Two puzzles in the beauty sector of QCD

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I will discuss two related problems in the beauty sector of QCD: the b-quark mass and its determinations and the extraction of decay constants of beauty mesons. In particular, two recent puzzles will be addressed:

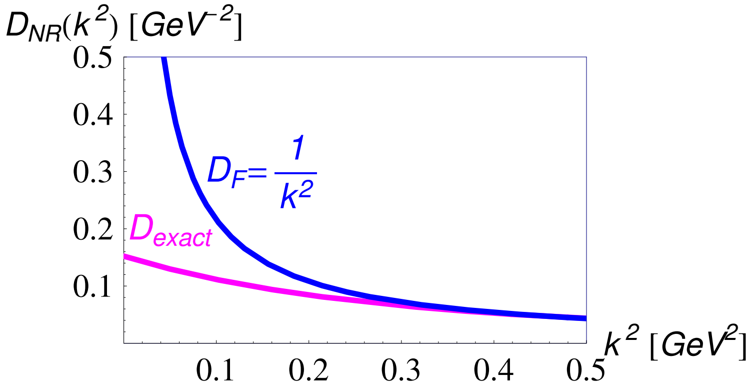
1. A tension between $\bar{m}_b(\bar{m}_b)$ as extracted from heavy-heavy and heavy-light QCD correlation functions

2. Unexpected results on f_B^*/f_B which suggests that our understanding of the heavy-quark expansion might be indufficient

QCD - theory of strong interactions: SU(3) gauge theory with Lagrangian based on quarks and gluons as fundamental degrees of freedom.

- **Confinement: only hadrons - colorless bound states of quarks and gluons - are observed in nature.**
- $\alpha_s(\mu)$ **falls at large** μ **(asymptotic freedom) but rises as** μ **decreases.**
- **For the description of theory at low scales, quarks and gluons are "irrelevant" degrees of freedom; one should desribe nature in terms of hadron degrees of freedom: ChPT.**

<u>Full propagator of a confined particle:</u>

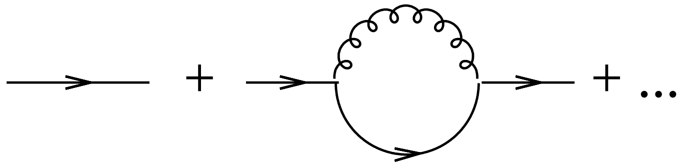


NR particle $k^2 \equiv \vec{k}^2$ **and** $E = 0$ **in a harmonic-oscillator potential (Coulomb term is switched-off).**

- **pole in the propagator disappears**
- **full propagator in the IR differs from Feynman propagator; at large** k^2 **they are equal**

Mass of a confined quark

Pole quark mass



In each order of the perturbation theory, quark propagator has a pole. The location of this pole is the *pole mass* of a particle.

- **IR finite**
- **Gauge-independent**
- **Renormalization scheme and scale independent**

However: in a confined theory no pole in the propagator; therefore this quantity is not fully consistent; price to pay in the ambiguity of the pole mass of a heavy quark of order $\Lambda_{\rm QCD} \simeq 200$ MeV ("IR sensitive").

Short-distance masses (e.g. $\bar{m}(\bar{m}) \equiv m_b(m_b)$)

- **IR insensitive (free from renormalon ambiguities)**
- **Scheme-scale dependent**

Other definitions of heavy-quark mass:

(Potential-subtracted mass; Kinetic mass; etc)

recent $\overline{m}_b(\overline{m}_b)$ determinations

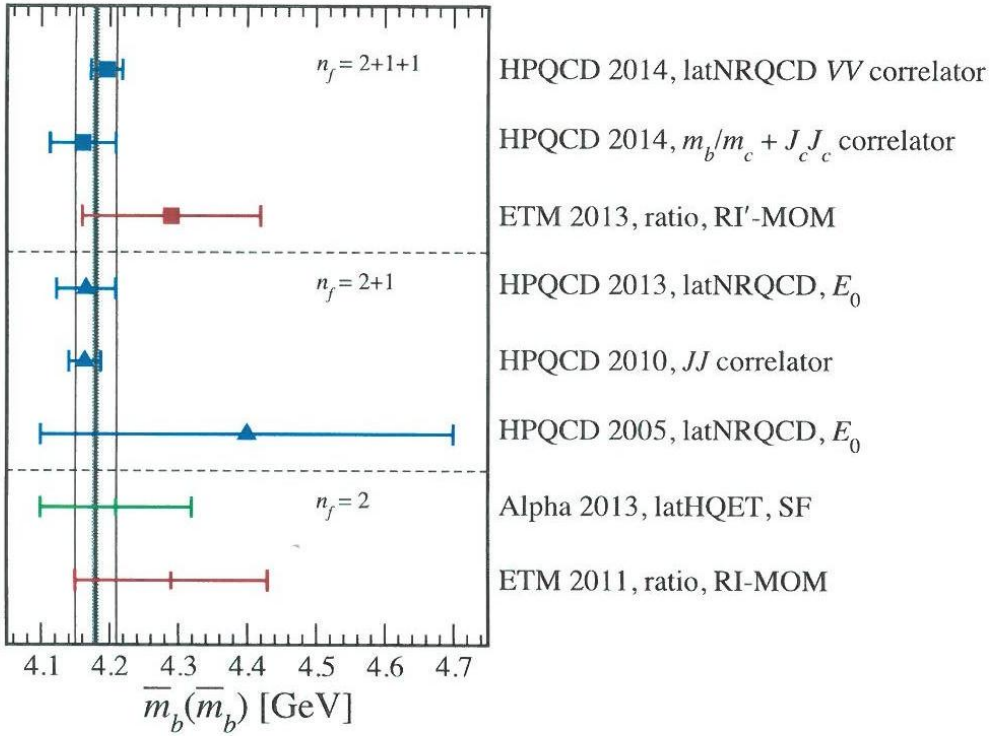
$\overline{m}_b$ [MeV]	approach	observables	group	arXiv
4196 ± 23	lattice ($n_f = 4$)	$\Gamma(\Upsilon, \Upsilon' \to e^+e^-)$	HPQCD	1408.5768
4174 ± 24	lattice ($n_f = 4$)	PS current	HPQCD	1408.4169
4201 ± 43	N^3LO PQCD	M_Υ	Ayala et al	1407.2128
4169 ± 9	15th moment SR	Υ(1S-6S)	Penin, Zerf	1401.7035
4247 ± 34	Borel SR	f_B, f_{Bs}	Lucha et al	1305.7099
4166 ± 43	lattice + PQCD	M_Υ, M_{Bs}	HPQCD	1302.3739
4235 ± 55	10th moment SR	Υ(1S-4S), R	Hoang et al	1209.0450
4171 ± 9	optimized SR	Υ(1S-4S), R	Bodenstein et al	1111.5742
4177 ± 11	exponential SR	Υ(1S-6S)	Narison	1105.5070
4180^{+50}_{-40}	lattice + PQCD	static potential	Laschka et al	1102.0945
4163 ± 16	2nd moment SR	Υ(1S-4S), R	Chetyrkin et al	1010.6157

