# Weak-annihilation rare radiative decays of B-mesons

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In this talk, we present the predictions [1] for a number of radiative decays $B_{(s)} \to V\gamma$, with the vector meson in the final state, which proceed through the weak-annihilation mechanism. Within the factorization approximation, we take into account the photon emission from the $B$-meson loop and from the vector-meson loop. The highest branching ratios for the weak-annihilation reactions reported here are $\mathcal{B}(\overline{B}_s^0 \to J/\psi\gamma) = 1.5 \cdot 10^{-7}$ and $\mathcal{B}(B^- \to \overline{D}_s^{*-}\gamma) = 1.7 \cdot 10^{-7}$, the estimated accuracy of these predictions being at the level of 20%.

## 1 Introduction

The investigation of rare $B$ decays forbidden at the tree level in the Standard Model provides the possibility to probe the electroweak sector at large mass scales. Interesting information about the structure of the theory is contained in the Wilson coefficents entering the effective Hamiltonian which take different values in different theories with testable consequences in rare $B$ decays.

There is an interesting class of rare radiative $B$-decays which proceed merely through the weak-annihilation mechanism. These processes have very small probabilities and have not been observed. So far, only upper limits on the branching ratios of these decays have been obtained: In 2004, the BaBar Collaboration provided the upper limit $\mathcal{B}(B^0 \to J/\psi\gamma) < 1.6 \cdot 10^{-6}$ [2]. Very recently, the LHCb Collaboration reached the same sensitivity to the $B^0$-decay and set the limit on the $B_s^0$ decay: $\mathcal{B}(B^0 \to J/\psi\gamma) < 1.7 \cdot 10^{-6}$ and $\mathcal{B}(B_s^0 \to J/\psi\gamma) < 7.4 \cdot 10^{-6}$ at 90% CL [3]. Obviously, with the increasing statistics, the prospects to improve the limits on the branching ratios by one order of magnitude or eventually to observe these decays in the near future seem very favourable.

The annihilation-type $B$-decays are promising from the perspective of obtaining theoretical predictions since the QCD dynamics of these decays is relatively simple [4, 5]. These decays have been addressed in the literature but — in spite of their relative simplicity — the available theoretical predictions turned out to be rather uncertain; for instance, the predictions for $\mathcal{B}(B_s^0 \to J/\psi\gamma)$ decay vary from $5.7 \cdot 10^{-8}$ [6] to $5 \cdot 10^{-6}$ [7]. The situation is clearly unsatisfactory and requires clarification. We did not find any of these results convincing and present in this paper a more detailed analysis of the $B \to V\gamma$ decays.

The annihilation type $B \to V\gamma$ decays proceed through the four-quark operators of the effective weak Hamiltonian. In the factorization approximation, the amplitude can be represented as the product of meson leptonic decay constants and matrix elements of the weak current between meson and photon; the latter contain the meson-photon transition form factors. The photon can be emitted from the loop containing the $B$-meson (Fig. 1a), this contribution is described by the $B\gamma$ transition form factors. The photon can be also emitted from the vector-meson $V$-loop (Fig 1b); this contribution is described by the $V\gamma$ transition form factors. The latter were erroneuosly believed to give small contribution to the amplitude and have not

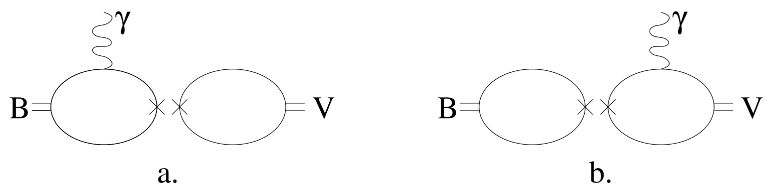


Figure 1: Diagrams describing the weak annihilation process for $B \to V\gamma$ in the factorization approximation: (a) The photon is emitted from the $B$-loop, (b) The photon is emitted from the vector-meson $V$-loop.

been considered in the previous analyses. We calculate the $B\gamma$ and $V\gamma$ form factors within the relativistic dispersion approach based on the constituent quark picture [8]. As shown in [9], the form factors from this approach satisfy all rigorous constrains which emerge in QCD in the limit of heavy-to-heavy and heavy-to-light transitions; as demonstrated in [10–12], the numerical results for the weak transition form factors from this approach exhibit an excellent agreement with the results from lattice QCD and QCD sum rules.

