

# Prospects to observe double parton interactions in associated $W^\pm D^{(*)}$ production at the LHC

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Associated production of charged gauge bosons and charged charmed mesons at the LHC is considered in the framework of  $k_t$ -factorization approach. Theoretical predictions are compared with ATLAS data, and reasonably good agreement is found. Predictions on the same-sign  $W^\pm D^\pm$  configurations are presented including single parton scattering and double parton scattering contributions. The latter are shown to dominate over the former, thus giving evidence that the proposed process can serve as another indicator of double parton interactions.

## 1 Introduction

Having the LHC put into operation, one got access to a number of ‘rare’ processes which would have never been systematically studied at the accelerators of previous generations. In this talk based on our recent publication [1] we discuss the associated production of weak gauge bosons and charmed mesons. This process is interesting on its own as providing a complex test of perturbative QCD and our knowledge of parton distributions. Moreover, we argue that it can serve as an indicator of double parton interactions, nowadays widely discussed in the literature [2–4].

Our work was greatly stimulated by the measurements of the charm-associated  $W$  production cross sections reported by CMS [5] and ATLAS [6] Collaborations. In those studies, the interest was mainly focused on the properties of strange sea (see discussion below) and, therefore, in order to suppress other possible contributions (considered in this context as background), the authors have only presented the difference between the opposite-sign (OS) and same-sign (SS)  $WD$  production cross sections,  $\sigma^{OS-SS}(WD)$ . In particular, this excludes the Double Parton Scattering (DPS) processes which yield same-sign  $W^\pm D^{(*)\pm}$  and opposite-sign  $W^\pm D^{(*)\mp}$  combinations with equal probability.

On the contrary, we are more interested in just detecting the DPS events, and so, will lay emphasis on the SS states. Our article is organized as follows. First, we describe our theoretical approach and check its validity by comparing with ATLAS data on  $\sigma^{OS-SS}(WD)$ . Then we extend our consideration to the SS states and make predictions for the production cross sections and kinematic observables which could be useful in discriminating the SPS and DPS contributions.

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## 2 Theoretical framework. Comparison with ATLAS data.

At the parton level, the production of opposite-sign  $W^\pm D^{(*)\mp}$  states is dominated by the quark-gluon scattering

$$g + q \rightarrow W^- + c \quad \text{or} \quad g + \bar{q} \rightarrow W^+ + \bar{c} \quad (1)$$

followed by nonperturbative fragmentation of  $c$ -quarks into charmed mesons. Here the main contribution comes from strange quarks, while the contribution from  $d$ -quarks is suppressed by Cabibbo angle.

To calculate the physical cross sections, we employ the  $k_t$ -factorization approach [7,8]. Here we see certain advantages in the fact that, even with the leading-order (LO) matrix elements for hard subprocess, we can include a large piece of next-to-leading order (NLO) corrections taking them into account in the form of  $k_t$ -dependent parton densities. In this way we automatically incorporate the initial state radiation effects, which play important role in the event kinematics. Further on, the formally NLO subprocess

$$g + g \rightarrow W^- + c + \bar{s} \quad \text{or} \quad g + g \rightarrow W^+ + \bar{c} + s \quad (2)$$

no longer needs to be added because it is already contained in (1). Indeed, the quark-gluon coupling in subprocess (2) can also be regarded as part of the evolution of sea quark densities  $q(x)$  and  $\bar{q}(x)$  in (1).

On the technical side, our calculations follow standard QCD and electroweak theory Feynman rules, but the initial gluon spin density matrix is taken in the form [7,8]  $\epsilon_g^\mu \epsilon_g^{*\nu} = k_T^\mu k_T^\nu / |k_T|^2$ , where  $k_T$  is the component of the gluon momentum perpendicular to the beam axis. In the collinear limit, when  $k_T \rightarrow 0$ , this expression converges to the ordinary  $\epsilon_g^\mu \epsilon_g^{*\nu} = -\frac{1}{2} g^{\mu\nu}$ , while in the case of off-shell gluons it contains an admixture of longitudinal polarization.