From J. Erler, *Status of Precision Extractions of α_s and Heavy Quark Masses*, **arXiv:1412.4435**

It is common to recalculate all values to $m_b(m_b)$. This induced uncertainties.

- **The b-quark mass obtained from $\bar{b}b$ correlator is lower than from $\bar{b}q$ correlator.**

$m_b(m_b)$ from lattice QCD



A. Kronfeld, *'Quark masses from lattice QCD*, **April 2015**

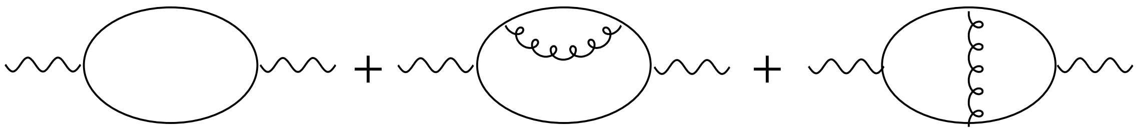
- **$m_b(m_b)$ does not intrinsically belong to lattice QCD; recalculation from bare or renormalized m_b on the lattice to $m_b(m_b)$ requires a renormalization constant calculated nonperturbatively. Large errors of purely lattice calculations.**

- **Accurate "lattice QCD" results are in practice combination of moments calculated in lattice QCD with moments calculated in pQCD (again relies on OPE)**

OPE for correlation functions in QCD

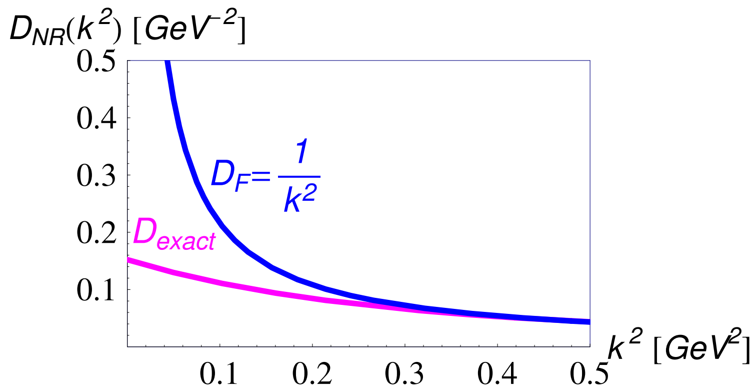
- **The basic object T-product of 2 quark currents:** $j_\alpha(x) = \bar{b}(x)\gamma_\alpha b(x)$**,**

$$i\int d^4x\, e^{ipx}\left\langle \Omega \middle| T\left(\bar{b}(x)\gamma_\alpha b(x), \bar{b}(0)\gamma_\beta b(0)\right)\middle| \Omega\right\rangle = \left(p_\alpha p_\beta - g_{\alpha\beta}p^2\right)\Pi(p^2), \qquad \Pi(p^2) = \frac{p^2}{\pi}\int_{4m^2}^{\infty}\frac{\mathrm{Im}\,\Pi(s)ds}{s(s-p^2)}$$



(1)

$$\int dk \frac{1}{k^2}\frac{1}{(p-k)^2}, \qquad k \sim \Lambda \to \frac{\Lambda^2}{p^2}$$



Regions of soft momenta in Feynman integrals (where the exact non-perturbative propagators differ strongly from Feynman propagators) lead to power-suppressed terms in correlators.

• **Wilsonian OPE - separation of distances:**

$$T\left(j(x)j^{\dagger}(0)\right) = C_0(x^2,\mu)\hat{1} + \sum_n C_n(x^2,\mu) : \hat{O}(x=0,\mu) :$$

$$\Pi(p^2) = \Pi_{\rm pert}(p^2,\mu) + \sum_n \frac{C_n}{(p^2)^n}\langle\Omega| : \hat{O}(x=0,\mu) : |\Omega\rangle$$

• **Physical QCD vacuum $|\Omega\rangle$ is complicated and differs from perturbative QCD vacuum $|0\rangle$.**

Condensates – nonzero expectation values of gauge-invariant operators over physical vacuum:

$$\boxed{\langle\Omega| : \hat{O}(0,\mu) : |\Omega\rangle \neq 0}$$

Gell-Mann Oakes Renner relation:

