Recent developments in Neutrino Physics

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QFTHEP2015

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Plan

Oscillation sector

- Standard 3-neutrino oscillation
- Anomalies in neutrino data

Flavor sector

- Problem of neutrino masses and mixings: the role of family symmetries

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Introduction

- New data from neutrino oscillation experiments have given precise results on mixing parameters
- Possible leptonic CP violation (<=5 y) (T2K, NoVA...)
- However:

not a unique extension of the Standard Model that allows to explain:

- origin of masses and mixing angles
- differences with respect to the quark sector

3-ν state formalism

- Neutrinos can be described in terms of mass or weak eigenstates

$$|\nu_\alpha\rangle = \Sigma_{i=1}^{3} U_{\alpha i} |\nu_i\rangle$$

neutrino matrix
matrix

- Simple time evolutions of the vector $\nu(t) = (\nu_e(t), \nu_\mu(t), \nu_\tau(t))$:

$$i \frac{d}{dt} |\nu(t)\rangle = H |\nu(t)\rangle$$

$$H = \frac{1}{2 E_\nu} U \, Diag[0, m_2^2 - m_1^2, m_3^2 - m_1^2] U^+$$

there exist a probability of a change of the neutrino flavour

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Theory of neutrino oscillation

- Flavour changing transitions

$$P(\nu_\alpha \rightarrow \nu_\beta) = |\langle \nu_\beta | \nu_\alpha(t) \rangle|^2 = \left| \Sigma_j U_{\beta j} e^{\frac{-i m_j^2 L}{2 E_\nu}} U^*_{\alpha j} \right|^2$$

$\alpha=\beta$ → disappearance

$\alpha \mathrel{!}= \beta$ → appearance

- In the case of two neutrinos only:

$$P(\nu_e \rightarrow \nu_\mu) = \sin^2 2\theta \ \sin^2\left(\frac{\Delta m^2 L}{4 E_\nu}\right)$$

→ distance source-detector

mixing angle

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

U = unitary matrix

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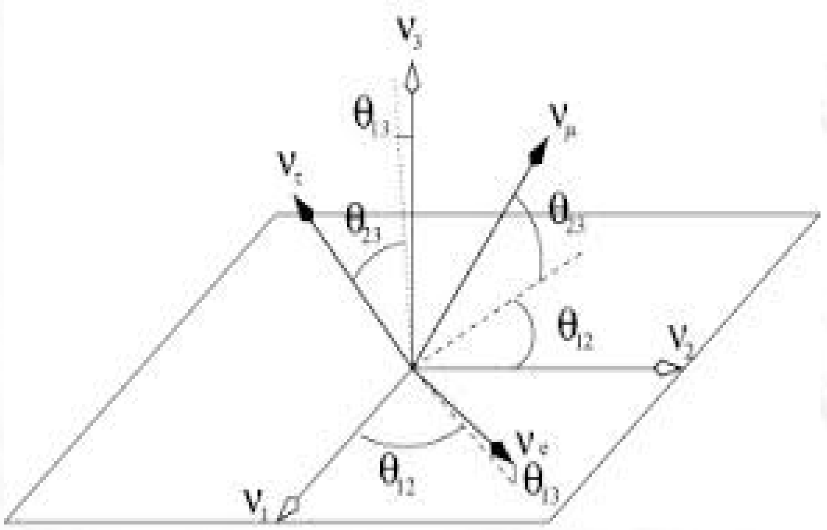
Theory of neutrino oscillation

- The neutrino mixing matrix depends on 4 real parameters

$$U_{PMNS}=\underbrace{\begin{pmatrix}1 & 0 & 0\\ 0 & c_{23} & s_{23}\\ 0 & -s_{23} & c_{23}\end{pmatrix}}_{atmospheric\ \ mixing}\times\underbrace{\begin{pmatrix}c_{13} & 0 & s_{13}e^{i\delta}\\ 0 & 1 & 0\\ -s_{13}e^{-i\delta} & 0 & c_{13}\end{pmatrix}}_{reactor\ \ mixing}\times\underbrace{\begin{pmatrix}c_{12} & s_{12} & 0\\ -s_{12} & c_{12} & 0\\ 0 & 0 & 1\end{pmatrix}}_{solar\ \ mixing}$$

- A more complicated mismatch between the ν bases



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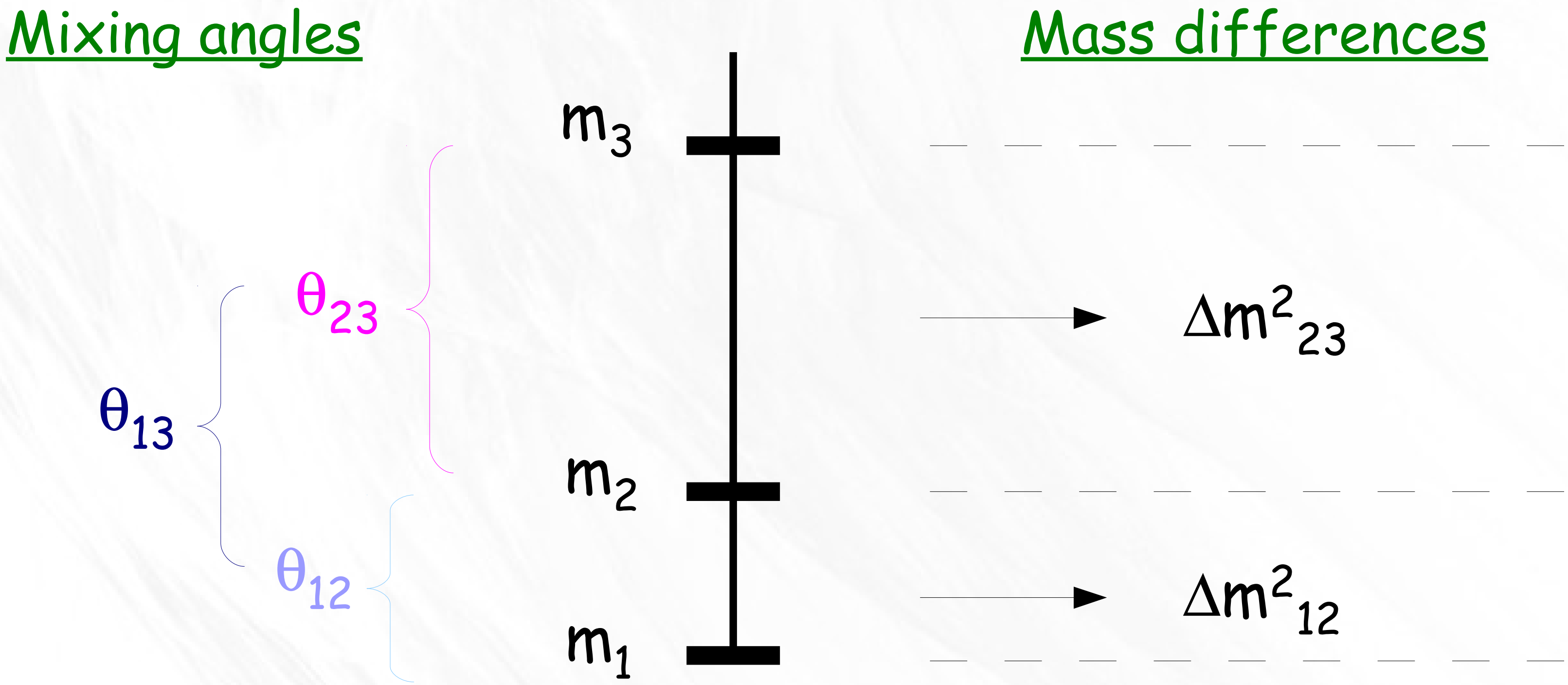
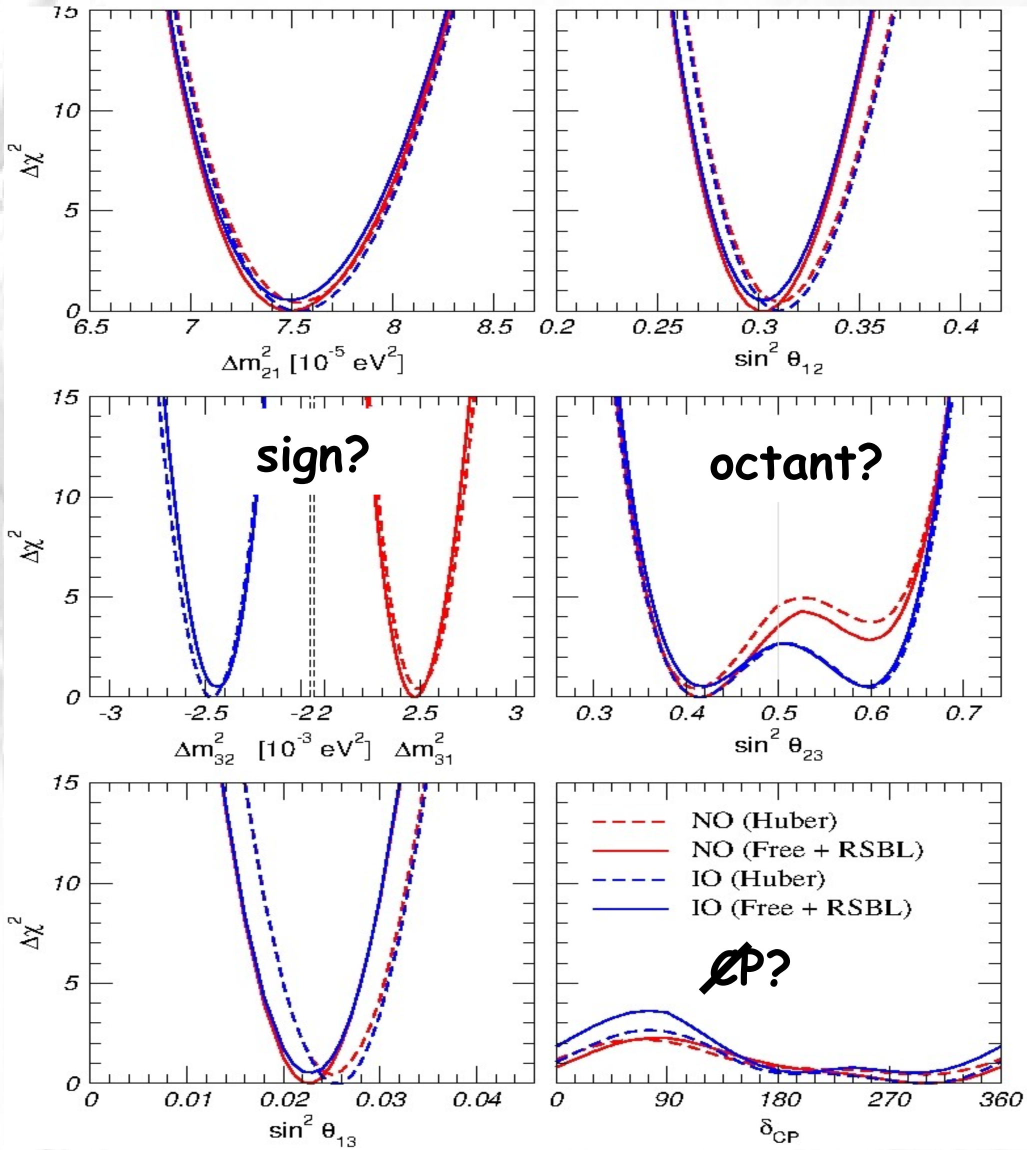
Experiments measure...



