum &  $v_2(p_T)$  of  $D$ -mesons with the *same freeze-out parameters* as for inclusive hadrons  $\Rightarrow$  significant part of  $D$ -mesons (*thermal component*) is in the kinetic equilibrium with the medium; *non-thermal component* is important at high  $p_T$  26

# D mesons at LHC (nuclear modification factor $R_{AA}$ )

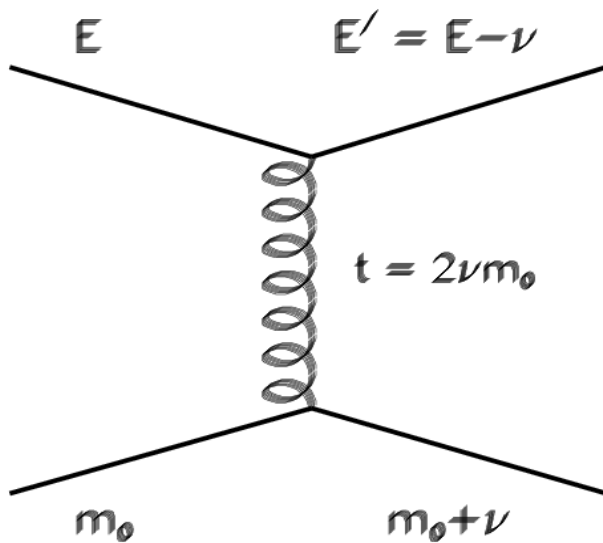
$$R_{AA} = \frac{\sigma_{pp}^{inel} \frac{d^2 N_{AA}}{dp_T d\eta}}{\langle N_{coll} \rangle \frac{d^2 \sigma_{pp}}{dp_T d\eta}} \sim \frac{\text{“QCD Medium”}}{\text{“QCD Vacuum”}} \left\{ \begin{array}{l} R_{AA} > 1: \text{enhancement} \\ R_{AA} = 1: \text{no medium effect} \\ R_{AA} < 1: \text{suppression} \end{array} \right.$$



HYDJET++ reproduces  $R_{AA}(p_T)$  of D-mesons up to very high  $p_T \Rightarrow$  treatment of heavy quark energy loss in hard component of HYDJET++ (PYQUEN) seems quite successful

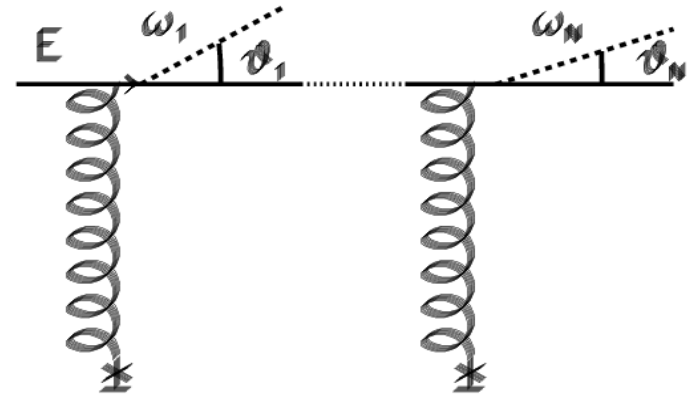
# Medium-induced partonic rescattering and energy loss («jet quenching»)

Collisional loss  
*(high momentum transfer approximation)*



+

Radiative loss  
*(BDMPS model, coherent radiation)*



# Angular structure of energy loss in PYQUEN

**Radiative loss**, three options (simple parametrizations) for angular distribution of in-medium emitted gluons:

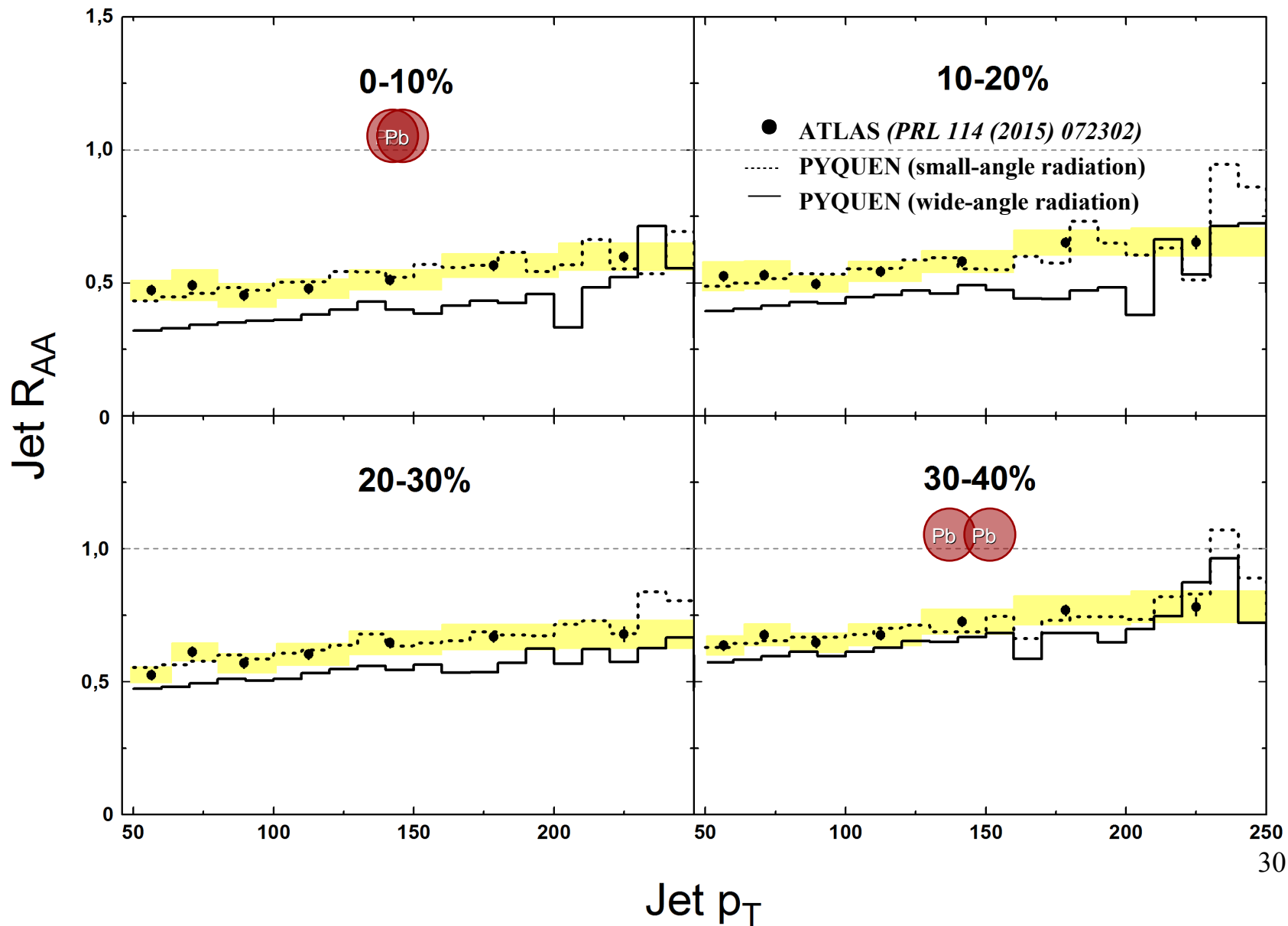
Collinear radiation  $\theta=0$

Small-angular radiation  $\frac{dN^g}{d\theta} \propto \sin\theta \exp\left(\frac{-(\theta-\theta_0)^2}{2\theta_0^2}\right), \quad \theta_0 \sim 5^\circ$

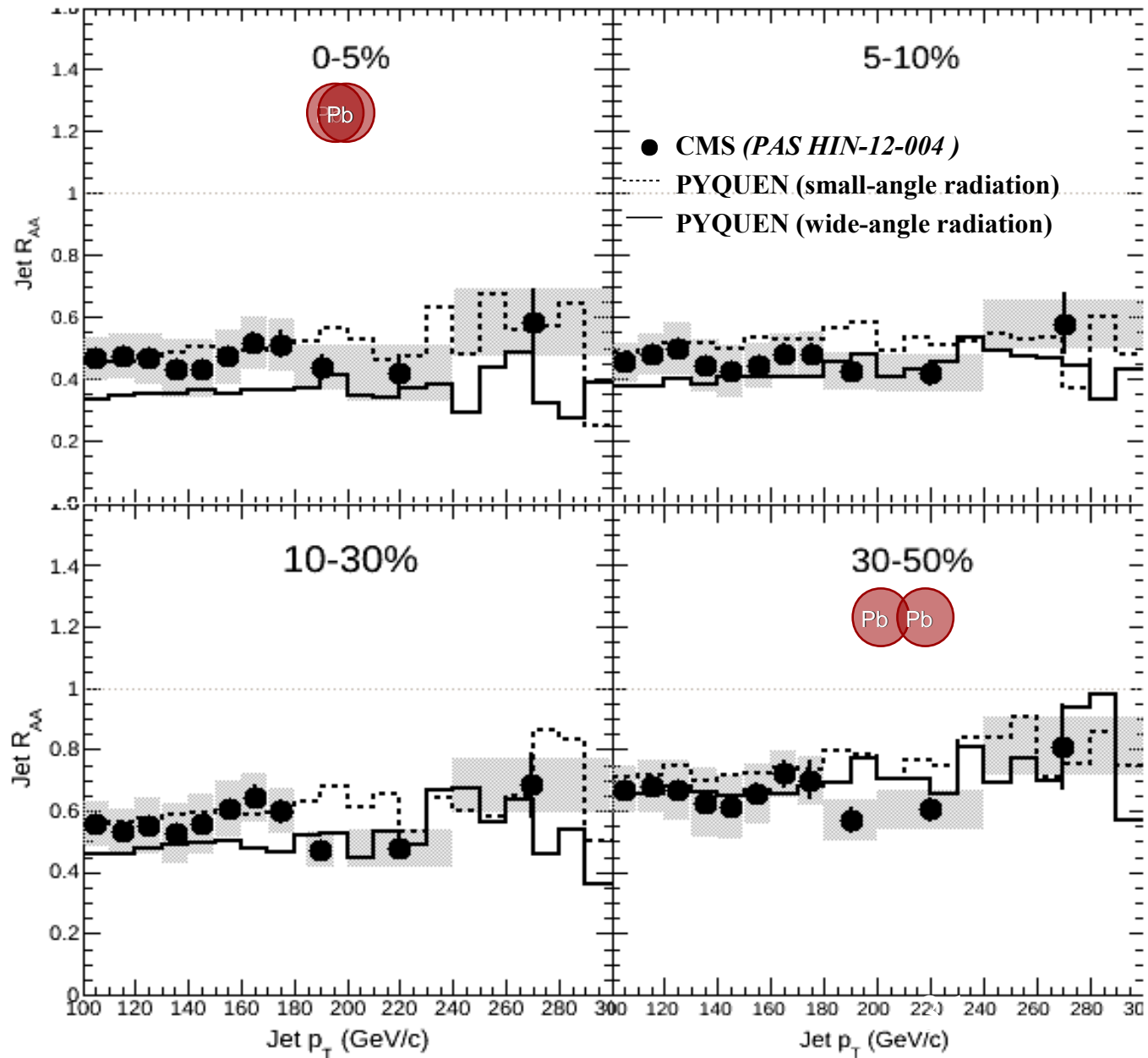
Wide-angular radiation  $\frac{dN^g}{d\theta} \propto \frac{1}{\theta}$