The unintegrated parton distributions were constructed using the Kimber-Martin-Ryskin (KMR) [9] formalism<sup>2</sup>. It returns the  $k_T$ -dependent parton densities  $f_{q,g}(x, \mathbf{k}_T^2, \mu^2)$  from a convolution of conventional parton densities  $q(x, \mu^2 = \mathbf{k}_T^2)$  and  $g(x, \mu^2 = \mathbf{k}_T^2)$  with the usual DGLAP splitting functions  $P_{gg}(z)$ ,  $P_{gq}(z)$ ,  $P_{qg}(z)$ , where the splitting scale  $\mu^2$  is interpreted as the  $\mathbf{k}_T^2$  of the resulting parton. The MSTW [11] parametrization was taken for the input set of collinear densities. In our numerical analysis we used running strong and electroweak coupling constants normalized to  $\alpha_s(m_Z^2) = 0.118$ ;  $\alpha(m_Z^2) = 1/128$ ;  $\sin^2 \Theta_W = 0.2312$ ; the factorization and renormalization scales were chosen as  $\mu_R^2 = \mu_F^2 = m_T^2(W) \equiv m_W^2 + p_T^2(W)$ ; the  $c$ -quark mass was set to  $m_c = 1.5$  GeV;  $c$ -quarks were converted into  $D^{(*)}$  mesons using Peterson fragmentation function [12] with  $\epsilon = 0.06$  and normalized to  $f(c \rightarrow D) = 0.268$  and  $f(c \rightarrow D^*) = 0.229$  [13].

Our results obtained with this parameter setting are summarized in Table 1, where we present the  $W^\pm D^{(*)\mp}$  production cross sections integrated over the fiducial phase space region described in Ref. [6]. We observe reasonable agreement with ATLAS data.

Table 1: Measured and predicted cross sections times the  $W \rightarrow lv$  branching ratio (in pb) integrated over the fiducial region  $p_T(l) > 20$  GeV,  $|\eta(l)| < 2.5$ ,  $p_T(\nu) > 25$  GeV,  $p_T(D^{(*)}) > 8$  GeV,  $|\eta(D^{(*)})| < 2.2$ .

Observable	Data	Theory	Observable	Data	Theory
$Br^{W \rightarrow lv} \sigma^{OS-SS}(W^+ D^-)$	17.8	17.7	$Br^{W \rightarrow lv} \sigma^{OS-SS}(W^- D^+)$	22.4	19.5
$Br^{W \rightarrow lv} \sigma^{OS-SS}(W^+ D^{*-})$	21.2	15.1	$Br^{W \rightarrow lv} \sigma^{OS-SS}(W^- D^{*+})$	22.1	16.8

<sup>2</sup>Originally this method traces back to Ref. [10], where it was formulated in the moment space as Eq. (5).

### 3 Same-sign $W^\pm D^\pm$ states and double parton interactions

Now, having our approach validated, we turn to double parton scattering. Detecting same-sign  $W^\pm D^{(*)\pm}$  configurations is certainly preferable here, because we are then free from SPS background due to subprocesses (1) or (2). There are, however, still many other background sources, both direct and indirect. Among the direct ones, we consider the quark-antiquark annihilation at  $\mathcal{O}(\alpha_s^2\alpha)$

$$u + \bar{d} \rightarrow W^+ + c + \bar{c} \quad \text{or} \quad d + \bar{u} \rightarrow W^- + c + \bar{c} \quad (3)$$

and quark-gluon scattering at  $\mathcal{O}(\alpha_s^3\alpha)$

$$g + u \rightarrow W^+ + d + c + \bar{c} \quad \text{or} \quad g + d \rightarrow W^- + u + c + \bar{c}. \quad (4)$$