- unknowns: leptonic CP violation and the ordering of the mass eigenstates

Global fit

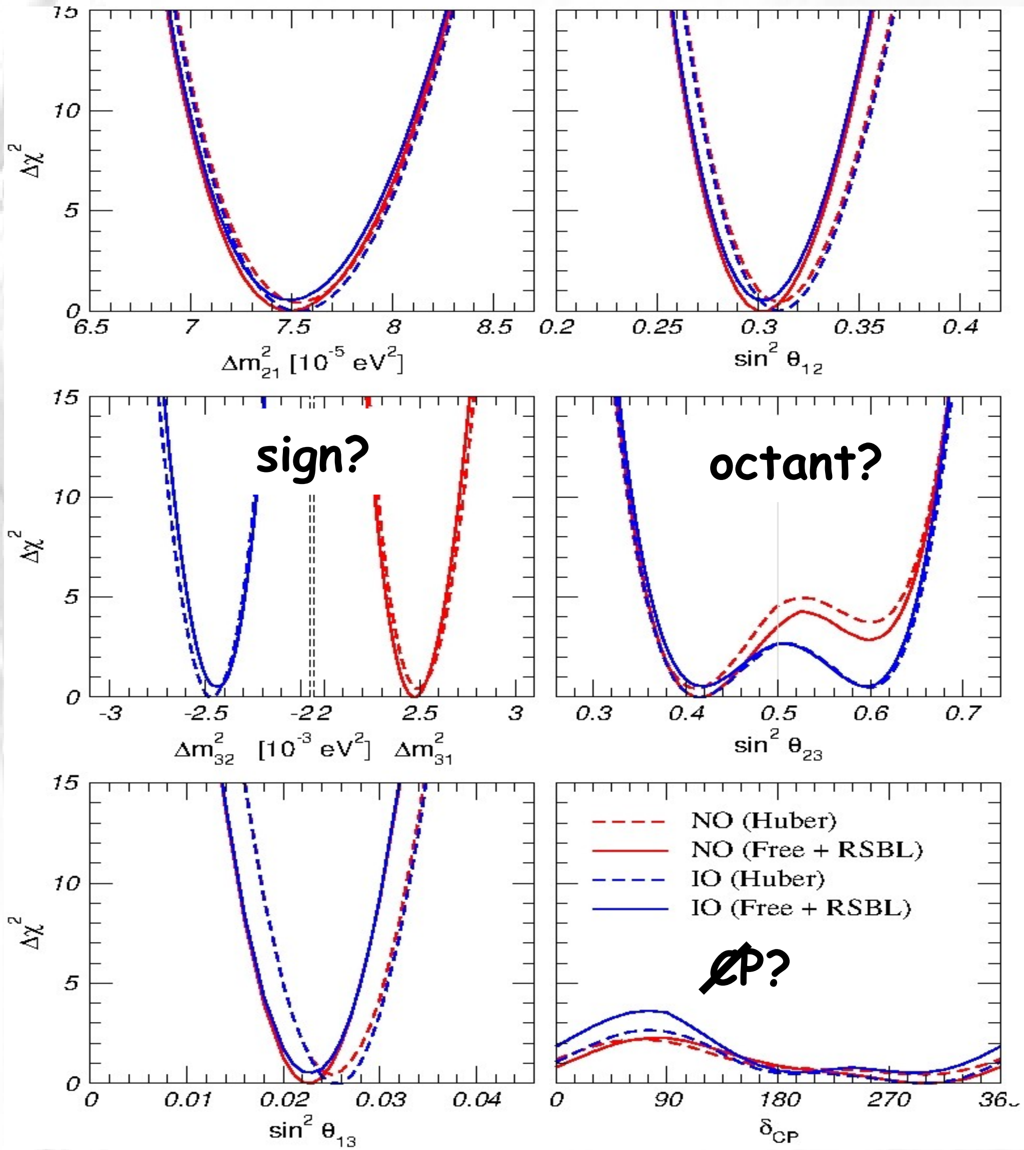


Gonzalez-Garcia et al. JHEP1212,(2012)123

Parameter	Result
θ_{12}	$33.36^{+0.81}_{-0.78}$
θ_{13}	$8.66^{+0.44}_{-0.46}$
θ_{23}	$40.0^{+2-1}_{-1.5}$
δ	300^{+66}_{-138}
Δm^2_{23} (10^{-3} eV^2)	$2.47^{+0.07}_{-0.07}$
Δm^2_{12} (10^{-5} eV^2)	$7.50^{+0.18}_{-0.19}$

- Masses @3%
- Angles between 5% and 10%

Global fit



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$$|U| = \begin{pmatrix} 0.795 \to 0.846 & 0.513 \to 0.585 & 0.126 \to 0.178 \\ 0.205 \to 0.543 & 0.416 \to 0.730 & 0.579 \to 0.808 \\ 0.215 \to 0.548 & 0.409 \to 0.725 & 0.567 \to 0.800 \end{pmatrix}$$

Neutrino oscillation anomalies

- LSND

  evidence for oscillations $\bar{\nu}_\mu \to \bar{\nu}_e$ with L/E~ 1 km/GeV (ν_e appearance)

- Anomalies in Gallium experiments (SAGE & GALLEX)

  they measured an electron neutrino flux from the Sun smaller than expected (ν_e disappearance)

- Anomalies due to new computations of reactor neutrino fluxes

  fluxes from reactor neutrinos are ~ 3.5% larger than in the past → experiments with L<= 100 m show deficit of neutrinos (ν_e disappearance- Bugey, Rovno...)