**Collisional loss** always “out-of-cone” (energy is absorbed by medium)

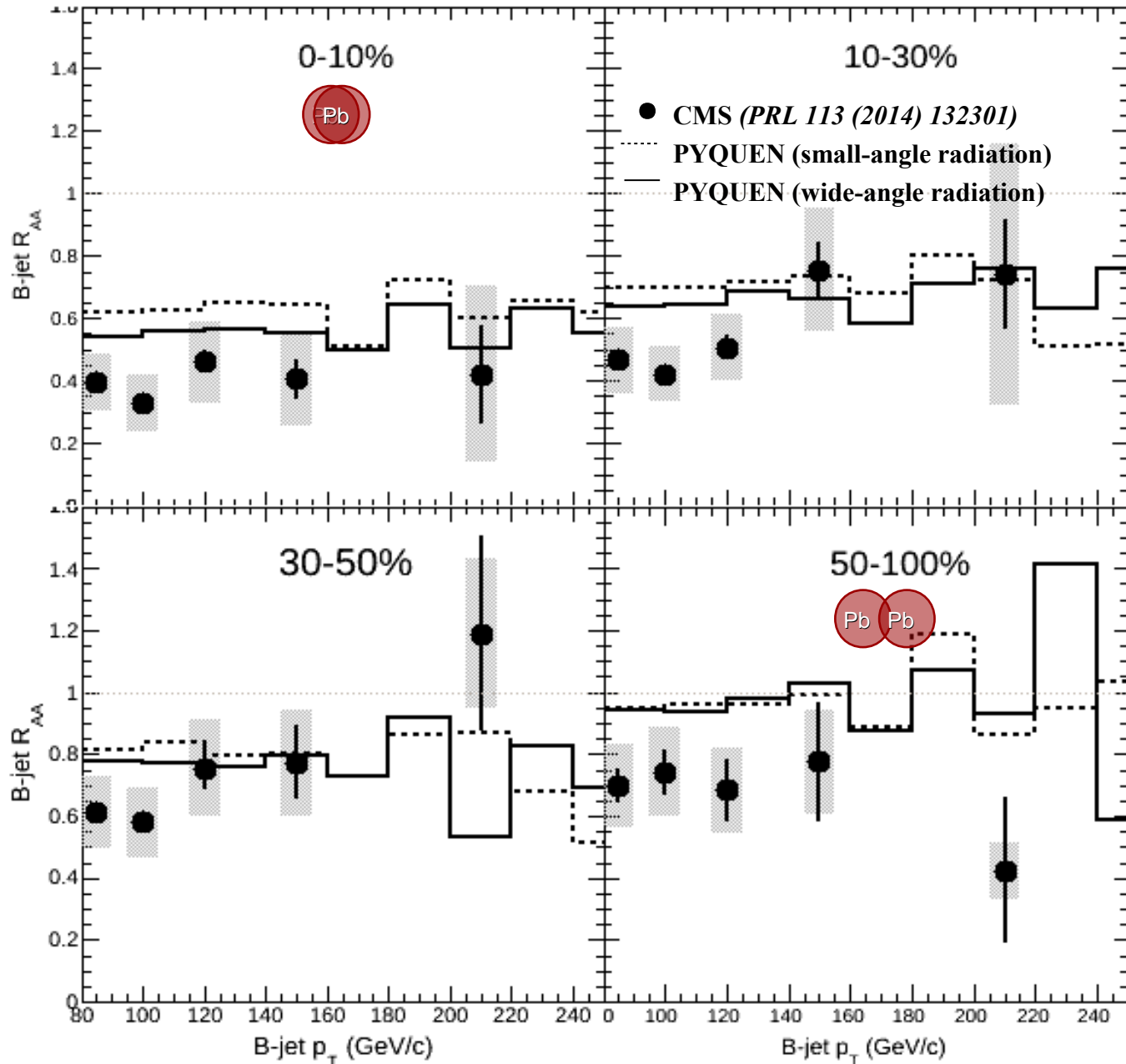
# Suppression factor of inclusive jets



# Suppression factor of inclusive jets

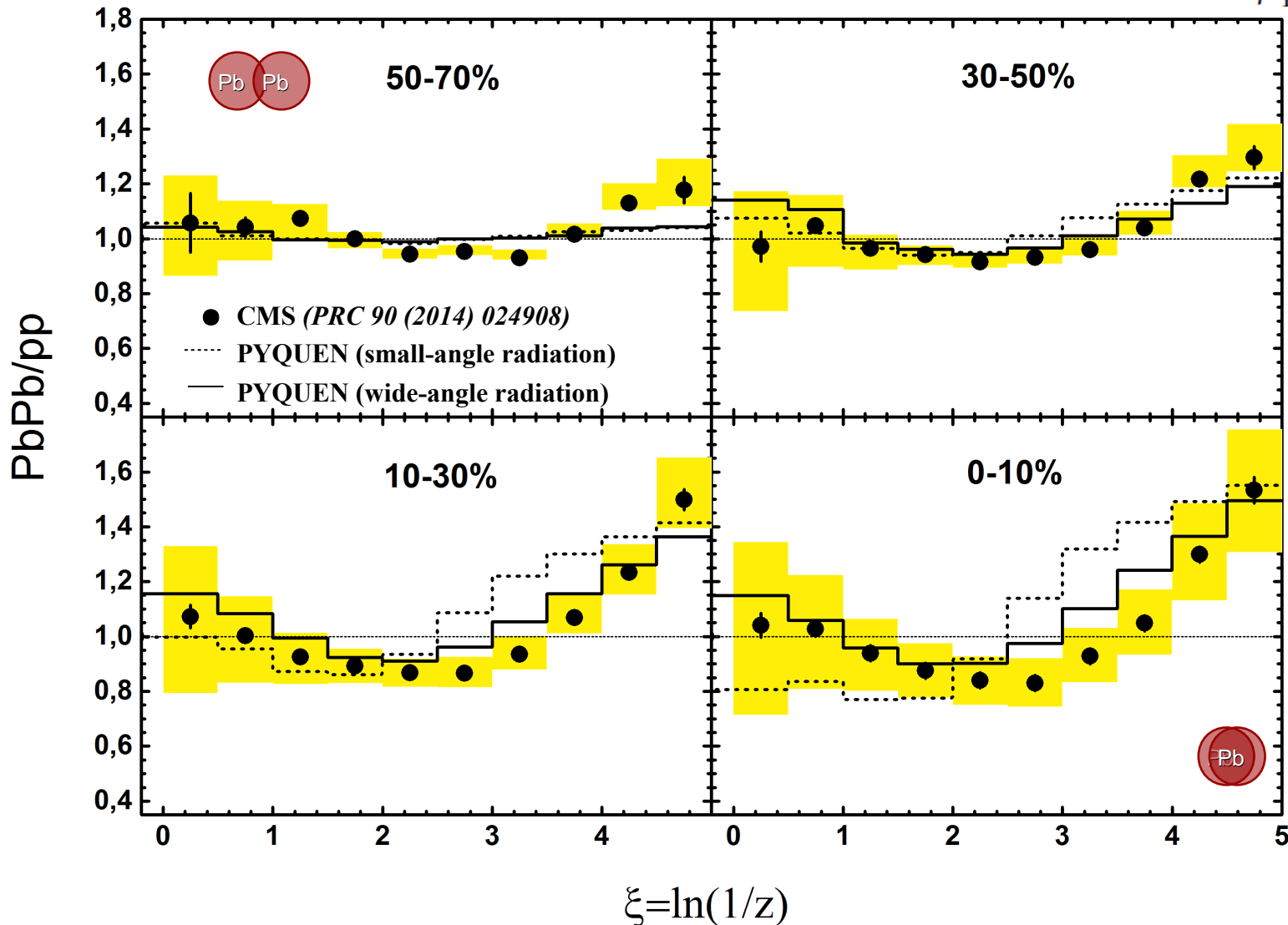


# Suppression factor of b-jets





# Jet fragmentation function $\xi = -\ln z = -\ln \frac{p_T^{track}}{p_T^{jet}}$



The modification of longitudinal jet profile ( $E_T^{jet} > 100$  GeV,  $R=0.3$ ):