Subprocess (4) has one extra  $\alpha_s$  in comparison with (3), but it employs gluons which are more abundant than antiquarks in the proton, and that is why may take over. Among the indirect sources we have gluon-gluon fusion

$$g + g \rightarrow W^- + c + \bar{b} \quad \text{or} \quad g + g \rightarrow W^+ + b + \bar{c} \quad (5)$$

followed by the decays  $b \rightarrow c + X$  or  $\bar{b} \rightarrow \bar{c} + X$ , and the production of top quark pairs

$$g + g \rightarrow t + \bar{t} \quad \text{and} \quad q + \bar{q} \rightarrow t + \bar{t} \quad (6)$$

followed by a long chain of decays, such as  $t \rightarrow W^+ + b$ ,  $W^+ \rightarrow c + \bar{s}$ ,  $b \rightarrow c + X$  or  $b \rightarrow c + \bar{c} + s$  (and the charge conjugated modes). Here same-sign  $W^+ D^{(*)+}$  configurations may be formed by a  $W^+$  boson coming from  $t$  and a  $c$ -quark coming from  $b$  coming from  $t$ , or a  $c$ -quark coming from  $\bar{b}$  coming from  $\bar{t}$ . Similarly, in the case of single top production

$$u + \bar{d} \rightarrow t + \bar{b} \quad \text{or} \quad d + \bar{u} \rightarrow \bar{t} + b \quad (7)$$

the  $W^+ D^{(*)+}$  configuration may be formed by a  $W^+$  boson coming from  $t$  and a  $c$ -quark coming from  $b$  coming from the same  $t$ , or a  $c$ -quark coming from  $\bar{b}$ . Note that the subprocesses (6) are purely strong, and so, may have large cross sections in spite of large  $t$ -quark mass. All other possible processes beyond (3)-(7) are expected to be suppressed by extra powers of coupling constants (already the case of (7)) or by Kobayashi-Maskawa mixing matrix (already the case of (5)).

A comment is needed on the choice of renormalization scale in (3). This process factorises into the production of  $W + g^*$  at  $\mu_R^2 = m_T^2(W)$  and the subsequent gluon splitting  $g^* \rightarrow c\bar{c}$ , for which the  $c\bar{c}$  invariant mass seems to be a more suitable measure. Note that using different  $\alpha_s$  values for these two different steps does not violate the overall gauge invariance. So, we calculate the resulting cross section with  $\alpha_s(m_T^2(W))\alpha_s(m_{c\bar{c}}^2)$ , regarding it as the pessimistic (the upper) limit for the background. By the same reasoning, we adopt  $\alpha_s^2(m_T^2(W))\alpha_s(m_{c\bar{c}}^2)$  for subprocess (4).

For the fiducial phase space of Ref. [6], we estimate the above contributions to  $W^+ D^+$  and  $W^- D^-$  states as

$$Br^{W \rightarrow l\nu} \sigma^{W^+ D^+} (u\bar{d} \rightarrow Wc\bar{c}) = 0.41 \text{ pb} \quad Br^{W \rightarrow l\nu} \sigma^{W^- D^-} (d\bar{u} \rightarrow Wc\bar{c}) = 0.29 \text{ pb} \quad (8)$$

$$Br^{W \rightarrow l\nu} \sigma^{W^+ D^+} (gu \rightarrow Wdc\bar{c}) = 1.0 \text{ pb} \quad Br^{W \rightarrow l\nu} \sigma^{W^- D^-} (gd \rightarrow Wuc\bar{c}) = 0.7 \text{ pb} \quad (9)$$

$$Br^{W \rightarrow l\nu} \sigma^{W^+ D^+} (gg \rightarrow Wb\bar{c}) = 0.002 \text{ pb} \quad Br^{W \rightarrow l\nu} \sigma^{W^- D^-} (gg \rightarrow Wb\bar{c}) = 0.002 \text{ pb} \quad (10)$$

$$Br^{W \rightarrow l\nu} \sigma^{W^+ D^+} (gg \rightarrow t\bar{t}) = 1.1 \text{ pb} \quad Br^{W \rightarrow l\nu} \sigma^{W^- D^-} (gg \rightarrow t\bar{t}) = 1.1 \text{ pb} \quad (11)$$