In addition there are *null results:* ν_μ disappearance (CDHS,SK, MINOS) e ν_e appearance (KARMEN, NOMAD, ICARUS, OPERA) which gave no signal

Neutrino oscillation anomalies

Joachim Kopp

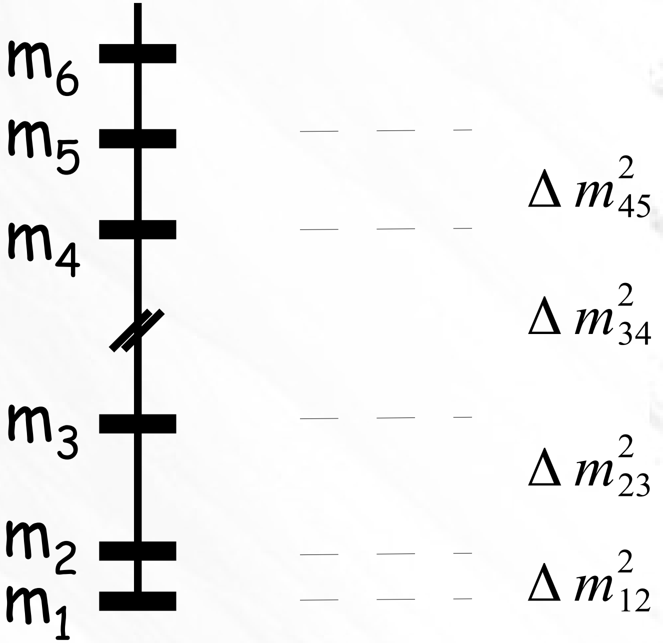
August 21, Aspen

- LSND

  evidence for oscillations
  $\bar{\nu}_e \rightarrow \bar{\nu}_\mu$ con L/E~ 1 km/GeV

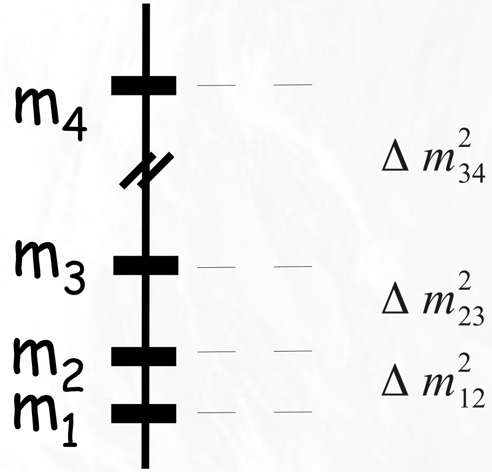
- MiniBooNE

  no significative excess of ν_e o $\bar{\nu}_e$ in the LSND preferred region but antinu results consistent with LSND



explanation in terms of sterile neutrinos

3+1 scheme



These states are considered as "degenearate"

m_4 is at a much higher scale, around 1 eV^2: effective description in terms of two-flavor

$$P_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta} = \delta_{\alpha\beta} - 4|U_{\alpha 4}|^2 \left(\delta_{\alpha\beta} - |U_{\beta 4}|^2\right) \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right)$$

$$\sin^2 2\vartheta_{e\mu} = 4|U_{e4}|^2|U_{\mu 4}|^2 \text{ of } \overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e \text{ transitions}$$

$$\sin^2 2\vartheta_{ee} = 4|U_{e4}|^2\left(1-|U_{e4}|^2\right) \text{ of } \overset{(-)}{\nu}_e \text{ disappearance}$$

$$\sin^2 2\vartheta_{\mu\mu} = 4|U_{\mu 4}|^2\left(1-|U_{\mu 4}|^2\right) \text{ of } \overset{(-)}{\nu}_\mu \text{ disappearance}$$

ν_e appearance

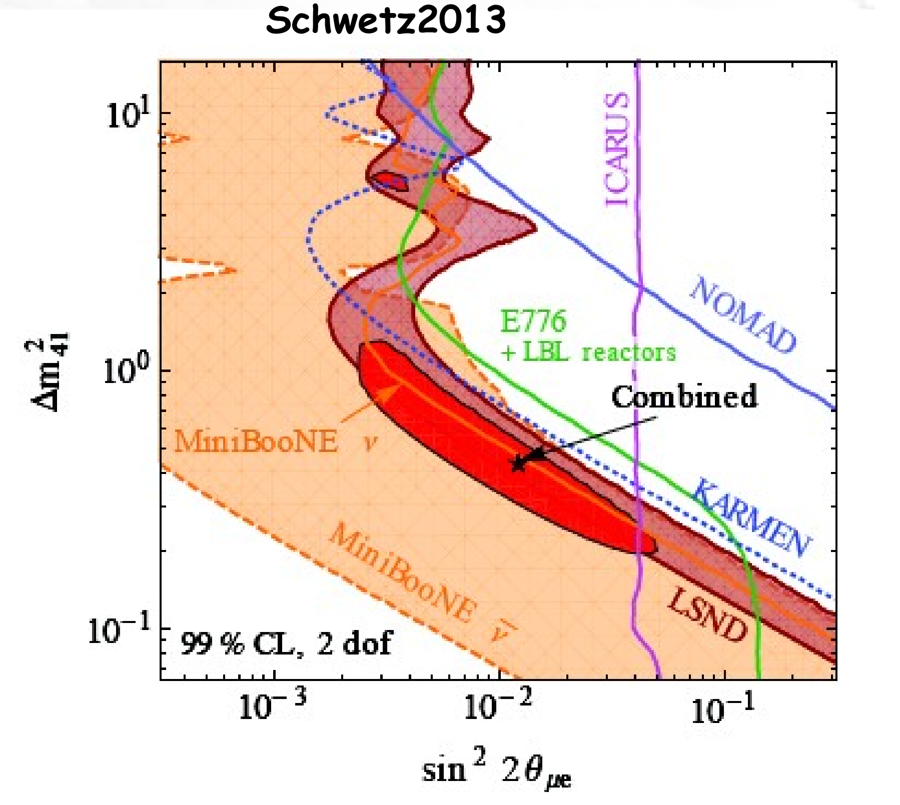
Global fit of nue appearance data are consistent

$\sin^2 2\theta_{\mu e} = 0.013$

$\Delta m^2_{41} = 0.42\ eV^2$

$\chi^2_{min}/dof = 87.9/66$

Kopp, Machado, Maltoni, Schwetz2013



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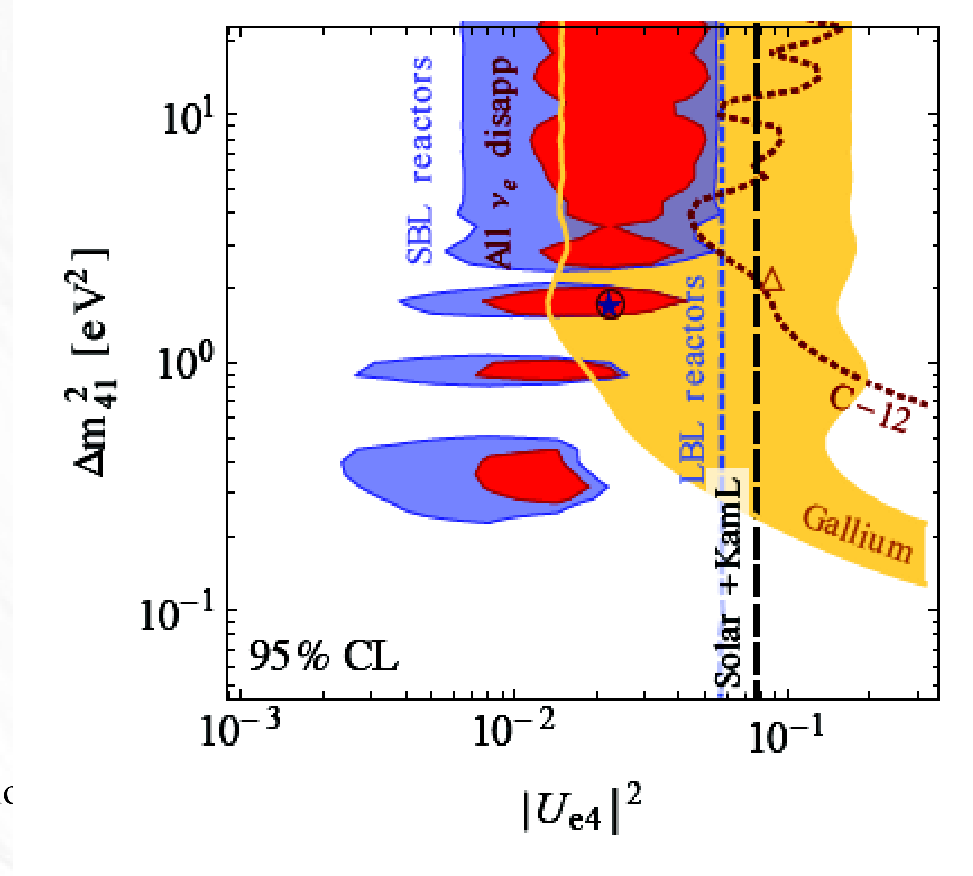
ν_e disappearance