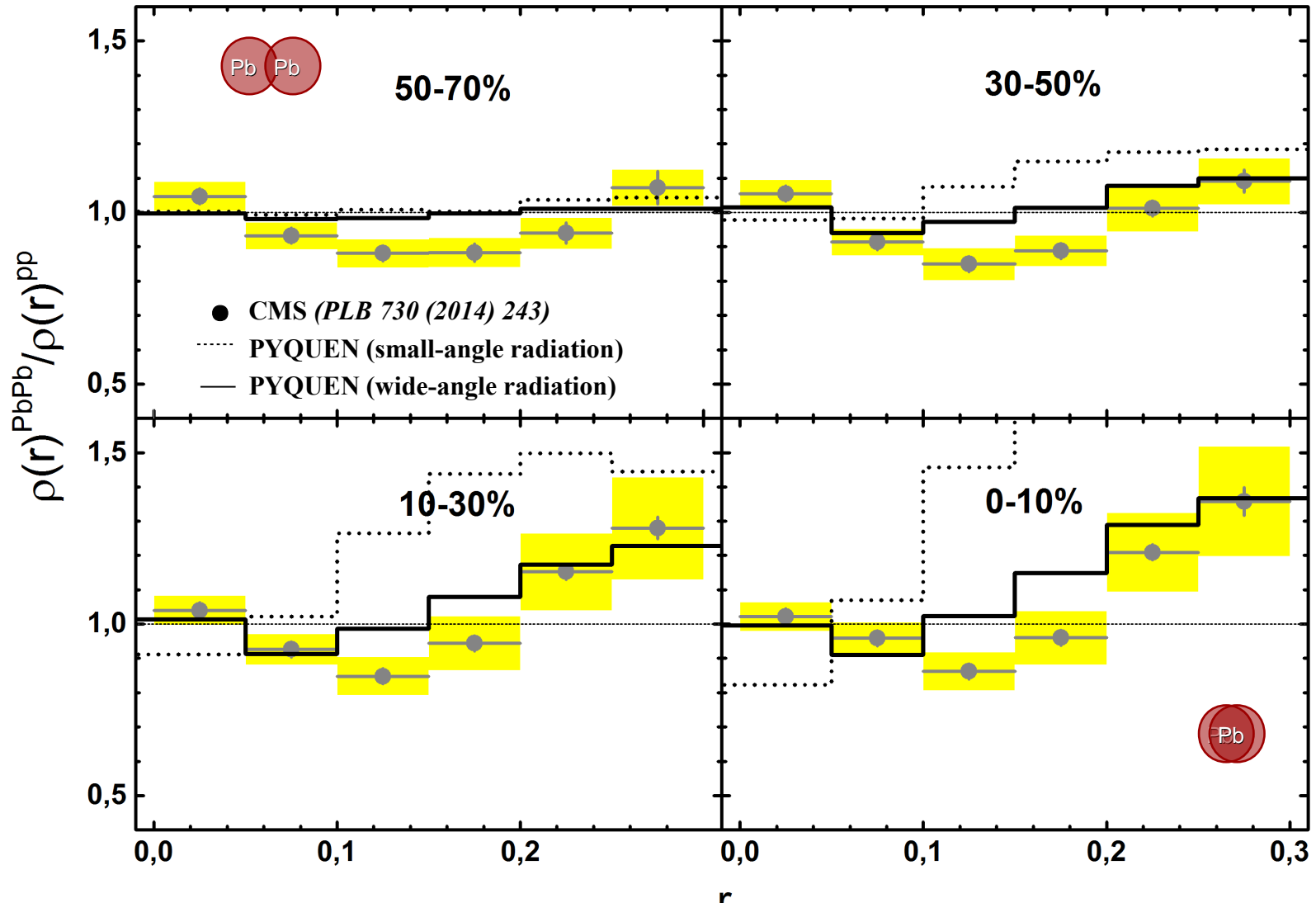
excess at low  $p_T$ ; suppression at intermediate  $p_T$ ; high  $p_T$  is slightly enhanced.

33

Reproduced well by PYQUEN with wide-angle radiative + collisional partonic energy loss.

# Jet shapes

$$\rho(r) \sim \frac{1}{\delta r} \frac{1}{N_{\text{jet}}} \sum_{\text{jets}} \frac{p_{\text{T}}(r - \delta r/2, r + \delta r/2)}{p_{\text{T}}^{\text{jet}}}$$



The modification of *radial jet profile* ( $E_{\text{T}}^{\text{jet}} > 100 \text{ GeV}$ ,  $R=0.3$ ):

excess at large radii; suppression at intermediate radii; core is unchanged.

Reproduced well by PYQUEN with *wide-angle radiative + collisional partonic energy loss*.

## Main publications (2011-2015)

- [1] I.P. Lokhtin, A.V. Belyaev, A.M. Snigirev, “Jet quenching pattern at LHC in PYQUEN model”, *Eur. Phys. J. C* 71 (2011) 1650
- [2] I.P. Lokhtin, A.V. Belyaev, L.V. Malinina, S.V. Petrushanko, E.P. Rogochaya, A.M. Snigirev, “Hadron spectra, flow and correlations in PbPb collisions at the LHC: interplay between soft and hard physics”, *Eur. Phys. J. C* 72 (2012) 2045
- [3] L.V. Bravina, B.H. Brusheim Johansson, G.Kh. Eyyubova, V.L. Korotkikh, I.P. Lokhtin, L.V. Malinina, S.V. Petrushanko, A.M. Snigirev, E.E. Zabrodin. “Hexagonal flow  $v_6$  as a superposition of elliptic  $v_2$  and triangular  $v_3$  flows”, *Phys. Rev. C* 89 (2014) 024909
- [4] L.V. Bravina, B.H. Brusheim Johansson, G.Kh. Eyyubova, V.L. Korotkikh, I.P. Lokhtin, L.V. Malinina, S.V. Petrushanko, A.M. Snigirev, E.E. Zabrodin. “Higher harmonics of azimuthal anisotropy in relativistic heavy ion collisions in HYDJET++ model”, *Eur. Phys. J. C* 74 (2014) 2807
- [5] I.P. Lokhtin, A.A. Alkin, A.M. Snigirev, “On jet structure in heavy ion collisions”, *arXiv 1410.0147*, submitted to *Eur. Phys. J. C*
- [6] G. Eyyubova, V.L. Korotkikh, I.P. Lokhtin, S.V. Petrushanko, A.M. Snigirev, L.V. Bravina, E.E. Zabrodin, “Angular dihadron correlations as interplay of elliptic and triangular flows”, *Phys. Rev. C* 91 (2015) 064907
- [7] I.P. Lokhtin, A.V. Belyaev, G.Kh. Eyyubova, G. Ponimatkin, E. Pronina, “Thermal and non-thermal charmed meson production in heavy ion collisions at the LHC”, *in preparation*
- [8] V.L. Korotkikh, I.P. Lokhtin, L.V. Malinina, E.N. Nazarova, S.V. Petrushanko, A.M. Snigirev, E.S. Fotina, “Anisotropic flow fluctuations in hydro-inspired freeze-out model for relativistic heavy ion collisions”, *in preparation*

# SUMMARY

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Two-component model of relativistic heavy ion collisions HYDJET++ reproduces basic physical observables measured in PbPb collisions at the LHC:

- multiplicity and momentum spectra of inclusive and identified hadrons
- anisotropic flow of inclusive and identified hadrons (including odd and higher harmonics)
- two-particle angular correlations of inclusive hadrons (including “ridge”)
- momentum spectra and elliptic flow of D-mesons
- femtoscopic correlation radii of pion pairs
- transverse momentum imbalance in dijet production
- suppression of hard hadron and jet yields (including b-jets)
- modification of internal jet structure (longitudinal and radial profiles)

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The pattern of multi-hadron and jet production in most central PbPb collisions at the LHC agrees with the formation of hot strongly-interacting matter with hydrodynamical properties (“quark-gluon fluid”), which absorbs energetic quarks and gluons due to their multiple scattering and wide-angle radiative and collisional medium-induced energy loss.

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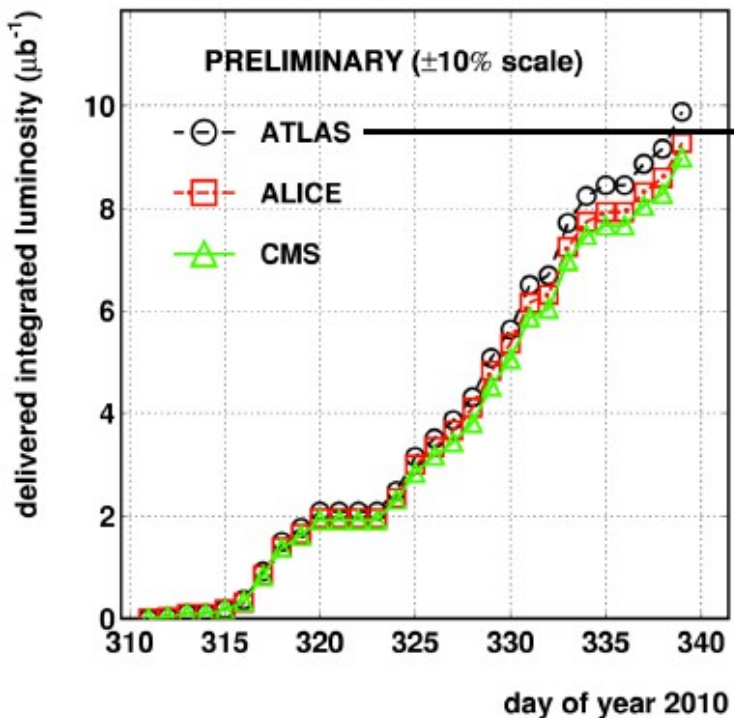
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Works in progress and near plans related to phenomenological analysis of LHC heavy ion data and the model improvements:

- event-by-event fluctuations of anisotropic flow
- azimuthal dependence of femtoscopic correlation radii
- momentum spectra and elliptic flow of  $J/\psi$ -mesons
- ...

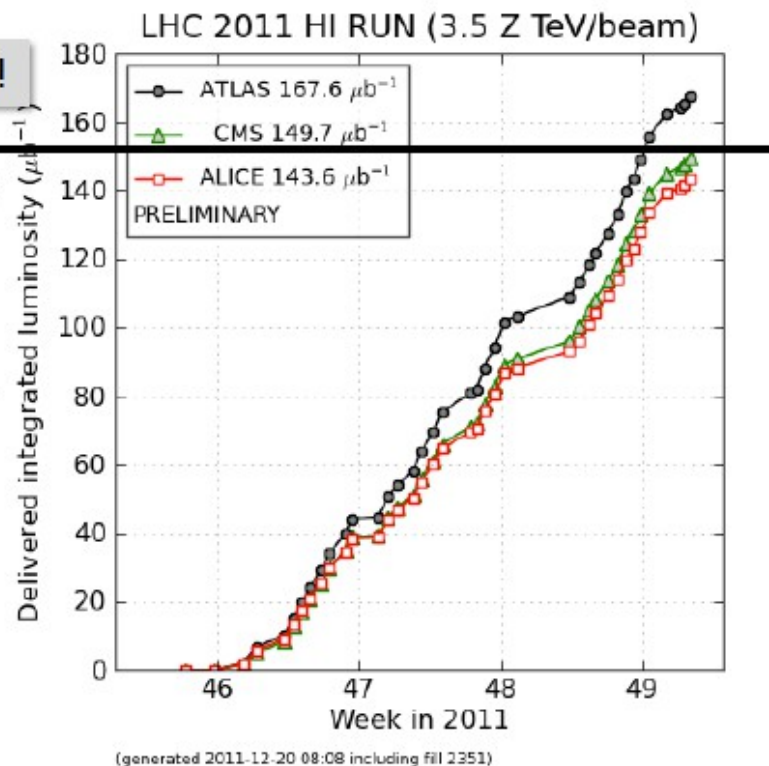
# BACKUP SLIDES





factor 16 improvement!

