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Similarity and Some Differences in Radion and Higgs Boson Production and Decay Processes Involving Off-Shell Fermions

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Stabilized Randall-Sundrum model as one of possible extensions of SM

- **brane world model**
- **one extra (5th) dimension**
- **two branes interacting with gravitation and the real scalar field**
- **the hierarchy problem is solved**

$$ds^2 = e^{-2A(y)}\eta_{\mu\nu}dx^{\mu}dx^{\nu} - dy^2 \equiv \gamma_{MN}(y)dx^M dx^N$$

$$\phi(x,y) = \hat{\phi}(y)$$

- **the SM fields are localized on one of the branes**
- **the stabilizing scalar field and gravitation**
- **<u>radion</u> is the lowest Kaluza-Klein mode of the 5D scalar field appearing from the fluctuations of the metric component corresponding to the extra dimension**

Radion couples to the trace of the energy–momentum tensor of SM

$$L = -\frac{r(x)}{\Lambda_r} T^{\mu}_{\mu},$$

$r(x)$ - the radion field, Λ_r - dimensional scale parameter,

$$T^{\mu}_{\mu} = \frac{\beta(g_s)}{2g_s} G^{ab}_{\rho\sigma} G^{\rho\sigma}_{ab} + \frac{\beta(e)}{2e} F_{\rho\sigma} F^{\rho\sigma} - m_Z^2 Z^{\mu} Z_{\mu} - 2m_W^2 W^{+}_{\mu} W^{-\mu} + \sum_f m_f \bar{f} f + \ldots$$

(anomaly terms are included)

The fermion part of Lagrangian for <u>on-shell</u> fermions is the same as for the Higgs boson (with the replacement $\Lambda_r \to \upsilon$):

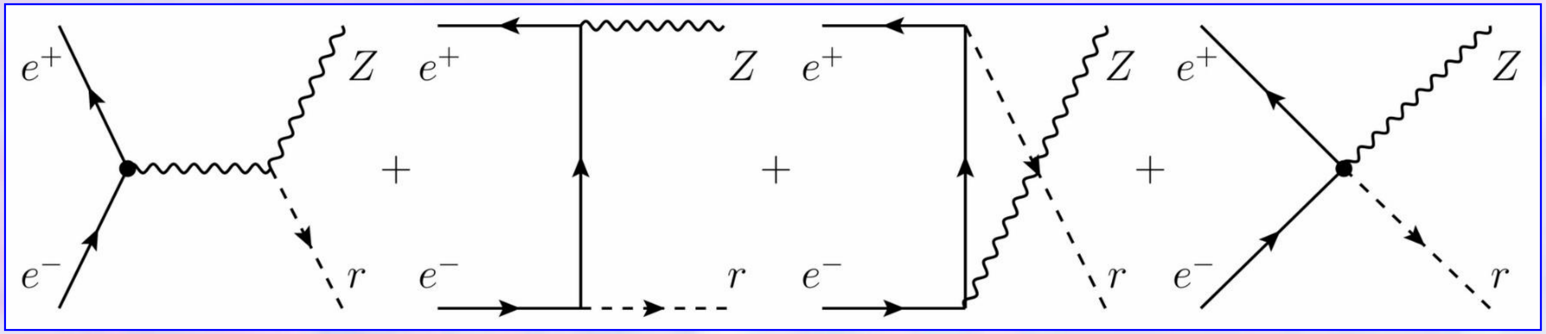
$$L = -\sum_f \frac{r}{\Lambda_r} m_f \bar{f} f$$

But for the case of <u>off-shell</u> fermions it is different:

$$L = -\sum_f \frac{r}{\Lambda_r} \left[\frac{3i}{2} \left((D_{\mu} \bar{f}) \gamma^{\mu} f - \bar{f} \gamma^{\mu} (D_{\mu} f) \right) + 4 m_f \bar{f} f \right] + \ldots$$

where D_{μ} are the SM covariant derivatives

The radion-strahlung as a simple example $\left(m_e \approx 0\right)$



$$M_1 = -2iC\,\bar{e}^+(p_2)\Gamma_\mu e^-(p_1)\frac{1}{p^2 - M_Z^2}M_Z^2\,\varepsilon^\mu(p_Z)r(p_r) \qquad C = \frac{1}{\Lambda_r}\frac{e}{2\sin\theta_W\cos\theta_W}$$

$$M_2 = -iC\,\bar{e}^+(p_2)\left[\frac{3}{2}\left(\not{k} + \not{p}_2\right)\right]\frac{\not{k}}{k^2}\Gamma_\mu e^-(p_1)\varepsilon^\mu(p_Z)r(p_r) = -\frac{3}{2}iC\,\bar{e}^+(p_2)\Gamma_\mu e^-(p_1)\varepsilon^\mu(p_Z)r(p_r)$$

$$M_3 = -iC\,\bar{e}^+(p_2)\Gamma_\mu\frac{\not{q}}{q^2}\left[\frac{3}{2}\left(\not{q} - \not{p}_1\right)\right]e^-(p_1)\varepsilon^\mu(p_Z)r(p_r) = -\frac{3}{2}iC\,\bar{e}^+(p_2)\Gamma_\mu e^-(p_1)\varepsilon^\mu(p_Z)r(p_r)$$

$$M_4 = +3iC\,\bar{e}^+(p_2)\Gamma_\mu e^-(p_1)\varepsilon^\mu(p_Z)r(p_r) = +3\,iC\,\bar{e}^+(p_2)\Gamma_\mu e^-(p_1)\varepsilon^\mu(p_Z)r(p_r)$$