- Global fit on nue disappearance data are consistent among themselves

Kopp, Machado, Maltoni, Schwetz2013



$\sin^2 2\theta_{ee} = 0.09$

$\Delta m^2_{41} = 1.78\ eV^2$

$\chi^2_{min}/dof = 403/427$

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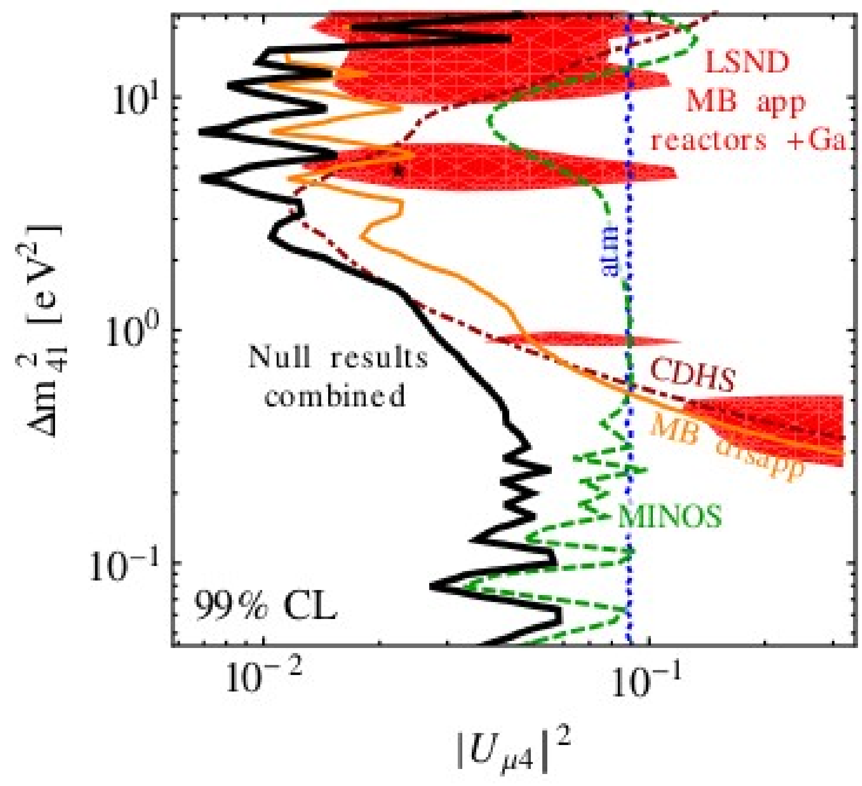
ν_μ disappearance

- Global fit on numu disappearance data:

Kopp, Machado, Maltoni, Schwetz2013

⇓

no signal → strong constraints on masses and mixing



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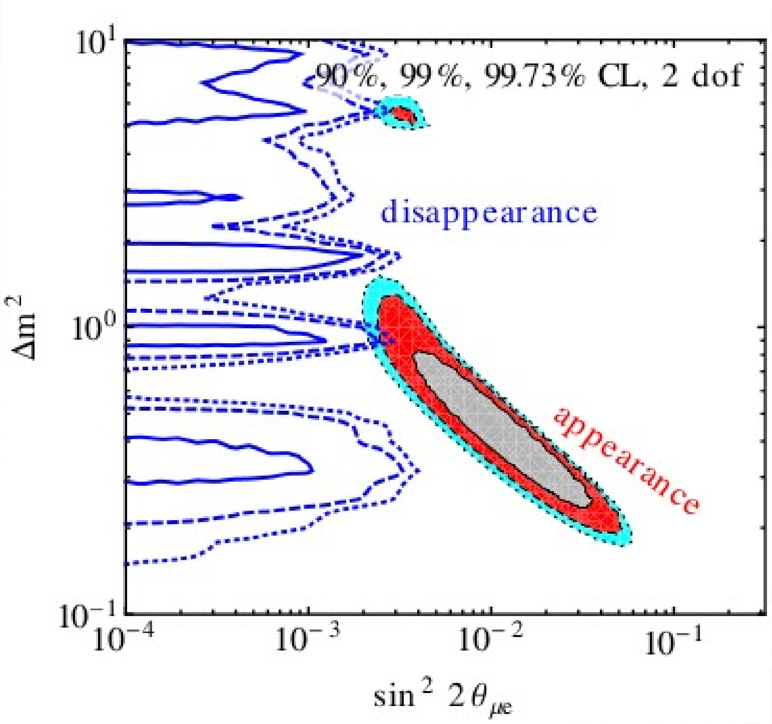
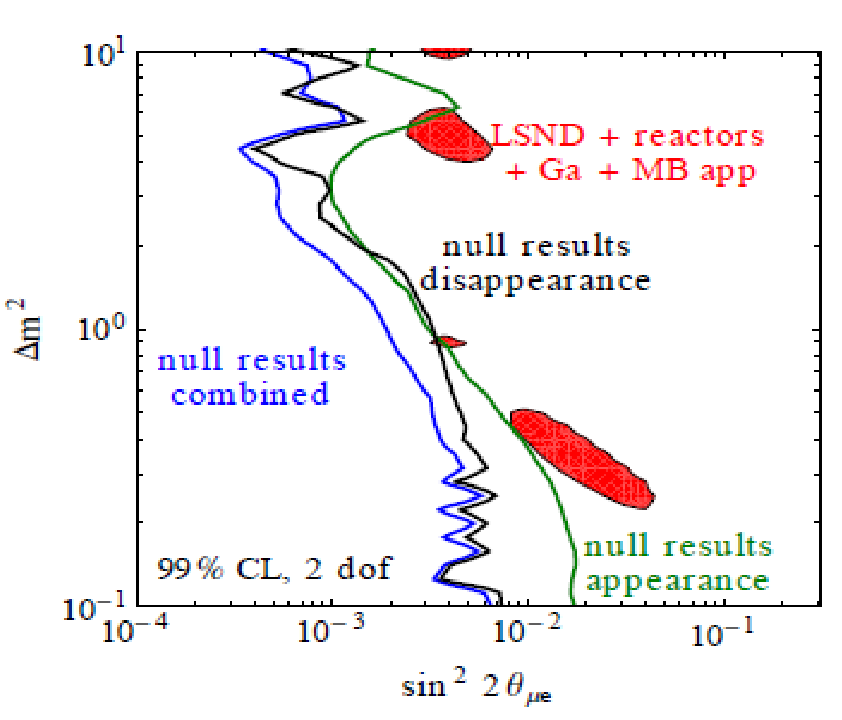
Global picture

Tension between appearance and disappearance



Tension between exp's with and without signal



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The (hard) job of a theorist

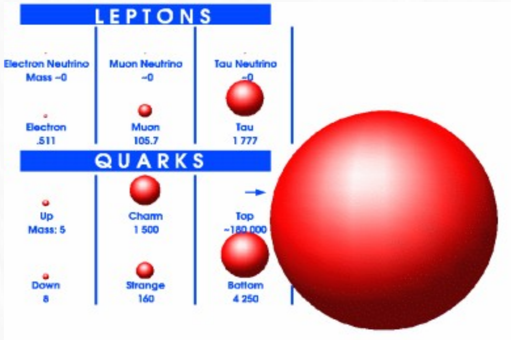
Take hints from experiments seriously
And explain:

Smallness of masses

Values of the mixing angles

- Some ideas...