2010/12/06 21:31



Period	Species	Energy	Lumi
Dec. 2010	Pb+Pb	2.76 TeV	7 $\mu\text{b}^{-1}$
Dec. 2011	Pb+Pb	2.76 TeV	150 $\mu\text{b}^{-1}$
Mar. 2011	p+p	2.76 TeV	230 $\text{nb}^{-1}$
Jan. 2013	p+Pb	5.02 TeV	35 $\text{nb}^{-1}$
Fev. 2013	p+p	2.76 TeV	5.4 <sup>41</sup> $\text{pb}^{-1}$

# PYQUEN: physics frames

## General kinetic integral equation:

$$\Delta E(L, E) = \int_0^L dx \frac{dP}{dx}(x) \lambda(x) \frac{dE}{dx}(x, E), \quad \frac{dP}{dx}(x) = \frac{1}{\lambda(x)} \exp(-x/\lambda(x))$$

### 1. Collisional loss and elastic scattering cross section:

$$\frac{dE}{dx} = \frac{1}{4T\lambda\sigma} \int_{\mu_D^2}^{t_{max}} dt \frac{d\sigma}{dt} t, \quad \frac{d\sigma}{dt} \simeq C \frac{2\pi\alpha_s^2(t)}{t^2}, \quad \alpha_s = \frac{12\pi}{(33-2N_f)\ln(t/\Lambda_{QCD}^2)}, \quad C = 9/4(gg), 1(gq), 4/9(qq)$$

### 2. Radiative loss (BDMPS):

$$\frac{dE}{dx}(m_q=0) = \frac{2\alpha_s C_F}{\pi\tau_L} \int_{E_{LPM} \sim \lambda_g \mu_D^2}^E d\omega \left[ 1 - y + \frac{y^2}{2} \right] \ln |\cos(\omega_1 \tau_1)|, \quad \omega_1 = \sqrt{i \left( 1 - y + \frac{C_F}{3} y^2 \right) \bar{k} \ln \frac{16}{\bar{k}}}, \quad \bar{k} = \frac{\mu_D^2 \lambda_g}{\omega(1-y)}, \quad \tau_1 = \frac{\tau_L}{2\lambda_g}, \quad y = \frac{\omega}{E}, \quad C_F = \frac{4}{3}$$

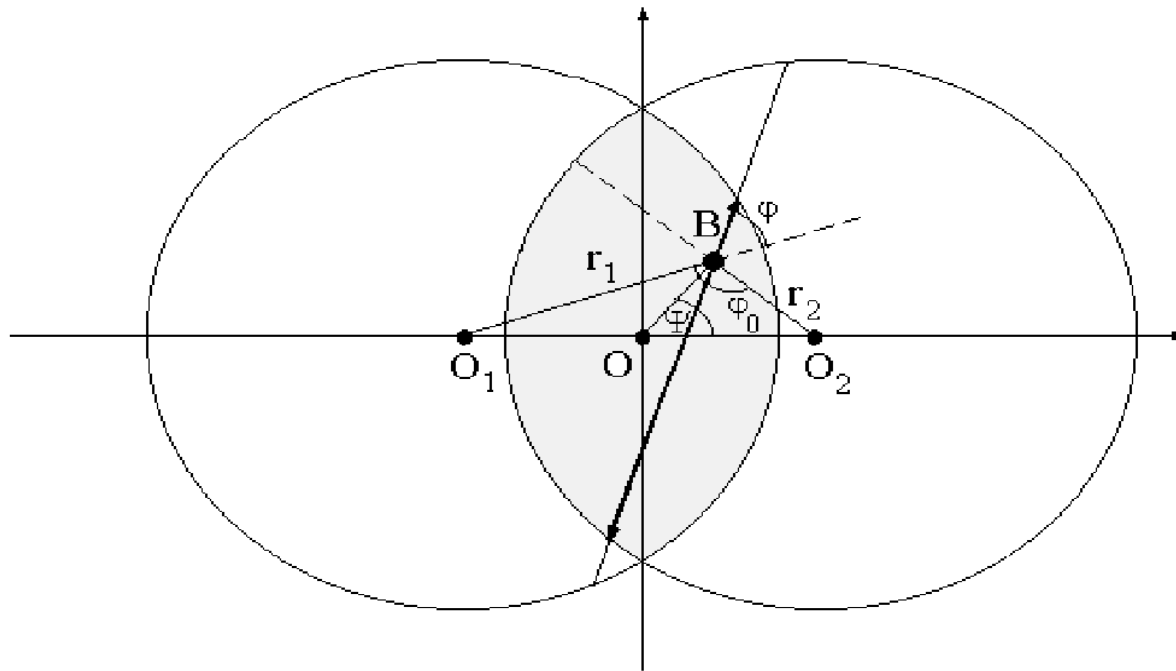
“dead cone” approximation for massive quarks:

$$\frac{dE}{dx}(m_q \neq 0) = \frac{1}{(1+(l\omega)^{3/2})^2} \frac{dE}{dx}(m_q=0), \quad l = \left( \frac{\lambda}{\mu_D^2} \right)^{1/3} \left( \frac{m_q}{E} \right)^{4/3}$$

# Nuclear geometry and QGP evolution

impact parameter  $b \equiv |O_1 O_2|$  - transverse distance between nucleus centers

$$\varepsilon(r_1, r_2) \propto T_A(r_1) * T_A(r_2) \quad (T_A(b) - \text{nuclear thickness function})$$



Space-time evolution of QGP, created in region of initial overlapping of colliding nuclei, is described by Lorenz-invariant Bjorken's hydrodynamics *J.D. Bjorken, PRD 27 (1983) 140*

# Monte-Carlo simulation of parton rescattering and energy loss in PYQUEN

- Distribution over jet production vertex  $V(r \cos \psi, r \sin \psi)$  at im.p.  $b$

$$\frac{dN}{d\psi dr}(b) = \frac{T_A(r_1) T_A(r_2)}{\int_0^{2\pi} d\psi \int_0^{r_{\max}} r dr T_A(r_1) T_A(r_2)}$$

- Transverse distance between parton scatterings  $l_i = (\tau_{i+1} - \tau_i) E/p_T$

$$\frac{dP}{dl_i} = \lambda^{-1}(\tau_{i+1}) \exp\left(-\int_0^{l_i} \lambda^{-1}(\tau_i + s) ds\right), \quad \lambda^{-1} = \sigma \rho$$

- Radiative and collisional energy loss per scattering

$$\Delta E_{tot,i} = \Delta E_{rad,i} + \Delta E_{col,i}$$

- Transverse momentum kick per scattering

$$\Delta k_{t,i}^2 = \left(E - \frac{t_i}{2m_{0i}}\right)^2 - \left(p - \frac{E}{p} \frac{t_i}{2m_{0i}} - \frac{t_i}{2p}\right)^2 - m_q^2$$

# HYDJET(soft): physics frames & simulation procedure

The final hadron spectrum are given by the superposition of thermal distribution and collective flow assuming Bjorken's scaling.

## 1. Thermal distribution of produced hadron in rest frame of fluid element

$$f(E_0) \propto E_0 \sqrt{E_0^2 - m^2} \exp(-E_0/T_f), \quad -1 < \cos \theta_0 < 1, \quad 0 < \phi_0 < 2\pi$$

## 2. Space position $r$ and local 4-velocity $u_\mu$

$$f(r) = 2r/R_f^2(R_A, b, \Phi) (0 < r < R_f), \quad f(\eta) \propto e^{\frac{-(\eta - Y_L^{max})^2}{2(Y_L^{max})^2}}, \quad 0 < \Phi < 2\pi$$

$$u_r = \sinh Y_T^{max} \cdot r / \sqrt{R_{eff}(R_A, b) \cdot R_A}, \quad u_t = \sqrt{1 + u_r^2} \cosh \eta, \quad u_z = \sqrt{1 + u_r^2} \sinh \eta$$

## 3. Boost of hadron 4-momentum $p_\mu$ in c.m. frame of the event

$$p_x = p_0 \sin \theta_0 \cos \phi_0 + u_r \cos \Phi [E_0 + (u^i p_0^i) / (u_t + 1)],$$

$$p_y = p_0 \sin \theta_0 \sin \phi_0 + u_r \sin \Phi [E_0 + (u^i p_0^i) / (u_t + 1)],$$

$$p_z = p_0 \cos \theta_0 + u_z [E_0 + (u^i p_0^i) / (u_t + 1)],$$

$$E = E_0 u_t + (u^i p_0^i), \quad (u^i p_0^i) = u_r p_0 \sin \theta_0 \cos(\Phi - \phi_0) + u_z p_0 \cos \theta_0$$

# Monte-Carlo simulation of hard component (including nuclear shadowing) in HYDJET/HYDJET++

- Calculating the number of hard NN sub-collisions  $N_{jet}(b, P_{tmin}, \sqrt{s})$  with  $P_t > P_{tmin}$  around its mean value according to the binomial distribution.
- Selecting the type (for each of  $N_{jet}$ ) of hard NN sub-collisions ( $pp$ ,  $np$  or  $nn$ ) depending on number of protons ( $Z$ ) and neutrons ( $A-Z$ ) in nucleus  $A$  according to the formula:  $Z = A / (1.98 + 0.015A^{2/3})$ .
- Generating the hard component by calling PYQUEN  $n_{jet}$  times.
- Correcting the PDF in nucleus by the accepting/rejecting procedure for each of  $N_{jet}$  hard NN sub-collisions: comparison of random number generated uniformly in the interval  $[0,1]$  with shadowing factor  $S(r1, r2, x1, x2, Q2) \leq 1$  taken from the adapted impact parameter dependent parameterization based on Glauber-Gribov theory (*K.Tywniuk et al., Phys. Lett. B 657 (2007) 170*).