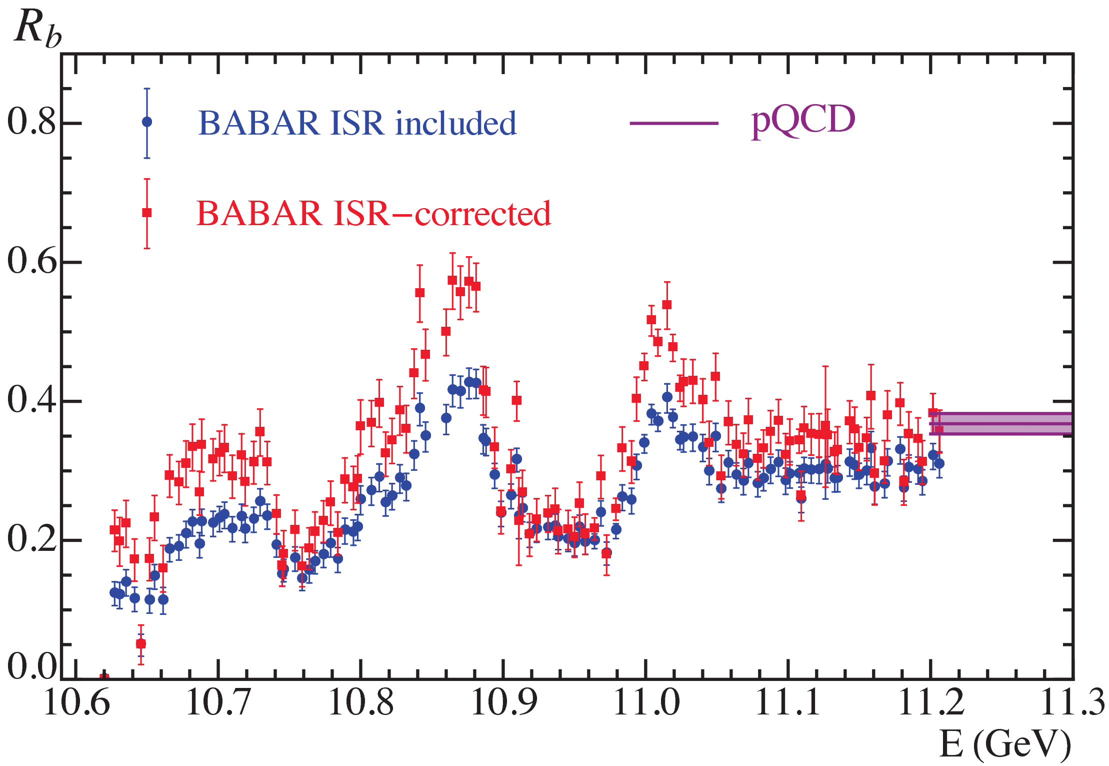
$$m_\pi^2 f_\pi^2 = \langle\Omega|\bar{q}q|\Omega\rangle(m_u + m_d) + O(m_{u,d}^2).$$

$\langle\Omega|\bar{q}q(2\ \mathrm{GeV}|\Omega\rangle \simeq 290$ **MeV.**

$$\langle\bar{q}q(2\ \mathrm{GeV})\rangle = (271 \pm 3\ \mathrm{MeV})^3, \qquad \left\langle\frac{\alpha_s}{\pi}GG\right\rangle = 0.012 \pm 0.006\ \mathrm{GeV}.$$

Heavy quark condensates

$$m_Q\langle\bar{Q}Q\rangle = -\frac{1}{12}\langle\frac{\alpha_s}{\pi}GG\rangle.$$



Sum rules for moments and m_b

For $\bar{Q}Q$ systems, mainly moment sum rules are used

$$M_n = \int \frac{ds}{s^{n+1}} \mathrm{Im}\ \Pi_{\bar{b}b}(s).$$

Moments are known to $O(\alpha_s^3)$ accuracy for several n. Moment SRs + experimental data or lattice QCD calculation of moments → *Quark masses*

$m_b(m_b) = 4.163 \pm 0.016$ **GeV (Chetyrkin et al, relativistic, i.e. low-n, moment sum rules**

$m_b(m_b) = 4.235 \pm 0.055_{pert} \pm 0.003_{exp}$ **GeV (Hoang et al, "nonrelativistic", or large n at NNLL).**

Properties of individual resonances

T-product of 2 quark currents currents $j_5(x) = (m_b + m)\,\bar{q}(x) i\gamma_5 b(x)$,

$$\Pi(p^2) = i \int d^4x\, e^{ipx} \left\langle \Omega \left| T\left(j_5(x) j_5^\dagger(0) \right) \right| \Omega \right\rangle$$

$$\Pi(\tau) = \int ds \exp(-s\tau)\rho(s) = f_B^2 M_B^4 e^{-M_B^2\tau} + \int\limits_{s_{\rm phys}}^{\infty} ds\, e^{-s\tau}\rho_{\rm hadr}(s) = \int\limits_{(m_b+m)^2}^{\infty} ds\, e^{-s\tau}\rho_{\rm pert}(s,\mu) + \Pi_{\rm power}(\tau,\mu).$$

Here $s_{\rm phys} = (M_{B^*} + M_P)^2$, and f_B is the decay constant defined by

$$(m_b + m)\langle 0|\bar{q} i\gamma_5 b|B\rangle = f_B M_B^2.$$

$$\Pi_{\rm power}(\tau, \mu = m_Q) = (m_Q + m)^2 e^{-m_Q^2\tau}$$
$$\times \left\{ -m_Q\langle\bar{q}q\rangle \left[1 + \frac{2C_F\alpha_s}{\pi}\left(1 - \frac{m_Q^2\tau}{2}\right) - \frac{m}{2m_Q}(1 + m_Q^2\tau) + \frac{m^2}{2}m_Q^2\tau^2 + \frac{m_0^2\tau}{2}\left(1 - \frac{m_Q^2\tau}{2}\right)\right] + \frac{1}{12}\left\langle\frac{\alpha_s}{\pi}GG\right\rangle \right\}.$$

To exclude the excited-state contributions, one adopts the *duality Ansatz*: all contributions of excited states are counterbalanced by the perturbative contribution above an *effective continuum threshold*, $s_{\rm eff}(\tau)$ which differs from the physical continuum threshold.

Applying the duality assumption yields:

$$f_B^2 M_B^4 e^{-M_B^2\tau} = \int\limits_{(m_b+m)^2}^{s_{\rm eff}(\tau)} ds\, e^{-s\tau}\rho_{\rm pert}(s,\mu) + \Pi_{\rm power}(\tau,\mu) \equiv \Pi_{\rm dual}(\tau, s_{\rm eff}(\tau)).$$

Even if the QCD inputs $\rho_{\rm pert}(s,\mu)$ and $\Pi_{\rm power}(\tau,\mu)$ are known the extraction of the decay constant requires $s_{\rm eff}(\tau)$.

Extraction of the decay constant

According to the standard procedures of QCD sum rules, one executes the following steps:

1. *The Borel window*

The working τ-window is chosen such that the OPE gives an accurate description of the exact correlator (i.e., all higher-order radiative and power corrections are under control) and at the same time the ground state gives a "sizable" contribution to the correlator.

2. *The effective continuum threshold*

The major part of hadron continuum is removed by applying the cut at $s_{\rm eff}$.