On the ν masses



Easy part: neutrino Yukawa couplings smaller than those of the other fermions
Neutrinos are Dirac fermions: we have to introduce a right-handed neutrino field

neutrinos: $Y_\nu \bar{\psi}_L \widetilde{H} \nu_R$ electrons: $Y_e \bar{\psi}_L H e^c$

$$\frac{Y_\nu}{Y_e} \sim 10^{-5}$$

But we want to go beyond this "unnatural" scheme...

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Neutrino mass terms

we assume the existence of ν_L and the SM singlet ν_R

must be conserved: $|\Delta I| = 0$

Weak isospin	ν_L	ν_R	H = (h+,h0)
I	1/2	0	1/2
I_3	1/2	0	(+1/2,-1/2)

	ν	$\bar{\nu}$
Lepton number	1	-1

- Dirac mass term

(same for quarks and leptons)

lepton number L is conserved

$$L_D = m_D \bar{\psi}_L \tilde{H} \nu_R$$

- Majorana mass term

lepton number L **is not** conserved

$$L_M = m_M \nu_R^T \nu_R$$

The see-saw mechanism

- Total lagrangian

$$L_m = m_D \bar{\psi}_L \tilde{H} \nu_R + m_M \nu_R^T \nu_R$$

Electroweak symmetry breaking → see-saw

$$\langle H \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix} \rightarrow m_\nu = \begin{pmatrix} 0 & m_D \\ m_D^T & m_M \end{pmatrix}$$

$$m_\nu = -m_D^T \frac{1}{m_M} m_D$$

The see-saw mechanism

- An indicative numerical example

$$m_\nu \sim m_D^2/m_M$$

for $m_D \sim 100$ GeV, $m_\nu \sim 0.05$ eV

↓

$$m_M \sim 10^{14}\text{-}10^{15}\ \text{GeV}$$

Probe into GUT !

Mixing angles

Two different approaches and equally (not?) promising

- Models with non-trivial dynamics: means that the structure of the mixing matrix is determined by discrete symmetries

  such symmetries are motivated by the fact that the data themselves suggest rotations with fixed special angles ($\frac{1}{2}$, 1/3...)

  - permutational groups like A_4, S_4 ...

- Models where the main idea is that there is no need of introducing additional symmetries to explain the mixing angles

  In such models, the *chance* plays the fundamental role (anarchical models and variants)

Special mixing matrices

- mixing angles are obtained from the diagonalization of the mass matrix

$$m_\nu^{Diag} = U^T m_\nu U$$

- Good starting point suggested by the data:

$$\sin^2\theta_{23} = \frac{1}{2} \quad \sin^2\theta_{13} = 0$$

Tri-Bimaximal Mixing

$$\sin^2\theta_{12} = \frac{1}{3} \quad (TBM)$$

Bimaximal Mixing

$$\sin^2\theta_{12} = \frac{1}{2} \quad (BM)$$

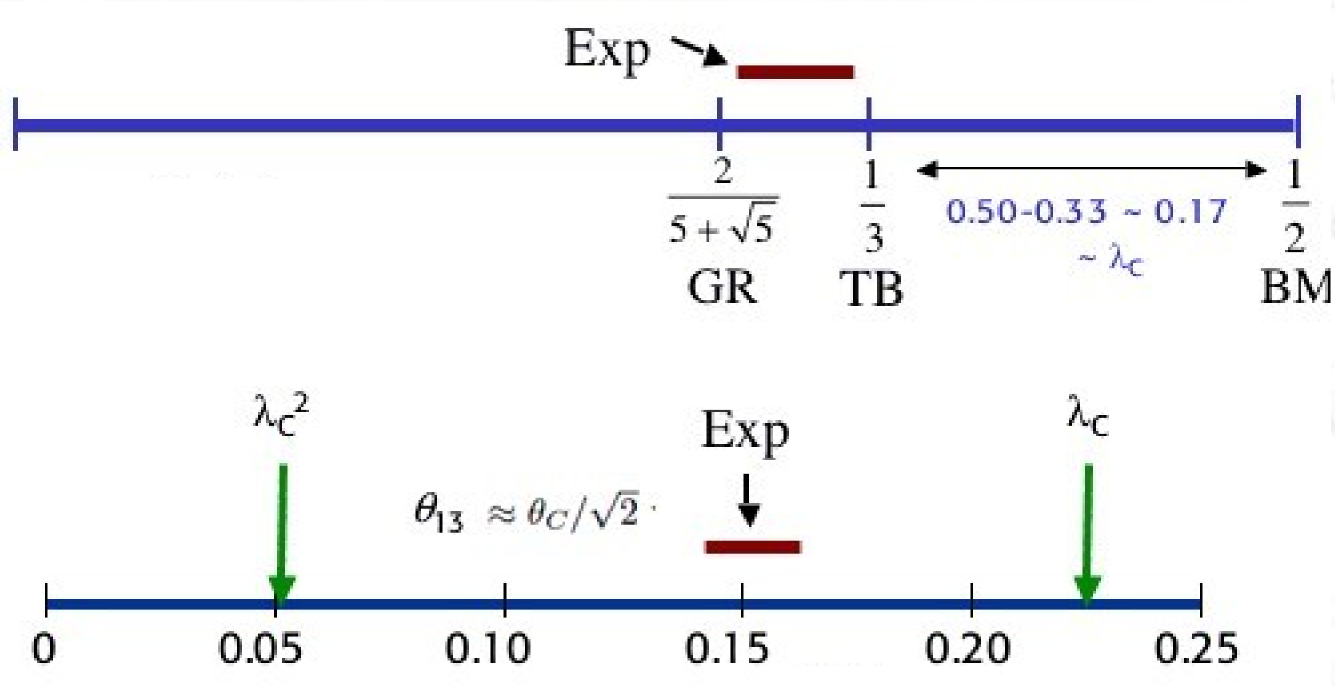
Golden Ratio

$$\sin^2\theta_{12} = \frac{2}{5+\sqrt{5}} \quad (GR)$$

Special mixing matrices

- How good are such starting points ?

$\sin^2\theta_{12}$



λ_c = Cabibbo angle

$\sin\theta_{13}$

Corrections are needed to stay on the experimental data

Special mixing matrices

- in models with no baroque dynamics, all mixing angles receive corrections of the same order of magnitude

TBM

$$\sin^2\theta_{12} = \frac{1}{3} + O(\lambda_C^2) \quad \sin^2\theta_{23} = \frac{1}{2} + O(\lambda_C^2) \quad \sin\theta_{13} = O(\lambda_C^2)$$

ok ok not really good

BM

$$\sin^2\theta_{12} = \frac{1}{2} + O(\lambda_C) \quad \sin^2\theta_{23} = \frac{1}{2} + O(\lambda_C) \quad \sin\theta_{13} = O(\lambda_C)$$

ok ok ok

This pattern seems to be favored

Special mixing matrices

Possible origin of corrections

- U_{PMNS} receives contributions from the charged lepton diagonalization

$$\nu_\alpha = U^\nu_{\alpha i}\,\nu_i \qquad\qquad l_\alpha = U^l_{\alpha i}\,l_i$$

diagonalizes the neutrino mass matrix

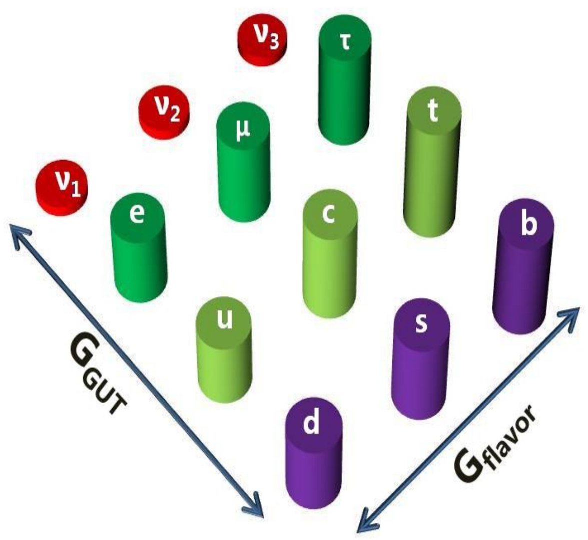
diagonalizes the charged lepton mass matrix

$$\bar{l}_\alpha \gamma_\mu \nu_\alpha W^\mu \rightarrow \underbrace{U^{+l}_{\alpha i} U^\nu_{\alpha j}}_{U_{PMNS}} \bar{l}_i \gamma_\mu \nu_j W^\mu$$