Using the Dirac equation $\not{p}_j e(p_j) = 0$ and $\not{q}\frac{\not{q}}{q^2} = 1$ one can see

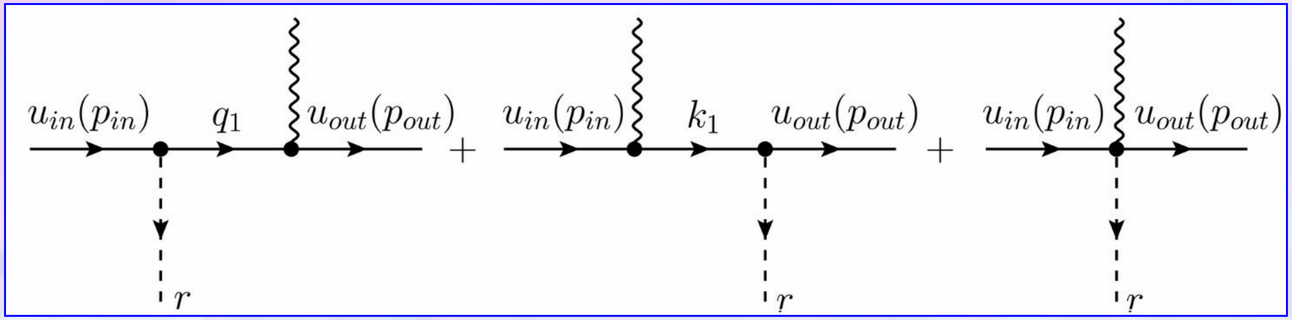
$$\frac{3}{2} + \frac{3}{2} - 3 = 0$$

$$\boxed{M_2 + M_3 + M_4 = 0}$$

i.e. $\left|M\right|^2 = \left|M_1\right|^2$ and the radion-strahlung process is absolutely the same as the Higgs-strahlung process (with the replacements $m_r \to m_h$ and $\Lambda_r \to \upsilon$)

The observed cancellation follows from the structure of the fermion current with the emission of the radion and a number of gauge bosons

1) One gauge boson:



Let's rewrite the fermion-radion vertex in the following way

$$\frac{i}{\Lambda_r}\left[\frac{3}{2}\left(\not{p}_{out}+\not{p}_{in}\right)-4m_f\right]=\frac{i}{\Lambda_r}\left[\frac{3}{2}\left(\not{p}_{out}-m_f\right)+\frac{3}{2}\left(\not{p}_{in}-m_f\right)-m_f\right]=$$

$$=\frac{i}{\Lambda_r}\left[\frac{3}{2}S^{-1}(p_{in})+\frac{3}{2}S^{-1}(p_{out})-m_f\right],$$ where $S(p)=\frac{\not{p}+m_f}{p^2-m_f^2}$ is the propagator.

$$D_1=-iC\bar{u}_{out}(p_{out})\Gamma_\mu S(k_1)\left[\frac{3}{2}\left(S^{-1}(k_1)+S^{-1}(p_{in})\right)-m_f\right]u_{in}(p_{in})$$

$$D_2=-iC\bar{u}_{out}(p_{out})\left[\frac{3}{2}\left(S^{-1}(p_{out})+S^{-1}(q_1)\right)-m_f\right]S(q_1)\Gamma_\mu u_{in}(p_{in})$$