## 2 The effective Hamiltonian, the amplitude, and the decay rate

We consider the weak-annihilation radiative $B \to V\gamma$ transition, where $V$ is the vector meson containing at least one charm quark, i.e. having the quark content $\bar{q}c$ ($q = u, d, s, c$). The corresponding amplitude is given by the matrix element of the effective Hamiltonian [13]

$$A(B \to V\gamma) = \langle \gamma(q_1)V(q_2)|H_{\rm eff}|B(p)\rangle, \tag{1}$$

where $p$ is the $B$ momentum, $q_2$ is the vector-meson momentum, and $q_1$ is the photon momentum, $p = q_1 + q_2$, $q_1^2 = 0$, $q_2^2 = M_V^2$, $p^2 = M_B^2$. The effective weak Hamiltonian relevant for the transition of interest has the form:

$$H_{\rm eff} \;\; = \;\; -\frac{G_F}{\sqrt{2}}\xi_{\rm CKM}\left(C_1(\mu)\mathcal{O}_1 + C_2(\mu)\mathcal{O}_2\right), \tag{2}$$

$G_F$ is the Fermi constant, $\xi_{\rm CKM} = V_{cd}^{*}V_{ub}$, $C_{1,2}(\mu)$ are the scale-dependent Wilson coefficients [13], and we only show the relevant four-quark operators

$$\begin{aligned} \mathcal{O}_1 &= \bar{d}_\alpha\gamma_\nu(1-\gamma_5)c_\alpha\, \bar{u}_\beta\gamma_\nu(1-\gamma_5)b_\beta, \\ \mathcal{O}_2 &= \bar{d}_\alpha\gamma_\nu(1-\gamma_5)c_\beta\, \bar{u}_\beta\gamma_\nu(1-\gamma_5)b_\alpha. \end{aligned} \tag{3}$$

We use notations $e = \sqrt{4\pi\alpha_{\rm em}}$, $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$, $\sigma_{\mu\nu} = i\left[\gamma_\mu, \gamma_\nu\right]/2$, $\epsilon^{0123} = -1$ and ${\rm Sp}\left(\gamma^5\gamma^\mu\gamma^\nu\gamma^\alpha\gamma^\beta\right) = 4i\epsilon^{\mu\nu\alpha\beta}$.

In the amplitude it is convenient to isolate the parity-conserving contribution which emerges from the product of the two equal-parity currents, and the parity-violating contribution which emerges from the product of the two opposite-parity currents. The amplitude may then be parametrized as follows

$$A(B \to V\gamma) = \frac{eG_F}{\sqrt{2}}\left[\epsilon_{q_1\epsilon_1^* q_2\epsilon_2^*}F_{\rm PC} + i\epsilon_2^{*\nu}\epsilon_1^{*\mu}\left(g_{\nu\mu}\, pq_1 - p_\mu q_{1\nu}\right)F_{\rm PV}\right], \tag{4}$$

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where $F_{\rm PC}$ and $F_{\rm PV}$ are the parity-conserving and parity-violating invariant amplitudes, respectively. Hereafter $\epsilon_2(\epsilon_1)$ is the vector-meson (photon) polarization vector. We use the short-hand notation $\epsilon_{abcd} = \epsilon_{\alpha\beta\mu\nu}a^\alpha b^\beta c^\mu d^\nu$ for any 4-vectors $a, b, c, d$.

For the decay rate one finds

$$\Gamma(B \to V\gamma) = \frac{G_F^2\,\alpha_{em}}{16} M_B^3 \left(1 - M_V^2/M_B^2\right)^3 \left(|F_{\rm PC}|^2 + |F_{\rm PV}|^2\right). \tag{5}$$

After neglecting the nonfactorizable soft-gluon exchanges, i.e. assuming vacuum saturation, it is convinient to parametrize the parity-violating and the parity-conserving amplitudes of (4) in the following way

$$F_{\rm PV} = \xi_{\rm CKM} a_{\rm eff}(\mu) \left[\frac{F_A}{M_B} f_V M_V + f_B H_S - \frac{2Q_B f_B f_V M_V}{M_B^2 - M_V^2}\right], \tag{6}$$

$$F_{\rm PC} = \xi_{\rm CKM} a_{\rm eff}(\mu) \left[\frac{F_V}{M_B} f_V M_V + f_B H_P\right]. \tag{7}$$

