Higgs field as the Goldberger-Wise field

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An extension of the Standard Model (SM) based on the Randall-Sundrum model with two branes is considered, where the SM fermion fields are supposed to be localized on a brane in a five-dimensional space-time. In order this SM extension be phenomenologically acceptable, the interbrane distance must be stabilized. In the present paper we consider the case, where the interbrane separation is stabilized by a five-dimensional two-component complex scalar field, which serves as the Higgs field on the brane where our world is supposed to be located. The corresponding five-dimensional gauge fields are also taken into account. The equations of motion for this model are found and an exact background solution is obtained for the metric and the five-dimensional Higgs field. The second variation Lagrangian for the fluctuations of the considered fields against the background is derived and the equations of motion for the fluctuations are found. The possible values of the model parameters are estimated, which give the correct value of the Higgs boson mass.

1 Introduction

The Randall-Sundrum model with two branes is a brane-world model, which can be considered as a possible SM extensions [1]. The model includes two branes with tension interacting with gravity in a five-dimensional space-time, the extra dimension forming the orbifold S^1/Z_2 and the branes being located at its fixed points. The metric of the background solution is not flat and has an exponential factor due to which the problem of the hierarchy of the gravitational interaction is solved. In the original formulation of the model the interbrane distance was a free parameter, and the fluctuations of this distance gave rise to a massless scalar mode called the radion. The radion would interact with matter on the branes with the same strength as gravity, and the presence of such a massless scalar field contradicts already the classical gravity. In order to solve this problem the Goldberger-Wise field was introduced into the model [2, 3]. It is a five-dimensional real scalar field with a potential in the bulk and additional potentials on the branes. The Goldberger-Wise field allows to stabilize the interbrane separation that leads to the radion gaining a mass. The stabilization is achieved due to the boundary conditions for the Goldberger-Wise field on the branes. The idea of this report is to stabilize the interbrane distance by a two-component complex scalar field propagating in the bulk and carrying the same representation $\underline{2}(\frac{1}{2})$ of the gauge group $SU(2) \times U(1)$ as the usual Higgs field. On the brane, where our world is supposed to be located, it will act as the Higgs field. The SM fermion fields, which are supposed to be localized on the brane, will get masses due to the interaction with the boundary value of this field. Thus, we introduce an object which can be called the five-dimensional Higgs field.

The question about the stabilization of the extra dimension size with the help of the five-dimensional Higgs field was previously raised in papers by L. Vecchi [4] and M. Geller et al. [5]. In the latter paper a perturbative solution for the Higgs field in the Randall-Sundrum background has been found, whereas the gauge

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fields have not been considered. However, since the gauge group $SU(2) \times U(1)$ acts in the five-dimensional space-time, the gauge invariance of the theory necessarily demands the presence of the corresponding multidimensional gauge fields in the model. In the present report we attempt to find an exact self-consistent background solution for the metric and the five-dimensional Higgs field and to take into account the presence of the five-dimensional gauge fields.

2 The background solution

Let us consider gravity interacting with two branes, a two-component complex scalar field $\phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$ and five-dimensional gauge fields A_M and B_M in the five-dimensional space-time $E = M_4 \times S^1/Z_2$. The field A_M corresponds to the $SU(2)$-component of gauge group $SU(2) \times U(1)$, and the field B_M corresponds to the $U(1)$-component. We denote the coordinates in E by $\{x^M\} \equiv \{x^\mu, y\}$, $M = 0,1,2,3,4$, where $\{x^\mu\}$, $\mu = 0,1,2,3$ – the coordinates in four-dimensional space-time, and $x^4 \equiv y$ parameterizing the extra dimension, $-L \le y \le L$. It forms the orbifold S^1/Z_2, which is realized as the circle of diameter $2L/\pi$ with the points y and $-y$ identified.

The action of the model can be written as

$$S = S_g + S_\phi + S_{gauge} + S_{brane+SM}, \tag{1}$$

where the gravitational action S_g is given by

$$S_g = 2M^3 \int d^4x \int\limits_{-L}^{L} dy R\sqrt{-g}, \tag{2}$$

action S_ϕ of the two-component complex scalar field

$$S_\phi = -M \int d^4x \int\limits_{-L}^{L} dy \left[(D_M\phi)^+ D^M\phi + V(\phi^+\phi) \right] \sqrt{-g}, \tag{3}$$

action S_{gauge} of the gauge fields

$$S_{gauge} = -\int d^4x \int\limits_{-L}^{L} dy \left[\frac{1}{4p^2} \overset{a}{A}_{MN} \overset{a}{A}{}^{MN} + \frac{1}{4q^2} B_{MN}B^{MN} \right] \sqrt{-g}, \tag{4}$$

and action $S_{brane+SM}$ of the branes and Standard Model is

$$S_{brane+SM} = -\int\limits_{y=0} d^4x\lambda_1 \left(\phi^+\phi\right) \sqrt{-\tilde{g}} + \int\limits_{y=L} d^4x \left[-\lambda_2 \left(\phi^+\phi\right) + L_{SM-HP}\left(\phi,\phi^+\right) \right] \sqrt{-\tilde{g}}. \tag{5}$$