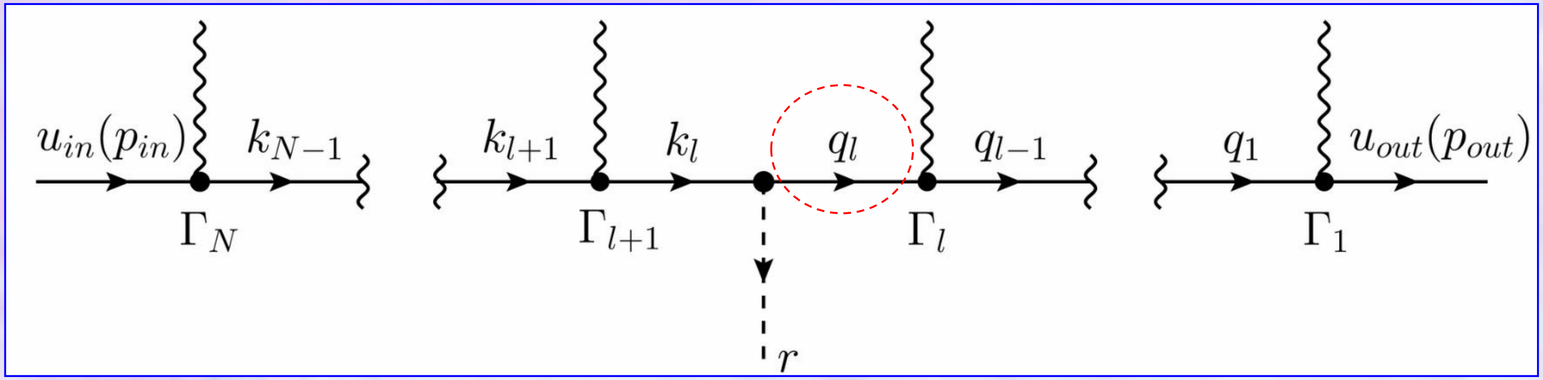
$$D_3=+i3C\bar{u}_{out}(p_{out})\Gamma_\mu u_{in}(p_{in})$$

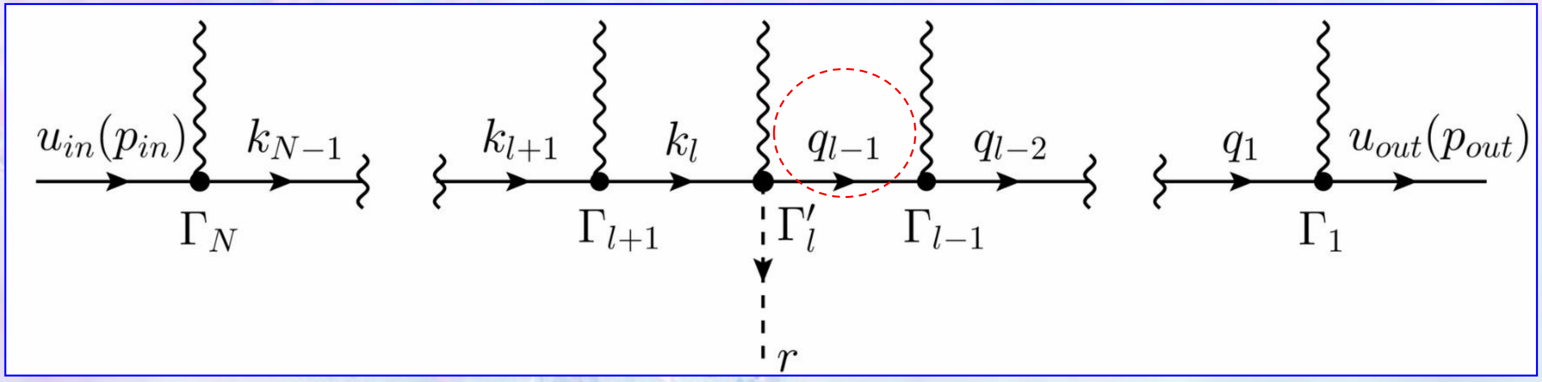
$$D_1+D_2+D_3\sim m_f$$

Higgs-like contribution only!

2) N gauge bosons:

$(N+1)$ vertices





N vertices

$$M_{N\,vector\ bosons} = M_0 + \sum_{l=1}^{N} \left(M_l + M_l' \right)$$

$$M_l \sim i^{2N+1}\bar{f}_{out}(p_{out})\left[\prod_{j=1}^{l}\Gamma^{j}_{\mu_j}S(q_j)\right]\left[-\frac{3}{2}\left(S^{-1}(q_l)+S^{-1}(k_l)\right)+m_{f_l}\right]\times$$

for $l=1,\ldots,N-1;$

$$\times\left[\prod_{j=l+1}^{N}S(k_{j-1})\Gamma^{j}_{\mu_j}\right]f_{in}(p_{in})$$

$$M_0 \sim i^{2N+1}\bar{f}_{out}(p_{out})\left[-\frac{3}{2}\left(S^{-1}(p_{out})+S^{-1}(k_0)\right)+m_{f_{out}}\right]\left[\prod_{j=1}^{N}S(k_{j-1})\Gamma^{j}_{\mu_j}\right]f_{in}(p_{in})$$

$$M_N \sim i^{2N+1}\bar{f}_{out}(p_{out})\left[\prod_{j=1}^{N}\Gamma^{j}_{\mu_j}S(q_j)\right]\left[-\frac{3}{2}\left(S^{-1}(q_N)+S^{-1}(p_{in})\right)+m_{f_{in}}\right]f_{in}(p_{in})$$

$$M_l' \sim i^{2N-1}\bar{f}_{out}(p_{out})\left[\prod_{j=1}^{l-1}\Gamma^{j}_{\mu_j}S(q_j)\right]\left[-3\Gamma^{l}_{\mu_l}\right]\left[\prod_{j=l+1}^{N}S(k_{j-1})\Gamma^{j}_{\mu_j}\right]f_{in}(p_{in})$$