In those cases where the bound-state mass M_B is known, one can use it and improve the accuracy of f_B.

Introduce the *dual invariant mass* $M_{\rm dual}$

$$M_{\rm dual}^2(\tau) \equiv -\frac{d}{d\tau}\log\Pi_{\rm dual}(\tau, s_{\rm eff}(\tau)).$$

The deviation of $M_{\rm dual}$ from M_B measures the contamination of the dual correlator by excited states.

Starting from a trial function for $s_{\rm eff}(\tau)$ and requiring a minimum deviation of $M_{\rm dual}$ from M_B in the τ-window generates a variational solution for $s_{\rm eff}(\tau)$. We consider polynomials in τ and obtain their paramaters by minimizing the squared difference between $M^2_{\rm dual}$ and M^2_B in the τ-window:

$$\chi^2 \equiv \frac{1}{N}\sum_{i=1}^{N}\left[M^2_{\rm dual}(\tau_i) - M^2_B\right]^2 .$$

Uncertainties in the extracted decay constant

The resulting f_B is sensitive to the input values of the OPE parameters — the *OPE-related error* — and to the adopted prescription for fixing the effective continuum threshold $s_{\rm eff}(\tau)$ — the *systematic error*.

OPE – related error

Gaussian distributions for all OPE parameters but the renormalization scales; for the latter, *uniform* distribution.

Systematic error

The systematic error, related to the limited intrinsic accuracy of the method of sum rules.

The band of results obtained from linear, quadratic, and cubic trial functions for $s_{\rm eff}(\tau)$, optimized by minimizing the deviation of the dual mass from the true mass may be regarded as a realistic estimate for the systematic uncertainty of the decay constant.

$$f_B^2 M_B^4 e^{-M_B^2\tau} = \int_{(m_b+m)^2}^{s_{\rm eff}(\tau)} ds\, e^{-s\tau}\rho_{\rm pert}(s,\mu) + \Pi_{\rm power}(\tau,\mu) \equiv \Pi_{\rm dual}(\tau, s_{\rm eff}(\tau)).$$

OPE

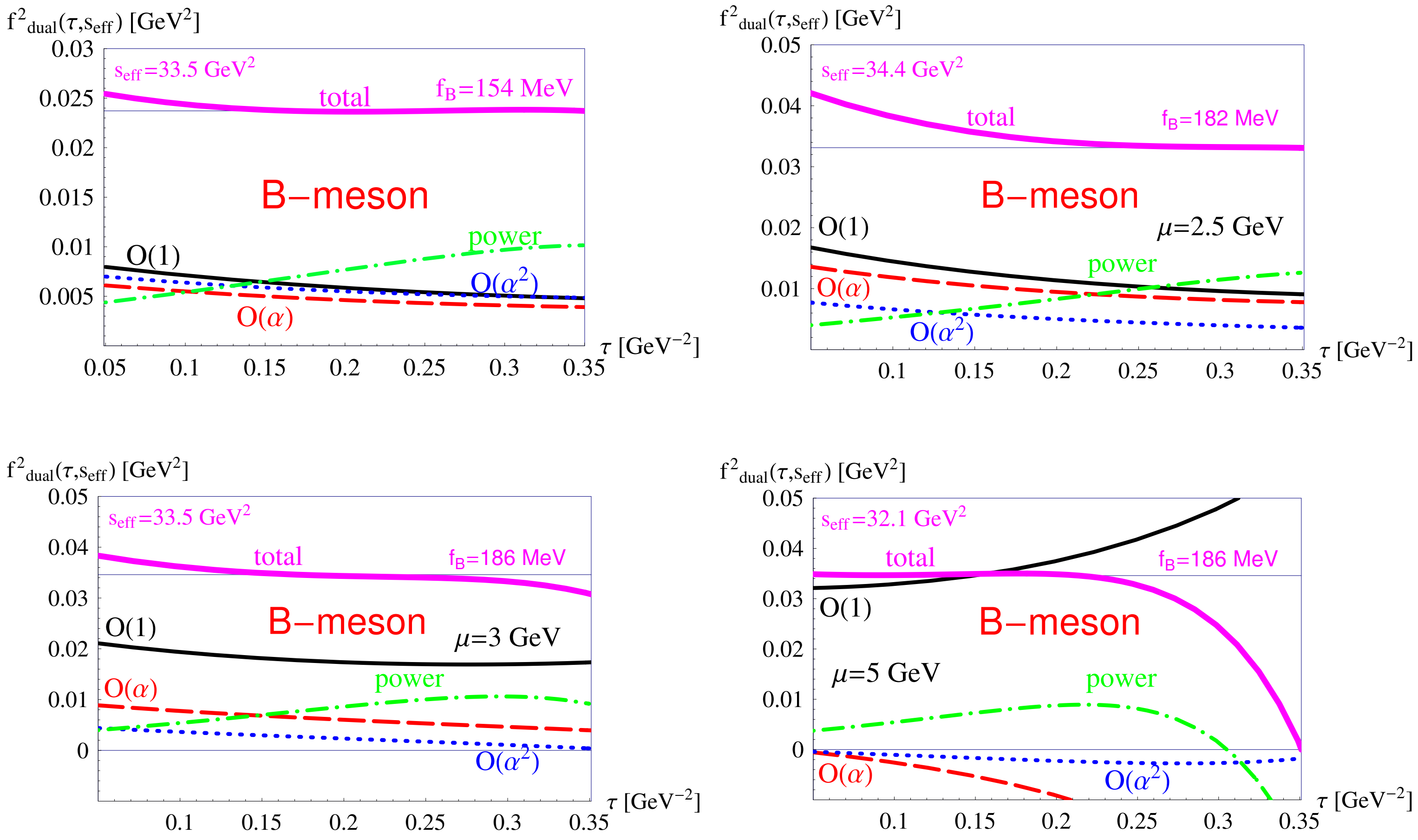
The best-known 3-loop calculations of the perturbative spectral density have been performed in form of an expansion in terms of the $\overline{\rm MS}$ strong coupling $\alpha_{\rm s}(\mu)$ and the pole mass M_b:

$$\rho_{\rm pert}(s,\mu) = \rho^{(0)}(s, M_b^2) + \frac{\alpha_{\rm s}(\mu)}{\pi}\rho^{(1)}(s, M_b^2) + \left(\frac{\alpha_{\rm s}(\mu)}{\pi}\right)^2 \rho^{(2)}(s, M_b^2, \mu) + \cdots .$$