Here V denotes the potential of the five-dimensional Higgs field in the bulk; $\lambda_{1,2}$ stand for the potentials of this field on the first and second branes; $g = \det g_{MN}$; $\tilde{g} = \det \tilde{g}_{\mu\nu}$, where $\tilde{g}_{\mu\nu}$ denotes the metric induced on the branes. The Lagrangian L_{SM-HP} is the Lagrangian of the SM without the Higgs potential. $M = \frac{1}{2}(4\pi\hat{G})^{-\frac{1}{3}}$ is the fundamental five-dimensional energy scale in this theory, $\hat{G}$ denoting the five-dimensional gravitational constant; p and q are the coupling constants of the gauge fields A_M and B_M, respectively. The signature of the metric g_{MN} is chosen to be $(-,+,+,+,+)$. The indices 1 and 2 label the branes. The SM fields are localized on the second brane, which has the coordinate $y = L$.

The standard ansatz for the metric, the scalar and gauge fields, which preserves the Poincar'e invariance in any four-dimensional subspace $y = const$, looks like

$$ds^2 = \gamma_{MN}dx^M dx^N = e^{-2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2, \qquad \phi(x,y) = \phi(y), \tag{6}$$

$$A_\mu(x,y) = 0, \quad A_4(x,y) = A_4(y), \qquad B_\mu(x,y) = 0, \quad B_4(x,y) = B_4(y). \tag{7}$$

If one substitutes this ansatz into the equations corresponding to action (1), one can get the following equations of motion:

$$\frac{1}{2}\left(\phi'^{+}\phi' + V + \frac{\lambda_1}{M}\delta(y) + \frac{\lambda_2}{M}\delta(y-L)\right) = 2M^2\left(3A'' - 6(A')^2\right), \tag{8}$$

$$12M^2(A')^2 + \frac{1}{2}\left(V - \phi'^{+}\phi'\right) = 0, \tag{9}$$

$$\frac{dV}{d\phi} + \frac{1}{M}\frac{d\lambda_1}{d\phi}\delta(y) + \frac{1}{M}\frac{d\lambda_2}{d\phi}\delta(y-L) = \phi''^{+} - 4A'\phi'^{+}, \tag{10}$$

$$\frac{dV}{d\phi^+} + \frac{1}{M}\frac{d\lambda_1}{d\phi^+}\delta(y) + \frac{1}{M}\frac{d\lambda_2}{d\phi^+}\delta(y-L) = \phi'' - 4A'\phi', \tag{11}$$

$$A_M = B_M = 0. \tag{12}$$

Here $' = \partial_4 \equiv \partial/\partial y$. We found that the gauge fields have no influence on the background solution.

Let us choose an ansatz for the potential V, the scalar field and the function A in the exponent in the form:

$$V = \frac{1}{4}\frac{dW}{d\phi}\frac{dW}{d\phi^+} - \frac{1}{24M^2}\left(W(\phi^+\phi)\right)^2, \qquad A'(y) = sign(y)\frac{1}{24M^2}W(\phi^+\phi) \tag{13}$$

$$\phi'(y) = sign(y)\frac{1}{2}\frac{dW}{d\phi^+}, \qquad \phi'^{+}(y) = sign(y)\frac{1}{2}\frac{dW}{d\phi}. \tag{14}$$

With this ansatz, the equations of motion are valid everywhere, except for the branes. In order that they are valid everywhere, one needs to finetune the brane potentials $\lambda_{1,2}$. Let us take the functions W and $\lambda_{1,2}$ as:

$$W = 24M^2k - 2u\phi^+\phi, \tag{15}$$

$$\lambda_1(\phi^+\phi) = MW(\phi^+\phi) + \beta_1\left(\phi^+\phi - \frac{v_1^2}{2}\right)^2, \tag{16}$$

$$\lambda_2(\phi^+\phi) = -MW(\phi^+\phi) + \beta_2\left(\phi^+\phi - \frac{v_2^2}{2}\right)^2. \tag{17}$$

Here k, u, β and v are the model parameters. If they are done dimensionless by using the fundamental five-dimensional energy scale M, they should be of the order of unity. Note that the potential λ_2 chosen in this simplest form has the same structure as the Higgs potential.