for $l=2,\ldots,N-1;$

$$M_1' \sim i^{2N-1}\bar{f}_{out}(p_{out})\left[-3\Gamma^{1}_{\mu_1}\right]\left[\prod_{j=2}^{N}S(k_{j-1})\Gamma^{j}_{\mu_j}\right]f_{in}(p_{in})$$

$$M_N' \sim i^{2N-1}\bar{f}_{out}(p_{out})\left[\prod_{j=1}^{N-1}\Gamma^{j}_{\mu_j}S(q_j)\right]\left[-3\Gamma^{N}_{\mu_N}\right]f_{in}(p_{in})$$

i^{2N+1} and i^{2N-1}

$\Downarrow$

(-1)

Using the Dirac equation for *in* and *out* fermion states and $S^{-1}(q_j)\times S(q_j)=1$ one can get

$$M_l = M_l^H - \frac{1}{2}M_l' - \frac{1}{2}M_{l+1}'$$

$$M_0 = M_0^H - \frac{1}{2}M_1'$$

$$M_N = M_N^H - \frac{1}{2}M_N'$$

where M_l^H, M_0^H, M_N^H are the Higgs-like contributions which are proportional to the fermion masses:

$$M_l^H \sim i^{2N+1}\bar{f}_{out}(p_{out})\left[\prod_{j=1}^{l}\Gamma_{\mu_j}^j S(q_j)\right]\left[m_{f_l}\right]\left[\prod_{j=l+1}^{N}S(k_{j-1})\Gamma_{\mu_j}^j\right]f_{in}(p_{in})$$

$$M_0^H \sim i^{2N+1}\bar{f}_{out}(p_{out})\left[m_{f_{out}}\right]\left[\prod_{j=1}^{N}S(k_{j-1})\Gamma_{\mu_j}^j\right]f_{in}(p_{in})$$

$$M_N^H \sim i^{2N+1}\bar{f}_{out}(p_{out})\left[\prod_{j=1}^{N}\Gamma_{\mu_j}^j S(q_j)\right]\left[m_{f_{in}}\right]f_{in}(p_{in})$$

$$\boxed{\sum_{l=0}^{N}M_l + \sum_{l=1}^{N}M_l' = \sum_{l=0}^{N}M_l^H}$$

the sum of all the contributions leads to only the Higgs-like type of the contribution and all the other parts are canceled out.

Generalization to the loop case

The boson lines in the diagrams could correspond to real particles or virtual propagators in the loops. In the case of a fermion loop there is an additional fermion propagator instead of the external spinors at the tree level.

$$M_l \sim i^{2N+1} Tr\left\{ \left[\prod_{j=1}^{l} \Gamma^j_{\mu_j} S(q_j) \right] \left[-\frac{3}{2}\left(S^{-1}(q_l) + S^{-1}(k_l) \right) + m_{f_l} \right] \left[\prod_{j=l+1}^{N} S(k_{j-1}) \Gamma^j_{\mu_j} \right] S(p) \right\}$$

for $l = 1, \ldots, N-1;$

$$M_N \sim i^{2N+1} Tr\left\{ \left[\prod_{j=1}^{N} \Gamma^j_{\mu_j} S(q_j) \right] \left[-\frac{3}{2}\left(S^{-1}(q_N) + S^{-1}(p_{in}) \right) + m \right] S(p) \right\}$$

$$M_l' \sim i^{2N-1} Tr\left\{ \left[\prod_{j=1}^{l-1} \Gamma^j_{\mu_j} S(q_j) \right] \left[-3\Gamma^l_{\mu_l} \right] \left[\prod_{j=l+1}^{N} S(k_{j-1}) \Gamma^j_{\mu_j} \right] S(p) \right\}$$

for $l = 2, \ldots, N-1;$

$$M_1' \sim i^{2N-1} Tr\left\{ \left[-3\Gamma^1_{\mu_1} \right] \left[\prod_{j=2}^{N} S(k_{j-1}) \Gamma^j_{\mu_j} \right] S(p) \right\}$$

$$M_N' \sim i^{2N-1} Tr\left\{ \left[\prod_{j=1}^{N-1} \Gamma^j_{\mu_j} S(q_j) \right] \left[-3\Gamma^N_{\mu_N} \right] S(p) \right\}$$