An alternative option is to reorganize the perturbative expansion in terms of the running $\overline{\rm MS}$ mass, $\overline{m}_b(\nu)$, by substituting M_b in the spectral densities $\rho^{(i)}(s, M_b^2)$ via its perturbative expansion in terms of the running mass $\overline{m}_b(\nu)$

$$M_b = \overline{m}_b(\nu)\left(1 + \frac{\alpha_s(\nu)}{\pi}\, r_1 + \left(\frac{\alpha_s(\nu)}{\pi}\right)^2 r_2 + \ldots\right).$$

(a) Pole-mass OPE for B (b) running-mass OPE at $\mu = 2.5$ GeV (c) at $\mu = 3$ GeV (d) at $\mu = 5$ GeV



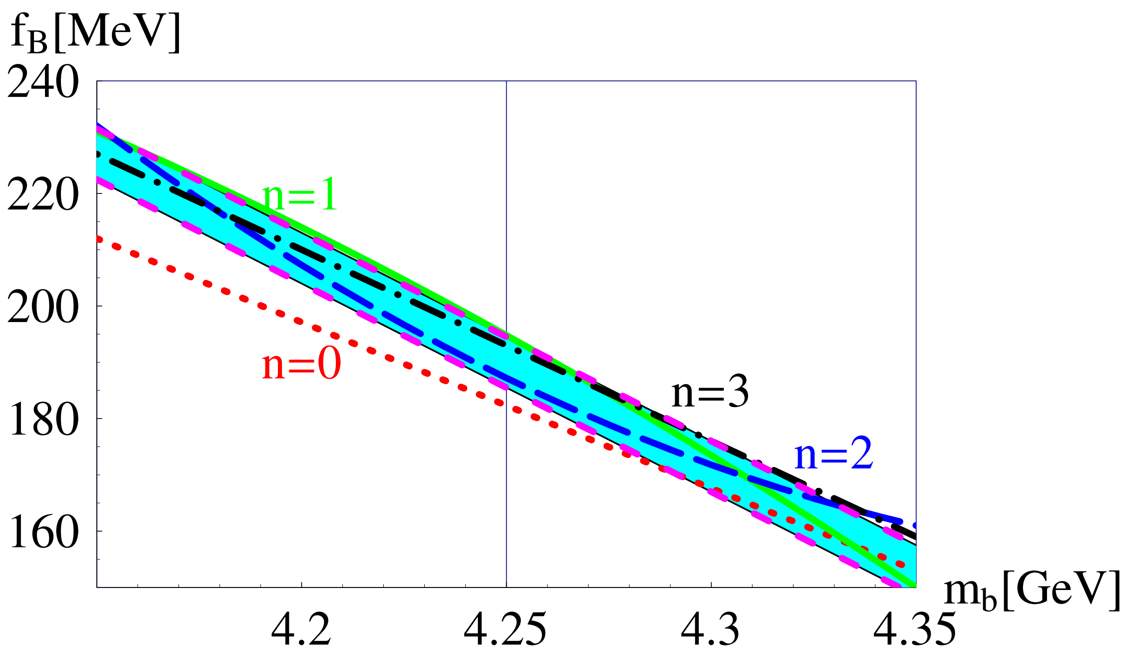
• **Result obtained on the basis of *pole-mass OPE* are not trustable: the pole-mass OPE shows no perturbative hierarchy. Reorganizing the OPE series in terms of the running mass improves the hierarchy; however induces an explicit scale-dependence.**

Correlation between $m_b(m_b)$ and f_B

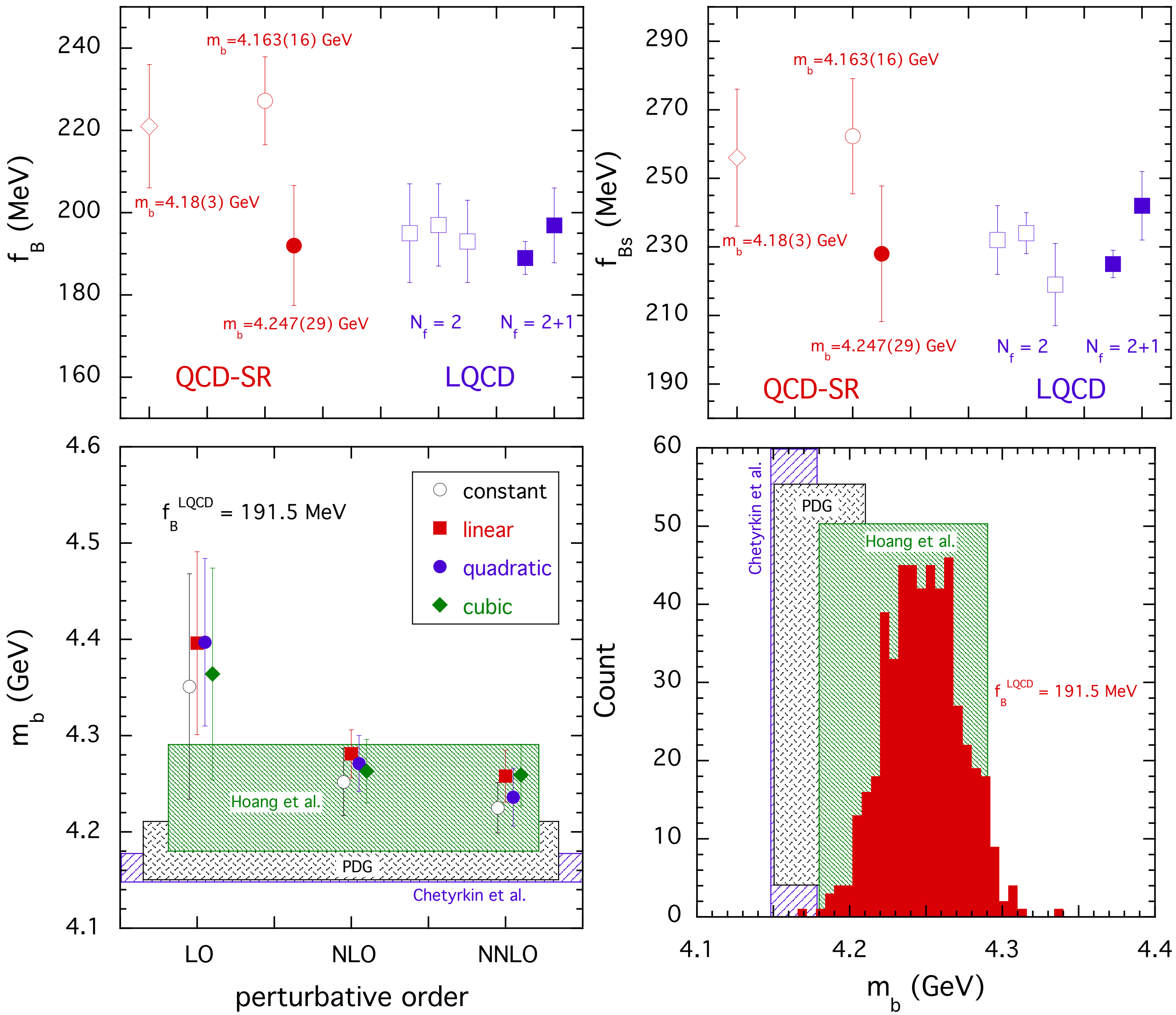
A strong correlation between m_b and the sum-rule result for f_B was observed