$$Br^{W \rightarrow l\nu} \sigma^{W^+ D^+} (q\bar{q} \rightarrow t\bar{t}) = 0.6 \text{ pb} \quad Br^{W \rightarrow l\nu} \sigma^{W^- D^-} (q\bar{q} \rightarrow t\bar{t}) = 0.6 \text{ pb} \quad (12)$$

$$Br^{W \rightarrow l\nu} \sigma^{W^+ D^+} (u\bar{d} \rightarrow t\bar{b}) = 0.06 \text{ pb} \quad Br^{W \rightarrow l\nu} \sigma^{W^- D^-} (d\bar{u} \rightarrow b\bar{t}) = 0.04 \text{ pb} \quad (13)$$

The results (8)-(9) were obtained assuming the already mentioned fragmentation probability  $f(c \rightarrow D) = 0.268$ . The results (10)-(13) were obtained under the assumption of 100% branching fraction for  $t \rightarrow bW$ , of equal fragmentation probabilities for  $b \rightarrow \bar{B}^0$  and  $b \rightarrow B^-$ , and using the inclusive branching fractions  $Br(\bar{B}^0 \rightarrow D^+ X) = 37\%$ ,  $Br(B^0 \rightarrow D^+ X) = 3\%$ ,  $Br(B^- \rightarrow D^+ X) = 10\%$ ,  $Br(B^+ \rightarrow D^+ X) = 2.5\%$  listed in the Particle Data Book [14]. The quark masses were set to  $m_t=175$  GeV and  $m_b=4.8$  GeV. We make no predictions for  $W^\pm D^{*\pm}$  states for the reason of not knowing the relevant  $B \rightarrow D^* X$  decay branchings. Variations in  $\mu_R^2$  and  $\mu_F^2$  within a factor of 2 around the default value make a factor of 1.6 increasing or decreasing effect on the estimated production rate. However, these effects mostly cancel out in the signal to background ratio.

Now we proceed to discussing the expected signal from double parton interactions. Under the hypothesis of having two independent hard partonic subprocesses  $A$  and  $B$  in a single  $pp$  collision, and under further assumption that the longitudinal and transverse components of generalized parton distributions factorize from each other, the inclusive DPS cross section reads (for details see, e.g., the recent review [2] and references therein)

$$\sigma_{\text{DPS}}^{\text{AB}} = \frac{m}{2} \frac{\sigma_{\text{SPS}}^A \sigma_{\text{SPS}}^B}{\sigma_{\text{eff}}}, \quad (14)$$

where  $\sigma_{\text{eff}}$  is a normalization cross section that encodes all "DPS unknowns" into a single parameter which can be experimentally measured. One can identify  $\sigma_{\text{eff}}$  with the inverse of the proton overlap functions squared:

$$\sigma_{\text{eff}} = \left[ \int d^2b (T(\mathbf{b}))^2 \right]^{-1}, \quad (15)$$

where  $T(\mathbf{b}) = \int f(\mathbf{b}_1) f(\mathbf{b}_1 - \mathbf{b}) d^2b_1$  is the overlap function that characterizes the transverse area occupied by the interacting partons, and  $f(\mathbf{b})$  is supposed to be a universal function of the impact parameter  $\mathbf{b}$  for all kinds of partons with its normalization fixed as

$$\int f(\mathbf{b}_1) f(\mathbf{b}_1 - \mathbf{b}) d^2b_1 d^2b = \int T(\mathbf{b}) d^2b = 1. \quad (16)$$

A numerical value of  $\sigma_{\text{eff}} \simeq 15$  mb has been obtained empirically from fits to  $p\bar{p}$  and  $pp$  data [15–19]. It will be used in our further analysis, although an estimate as low as  $\sigma_{\text{eff}} \simeq 5$  mb is also present in [20].