Charged current

Additional symmetries

- The previous patterns are easily obtained using *flavor symmetries*



- gauge symmetries act on members of particle multiplets

- flavor symmetries act on different families

Vantages: strong correlation among the entries of the mass matrices, so less free parameters → predictability

Additional symmetries at work

- The models work as follows:

Altarelli-Feruglio2012

Theory invariant under G_F

→ Flavor group

- Symmetry breaking of the flavor group: new scalar fields Φ in the theory with non vanishing vevs

Residual symmetry

given by a subset of the generators of G_F

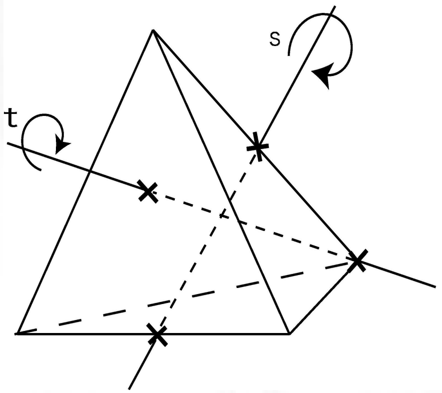
in the neutrino sector $G_\nu \rightarrow U_\nu$

in the charged lepton sector $G_l \rightarrow U_l$

$$U_{PMNS} = U_l^+ U_\nu$$

A possible flavor group: A4

A_4 : is the group of even permutation of 4 objects
(also the symmetry of a tetrahedron)



generators of the group

$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}, \qquad S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$$

The 12 elements are obtained considering all possible even permutations of 1234. They belong to 4 conjugacy classes... $given\ \ a\ \ of\ \ G, \{g^{-1} a g, \forall g \in G\}$

A_4 has 4 irreducible representations

- three singlets 1, 1' and 1''
- 1 triplet 3

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A possible flavor group: A4

After breaking of A_4

- charged lepton mass matrix (residual symmetry generated by T)

$$m_e^{(0)} = v_d \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix} \eta$$

$$U_l = I$$

- neutrino mass matrix (generated by a **non**-diagonal generator S of A4)

$$m_\nu^{(0)} = \begin{pmatrix} x & y & y \\ y & x+z & y-z \\ y & y-z & x+z \end{pmatrix}$$

$$U_\nu = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}$$

$$\sin^2\theta_{23} = \frac{1}{2} \quad \sin^2\theta_{12} = \frac{1}{3} \quad \sin^2\theta_{13} = 0$$

A possible flavor group: A4

After breaking of T and S

- Charged lepton rotation

$$U_e = \begin{pmatrix} 1 & c^e_{12}\,\xi & c^e_{13}\,\xi \\ -c^{e*}_{12}\,\xi & 1 & c^e_{23}\,\xi \\ -c^{e*}_{13}\,\xi & -c^{e*}_{23}\,\xi & 1 \end{pmatrix}$$

- neutrino rotation

$$U_\nu = \begin{pmatrix} 1 & c^\nu_{12}\,\xi' & c^\nu_{13}\,\xi' \\ -c^{\nu*}_{12}\,\xi' & 1 & c^\nu_{23}\,\xi' \\ -c^{\nu*}_{13}\,\xi' & -c^{\nu*}_{23}\,\xi' & 1 \end{pmatrix}$$

$$\sin^2\theta_{23} = \frac{1}{2} + \mathcal{R}e(c^e_{23})\,\xi + \frac{1}{\sqrt{3}}\left(\mathcal{R}e(c^\nu_{13}) - \sqrt{2}\,\mathcal{R}e(c^\nu_{23})\right)\xi \longrightarrow \frac{\langle\Phi\rangle}{\Lambda} \sim O(0.1)$$

$$\sin^2\theta_{12} = \frac{1}{3} - \frac{2}{3}\mathcal{R}e(c^e_{12} + c^e_{13})\,\xi + \frac{2\sqrt{2}}{3}\mathcal{R}e(c^\nu_{12})\,\xi$$

$$\sin\theta_{13} = \frac{1}{6}\left|3\sqrt{2}\,(c^e_{12} - c^e_{13}) + 2\sqrt{3}\left(\sqrt{2}\,c^\nu_{13} + c^\nu_{23}\right)\right|\xi\,.$$

Altarelli, Feruglio, Merlo, Stamou '12

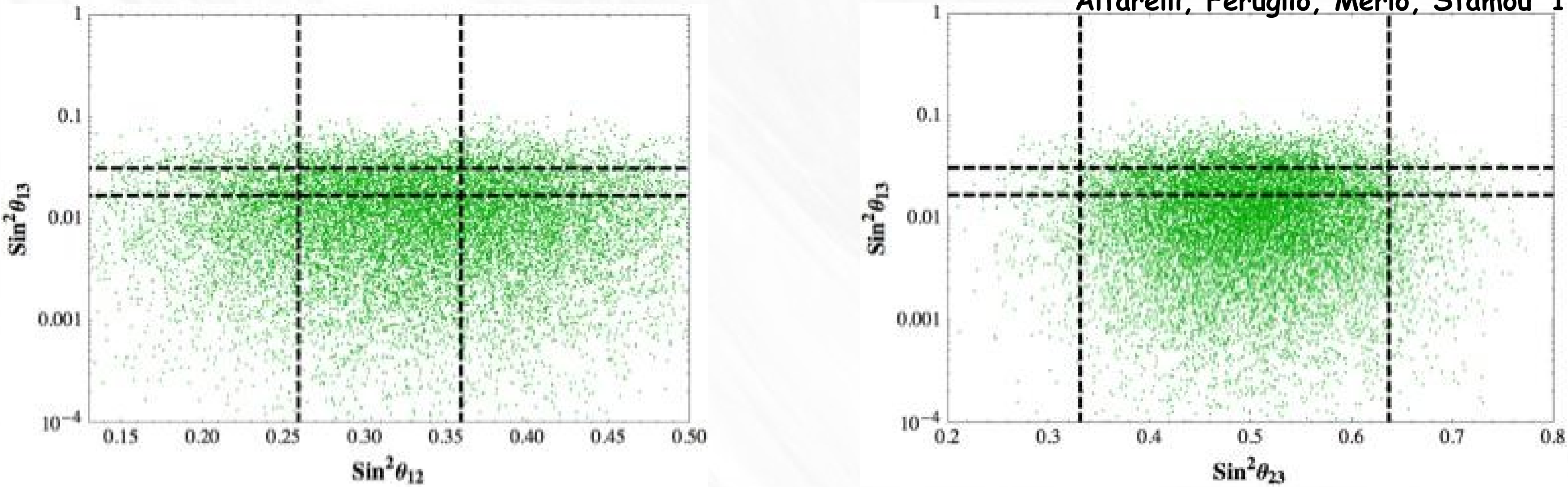
from charged lepton rotation

from neutrino rotation

Typical predictions of A4 models

c_{ij} = random complex with abs. value gaussian around 1 with variance 0.5

Altarelli, Feruglio, Merlo, Stamou '12



Models with no special dynamics

- The chance is the basis of the success
- Only abelian U(1) to generate the hierarchies among fermions

- fields tranform as: $\psi \rightarrow e^{i\, q_\psi}\, \psi$

- so a mass term transforms as:

$$y \bar{\psi}_L H \psi_R \rightarrow e^{i(-q_{\psi_R}+q_{\psi_L}+q_H)}\, y \bar{\psi}_L H \psi_R$$

If $(-q_{\psi_R}+q_{\psi_L}+q_H)=0$ the mass term is allowed, otherwise we need a new scalar field θ with charge q_θ and vev v_θ:

$$y \bar{\psi}_L H \psi_R \left(\frac{\theta}{\Lambda}\right)^k \rightarrow e^{i(-q_{\psi_R}+q_{\psi_L}+q_H+k\, q_\theta)}\, y \left(\frac{v_\theta}{\Lambda}\right)^k \bar{\psi}_L H \psi_R$$