# HYDJET: model parameters

## Minimal external input

**A** - beam and target nucleus atomic weight;  
**energy** - c.m.s. energy per nucleon pair;  
**ifb**, **bmin**, **bmax**, **bfix** – parameters to fix event centrality selection;  
**nh**- total mean multiplicity of primary hadrons for soft component (PbPb, b=0);  
(multiplicity for other centralities and atomic weights is calculated automatically).

## Parameter can be varied by user

**ytf1** - maximum transverse collective rapidity, controls slope of low-pt spectra;  
**ylf1** - maximum longitudinal collective rapidity, controls width of  $\eta$ -spectra;  
**Tf** – hadron thermal freeze-out temperature;  
**fpart** - fraction of soft multiplicity proportional to # of participants ( $fpart(D)=1$ );  
**sign** – inelastic NN cross-section (calculated by PYTHIA by default);  
**ptmin** - minimal transverse momentum of “non-thermalized” initial parton-parton scatterings ( $=ckin(3)$  in PYTHIA; other PYTHIA parameters also can be varied);  
**T0**, **tau0**, **nf**, **ienglu**, **ianglu** – PYQUEN parameters;  
**nhsel** - flag to switch on/off jet production and jet quenching;  
**ishad** - flag to switch on/off nuclear shadowing.

## Internal sets for soft component

poison multiplicity distribution; thermal particle ratios.

# HYDJET++ (soft): main physics assumptions

A hydrodynamic expansion of the fireball is supposed **ends by a sudden system breakup** at given  $T$  and chemical potentials. Momentum distribution of produced hadrons keeps the thermal character of the equilibrium distribution.

**Cooper-Frye formula:** 
$$p^0 \frac{d^3 N_i}{d^3 p} = \int_{\sigma(x)} d^3 \sigma_\mu(x) p^\mu f_i^{eq}(p^\nu u_\mu(x); T, \mu_i)$$

- HYDJET++ avoids straightforward 6-dimensional integration by using the special simulation procedure (like HYDJET): momentum generation in the rest frame of fluid element, then Lorentz transformation in the global frame  $\rightarrow$  uniform weights  $\rightarrow$  effective von-Neumann rejection-acceptance procedure.

## Freeze-out surface parameterizations

1. The Bjorken model with hypersurface

$$\tau = (t^2 - z^2)^{1/2} = \text{const}$$

2. Linear transverse flow rapidity profile

$$\rho_u = \frac{r}{R} \rho_u^{\max}$$

3. The total effective volume for particle production at

$$- V_{\text{eff}} = \int_{\sigma(x)} d^3 \sigma_\mu(x) u^\mu(x) = \tau \int_0^R \gamma_r r dr \int_0^{2\pi} d\phi \int_{\eta_{\min}}^{\eta_{\max}} d\eta = 2\pi\tau\Delta\eta \left( \frac{R}{\rho_u^{\max}} \right)^2 (\rho_u^{\max} \sinh \rho_u^{\max} - \cosh \rho_u^{\max} + 1)$$



# HYDJET++ (soft): hadron multiplicities

1. The hadronic matter created in heavy-ion collisions is considered as a hydrodynamically expanding fireball with EOS of an ideal hadron gas.

2. “Concept of effective volume”  $T=\text{const}$  and  $\mu=\text{const}$ : the total yield of particle species is  $N_i = \rho_i(T, \mu_i)V_{eff}$  .

3. Chemical freeze-out :  $T, \mu_i = \mu_B B_i + \mu_S S_i + \mu_c C_i + \mu_Q Q_i$ ;  $T, \mu_B$  –can be fixed by particle ratios, or by phenomenological formulas

$$T(\mu_B) = a - b\mu_B - c\mu_B^4; \mu_B(\sqrt{s_{NN}}) = \frac{d}{1 + e\sqrt{s_{NN}}}$$

4. Chemical freeze-out: all macroscopic characteristics of particle system are determined via a set of equilibrium distribution functions in the fluid element rest frame:

$$f_i^{eq}(p^{0*}; T, \mu_i) = \frac{1}{(2\pi)^3} \frac{g_i}{\exp([p^{0*} - \mu_i]/T) \pm 1}$$

$$\rho_i^{eq}(T, \mu_i) = \int_0^\infty d^3 \vec{p}^* f_i^{eq}(p^{0*}; T(x^*), \mu(x^*)_i) = 4\pi \int_0^\infty dp^* p^{*2} f_i^{eq}(p^{0*}; T, \mu_i)$$

# HYDJET++ (soft): thermal and chemical freeze-outs

1. The particle densities at the **chemical freeze-out** stage are too high to consider particles as free streaming and to associate this stage with the **thermal freeze-out**
2. Within the **concept of chemically frozen evolution**, assumption of the conservation of the particle number ratios from the chemical to thermal freeze-out :

$$\frac{\rho_i^{eq}(T^{ch}, \mu_i^{ch})}{\rho_\pi^{eq}(T^{ch}, \mu_\pi^{ch})} = \frac{\rho_i^{eq}(T^{th}, \mu_i^{th})}{\rho_\pi^{eq}(T^{th}, \mu_\pi^{th})}$$

3. The absolute values  $\rho_i^{eq}(T^{th}, \mu_i^{th})$  are determined by the choice of the **free parameter of the model: effective pion chemical potential**  $\mu_\pi^{eff,th}$  at  $T^{th}$ . Assuming for the other particles (heavier than pions) the Boltzmann approximation :

$$\mu_i^{th} = T^{th} \ln \left( \frac{\rho_i^{eq}(T^{ch}, \mu_i^{ch})}{\rho_i^{eq}(T^{th}, \mu_i = 0)} \frac{\rho_\pi^{eq}(T^{th}, \mu_\pi^{eff,th})}{\rho_\pi^{eq}(T^{ch}, \mu_i^{ch})} \right)$$

**Particle momentum spectra** are generated on the **thermal freeze-out hypersurface**, the hadronic composition at this stage is defined by the parameters of the system at chemical freeze-out

# HYDJET++ (soft): input parameters

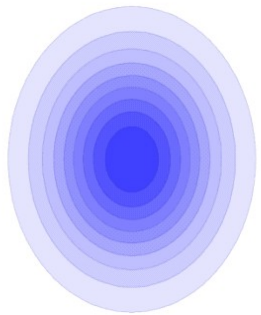
- 1-5. Thermodynamic parameters at chemical freeze-out:  $T^{\text{ch}}$ ,  $\{\mu_B, \mu_S, \mu_C, \mu_Q\}$  (option to calculate  $T^{\text{ch}}$ ,  $\mu_B$  and  $\mu_S$  using phenomenological parameterization  $\mu_B(\sqrt{s})$ ,  $T^{\text{ch}}(\mu_B)$  is foreseen).
- 6-7. Strangeness suppression factor  $\gamma_S \leq 1$  and charm enhancement factor  $\gamma_C \geq 1$  (options to use phenomenological parameterization  $\gamma_S(T^{\text{ch}}, \mu_B)$  and to calculate  $\gamma_C$  are foreseen).
- 8-9. Thermodynamical parameters at thermal freeze-out:  $T^{\text{th}}$ , and  $\mu_\pi$  - effective chemical potential of positively charged pions.
- 10-12. Volume parameters at thermal freeze-out: proper time  $\tau_f$ , its standard deviation (emission duration)  $\Delta\tau_f$ , maximal transverse radius  $R_f$ .
13. Maximal transverse flow rapidity at thermal freeze-out  $\rho_u^{\text{max}}$ .
14. Maximal longitudinal flow rapidity at thermal freeze-out  $\eta^{\text{max}}$ .
15. Flow anisotropy parameter:  $\delta(\mathbf{b}) \rightarrow u^\mu = u^\mu(\delta(\mathbf{b}), \varphi)$
16. Coordinate anisotropy:  $\varepsilon(\mathbf{b}) \rightarrow R_f(\mathbf{b}) = R_f(0) [V_{\text{eff}}(\varepsilon(0), \delta(0)) / V_{\text{eff}}(\varepsilon(\mathbf{b}), \delta(\mathbf{b}))]^{1/2} [N_{\text{part}}(\mathbf{b}) / N_{\text{part}}(0)]^{1/3}$

**For impact parameter range bmin-bmax:**

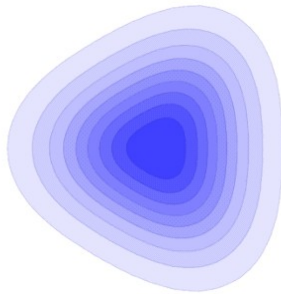
$$V_{\text{eff}}(\mathbf{b}) = V_{\text{eff}}(0) N_{\text{part}}(\mathbf{b}) / N_{\text{part}}(0), \quad \tau_f(\mathbf{b}) = \tau_f(0) [N_{\text{part}}(\mathbf{b}) / N_{\text{part}}(0)]^{1/3}$$

# Higher harmonic flow

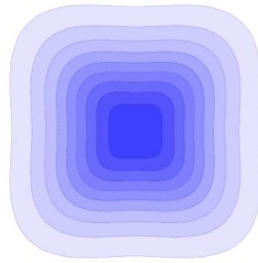
Non-zero high Fourier coefficients carry information about the details of the space-time evolution of QCD-matter and initial state fluctuations.