We use the condition that the background solution for the multidimensional Higgs field on "our" brane must give the known vacuum value of the four-dimentional Higgs field and the condition that the metric on "our" brane must be the flat Minkowski metric. Finally we get the answer:

$$\phi(y) = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}}e^{-u(|y|-L)} \end{pmatrix}, \tag{18}$$

$$A(y) = k\left(|y| - L\right) + \frac{v^2}{96M^2}\left(e^{-2u(|y|-L)} - 1\right). \tag{19}$$

v is the vacuum value of the Higgs field. In this case all the Standard Model fermion fields localized on the brane get the same masses as in the interaction with the usual Higgs field.

The interbrane distance is defined by the boundary conditions for the scalar field and is expressed in terms of the parameters of the model by the relation: $L = \frac{1}{u} \ln\left(\frac{v_1}{v}\right)$. The size of the extra dimension becomes stabilized.

3 Equations for the field fluctuations

In order to obtain the equations for the fluctuations we represent the metric, the five-dimensional Higgs field and the gauge fields in the form "the vacuum solution + a deviation"

$$g_{MN}(x,y) = \gamma_{MN}(y) + \frac{1}{\sqrt{2M^3}} h_{MN}(x,y), \qquad \phi(x,y) = \phi_0(y) + f(x,y), \tag{20}$$

$$\overset{a}{A}_M(x,y) = 0 + \overset{a}{A}_M(x,y), \qquad B_M(x,y) = 0 + B_M(x,y), \tag{21}$$

where $\phi_0(y) \equiv \phi(y)$ denotes the background solution given by (18). Substituting this representation into action (1) and keeping the terms of the second order in $h_{MN}, f, \overset{a}{A}_M$ and B_M we get the so-called second variation Lagrangian.

Varying the action built with this Lagrangian and taking into account the background field equations we arrive at the equations of motion for the fluctuations. We impose the gauge conditions

$$f = \begin{pmatrix} 0 \\ f_2 \end{pmatrix}, \qquad B_4 = 0. \tag{22}$$

where $f_2 \in \mathbf{R}$. Let us consider only the scalar degrees of freedom: $\tilde{h}$, h_{44}, $\overset{a}{A}_4$ and f. Finally we obtain the equations of motion for the scalar fluctuations as follows:

$$\overset{a}{A}_4 = 0, \tag{23}$$

the fluctuation of the $\mu\nu$-component of the metric:

$$\begin{aligned} &\frac{1}{4}\left(\partial_\mu\partial_\nu\tilde{h} + 2\partial_\mu\partial_\nu h_{44}\right) - \frac{1}{4}\gamma_{\mu\nu}\left(\partial_\sigma\partial^\sigma\tilde{h} + 2\partial_\sigma\partial^\sigma h_{44} + \frac{3}{2}\partial_4\partial_4\tilde{h}\right) + \frac{1}{2}\gamma_{\mu\nu}A'\left(4\partial_4\tilde{h} - 3\partial_4 h_{44}\right) + \\ &+\frac{1}{2}\gamma_{\mu\nu}\left(A'\right)^2\left(12h_{44} - \tilde{h}\right) + \frac{1}{4}\gamma_{\mu\nu}A''\left(\tilde{h} - 6h_{44}\right) - \frac{1}{\sqrt{2M}}\gamma_{\mu\nu}\left[\phi_0'^{+}f' + \left(\phi''^{+} - 4A'\phi'^{+}\right)f\right] = 0; \end{aligned} \tag{24}$$

the fluctuation of the $\mu 4$-component of the metric:

$$\frac{3}{4}\partial_4\partial_\mu\tilde{h} + 3A'\partial_\mu h_{44} + \sqrt{\frac{2}{M}}\phi_0'^{+}\partial_\mu f = 0; \tag{25}$$

the fluctuation of the 44-component of the metric:

$$-\frac{3}{4}\partial_\mu\partial^\mu\tilde{h} + 3A'\partial_4\tilde{h} - \frac{1}{2M^2}Vh_{44} + \sqrt{\frac{2}{M}}\left[\phi_0'^{+}f' - \frac{1}{2}\left(\frac{dV}{d\phi}f + f^{+}\frac{dV}{d\phi^{+}}\right)\right] = 0; \tag{26}$$

the fluctuation of the field f:

$$M\left(\partial^M\partial_M f^+ - 4A'f'^+ - \frac{d^2V}{(d\phi)^2}f - f^+\frac{d^2V}{d\phi^+ d\phi}\right) + \frac{1}{\sqrt{2M}}\left[\frac{1}{2}\phi_0'^+\left(\partial_4\tilde{h} - \partial_4 h_{44}\right) - \left(\phi_0''^+ - 4A'\phi_0'^+\right)h_{44}\right] -$$
$$-\left(\frac{d^2\lambda_1}{(d\phi)^2}f + f^+\frac{d^2\lambda_1}{d\phi^+ d\phi}\right)\delta(y) - \left(\frac{d^2\lambda_2}{(d\phi)^2}f + f^+\frac{d^2\lambda_2}{d\phi^+ d\phi}\right)\delta(y-L) +$$
$$+\frac{1}{2\sqrt{2M^3}}\left[\frac{d\lambda_1}{d\phi}\delta(y) + \frac{d\lambda_2}{d\phi}\delta(y-L)\right]h_{44} = 0; \tag{27}$$