$$\frac{\delta f_B}{f_B} \approx -8\,\frac{\delta m_b}{m_b}.$$

Making use of the PDG $m_b = 4.18$ GeV leads to $f_B > 210$ MeV, in clear tention with the recent lattice QCD results for $f_B \sim 190$ MeV. Combining our sum-rule analysis with the latest results for f_B and f_{B_s} from lattice QCD yields



Extraction of $m_b(m_b)$ from lattice results for f_B

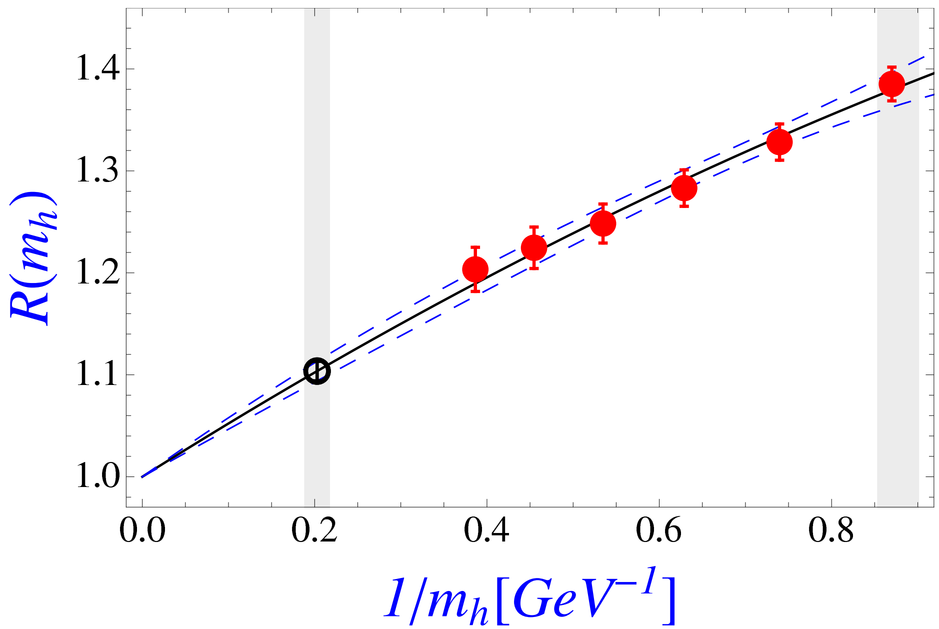


Decay constant ratio f_{B^*} / f_B

Expectations for f_{B^*}/f_B extrapolating the charm results

$$\frac{f_{V_Q}}{f_{P_Q}} = \left(1 - \frac{2\alpha_s(m_Q)}{3\pi}\right)[1 + \delta/m_Q]\,.$$