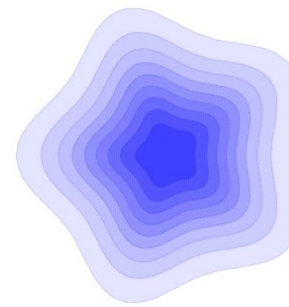
$n = 2$



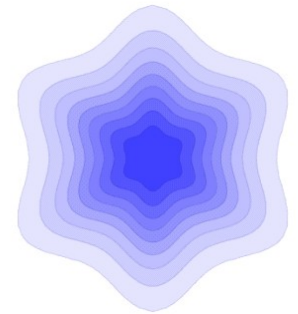
$n = 3$



$n = 4$



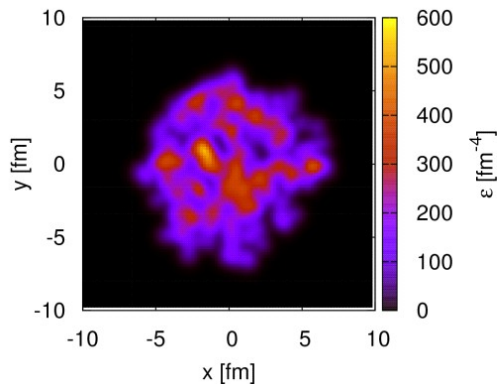
$n = 5$



$n = 6$

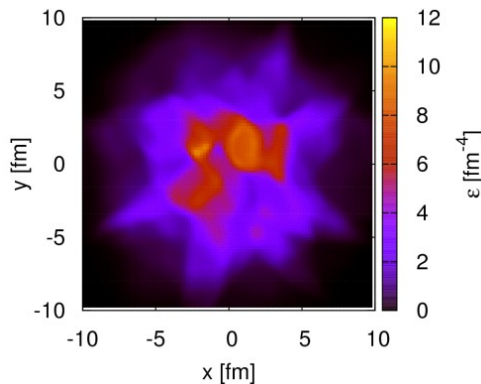
## initial

$\tau = 0.4 \text{ fm}/c$



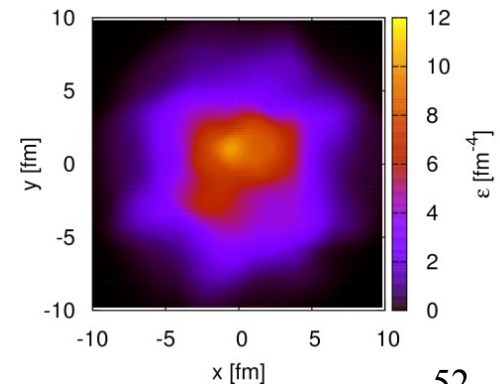
## ideal

$\tau = 6.0 \text{ fm}/c, \text{ ideal}$

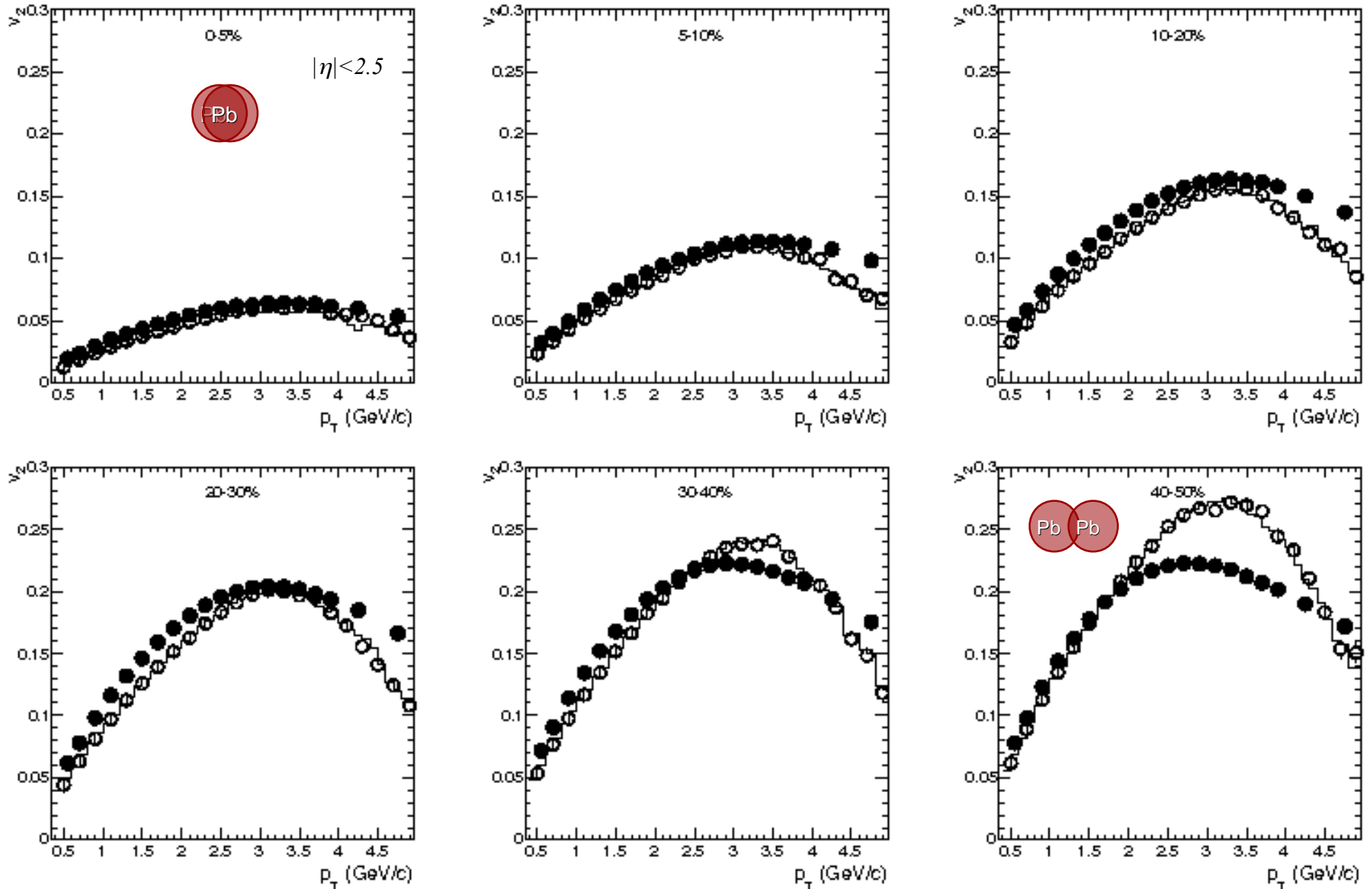


## viscous

$\tau = 6.0 \text{ fm}/c, \eta/s = 0.16$



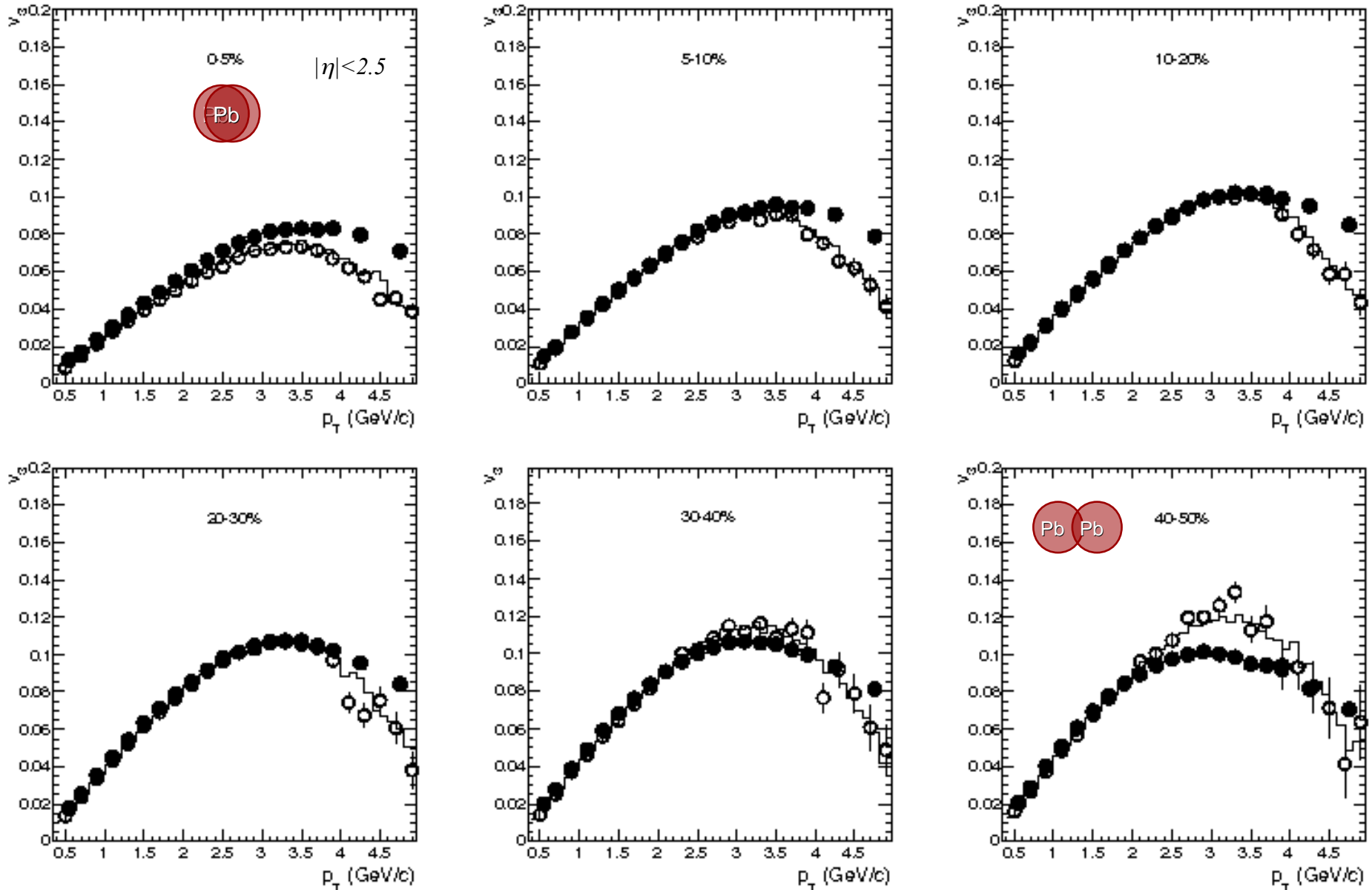
# Elliptic flow of inclusive charged hadrons



Closed circles: ATLAS data  $v_2\{\text{EP}\}$  (*PRC* 86 (2012) 014907);

histograms and open circles: HYDJET++ ("true"  $v_2(\psi_2)$  &  $v_2\{\text{EP}\}$ )

