Flavour puzzle or
Why Neutrinos Are Different?

Maxim Libanov
INR RAS, Moscow

In collaboration with
J.-M. Frere (ULB), F.-S. Ling (ULB),
E. Nugaev (INR), S. Troitsky (INR)

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The Flavour Puzzle In A Nutshell

✓ Why three families in the SM?
  - Hierarchical masses + small mixing angles

✓ Why massive neutrinos?
  - Tiny masses + two large mixing angles

✓ Why very suppressed FCNC?
  - Strong limits on a TeV scale extension of the SM

Proposed solution:

A model of family replication in 6D
3 Families In 4D From 1 Family In 6D

- Our 3D World is a core of Abrikosov-Nielsen-Olesen vortex:
  \[ U_g(1) \text{ gauge field } A + \text{scalar } \Phi \]

- There is only single vector-like fermionic generation in 6D

- Chiral fermionic zero modes are trapped in the core due to specific interaction with the \( A \) and \( \Phi \). Specific choice of \( U_g(1) \) fermionic gauge charges ⇒
  
  Number of zero modes = 3

- Zero modes \( \iff \) 4D fermionc families
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Hierarchical Dirac Masses

3 zero modes have different shapes, and different angular momenta $n = 0, 1, 2$

\[ \hat{J}\psi_n \equiv - \left( i\dot{\varphi} + 3 \frac{1 + \Gamma_7}{2} \right) \psi_n = n\psi_n \]

\[ \psi_n(r \to 0) \sim r^n \]

\[ m_{nm} \propto \int_0^{2\pi} d\varphi \int_0^R dr \bar{\psi}_n \psi_m H_X \text{or } \Phi \sim \sigma^{2n(-1)}\delta_{nm}(\pm 1) \]

- $\sigma$ depends on the parameters of the model. Hierarchy arises at $\sigma \sim 0.1$

\[ m_2 : m_1 : m_0 \sim \sigma^4 : \sigma^2 : 1 \sim 10^{-4} : 10^{-2} : 1 \]

\[ U^{CKM} \sim \begin{pmatrix} 1 & \sigma & \sigma^4 \\ \sigma & 1 & \sigma \\ \sigma^2 & \sigma & 1 \end{pmatrix} \]

Generation number $\Leftrightarrow$ Angular momentum

✓ The scheme is very constrained, as the profiles are dictated by the equations
Neutrinos masses. Why is it different?

- N -- additional neutral spinor

  ⇒ Free propagating in the extra dim (up to dist. \( R \sim (10^{-10} \text{TeV})^{-1} \)).
  ⇒ Majorano-like 6D mass term

\[
\frac{M}{2} \bar{N}cN + \text{h.c.}
\]

  ⇒ Kaluza-Klein tower in 4D (no zero mode)
  ⇒ Effective 6D couplings with leptons allowed by symmetries

\[
\sum_{S_+} H_S \bar{L} \frac{1 + \Gamma_7}{2} N + \sum_{S_-} H_S \bar{L} \frac{1 - \Gamma_7}{2} N + \text{h.c.}
\]

- \( S_+ = X^*, \Phi^*, X^* \Phi, \ldots \)
- \( S_- = X^2, X \Phi, \Phi^2, \ldots \)

⇒ Non-zero windings ⇒ more composite structure of the mass matrix

⇒ 4D Majorano neutrinos masses are generated by See-saw mechanism

Why Neutrinos Are Different?  Maxim Libanov
Neutrinos:

\[ m_{mn}^\nu \sim \int_0^{2\pi} d\varphi \int_0^R dr F(r, \varphi) [cL \propto LL] \]

\[ \sim \int_0^{2\pi} d\varphi e^{i(4-n-m+...)\varphi} \sim \delta_{4+\ldots,m+n} \]

\[ \sim \begin{pmatrix} \cdot & \cdot & 1 \\ \cdot & \sigma^2 & \cdot \\ 1 & \cdot & \cdot \end{pmatrix} \]

\[ U_v^\dagger m_v U_v^* \sim \text{diag}(-m, m, m\sigma^2) \]

\[ U_v \sim \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & \sigma \\ \sigma & \sigma & 1 \\ -1/\sqrt{2} & 1/\sqrt{2} & \sigma \end{pmatrix} \]

Charged fermions:

\[ m_{mn}^{\text{charged}} \sim \int_0^{2\pi} d\varphi \int_0^R dr F(r, \varphi) [\bar{\psi} \psi \propto \psi^* \psi] \]

\[ \sim \int_0^{2\pi} d\varphi e^{i(n-m+...)\varphi} \sim \delta_{n,m-} \]

\[ \sim \begin{pmatrix} \sigma^4 & \cdot & \cdot \\ \cdot & \sigma^2 & \cdot \\ \cdot & \cdot & 1 \end{pmatrix} \]

\[ m_{\text{charged}}^{\text{diag}} \sim \text{diag}(\mu\sigma^4, \mu\sigma^2, \mu) \]

\[ U_{\text{CKM}} \sim \begin{pmatrix} 1 & \sigma & \sigma^4 \\ \sigma & 1 & \sigma \\ \sigma^2 & \sigma & 1 \end{pmatrix} \]
Consequences of this structure