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→ Suppression factor

A GUT example

- Standard Model particles in the 10 and $\bar{5}$ representations (3 copies)

$$\bar{5} = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e \\ -\nu \end{pmatrix}_L , \qquad 10 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & u_3^c & -u_2^c & u_1 & d_1 \\ -u_3^c & 0 & u_1^c & u_2 & d_2 \\ u_2^c & -u_1^c & 0 & u_3 & d_3 \\ -u_1 & -u_2 & -u_3 & 0 & e^c \\ -d_1 & -d_2 & -d_3 & -e^c & 0 \end{pmatrix}_L$$

1 = right-handed neutrino

- SU(5) mass terms:

$$m_{up} \sim 10 \times 10$$

$$m_d = m_e^T \sim 10 \times \bar{5}$$

$$m_{\nu_D} \sim \bar{5} \times 1$$

$$m_M \sim 1 \times 1$$

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Models with no special dynamics

Choosing appropriate U(1) charges we can get several mass matrices structures:

- Anarchycal models (A)

$$q_{\bar{5}}=(0,0,0) \quad q_{10}=(3,2,0) \quad q_1=(0,0,0)$$

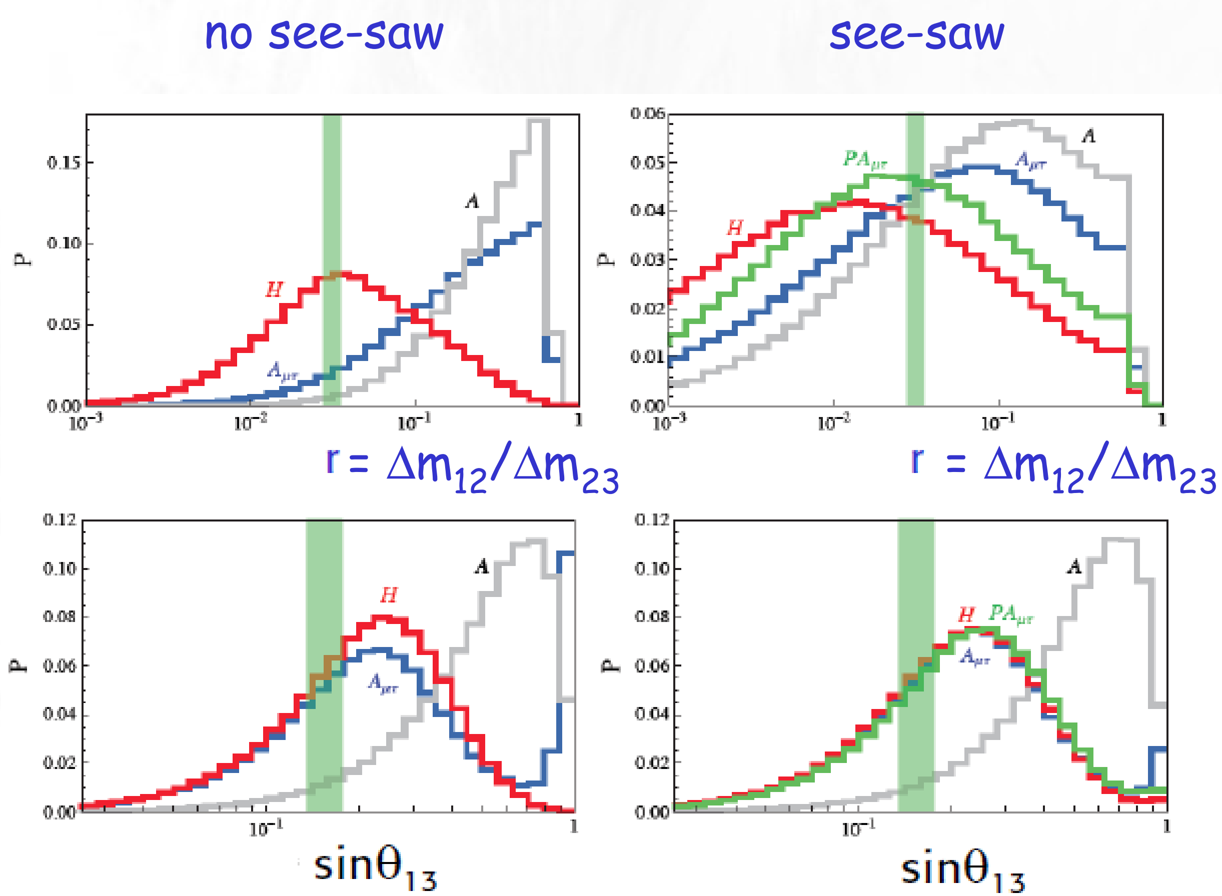
$$m_l=\begin{pmatrix} \lambda^3 & \lambda^3 & \lambda^3 \\ \lambda^2 & \lambda^2 & \lambda^2 \\ 1 & 1 & 1 \end{pmatrix}, \quad m_\nu=\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

- Hierachycal model (H)

$$q_{\bar{5}}=(2,1,0) \quad q_{10}=(5,3,0) \quad q_1=(2,1,0)$$

$$m_l=\begin{pmatrix} \lambda^7 & \lambda^6 & \lambda^5 \\ \lambda^5 & \lambda^4 & \lambda^3 \\ \lambda^2 & \lambda & 1 \end{pmatrix}, \quad m_\nu=\begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^2 \\ \lambda^3 & \lambda^2 & \lambda \\ \lambda^2 & \lambda & 1 \end{pmatrix}$$

Models with no special dynamics



message: H performs better than A

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The future (personal view)

Oscillation sector

- Better determination of the oscillation parameters and the mass pattern
- Check for new physics effects

Flavor sector

- Interplay of flavor symmetries and realistic GUT theories
- Differences among quarks and leptons

backup

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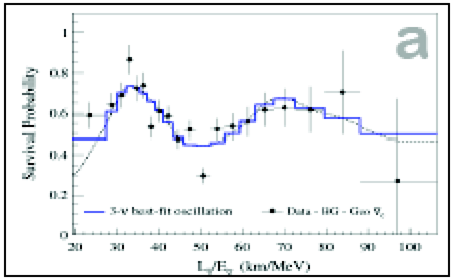
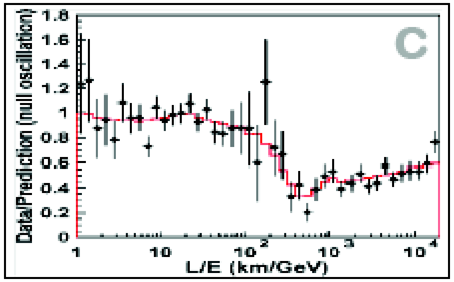
Global fit

Egidio Lisi, talk a Moriond2015

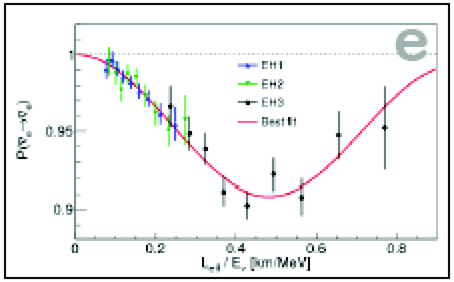
$e \to e$ $(\delta m^2, \theta_{12})$



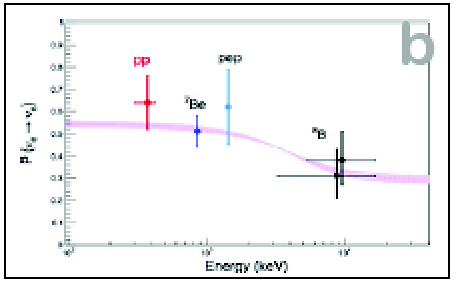
$\mu \to \mu$ $(\Delta m^2, \theta_{23})$



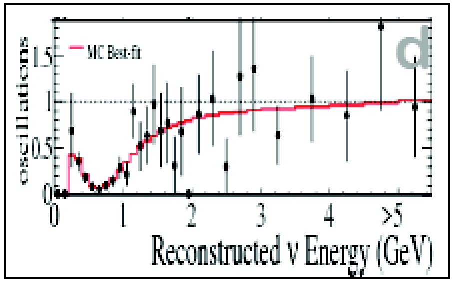
$e \to e$ $(\Delta m^2, \theta_{13})$



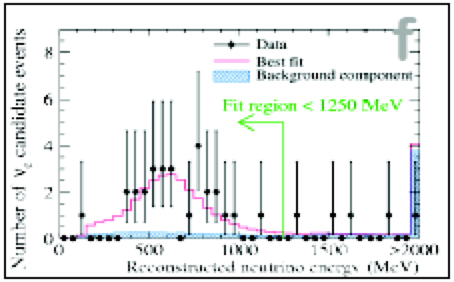
$e \to e$ $(\delta m^2, \theta_{12})$



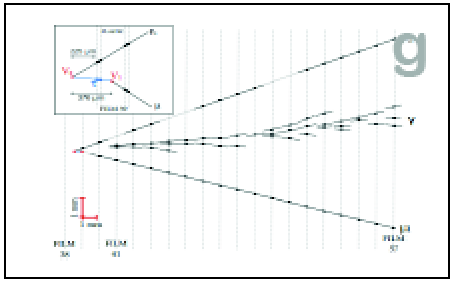
$\mu \to \mu$ $(\Delta m^2, \theta_{23})$



$\mu \to e$ $(\Delta m^2, \theta_{13}, \theta_{23})$



$\mu \to \tau$ $(\Delta m^2, \theta_{23})$



Data from various types of neutrino experiments: (a) solar, (b) long-baseline reactor, (c) atmospheric, (d) long-baseline accelerator, (e) short-baseline reactor, (f,g) long baseline accelerator (and, in part, atmospheric).