The inclusive SPS cross sections  $\sigma_{\text{SPS}}^A$  and  $\sigma_{\text{SPS}}^B$  for the individual partonic subprocesses  $A$  and  $B$  can be calculated in a usual way using the ordinary parton distribution functions. The symmetry factor  $m$  equals to 1 for identical subprocesses and 2 for the differing ones.

In our present case, the inclusive production cross sections  $\sigma(D^\pm)$  and  $\sigma(W^\pm)$  have been calculated in accordance with Refs. [13] and [21], respectively. For the considered fiducial phase space our expectations read

$$\sigma_{\text{incl}}(D^+) = \sigma_{\text{incl}}(D^-) = 11.4 \mu\text{b}, \quad (17)$$

$$Br^{W \rightarrow l\nu} \sigma_{\text{incl}}(W^+) = 3.5 \text{ nb}, \quad (18)$$

$$Br^{W \rightarrow l\nu} \sigma_{\text{incl}}(W^-) = 2.5 \text{ nb}, \quad (19)$$

where the estimates (18)-(19) are supported by direct recent measurement [22], and so,

$$Br^{W \rightarrow l\nu} \sigma_{\text{DPS}}(W^+ D^+) = 2.7 \text{ pb} \quad Br^{W \rightarrow l\nu} \sigma_{\text{DPS}}(W^- D^-) = 1.9 \text{ pb}. \quad (20)$$

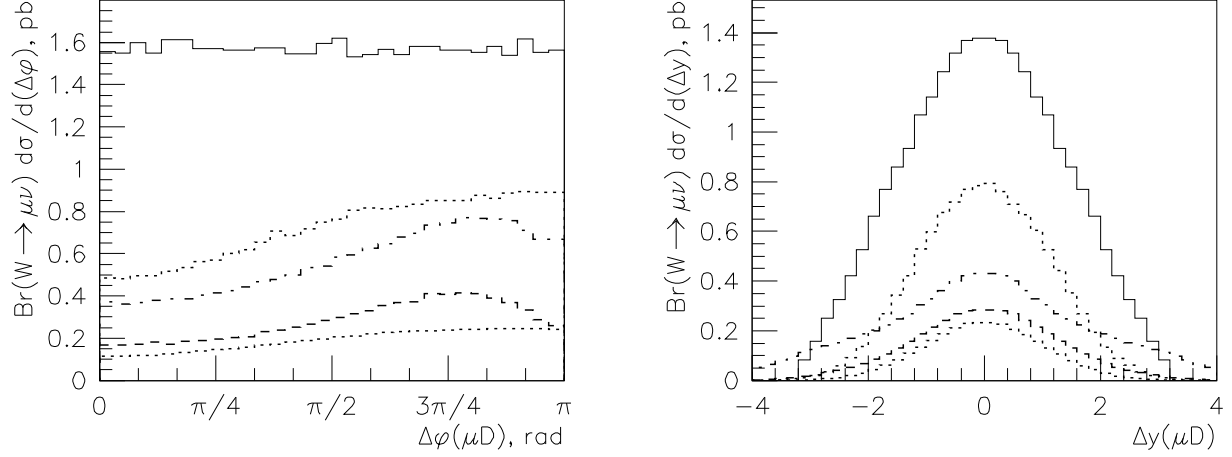


Figure 1: Kinematic correlations between muons and  $D$ -mesons in same-sign events ( $\mu^\pm D^\pm$ ) at ATLAS conditions: distributions in the azimuthal angle difference  $\Delta\phi$  (left panel) and rapidity difference  $\Delta y$  (right panel). The different contributions are represented by: solid curve, double parton scattering; upper dotted curve,  $gg \rightarrow t\bar{t}$ ; lower dotted curve,  $q\bar{q} \rightarrow t\bar{t}$ ; dashed curve,  $u\bar{d} \rightarrow Wc\bar{c}$  and  $d\bar{u} \rightarrow Wc\bar{c}$ ; dash-dotted curve,  $gu \rightarrow Wdc\bar{c}$  and  $gd \rightarrow Wuc\bar{c}$ .