Summing up this Section, within the factorization approximation the weak annihilation amplitude can be expressed in terms of four form factors: $F_A$, $F_V$, $H_P$ and $H_S$. It should be emphasized that each of the form factors $F_A$, $F_V$, $H_P$ and $H_S$ actually depends on two variables: The $B$-meson transition form factors $F_A$, $F_V$ depend on $q_1^2$ and $q_2^2$, and $F_{A,V}(q_1^2, q_2^2)$ should be evaluated at $q_1^2 = 0$ and $q_2^2 = M_V^2$. The vector-meson transition form factors $H_P$ and $H_S$ depend on $q_1^2$ and $p^2$, and $H_{S,P}(q_1^2, p^2)$ should be evaluated at $q_1^2 = 0$ and $p^2 = M_B^2$.

## 3 Photon emission from the $B$-meson loop and the form factors $F_A$ and $F_V$.

In this section we calculate the form factors $F_{A,V}$ within the relativistic dispersion approach to the transition form factors based on constituent quark picture. This approach has been formulated in detail in [9] and applied to the weak decays of heavy mesons in [10].

The pseudoscalar meson in the initial state is described in the dispersion approach by the following vertex [8]: $\bar{q}_1(k_1)\ i\gamma_5 q(-k_2)\ G(s)/\sqrt{N_c}$, with $G(s) = \phi_P(s)(s - M_P^2)$, $s = (k_1 + k_2)^2$, $k_1^2 = m_1^2$ and $k_2^2 = m_2^2$. The pseudoscalar-meson wave function $\phi_P$ is normalized according to the relation [8]

$$\frac{1}{8\pi^2} \int\limits_{(m_1+m_2)^2}^{\infty} ds \phi_P^2(s) \left(s - (m_1 - m_2)^2\right) \frac{\lambda^{1/2}(s, m_1^2, m_2^2)}{s} = 1. \tag{8}$$

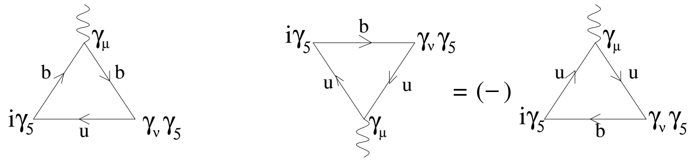
The decay constant is represented through $\phi_P(s)$ by the spectral integral

$$f_P = \sqrt{N_c} \int\limits_{(m_1+m_2)^2}^{\infty} ds \phi_P(s)(m_1 + m_2) \frac{\lambda^{1/2}(s, m_1^2, m_2^2)}{8\pi^2 s} \frac{s - (m_1 - m_2)^2}{s}. \tag{9}$$

Here $\lambda(a, b, c) = (a - b - c)^2 - 4bc$ is the triangle function.

The form factor $F_A$ is given by the diagrams of Fig 2. Fig 2a shows $F_A^{(b)}$, the contribution to the form factor of the process when the $b$ quark interacts with the photon; Fig 2b describes the contribution of the process when the quark $u$ interacts while $b$ remains a spectator.

It is convenient to change the direction of the quark line in the loop diagram of Fig 2b. This is done by performing the charge conjugation of the matrix element and leads to a sign change for the $\gamma_\nu\gamma_5$ vertex, so that both diagrams of Fig 2 can reduced to one diagram. The same is done for the $F_V$ form factor. The resulting diagrams are represented on Fig 3:




a. b.

Figure 2: Diagrams for the form factor $F_A$: a) $F_A^{(b)}$, b) $F_A^{(u)}$.

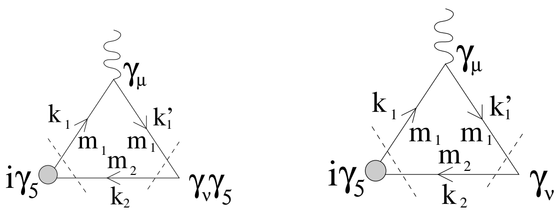



Figure 3: The triangle diagram for $F_A^{(1)}(m_1, m_2)$ and $F_V^{(1)}(m_1, m_2)$ form. The cuts correspond to calculating the double spectral density in $p^2$ and $q_2^2$.