In the same manner as in the previous paragraph one can get

$$M_l = M_l^H - \frac{1}{2} M_l' - \frac{1}{2} M_{l+1}'$$

$$M_N = M_N^H - \frac{1}{2} M_N' - \frac{1}{2} M_1'$$

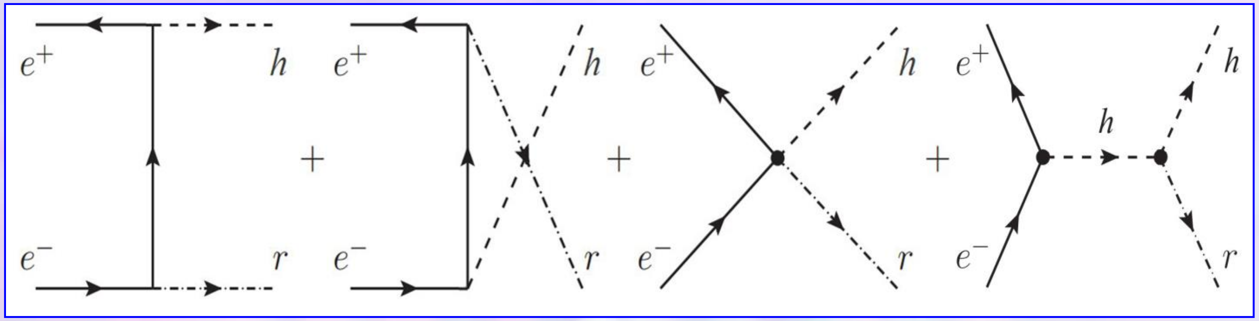
$$\sum_{l=1}^{N} M_l = \sum_{l=1}^{N-1} M_l + M_N = \sum_{l=1}^{N} M_l^H - \sum_{l=1}^{N} M_l'$$

$$\boxed{\sum_{l=1}^{N} M_l + \sum_{l=1}^{N} M_l' = \sum_{l=1}^{N} M_l^H}$$

which demonstrates that all the contributions except for the Higgs-like type are canceled out in the case of a fermion loop too.

We have shown that the additional fermion-radion terms in the interaction Lagrangian do not alter any production and decay properties of a <u>single</u> radion compared to those of the Higgs boson. Let's now consider the case of associated Higgs boson - radion production.

Associated Higgs boson - radion production



$$M_1 = \bar{v}^r(p_1)\, i\frac{-m_f}{v} h(p_h)\, i\frac{\not{k}' + m_f}{k'^2 - m_f{}^2}\frac{-i}{\Lambda_r}\left\{\frac{3}{2}\left[\left(-\not{k}' + m_f\right) - \left(\not{p}_2 - m_f\right)\right] + m_f\right\} r(p_r) u^s(p_2) =$$

$$= -i\bar{v}^r(p_1)\frac{1}{\Lambda_r}\frac{m_f}{v}\left\{-\mathbf{\frac{3}{2}} + m_f\frac{\not{k}' + m_f}{k'^2 - m_f{}^2}\right\} r(p_r) u^s(p_2) h(p_h)$$

$$M_2 = \bar{v}^r(p_1)\frac{-i}{\Lambda_r}\left\{\frac{3}{2}\left[\left(\not{p}_1 + m_f\right) - \left(\not{k} - m_f\right)\right] + m_f\right\} r(p_r)\, i\frac{\not{k} + m_f}{k^2 - m_f{}^2}\, i\frac{-m_f}{v} u^s(p_2) h(p_h) =$$

$$= -i\bar{v}^r(p_1)\frac{1}{\Lambda_r}\frac{m_f}{v}\left\{-\mathbf{\frac{3}{2}} + m_f\frac{\not{k} + m_f}{k^2 - m_f{}^2}\right\} r(p_r) u^s(p_2) h(p_h)$$

$$M_3 = \bar{v}^r(p_1)\frac{-i}{\Lambda_r}\frac{4m_f}{v} h(p_h) r(p_r) u^s(p_2) = -i\bar{v}^r(p_1)\frac{1}{\Lambda_r}\frac{m_f}{v}\left\{\quad \mathbf{4} \quad\right\} r(p_r) u^s(p_2) h(p_h)$$

$$M_4 = \bar{v}^r(p_1)\frac{-im_f}{v} u^s(p_2)\frac{i}{k_h{}^2 - m_h^2}\frac{-i}{\Lambda_r}\left(-2k_h p_h + 4m_h^2\right) r(p_r) h(p_h)$$

Using the simple kinematics computation

$$k_h = p_h + p_r \Rightarrow p_h = k_h - p_r$$

$$2k_h p_h = 2(p_h + p_r)p_h = (p_h + p_r)^2 + p^2{}_h - p^2{}_r = k^2{}_h + p^2{}_h - p^2{}_r = k^2{}_h + m_h^2 - m_r^2$$

one can get $\frac{-(2k_h p_h) + 4m_h^2}{k_h{}^2 - m_h^2} = \frac{-k^2{}_h - m_h^2 + m_r^2 + 4m_h^2}{k_h{}^2 - m_h^2} = -1 + \frac{m_r^2 + 2m_h^2}{k_h{}^2 - m_h^2}$

and rewrite M_4 as follows

$$M_4 = -i\frac{m_f}{\Lambda_r v}\bar{v}^r r(p_r)h(p_h)(p_1)\left\{-1 + \frac{m_r^2 + 2m_h^2}{k_h{}^2 - m_h^2}\right\}u^s(p_2)$$