- Inverted hierarchy:
  \[ \Delta m_{12}^2 = \Delta m_{13}^2 \]
  \[ \frac{\Delta m_{12}^2}{\Delta m_{13}^2} \sim \sigma^2 \]

- Pseudo-Dirac structure \( \Rightarrow \) O\(\nu\)\(\beta\)\(\beta\) decay

- Partial suppression

\[ |\langle m_{\beta\beta} \rangle| \sim \frac{1}{3} \sqrt{\Delta m_{12}^2} \]
Semi-realistic numerical example

\[ m_v^{\text{diag}} = \begin{pmatrix} -50.03 & 0 & 0 \\ 0 & 50.79 & 0 \\ 0 & 0 & 0.7089 \end{pmatrix} \text{[meV]}, \quad U_{\text{MNS}} = \begin{pmatrix} 0.808 & 0.559 & 0.186 \\ -0.286 & 0.660 & -0.693 \\ -0.514 & 0.502 & 0.696 \end{pmatrix} \]

\[ \Delta m^2_{12} = 7.63 \times 10^{-5} \text{eV}^2 \]
\[ \Delta m^2_{13} = 2.50 \times 10^{-3} \text{eV}^2 \]

\[ \tan^2 \theta_{12} = 0.471 \left( 0.47^{+0.14}_{-0.10} \right) \]
\[ \tan^2 \theta_{23} = 0.997 \left( 0.9^{+1.0}_{-0.4} \right) \]
\[ \sin^2 \theta_{13} = 3.46 \cdot 10^{-2} (\leq 0.036) \]

Consequence for $0\nu\beta\beta$ decay

\[ |\langle m_{\beta\beta} \rangle| = \left| \sum_i m_i U_{ei}^2 \right| = 17.0 \text{ meV} \]
Like in the UED, vector bosons can travel in the bulk of space. From the 4D point of view:

1 massless vector boson in 6D =

1 massless vector boson (zero mode) + $\text{KK tower of massive vector bosons } M_n \sim \frac{n}{R}$ ⇒ FCNC

+ $\text{KK tower of massive scalar bosons in 4D}$ ⇒ $\text{KK scalar modes do not interact with fermion zero modes}$
KK vector modes carry angular momentum = family number. In the absence of fermion mixings, family number is an exactly conserved quantity \( \Rightarrow \) processes with \( \Delta G = \Delta J \neq 0 \) are suppressed by mixing.

\[ \kappa^0_L \rightarrow \mu^\pm e^\mp \]
\[ \Delta G = \Delta J = 0 \]

\[ J_{Z'} = 1 \]
\[ J_{\mu e} = 1 \]
\[ J_{d\bar{s}} = 1 \]
\[ J_{\bar{s}} \]

\[ \mu \rightarrow ee\bar{e} \]
\[ \Delta G = \Delta J = 1 \]

\[ J_{e\bar{e}} = 0 \]
\[ J_{\bar{e}} \]

\[ J_{Z'} = 0 \]
\[ J_{e \bar{e}} = 2 \]
\[ \kappa^2 \sim \frac{\kappa^2}{M_{Z'}^2} \]

✓ \( \kappa = 1 \) for the particular model, but may be \( \ll 1 \) for extensions
Rare processes:

- $\Delta G = 0$: $K_L^0 \rightarrow \mu e, K^+ \rightarrow \pi^+ \mu^+ e^-$
- $\Delta G = 1$: $\mu \rightarrow eee, \mu e$-conversion, $\mu \rightarrow ey$
- $\Delta G = 2$: mass difference $K_L - K_S$, CP-violation

Bound on $M_Z' \gtrsim \kappa \cdot 100 \cdot \text{TeV}$

$NB$: A clear signature of the model would be an observation $K_L^0 \rightarrow \mu e$ without observation other FCNC-processes at the same precision level.
Search at LHC

- Search for an «ordinary» massive $Z'(W', g', \nu')$
- Search for $pp \rightarrow \mu^+\mu^- + \ldots$
- Search for $pp \rightarrow \mu^-\mu^+ + \ldots$ --- one order below due to quark content of protons
- Search for $pp \rightarrow \bar{t} + c + \ldots$ or $pp \rightarrow \bar{b} + s + \ldots$ --- expect a few 1000's events, but must consider background!

LHC thus has the potential (in a specific model) to beat even the very sensitive fixed target $K^0 \rightarrow \mu\nu$ limit!
Conclusions

- Family replication model in 6D: elegant solution to the flavour puzzle
  - Hierarchical Dirac masses + small mixing angles
  - Neutrinos are different: See-saw + Majorano-like mass for the bulk neutral fermion can fit neutrino data
  - Family/lepton number violating FCNC suppressed by small fermion mixings

- Predictions for neutrinos
  - Inverted hierarchy
  - Reactor angle $\sim 0.1$
  - Partially suppressed neutrinoless $\beta\beta$ decay

- Other predictions
  - $K \rightarrow \mu e$ will show up earlier than other FCNC-processes
  - Massive gauge bosons with mass $\sim$ TeV or higher
  - Search for $pp \rightarrow \mu^+e^-$ at LHC can beat fixed target
  - Constraint on B-E-H boson: should be LIGHT