Setting $m_1 = m_b$, $m_2 = m_u$ gives $F_A^{(b)}$ and $F_V^{(b)}$, while setting $m_1 = m_u$, $m_2 = m_b$ gives $F_A^{(u)}$ and $F_V^{(u)}$ such that

$$F_A = Q_b F_A^{(b)} - Q_u F_A^{(u)}, \qquad F_V = Q_b F_V^{(b)} + Q_u F_V^{(u)}. \tag{10}$$

This expression may be cast in the form of a single dispersion integral

$$\frac{1}{M_B} F_A^{(1)}(m_1, m_2) = \frac{\sqrt{N_c}}{4\pi^2} \int\limits_{(m_1+m_2)^2}^{\infty} \frac{ds\ \phi_B(s)}{(s - M_V^2)} \left( \rho_+(s, m_1, m_2) + 2\frac{m_1 - m_2}{M_B^2 - M_V^2} \rho_{k_\perp^2}(s, m_1, m_2) \right), \tag{11}$$

$$\frac{1}{M_B} F_V^{(1)}(m_1, m_2) = -\frac{\sqrt{N_c}}{4\pi^2} \int\limits_{(m_1+m_2)^2}^{\infty} \frac{ds\phi_B(s)}{(s - M_V^2)} \rho_+(s, m_1, m_2). \tag{12}$$

where

$$\rho_+(s, m_1, m_2) = (m_2 - m_1)\frac{\lambda^{1/2}(s, m_1^2, m_2^2)}{s} + m_1 \log\left( \frac{s + m_1^2 - m_2^2 + \lambda^{1/2}(s, m_1^2, m_2^2)}{s + m_1^2 - m_2^2 - \lambda^{1/2}(s, m_1^2, m_2^2)} \right), \tag{13}$$

$$\rho_{k_\perp^2}(s, m_1, m_2) = \frac{s + m_1^2 - m_2^2}{2s} \lambda^{1/2}(s, m_1^2, m_2^2) - m_1^2 \log\left( \frac{s + m_1^2 - m_2^2 + \lambda^{1/2}(s, m_1^2, m_2^2)}{s + m_1^2 - m_2^2 - \lambda^{1/2}(s, m_1^2, m_2^2)} \right). \tag{14}$$

# 4 Photon emission from the vector meson loop. The form factors $H_S$ and $H_P$.

We now calculate the form factors $H_{P,S}$ using the relativistic dispersion approach. The vector meson in the final state is described in this approach by the vertex $\bar{q}_2(-k_2)\Gamma_\beta q_1(k'_1)$, $\Gamma_\beta = \left(-\gamma_\beta + \frac{(k'_1-k_2)_\beta}{\sqrt{s}+m_1+m_2}\right) G(s)/\sqrt{N_c}$, with $G(s) = \phi_V(s)(s - M_V^2)$, $s = (k'_1 + k_2)^2$, $k'^2_1 = m_1^2$ and $k_2^2 = m_2^2$. The vector-meson wave function $\phi_V$ is normalized according to [9]

$$\frac{1}{8\pi^2} \int\limits_{(m_1+m_2)^2}^{\infty} ds\phi_V^2(s) \left(s - (m_1 - m_2)^2\right) \frac{\lambda^{1/2}(s, m_1^2, m_2^2)}{s} = 1. \tag{15}$$

Its decay constant is represented through $\phi_V(s)$ by the spectral integral

$$f_V = \sqrt{N_c} \int\limits_{(m_1+m_2)^2}^{\infty} ds\phi_V(s) \frac{2\sqrt{s} + m_1 + m_2}{3} \frac{\lambda^{1/2}(s, m_1^2, m_2^2)}{8\pi^2 s} \frac{s - (m_1 - m_2)^2}{s}. \tag{16}$$

Now the form factors $H_{S,P}$ describe the transition of the current with momentum $p$, $p^2 = M_B^2$, to the photon with momentum $q_1$, $q_1^2 = 0$, and the vector meson with the momentum $q_2$, $q_2^2 = M_V^2$. Similar to the previous section, we derive the double spectral representations for the form factor in $p^2$ and $q_2^2$.