So the sum of the amplitudes yields

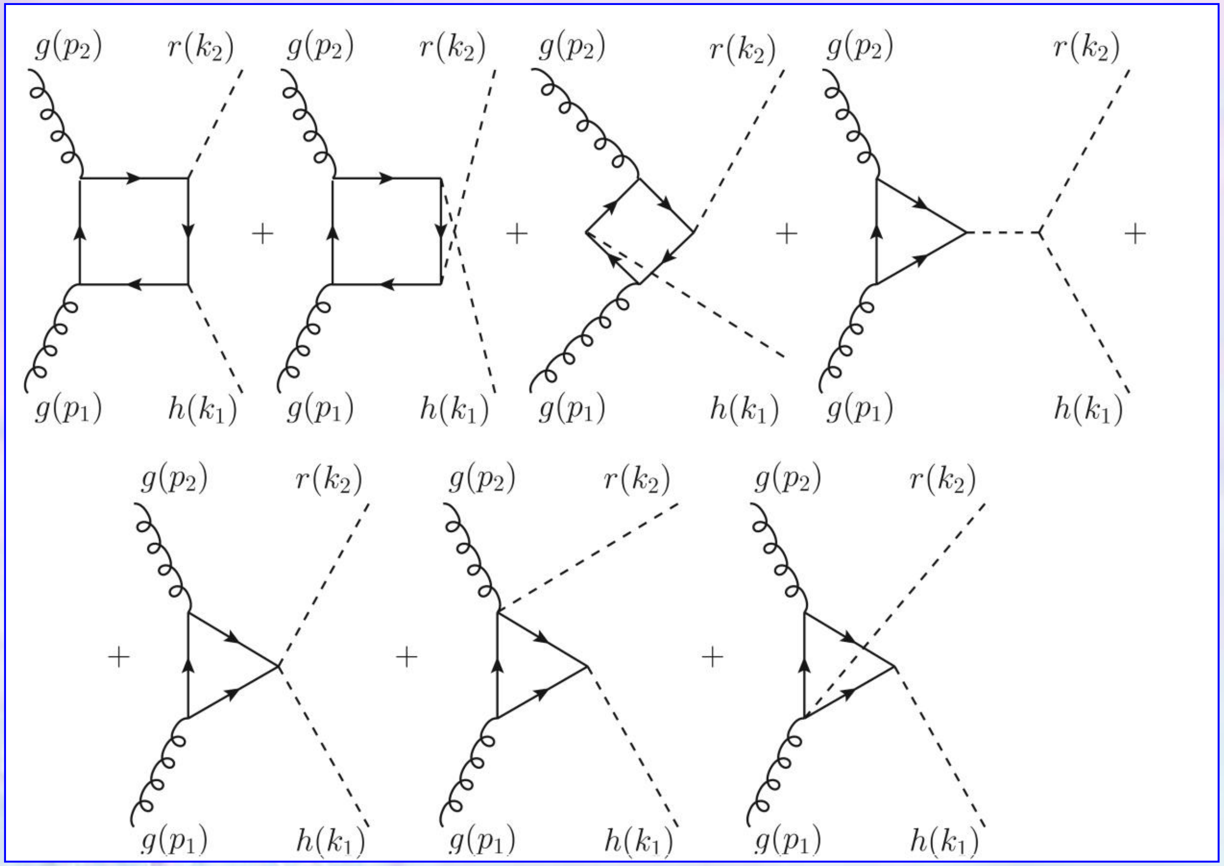
$$M_{rh} = -i\frac{m_f}{\Lambda_r v} r(p_r)h(p_h)\bar{v}^r(p_1)\left\{m_f\frac{\not{k} + m_f}{k^2 - m_f{}^2} + m_f\frac{\not{k}' + m_f}{k'^2 - m_f{}^2} - \frac{3}{2} - \frac{3}{2} + 4 - 1 + \frac{m_r^2 + 2m_h^2}{k_h{}^2 - m_h^2}\right\}u^s(p_2)$$