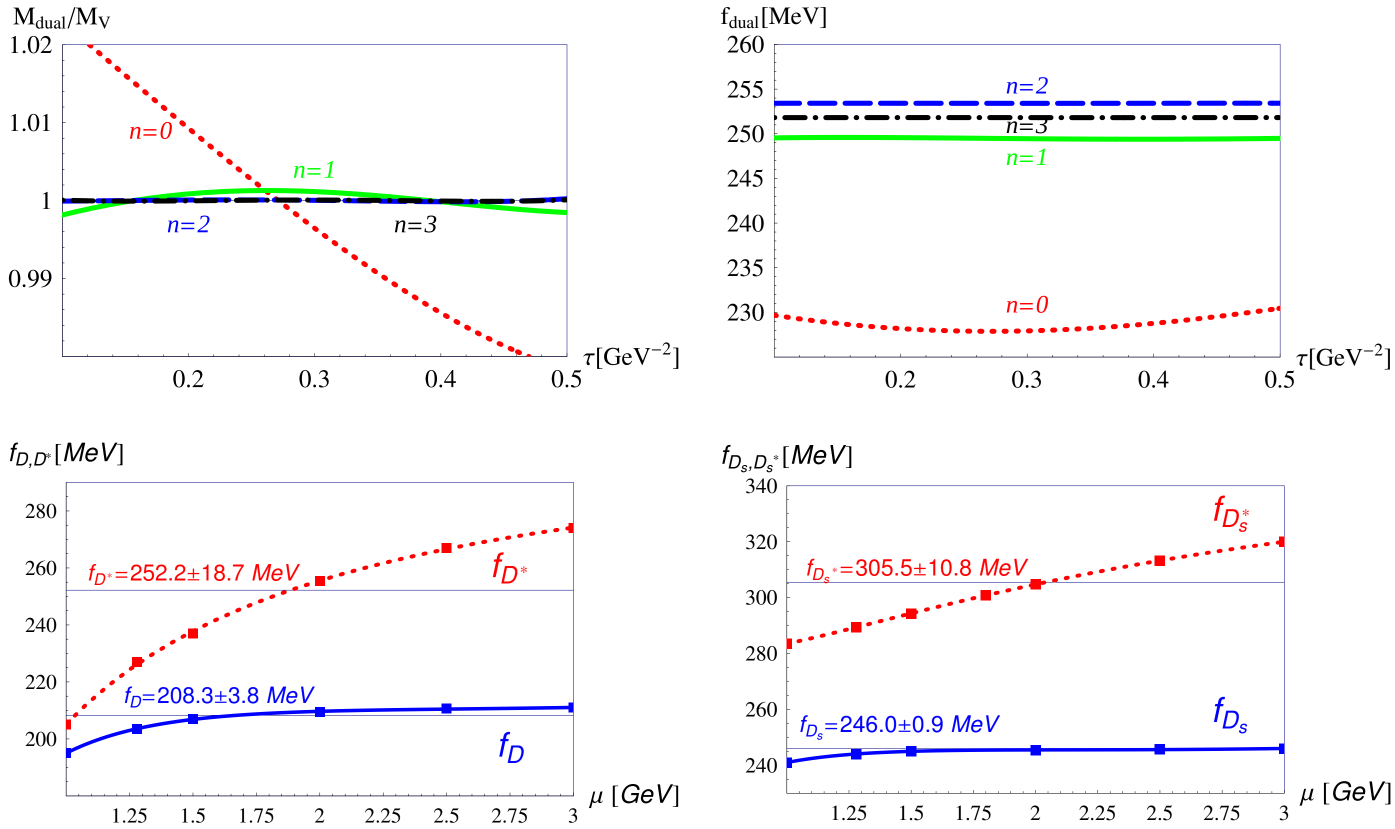
Extrapolating lattice results



$f_{B^*}/f_B = 1.042 \pm 0.014$ **(Becirevic et al)**

Charm sector: extraction for D^* and the ratio f_{D^*}/f_D

$m_c(m_c) = 1.279 \pm 0.013$ **GeV.**



$f_D = (208.3 \pm 7.3_{\mathrm{OPE}} \pm 5_{\mathrm{syst}})$ MeV

$f_{D_s} = (246.0 \pm 15.7_{\mathrm{OPE}} \pm 5_{\mathrm{syst}})$ MeV **(OPE error mainly due to $\langle \bar{s}s \rangle$)**

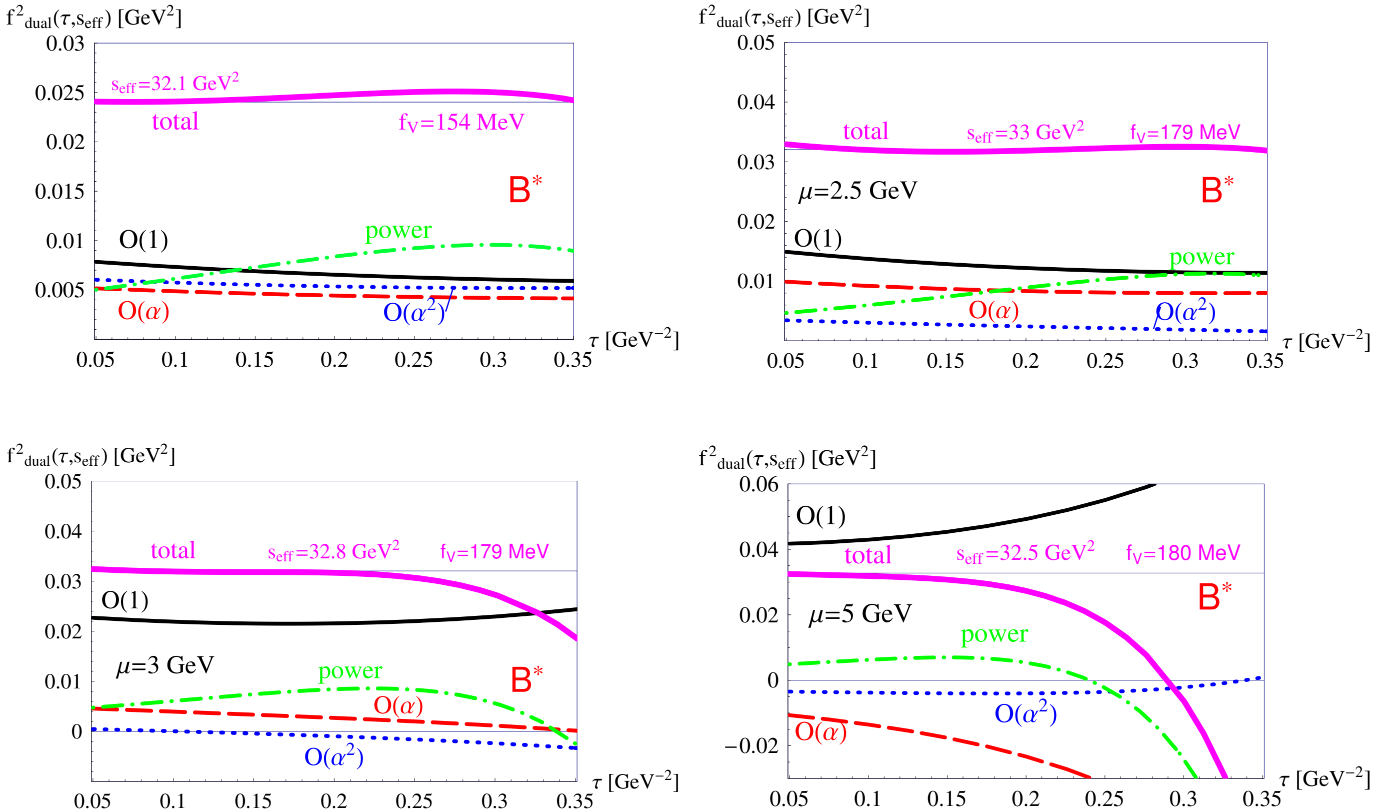
$f_{D^*} = (252.2 \pm 22.3_{\mathrm{OPE}} \pm 4_{\mathrm{syst}})$ MeV **(OPE error mainly due to $\langle \bar{s}s \rangle$ + scale-dependence)**