The form factors $H_S$ and $H_P$ are given by the diagrams of Fig 4.

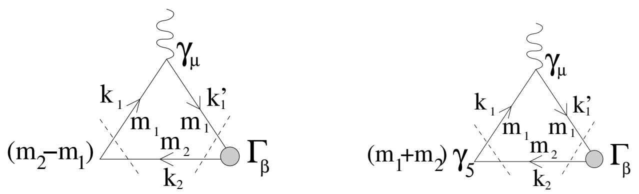



Figure 4: The triangle diagrams for $H_S^{(1)}(m_1, m_2)$ and $H_P^{(1)}(m_1, m_2)$.

Each diagram contains the contributions of photon emission both from the light and the heavy quarks:

$$H_S = Q_d H_S^{(d)} - Q_c H_S^{(c)}, \quad H_P = Q_d H_P^{(d)} + Q_c H_P^{(c)}. \tag{17}$$

The corresponding single dispersion integrals has the form

$$H_S^{(1)}(m_1, m_2) = \frac{\sqrt{N_c}}{4\pi^2} \int\limits_{(m_1+m_2)^2}^{\infty} \frac{ds\ \phi_V(s)}{(s - p^2 - i0)} (m_2 - m_1) \left(\rho_+(s, m_1, -m_2) + \frac{2\sqrt{s}}{p^2 - M_V^2} \rho_{k_\perp^2}(s, m_1, m_2)\right), \tag{18}$$

$$H_P^{(1)}(m_1, m_2) = \frac{\sqrt{N_c}}{4\pi^2} \int\limits_{(m_1+m_2)^2}^{\infty} \frac{ds\phi_V(s)}{(s - p^2 - i0)} (m_1 + m_2) \left(\rho_+(s, m_1, m_2) + \frac{\rho_{k_\perp^2}}{\sqrt{s} + m_1 + m_2}\right), \tag{19}$$

where $\rho_+(s, m_1, m_2)$ and $\rho_{k_\perp^2}(s, m_1, m_2)$ are determined earlier in (13) and (14).

For the $B$-decays of interest, we need the value of the form factors $H_{P,S}(p^2, q_1^2 = 0)$ at $p^2 = M_B^2$, which lies above the threshold $(m_c + m_q)^2$. The spectral representations for $H_{P,S}(p^2 = M_B^2)$ develop the imaginary parts which occur due to the quark-antiquark intermediate states in the $p^2$-channel. It should be emphasized that no anomalous cuts emerge in the double spectral representation at $q_1^2 \le 0$ [18]. In all cases considered in this paper, the value of $p^2 = M_B^2$ lies far above the region of resonances which occur in the quark-antiquark channel. Far above the resonance region local quark-hadron duality works well and the calculation of the imaginary part based on the quark diagrams is trustable. The imaginary part turns out to be orders of magnitude smaller than the real part of the form factor and for the practical purpose of the decay rate calculation may be safely neglected.

# 5 Numerical results

The derived spectral representations for the form factors allow one to obtain numerical predictions for the form factors of interest as soon as the parameters of the model – the meson wave finctions and the quark masses – are fixed.

The wave function $\phi_i(s)$, $i = P, V$ can be written as

$$\phi_i(s) = \frac{\pi}{\sqrt{2}} \frac{\sqrt{s^2 - (m_1^2 - m^2)^2}}{\sqrt{s - (m_1 - m)^2}} \frac{w_i(k^2)}{s^{3/4}}, \qquad k^2 = \lambda(s, m_1^2, m^2)/4s, \tag{20}$$

with $w_i(k^2)$ normalized as follows

$$\int w_i^2(k^2) k^2 dk = 1. \tag{21}$$

The meson weak transition form factors from dispersion approach reproduce correctly the structure of the heavy-quark expansion in QCD for heavy-to-heavy and heavy-to-light meson transitions, as well as for the meson-photon transitions, if the radial wave functions $w(k^2)$ are localized in a region of the order of the confinement scale, $k^2 \le \Lambda^2$ [9]. Following [10] we assume a simple gaussian parameterization of the radial wave function

$$w_i(k^2) \propto \exp(-k^2/2\beta_i^2), \tag{22}$$

which satisfies the localization requirement for $\beta \simeq \Lambda_{QCD}$ and proved to provide a reliable picture of a large family of the transition form factors [10].