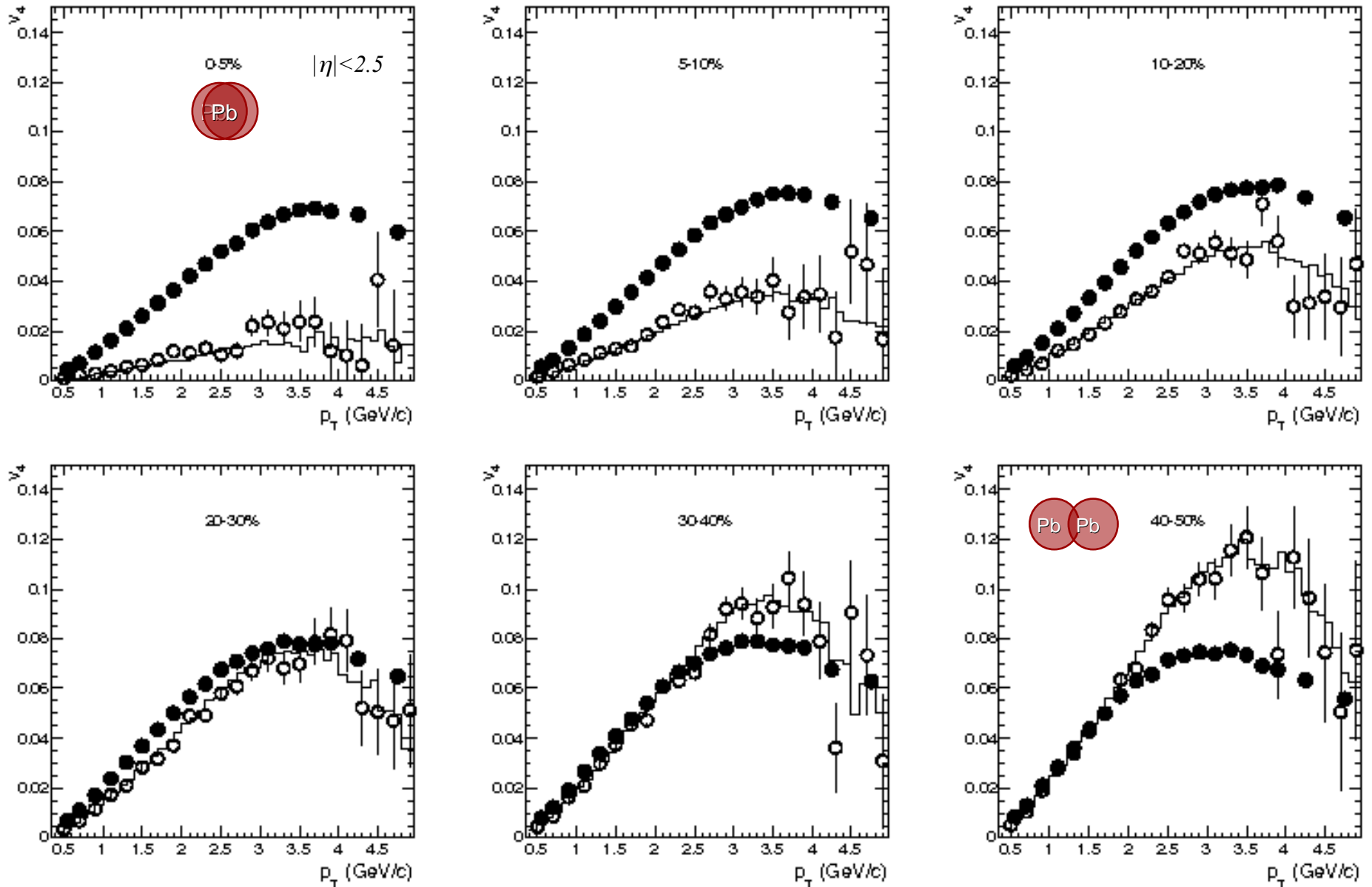
# Triangular flow of inclusive charged hadrons



Closed circles: ATLAS data  $v_3\{\text{EP}\}$  (*PRC 86 (2012) 014907*);

histograms and open circles: HYDJET++ (“true”  $v_3(\psi_3)$  &  $v_3\{\text{EP}\}$ )

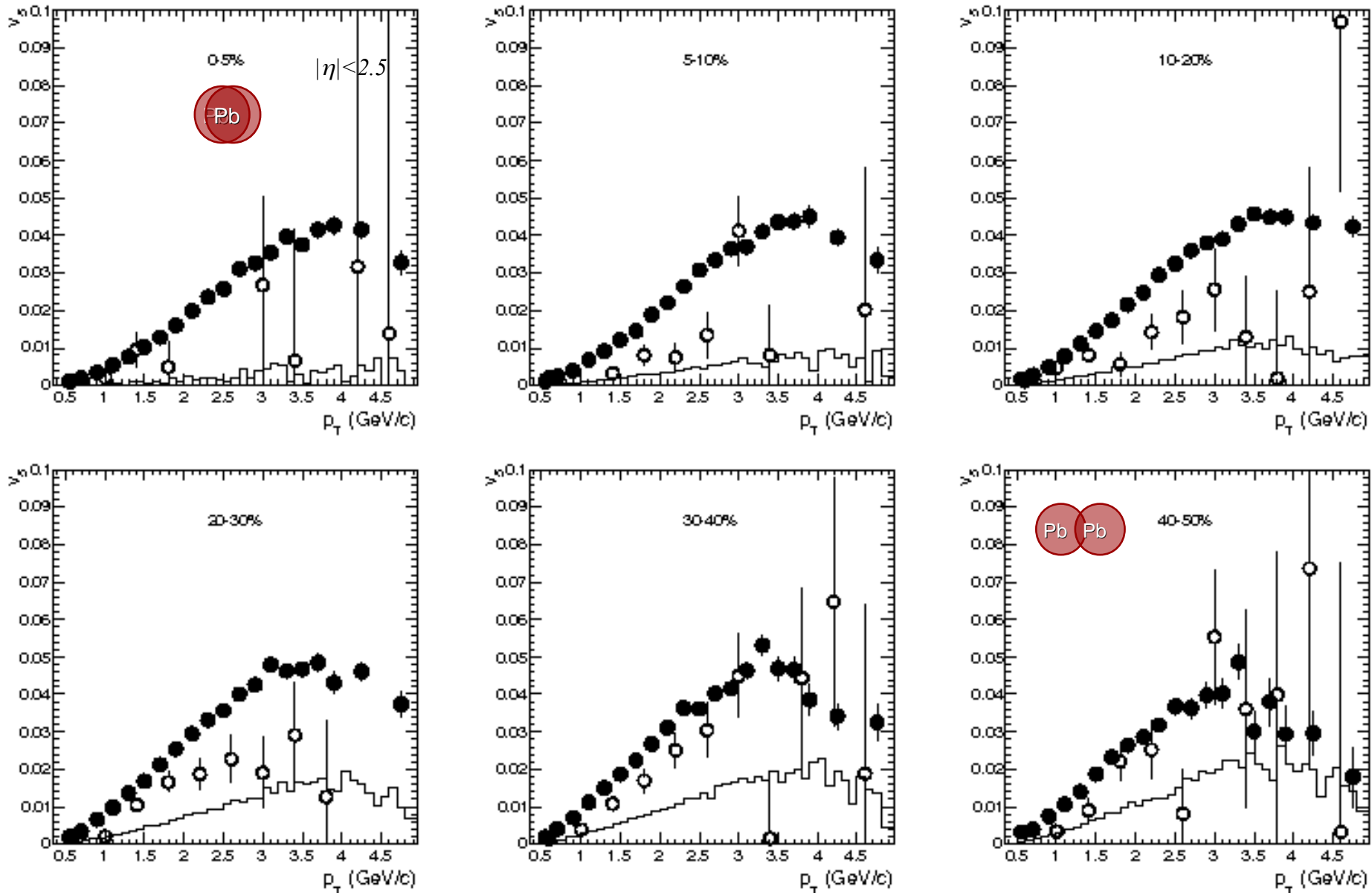
# Quadrangular flow of inclusive charged hadrons



Closed circles: ATLAS data  $v_4\{EP\}$  (*PRC* 86 (2012) 014907);

histograms and open circles: HYDJET++ (“true”  $v_4(\psi_2)$  &  $v_4\{EP\}$ )