the fluctuation of the field f^+:

$$M\left(\partial_M\partial^M f - 4A'f' - \frac{d^2V}{d\phi^+ d\phi}f - f^+\frac{d^2V}{(d\phi^+)^2}\right) + \frac{1}{\sqrt{2M}}\left[\frac{1}{2}\phi_0'\left(\partial_4\tilde{h} - \partial_4 h_{44}\right) - \left(\phi_0'' - 4A'\phi_0'\right)h_{44}\right] -$$
$$-\left(\frac{d^2\lambda_1}{d\phi^+ d\phi}f + f^+\frac{d^2\lambda_1}{(d\phi^+)^2}\right)\delta(y) - \left(\frac{d^2\lambda_2}{d\phi^+ d\phi}f + f^+\frac{d^2\lambda_2}{(d\phi^+)^2}\right)\delta(y-L) +$$
$$+\frac{1}{2\sqrt{2M^3}}\left[\frac{d\lambda_1}{d\phi^+}\delta(y) + \frac{d\lambda_2}{d\phi^+}\delta(y-L)\right]h_{44} = 0. \tag{28}$$

4 Higgs boson mass and an estimate of the model parameters

Using equations of motion found in the previous section, one can obtain the following differential equation and boundary conditions on the branes that define the mass spectrum of the Kaluza-Klein tower of the field ϕ:

$$\frac{d}{dy}\left(\frac{e^{2A}}{(\phi_2')^2}g_n'\right) - \frac{e^{2A}}{6M^2}g_n = -\mu_n^2 g_n\frac{e^{4A}}{(\phi_2')^2}, \tag{29}$$

$$\left(\frac{1}{4M}\frac{d^2\lambda_1}{d\phi_2^2} - \frac{\phi_2''}{\phi_2'}\right)g_n' + \mu_n^2 e^{2A}g_n\bigg|_{y=+0} = 0, \qquad \left(\frac{1}{4M}\frac{d^2\lambda_2}{d\phi_2^2} + \frac{\phi_2''}{\phi_2'}\right)g_n' - \mu_n^2 e^{2A}g_n\bigg|_{y=L-0} = 0. \tag{30}$$

ϕ_2 is the lower component of the vacuum solution (18) for the two-component scalar field.

As it was shown in paper [6], in the case $uL << 1$ we can get an expression for the mass of the lower excitation of the scalar field which is identified as the Higgs boson:

$$m_H^2 = \frac{v^2u^2}{3M^2}\frac{\beta_2 v^2 - uM}{\beta_2 v^2 + uk}. \tag{31}$$

It allows us to estimate the model parameters under the following conditions: $M = 2TeV$ and $\beta_2 \to \infty$, consequently, $u \simeq 1.76\ TeV,\ \phi_1 = 345\ TeV,\ k \simeq 186\ TeV,\ L = 0.2\ TeV^{-1} \simeq 2\cdot 10^{-18}\ cm$. The Higgs boson in this theory is the radion at the same time. It can interact with the trace of the energy-momentum tensor. The coupling constant of the Higgs boson to the trace of the energy-momentum tensor is given by the equation [6] $\epsilon_H = -\sqrt{\frac{k}{24M^3}}$ and turns to be of the order of 1 inverse TeV. It is rather strong and should significantly affect the properties of the Higgs boson in this model. The next excitations of the field ϕ have masses of the order of hundreds of TeV and cannot be observed at the existing colliders.

5 Conclusion

The stabilization of the extra dimension size in the Randall-Sundrum model and the spontaneous symmetry breaking on "our" brane are explained simultaneously with the help of the five-dimensional Higgs field. The equation of motion for this field is found and an exact solution that gives rise to spontaneous symmetry breaking is obtained. It was shown that the multidimensional gauge fields are identically equal to zero in the vacuum state and do not affect the background solution for the metric and scalar field. In this theory the Higgs boson is the radion at the same time, and it now has an interaction with the trace of the energy-momentum tensor that can affect its properties significantly. The radion as a separate particle does not exist. The second variation Lagrangian is obtained and the equations of motion for the fluctuations of the considered fields are found with its help. It was found that the fluctuations of the scalar components of the gauge fields vanish simultaneously in an appropriate gauge and due to the background field equations. The values of the model parameters are estimated, which give the correct value of the Higgs boson mass in the approximation of a small deviation of the metric of the stabilized model from the metric of the unstabilized model.

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