We use the same values of the constituent quark masses and the quark couplings as have been obtained in [10]

$$m_d = m_u = 0.23 \text{ GeV}, \quad m_s = 0.35 \text{ GeV}, \quad m_c = 1.45 \text{ GeV}, \quad m_b = 4.85 \text{ GeV}. \tag{23}$$

We consider several annihilation-type $B$-decays which have the highest probabilities; the weak-annihilation quark diagrams which induce these decays are shown in Fig. 5.

The corresponding $F_{PC}$ and $F_{PV}$ and the decay rates are summarized in Table 1. To highlight the contribution to the amplitudes coming from the photon emission from the $V$-meson loop, we multiply it by a coefficient $r$ which is set to unity in the decay-rate calculations. Obviously, for some modes the photon emission from the vector-meson loop is comparable or even exceeds the photon emission from the $B$-meson loop and thus should be taken into account.

For the scale-dependent Wilson coefficients $C_i(\mu)$ and $a_{1,2}(\mu)$ at the renormalization scale $\mu \simeq 5$ GeV we use the following values [13]: $C_1 = 1.1$, $C_2 = -0.241$, $a_1 = C_1 + C_2/N_c = 1.02$, and $a_2 = C_2 + C_1/N_c = 0.15$.

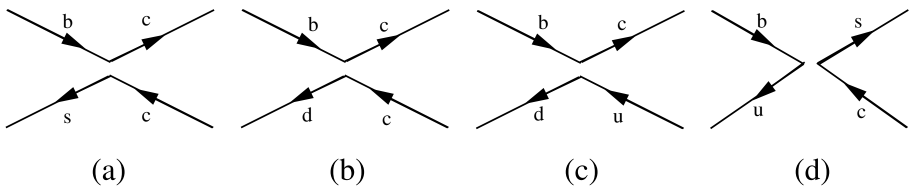



Figure 5: Four-quark operators inducing the annihilation $B$-decays listed in Table 1. (a) $\overline{B}_s^0 \to J/\psi\gamma$; (b) $\overline{B}_d^0 \to J/\psi\gamma$; (c) $\overline{B}_d^0 \to D^{*0}\gamma$; (d) $B^- \to D_s^{*-}\gamma$.

Table 1: The amplitudes and the branching ratio for the annihilation-type decay of $B$ and $B_s$.

| Reaction | CKM-factor | $F_{PC}$ [GeV] | $F_{PV}$ [GeV] | Br |
|---|---|---|---|---|
| $\overline{B}_s^0 \to J/\psi\gamma$ | $a_2 V_{cb} V_{cs}^*$ | $0.036 - 0.052r$ | $0.020$ | $1.43 \cdot 10^{-7} \left(\frac{a_2}{0.15}\right)^2$ |
| $\overline{B}_d^0 \to J/\psi\gamma$ | $a_2 V_{cb} V_{cd}^*$ | $0.035 - 0.050r$ | $0.021$ | $7.54 \cdot 10^{-9} \left(\frac{a_2}{0.15}\right)^2$ |
| $\overline{B}_d^0 \to D^{*0}\gamma$ | $a_2 V_{cb} V_{ud}^*$ | $0.012 - 0.014r$ | $0.007 + 0.002r$ | $4.33 \cdot 10^{-8} \left(\frac{a_2}{0.15}\right)^2$ |
| $B^- \to D_s^{*-}\gamma$ | $a_1 V_{ub} V_{cs}^*$ | $-0.025 + 0.001r$ | $-0.014 + 0.002r$ | $1.68 \cdot 10^{-7} \left(\frac{a_1}{1.02}\right)^2$ |

Similar values are used for numerical etimates in [6]: e.g., for $B_{(s)} \to J/\psi\gamma$ decay, $a_2 = 0.15$ in our analysis corresponds to the effective Wilson coefficient $\overline{a}_q = 0.163$.

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