# Pentagonal flow of inclusive charged hadrons

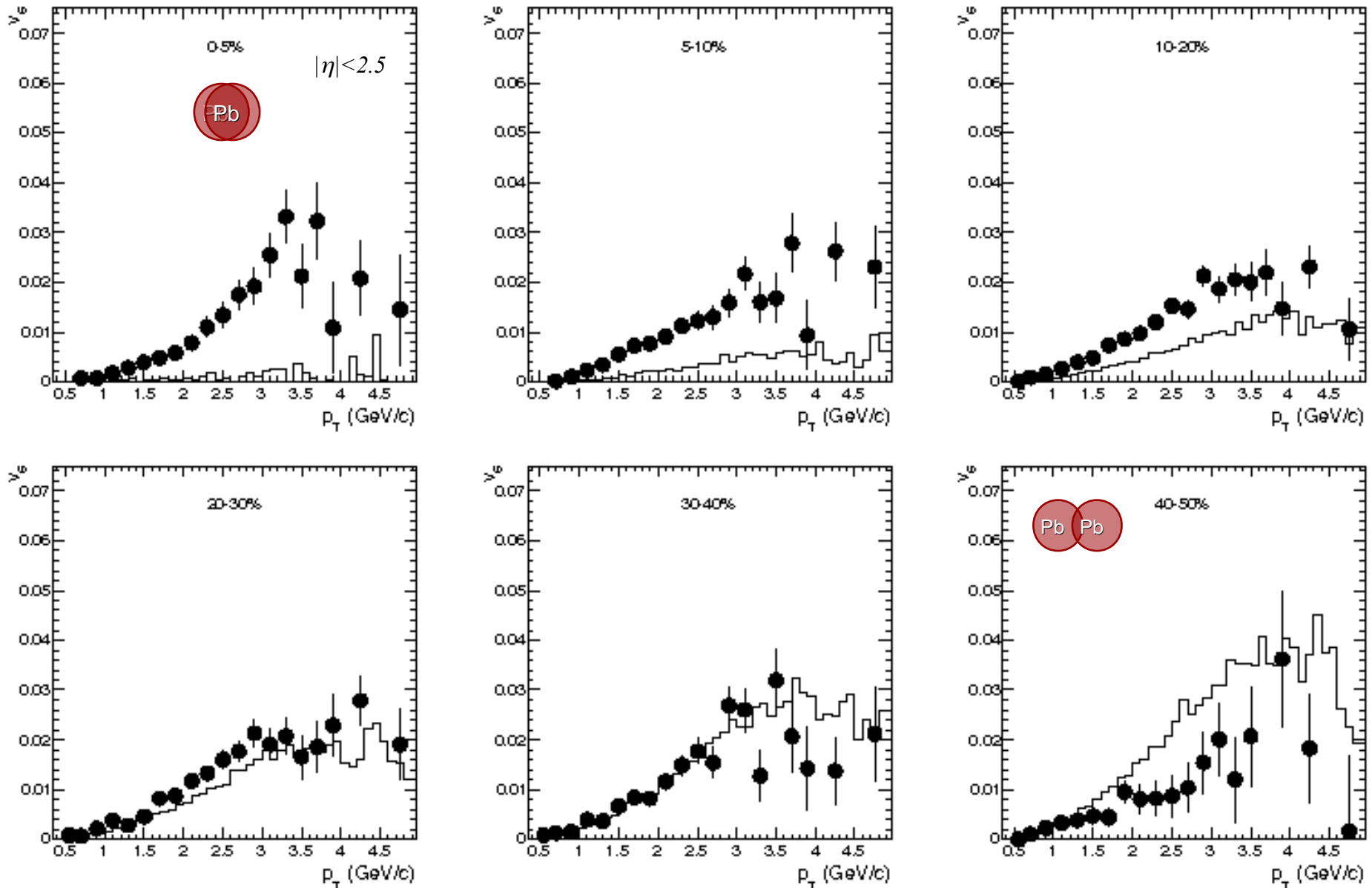


Closed circles: ATLAS data  $v_5\{\text{EP}\}$  (PRC 86 (2012) 014907);

histograms and open circles: HYDJET++ (“true”  $v_5(\psi_3)$  &  $v_5\{\text{EP}\}$ )



# Hexagonal flow of inclusive charged hadrons

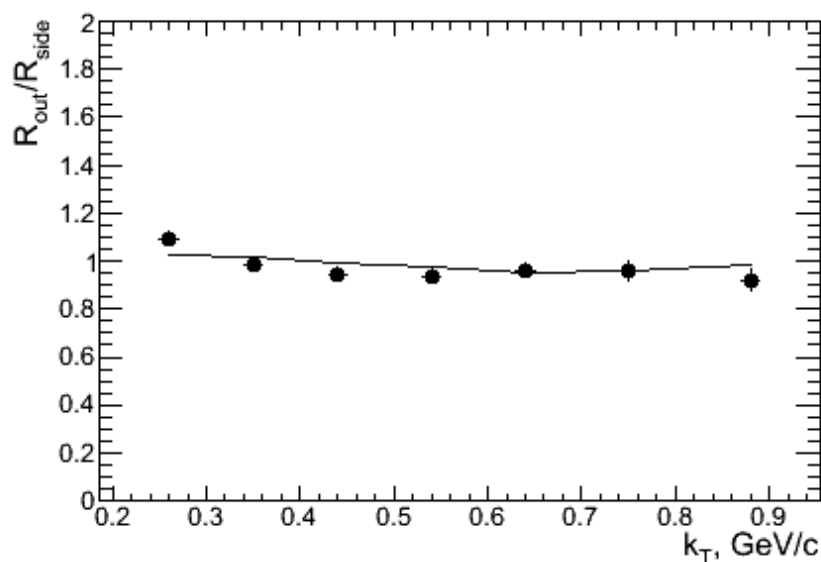
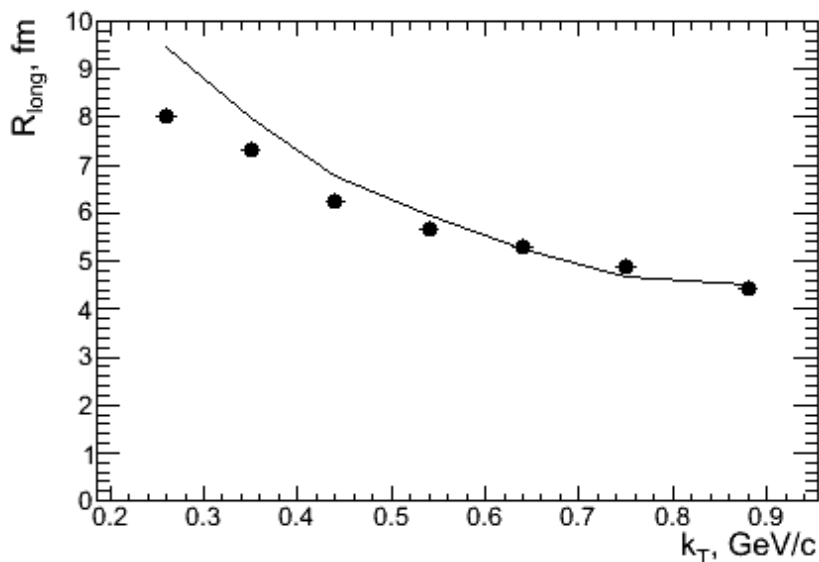
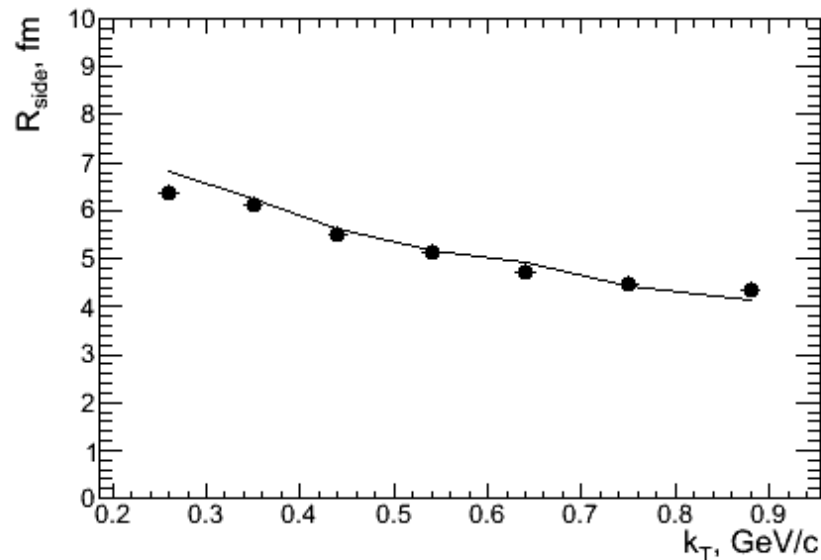
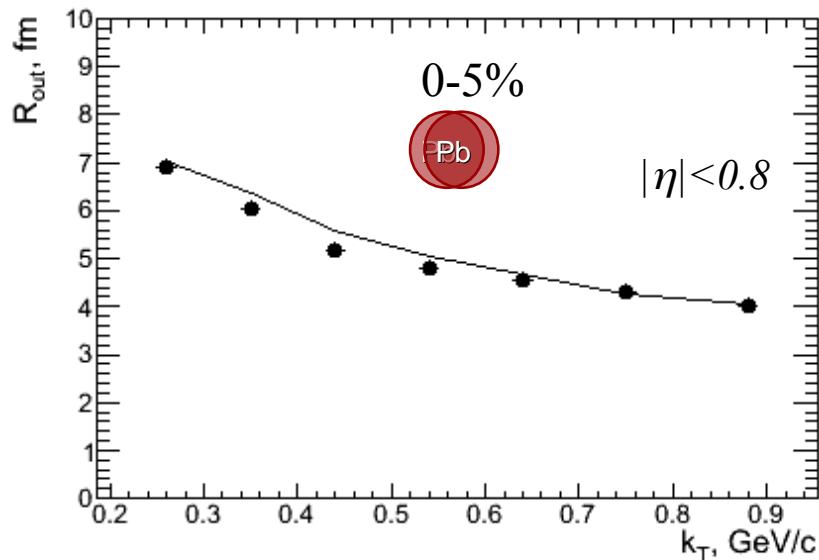


Closed circles: ATLAS data  $v_6\{EP\}$  (*PRC 86 (2012) 014907*);

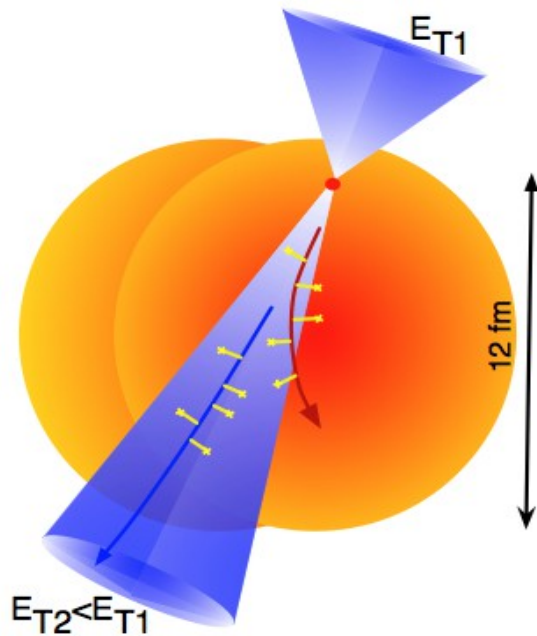
histograms: HYDJET++ (“true”  $v_6(\psi_2)$ )

# Femtoscopic momentum correlations (pion pairs)

$$CF = 1 + \lambda \exp(-R_o^2 q_o^2 - R_s^2 q_s^2 - R_l^2 q_l^2 - 2R_{ol}^2 q_o q_l)$$



One of first new LHC results from lead-lead collisions at  $\sqrt{s}=2.76$  A TeV was the observation of transverse energy asymmetry for dijet production in most central events. It is interpreted as a signal of partonic jet absorption in hot quark-gluon matter.



$$A_J = \frac{E_T^{j1} - E_T^{j2}}{E_T^{j1} + E_T^{j2}}$$