Comparing this with the result for the same process with double Higgs production

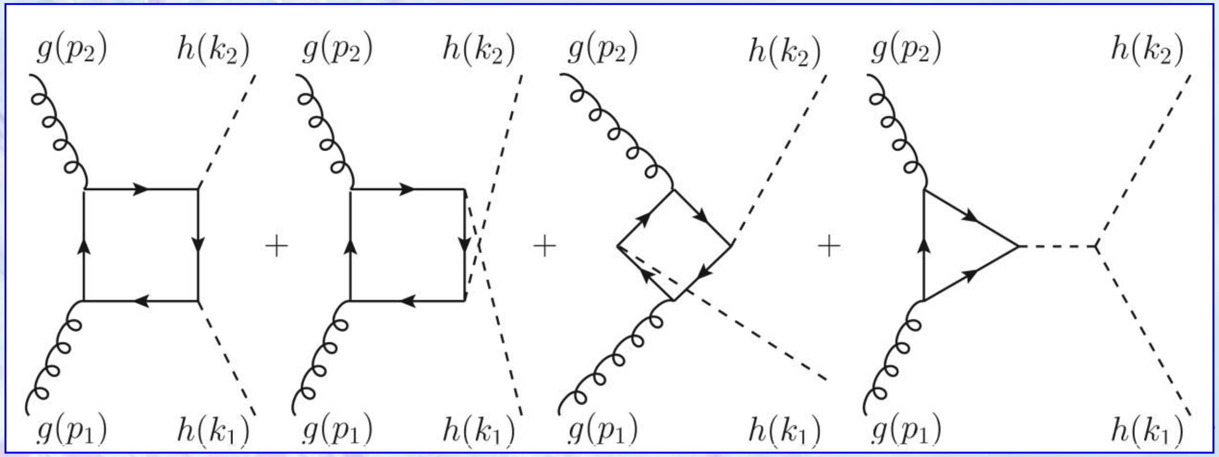
$$M_{hh} = -i\frac{m_f}{v^2}h(p_{h1})h(p_{h2})\bar{v}^r(p_1)\left\{m_f\frac{\not{k} + m_f}{k^2 - m_f{}^2} + m_f\frac{\not{k}' + m_f}{k'^2 - m_f{}^2} + \frac{3m_h^2}{k_h{}^2 - m_h^2}\right\}u^s(p_2)$$

one can see the similarity (with the replacements $m_r \to m_h$ and $\Lambda_r \to \upsilon$)

$gg \to rh$



$gg \to hh$

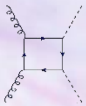


All the amplitudes have the following structure

$$M_i = \frac{g_c^2}{v^2}\varepsilon(p_1)_\mu \varepsilon(p_2)_\nu h(k_1)h(k_2)\int \frac{d^d l}{(2\pi)^d} X_i^{\mu\nu}\left(p_1, p_2, k_1, k_2\right)$$ - for $gg \to hh$ $\left(i=\overline{1,4}\right)$

$$M_i = \frac{g_c^2}{v\Lambda_r}\varepsilon(p_1)_\mu \varepsilon(p_2)_\nu h(k_1)r(k_2)\int \frac{d^d l}{(2\pi)^d} X_i^{\mu\nu}\left(p_1, p_2, k_1, k_2\right)$$ - for $gg \to rh$ $\left(i=\overline{1,7}\right)$

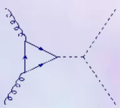
where



$$X_1^{\mu\nu} \equiv Sp\left[\gamma^\mu S_1^{-1}\gamma^\nu S_2^{-1}\Gamma_{2,3}S_3^{-1}(-m)S_4^{-1}\right]$$

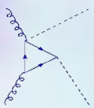
$$X_2^{\mu\nu} \equiv Sp\left[\gamma^\mu S_1^{-1}\gamma^\nu S_2^{-1}(-m)S_5^{-1}\Gamma_{5,4}S_4^{-1}\right]$$

$$X_3^{\mu\nu} \equiv Sp\left[\gamma^\mu S_1^{-1}(-m)S_6^{-1}\gamma^\nu S_5^{-1}\Gamma_{5,4}S_4^{-1}\right]$$



$$X_4^{\mu\nu} \equiv Sp\left[\gamma^\mu S_1^{-1}\gamma^\nu S_2^{-1}(-m)S_4^{-1}\right]D^{-1}\Gamma'$$

$$X_5^{\mu\nu} \equiv Sp\left[\gamma^\mu S_1^{-1}\gamma^\nu S_2^{-1}(+4m)S_4^{-1}\right]$$



$$X_6^{\mu\nu} \equiv Sp\left[\gamma^\mu S_1^{-1}(-3\gamma^\nu)S_3^{-1}(-m)S_4^{-1}\right]$$

$$X_7^{\mu\nu} \equiv Sp\left[(-3\gamma^\mu)S_1^{-1}\gamma^\nu S_2^{-1}(-m)S_5^{-1}\right]$$

$$\Gamma_{jl} = \begin{cases} -m \\ \frac{3}{2}S_j + \frac{3}{2}S_l - m \end{cases}$$

$$q_1 = 0$$
$$q_2 = -p_2$$
$$q_3 = -p_2 + k_2$$
$$q_4 = -p_2 + k_2 + k_1$$
$$q_5 = -p_2 + k_1$$
$$q_6 = k_1$$

$$\boxed{S_j = (\not{l} - \not{q}_j) - m}$$

$$D = (k_1 + k_2)^2 - m^2$$

After simple transformations one can get

$$X_1{}^{\mu\nu} = m^2 Sp\left[\gamma^\mu S_1{}^{-1}\gamma^\nu S_2{}^{-1}S_3{}^{-1}S_4{}^{-1}\right] - \frac{3}{8}X_5{}^{\mu\nu} - \frac{1}{2}X_6{}^{\mu\nu}$$

$$X_2{}^{\mu\nu} = m^2 Sp\left[\gamma^\mu S_1{}^{-1}\gamma^\nu S_2{}^{-1}S_5{}^{-1}S_4{}^{-1}\right] - \frac{3}{8}X_5{}^{\mu\nu} - \frac{1}{2}X_7{}^{\mu\nu}$$

$$X_3{}^{\mu\nu} = m^2 Sp\left[\gamma^\mu S_1{}^{-1}S_6{}^{-1}\gamma^\nu S_5{}^{-1}S_4{}^{-1}\right] - \frac{1}{2}X_6{}^{\mu\nu} - \frac{1}{2}X_7{}^{\mu\nu}$$