$f_{D_s^*} = (305.5 \pm 26.8_{\mathrm{OPE}} \pm 5_{\mathrm{syst}})$ MeV**.**

$f_{D^*}/f_D = 1.221 \pm 0.080_{\mathrm{OPE}} \pm 0.008_{\mathrm{syst}}$ **(lattice $f_{D^*}/f_D = 1.20 \pm 0.02$)**

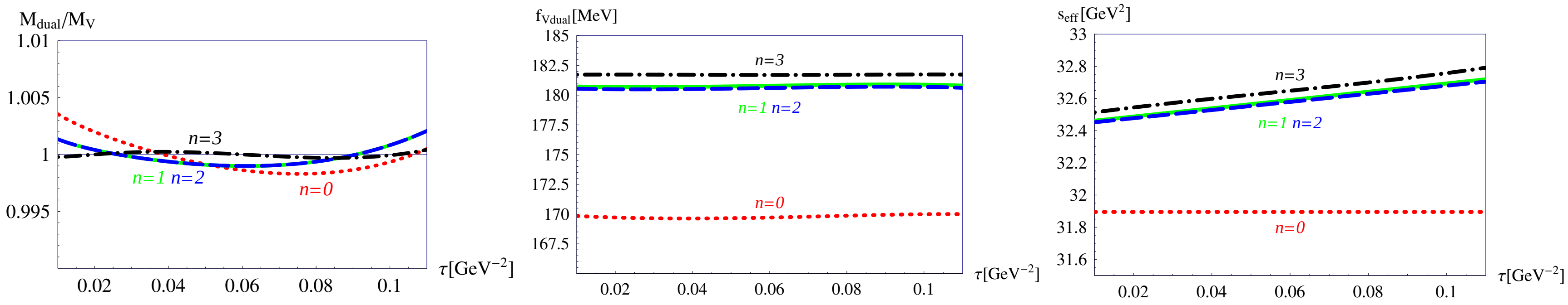
Beauty sector: vector correlator and decay constant of B^*-meson

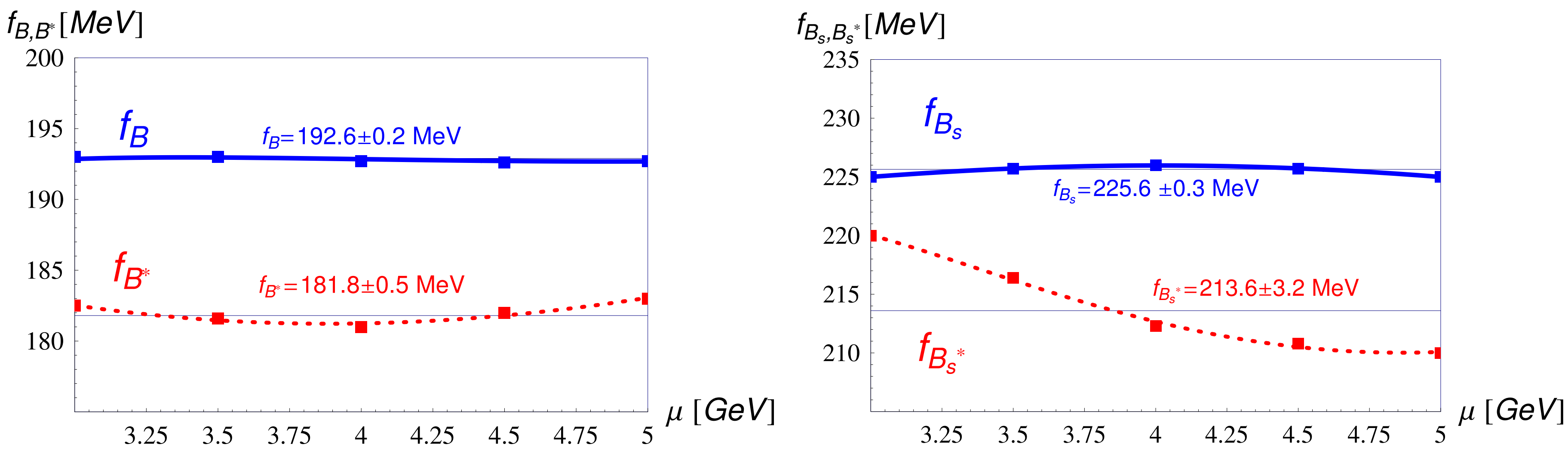
OPE via pole and $\overline{\rm MS}$ running mass at different scales:



1. **No perturbative hierarchy in terms of the pole mass.**
2. **Results for $\overline{\rm MS}$ mass depend on μ; playing with μ-choice one can acquire hierarchy.**
3. **Specific for B^*: τ-window choice should be correlated with μ to provide a reasonable stability.**

Extraction of decay constant of the $B*$ meson





Summary

• **Combining OPE results for heavy-heavy correlators with experimental data/lattice QCD, moment QCD sum rules report the most accurate value** $m_b = 4.163 \pm 0.016$ GeV

• **OPE results for heavy-light correlators complemented by duality concept/assumption lead to a strong correlation between $m_b(m_b)$ and f_B. Using the latest results for f_B and f_{B_s} from lattice QCD yields**

$$m_b = 4.247 \pm 0.027_{(OPE)} \pm 0.018_{(exp)} \pm 0.011_{syst}\ \mathrm{GeV}$$

• **Tension between the b-quark mass extracted from heavy-heavy and heavy-light correlation functions (OPE in danger?). Puzzle lies in the fact that for charm all results agree nicely.**

Truly lattice calculation of $m_b(m_b)$ is needed.

• **Borel QCD sum rules give rather unexpected but solid prediction (PRD91,2015)**

$$f_{B^*}/f_B = 0.944 \pm 0.011_{\mathrm{OPE}} \pm 0.018_{\mathrm{syst}}, \qquad f_{B_s^*}/f_{B_s} = 0.947 \pm 0.023_{\mathrm{OPE}} \pm 0.020_{\mathrm{syst}},$$

Fully agree with very recent lattice QCD (PRD91,2015)

$$f_{B^*}/f_B = 0.941 \pm 0.026, \qquad f_{B_s^*}/f_{B_s} = 0.953 \pm 0.023.$$

These results might suggest an unexpected structure of $1/m_Q$-expansion