(a) KamLAND [plot]; (b) Borexino [plot], Homestake, Super-K, SAGE, GALLEX/GNO, SNO; (c) Super-K atmosph. [plot], MACRO, MINOS etc.; (d) T2K (plot), MINOS, K2K; (e) Daya Bay [plot], RENO, Double Chooz; (f) T2K [plot], MINOS; (g) OPERA [plot], Super-K atmospheric.

New results from Planck

For T < me , radiation content of the Univers is

$$\varepsilon_R = \varepsilon_\gamma + \varepsilon_\nu + \varepsilon_x$$

Ninetta Saviano,
talk a Moriond2015

non-elettromagnetic contribution is parametrized in terms of effective neutrino species Neff

$$\varepsilon_\nu + \varepsilon_x = \frac{7}{8}\frac{\pi^2}{15}T_\nu^4 N_{\text{eff}} = \frac{7}{8}\frac{\pi^2}{15}T_\nu^4 (N_{\text{eff}}^{\text{SM}} + \Delta N)$$

3.046
(relativistic degrees of freedom)

Extra radiation, for example from sterile neutrinos

Planck 2015:

Neff = 3.15 ± 0.46

Not a large room for sterile states!

D.Meloni

A peculiarity of the neutrino

- For electrons: 4 different helicity states and all of them are needed

spin ←
e^- p →

left-handed particle: e_L^-

moving faster:
the boosted observer
see a RH object

charge is conserved

spin ←
e^- p ←

right-handed particle: e_R^-

- For neutrinos: from experiments we have identified n_L and $\bar{n}_R$ only

left-handed particle: ν_L

moving faster:
we see again a RH object

ν_R Dirac particle

$\bar{\nu}_R$ **Majorana particle**

if $\nu_L \sim \bar{\nu}_R$ no additive quantum numbers are conserved

A4

take this as an example

- A_4 is the discrete group of even permutations of 4 objects (4!/2 = 12 elements) generated by S and T

$$S^2=T^3=(ST)^3=1$$

The action of the generators S and T can be assigned as follows:

S: (1234) → (4321)   T:(1234) → (2314)

- irreducible representations:

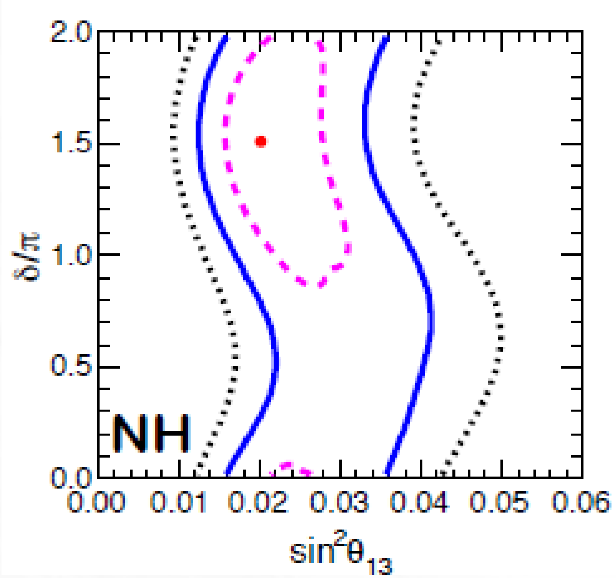
  a triplet and 3 different singlets 3, 1, 1′, 1″ (promising for 3 generations)

- invariance under S and T is automatic while A_{23} is not contained in A_4

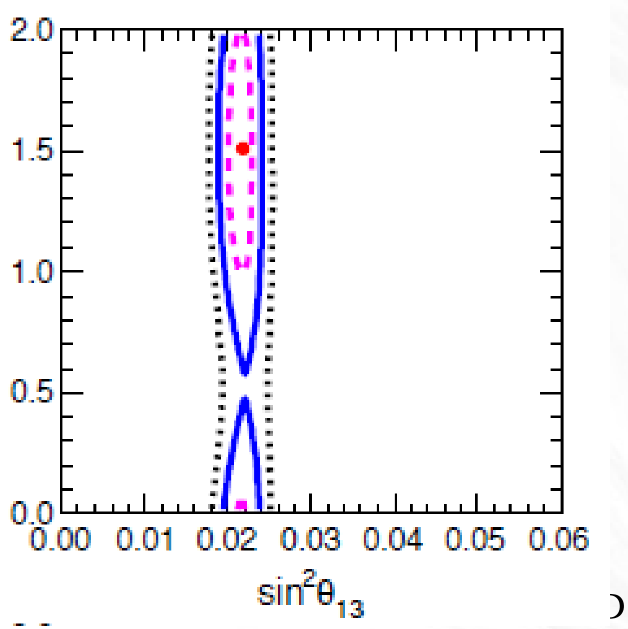
  (2-3 symmetry happens in A_4 if 1′ and 1″ symm. breaking flavons are absent or have equal VEV's)

A comment on the CP violating phase

Egidio Lisi, Moriond2015



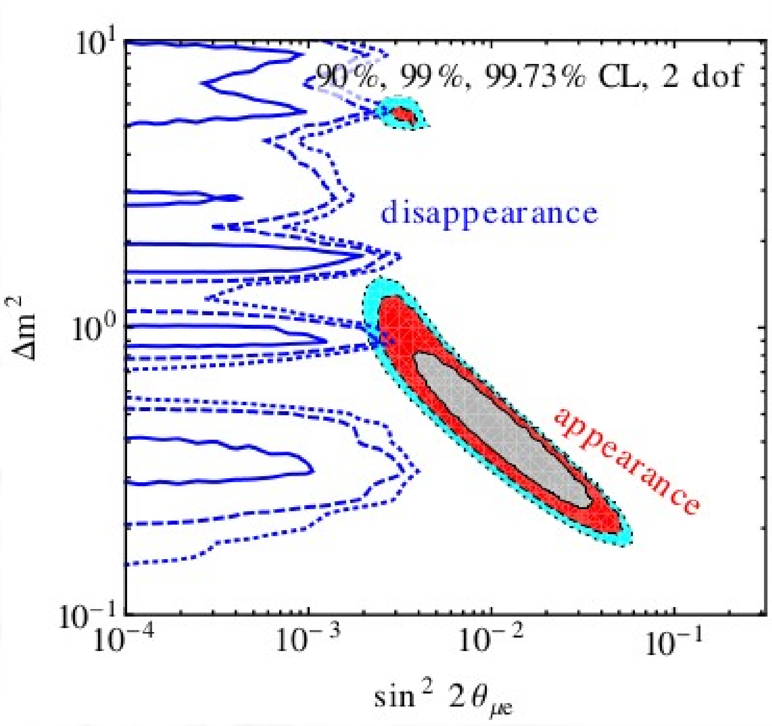
- Long Baseline experiments (T2K) indicates $\delta \sim 3/2\,\pi$



- Reactor experiments model the CL form for $\sin^2\theta_{13} \sim 0.02$

Appearance-disappearance tension

$$\sin^2 2\theta_{\mu e} \approx \frac{1}{4} \sin^2 2\theta_{ee} \sin^2 2\theta_{\mu\mu}$$



D.Meloni