So the sum comes up to

$$X_1{}^{\mu\nu} + X_2{}^{\mu\nu} + X_3{}^{\mu\nu} + X_5{}^{\mu\nu} + X_6{}^{\mu\nu} + X_7{}^{\mu\nu} = \frac{1}{4}X_5{}^{\mu\nu} +$$

$$+ m^2 Sp\left[\gamma^\mu S_1{}^{-1}\gamma^\nu S_2{}^{-1}S_3{}^{-1}S_4{}^{-1}\right] + m^2 Sp\left[\gamma^\mu S_1{}^{-1}\gamma^\nu S_2{}^{-1}S_5{}^{-1}S_4{}^{-1}\right] + m^2 Sp\left[\gamma^\mu S_1{}^{-1}S_6{}^{-1}\gamma^\nu S_5{}^{-1}S_4{}^{-1}\right]$$

Next one can rewrite the h^3-vertex in the following way

$$\Gamma' = 2\left\{(k_1+k_2)_\mu k_1{}^\mu - 2m_H^2\right\} = (k_1+k_2)^2 + k_1{}^2 - k_2{}^2 - 4m_H^2 = \left[(k_1+k_2)^2 - m_H^2\right] - m_r{}^2 - 2m_H^2$$

and multiply it with the reverse propagator $D^{-1}\Gamma' = 1 - \dfrac{m_r{}^2 + 2m_H{}^2}{(k_1+k_2)^2 - m_H^2}$

As a result one has

$$X_4{}^{\mu\nu} = -\frac{1}{4}X_5{}^{\mu\nu} + mSp\left[\gamma^\mu S_1{}^{-1}\gamma^\nu S_2{}^{-1}S_4{}^{-1}\right]\frac{m_r{}^2 + 2m_H{}^2}{(k_1+k_2)^2 - m_H^2}$$

Finally the sum of all $X_i^{\mu\nu}$ for ***gg→rh*** yields

$$\sum_{i=1}^{7} X_i^{\mu\nu} =$$

$$= m^2 Sp\left[\gamma^\mu S_1^{-1}\gamma^\nu S_2^{-1}S_3^{-1}S_4^{-1}\right] + m^2 Sp\left[\gamma^\mu S_1^{-1}\gamma^\nu S_2^{-1}S_5^{-1}S_4^{-1}\right] + m^2 Sp\left[\gamma^\mu S_1^{-1}S_6^{-1}\gamma^\nu S_5^{-1}S_4^{-1}\right] +$$

$$+ mSp\left[\gamma^\mu S_1^{-1}\gamma^\nu S_2^{-1}S_4^{-1}\right]\frac{m_r^{\ 2} + 2m_H^{\ 2}}{(k_1+k_2)^2 - m_H^2}$$

The same expression for ***gg→hh*** is

$$m^2 Sp\left[\gamma^\mu S_1^{-1}\gamma^\nu S_2^{-1}S_3^{-1}S_4^{-1}\right] + m^2 Sp\left[\gamma^\mu S_1^{-1}\gamma^\nu S_2^{-1}S_5^{-1}S_4^{-1}\right] + m^2 Sp\left[\gamma^\mu S_1^{-1}S_6^{-1}\gamma^\nu S_5^{-1}S_4^{-1}\right] +$$

$$+ mSp\left[\gamma^\mu S_1^{-1}\gamma^\nu S_2^{-1}S_4^{-1}\right]\frac{3m_H^{\ 2}}{(k_1+k_2)^2 - m_H^2}$$

Thus we get that these two processes coincide up to the constants (masses, vacuum expectation values) in the case of the anomalies being ignored.

Conclusion

We have shown several examples of the radion-Higgs similarity in single and associated production processes.

The same result can follow from the model without radion but with the <u>modified</u> factor in the Higgs backreaction.
In the case of $m_r = m_H$ this model can't be distinguished from SM.

$$L = -\frac{1}{2}\left(\partial_\mu h\right)\left(\partial^\mu h\right) - \frac{1}{2} m_H^2 h^2 - \frac{\xi}{2}\frac{m_H^2}{v} h^3 - \frac{1}{8}\frac{m_H^2}{v^2} h^4$$

where $\xi = \begin{cases} 1 & \text{- for SM,} \\ 1 + \dfrac{m_r^2 - m_H^2}{3m_H^2} & \end{cases}$