These numbers are close to the combined SPS contribution. This means that the excess brought by DPS to the visible  $W^\pm D^\pm$  cross-sections is not large enough to unambiguously testify for its presence: the DPS signal is large, but the background uncertainties are also large. Moreover, the shapes of the DPS and SPS kinematic distributions are rather similar: the decays of heavy  $t$ -quarks and  $W$  bosons make the final state distributions broad and smooth. Selection cuts on the azimuthal angle difference  $\Delta\phi$  or rapidity difference  $\Delta y$ , so promising in other reactions [23], remain practically useless in the present case. This is illustrated in Fig. 1 where we show correlations between  $D^\pm$  mesons and muons coming from  $W^\pm$  bosons in same-sign events at ATLAS conditions.

Fortunately, the indirect contributions can be significantly reduced (if not rejected completely) using a well-known experimental technique based on the property that the secondary  $b$ -decay vertex is displaced with respect to the primary interaction vertex. We are then left with direct background (8)-(9) lying well below the DPS level, even with conservative estimate of  $\sigma_{\text{eff}}=15$  mb and with 'pessimistic' choice of  $\mu_R$  as discussed above. In fact, our numbers represent the upper edge of the background uncertainty band. A similar relation is seen for  $D^*$  mesons:

$$Br^{W \rightarrow l\nu} \sigma_{DPS}(W^+ D^{*+}) = 2.3 \text{ pb} \quad Br^{W \rightarrow l\nu} \sigma_{DPS}(W^- D^{*-}) = 1.6 \text{ pb} \quad (21)$$

$$Br^{W \rightarrow l\nu} \sigma^{W^+ D^{*+}}(u\bar{d} \rightarrow Wc\bar{c}) = 0.35 \text{ pb} \quad Br^{W \rightarrow l\nu} \sigma^{W^- D^{*-}}(d\bar{u} \rightarrow Wc\bar{c}) = 0.25 \text{ pb} \quad (22)$$

$$Br^{W \rightarrow l\nu} \sigma^{W^+ D^{*+}}(g u \rightarrow W d c \bar{c}) = 0.85 \text{ pb} \quad Br^{W \rightarrow l\nu} \sigma^{W^- D^{*-}}(g d \rightarrow W u c \bar{c}) = 0.60 \text{ pb} \quad (23)$$

We thus come to an important conclusion that the production of same-sign  $W^\pm D^{*\pm}$  states is very indicative as DPS signal. This situation is close to the production of same-sign  $W^\pm W^\pm$  pairs proposed earlier in Ref. [24]. However, the  $W^\pm W^\pm$  events occur at a much lower rate and are then less convenient for analysis.

To carry out a practical phenomenological search at particular experimental conditions, one is advised to use the full event Monte-Carlo generator CASCADE [25] which is based on the  $k_t$ -factorization approach and incorporates a library of unintegrated parton densities [26]. The variety of the latter can further be

extended by addressing to such codes as TMDlib [27] and uPDFevolv [28].<sup>3</sup>

## 4 Conclusions

We have considered the production of a W boson in association with a charmed meson in  $pp$  collisions at the LHC and made a comparison with experimental results. Our theoretical calculations have shown reasonable agreement with ATLAS data on  $\sigma^{OS-SS}(WD)$ . We have extended our consideration to the same-sign  $W^\pm D^\pm$  configurations and found that after rejecting the  $b$ -decays the DPS signal clearly dominates over SPS background. Thus, we come to an important conclusion that the production of same-sign  $W^\pm D^\pm$  states can serve as a new reliable indicator of double parton scattering.

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<sup>3</sup>We note that there exist unintegrated sea distributions that go beyond our approximations (and for example use splitting functions which also depend on transverse momentum), and that these quark distributions may be practically useful in the future analysis; but the approximations which we use here are well sufficient for the present purpose as they fit the data. The goal of our note is not in giving a detailed description of the process prior to measurement, but is in showing that this kind of measurement is worth doing.

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