The XXIV International Workshop
High Energy Physics
and
Quantum Field Theory

Sochi, 2019

Invariant structures in extended Higgs sectors

Mikhail Dolgopolov, Lev Kuznetsov,
Elza Rykova, Albina Gurskaya

Introduction

In works concerning the physics of Higgs bosons, the specific basis in the space of isodoublets of complex scalar fields is chosen.
Higgs fields are considered near their vacuum expectations:

$$\langle \Phi_a^0 \rangle = \frac{v_a}{\sqrt{2}}, \qquad (a = 1, 2)$$

Parameter $\tan\beta = v_2/v_1$ plays an important role in phenomenology.
However, it cannot be unambiguously defined.
The meaning is lost upon transition to the Higgs basis in which the nontrivial vacuum expectation is in one of the doublets only.

Higgs basis

$$\langle \Phi_a^0 \rangle = \frac{v}{\sqrt{2}} \qquad \langle \Phi_b^0 \rangle = 0$$

is obtained from the general (arbitrary) basis by rotation in the space of scalar doublets:

$$\Phi_a = \Phi_1 c_\beta + e^{-i\theta} \Phi_2 s_\beta, \qquad \Phi_b = -\Phi_1 s_\beta + e^{i\theta} \Phi_2 c_\beta \tag{1}$$

$$\Phi_a = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v + \varphi_a^0 + iG^0) \end{pmatrix}, \qquad \Phi_b = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(\varphi_b^0 + iA) \end{pmatrix} \tag{2}$$

Potential in the form of the renormalized and gauge invariant polynomial:

$$V = Y_{a\bar{b}}\Phi^{\dagger}_{\bar{a}}\Phi_b + \frac{1}{2}Z_{a\bar{b}c\bar{d}}\left(\Phi^{\dagger}_{\bar{a}}\Phi_b\right)\left(\Phi^{\dagger}_{\bar{c}}\Phi_d\right) \quad (3)$$

The potential in Higgs basis:

$$U\big(\Phi_a, \Phi_b\big) = Y_1\big(\Phi_a^\dagger \Phi_a\big) + Y_2\big(\Phi_b^\dagger \Phi_b\big) + \big[Y_3\big(\Phi_a^\dagger \Phi_b\big) + \text{H.c.}\big] + \tag{4}$$

$$+\frac{1}{2} Z_1\big(\Phi_a^\dagger \Phi_a\big)^2 + \frac{1}{2} Z_2\big(\Phi_b^\dagger \Phi_b\big)^2 + Z_3\big(\Phi_a^\dagger \Phi_a\big)\big(\Phi_b^\dagger \Phi_b\big) + Z_4\big(\Phi_a^\dagger \Phi_b\big)\big(\Phi_b^\dagger \Phi_a\big) + \tag{5}$$

$$+\Big\{\frac{1}{2} Z_5\big(\Phi_a^\dagger \Phi_b\big)\big(\Phi_a^\dagger \Phi_b\big) + \big[Z_6\big(\Phi_a^\dagger \Phi_a\big) + Z_7\big(\Phi_b^\dagger \Phi_b\big)\big]\big(\Phi_a^\dagger \Phi_b\big) + \text{H.c.}\Big\}. \tag{6}$$

The form-invariants in an arbitrary basis:

$$
\begin{aligned}
&\mathrm{Y}_1 = -\mu_1^2 c_\beta^2 - \mu_2^2 s_\beta^2 - Re(\mu_{12}^2 e^{i\theta}) s_{2\beta},\\
&\mathrm{Y}_2 = -\mu_1^2 s_\beta^2 - \mu_2^2 c_\beta^2 - Re(\mu_{12}^2 e^{i\theta}) s_{2\beta},\\
&\mathrm{Y}_3 = \tfrac{1}{2}(\mu_1^2 - \mu_2^2) s_{2\beta} - Re(\mu_{12}^2 \mathbf{e} e^{i\theta}) c_{2\beta} - i Im(\mu_{12}^2 e^{i\theta}), \qquad (7)\\
&\text{and } \mathrm{Z}_1 = \lambda_1 c_\beta^4 + \lambda_2 s_\beta^4 + \tfrac{1}{2}\lambda_{345} s_{2\beta}^2 + 2 s_{2\beta}\big[c_\beta^2 Re(\lambda_6 e^{i\theta}) + s_\beta^2 Re(\lambda_7 e^{i\theta})\big],\\
&\mathrm{Z}_2 = \lambda_1 s_\beta^4 + \lambda_2 c_\beta^4 + \tfrac{1}{2}\lambda_{345} s_{2\beta}^2 + 2 s_{2\beta}\big[s_\beta^2 Re(\lambda_6 e^{i\theta}) + c_\beta^2 Re(\lambda_7 e^{i\theta})\big],\\
&\mathrm{Z}_3 = \tfrac{1}{4} s_{2\beta}^2[\lambda_1 + \lambda_2 - 2\lambda_{345}] + \lambda_3 - s_{2\beta} c_{2\beta} Re[(\lambda_6 - \lambda_7) e^{i\theta}],\\
&\mathrm{Z}_4 = \tfrac{1}{4} s_{2\beta}^2[\lambda_1 + \lambda_2 - 2\lambda_{345}] + \lambda_4 - s_{2\beta} c_{2\beta} Re[(\lambda_6 - \lambda_7) e^{i\theta}], \qquad (7)
\end{aligned}
$$

$$Z_5 = \frac{1}{4}s_{2\beta}^2[\lambda_1 + \lambda_2 - 2\lambda_{345}] + Re(\lambda_5 e^{2i\theta}) + ic_{2\beta}Im(\lambda_5 e^{2i\theta}) -$$
$$-s_{2\beta}c_{2\beta}Re[(\lambda_6 - \lambda_7)e^{i\theta}] - is_{2\beta}Im[(\lambda_6 - \lambda_7)e^{i\theta}],$$
$$Z_6 = \frac{1}{2}s_{2\beta}[\lambda_1 c_{2\beta}^2 - \lambda_2 s_{2\beta}^2 - \lambda_{345}c_{2\beta} - iIm(\lambda_5 e^{2i\theta})] + c_\beta c_{3\beta}Re(\lambda_6 e^{i\theta}) +$$
$$+s_\beta s_{3\beta}Re(\lambda_7 e^{i\theta}) + ic_\beta^2 Im(\lambda_6 e^{i\theta}) + is_\beta^2 Im(\lambda_7 e^{i\theta}),$$
$$Z_7 = \frac{1}{2}s_{2\beta}[\lambda_1 s_{2\beta}^2 - \lambda_2 c_{2\beta}^2 - \lambda_{345}c_{2\beta} - iIm(\lambda_5 e^{2i\theta})] + s_\beta s_{3\beta}Re(\lambda_6 e^{i\theta}) +$$
$$+c_\beta c_{3\beta}Re(\lambda_7 e^{i\theta}) + is_\beta^2 Im(\lambda_6 e^{i\theta}) + ic_\beta^2 Im(\lambda_7 e^{i\theta}), \qquad (7)$$

The condition of extremum of scalar potential:

$$\hat{v}^*_{\bar{a}} \left[Y_{a\bar{b}} + \frac{1}{2} v^2 Z_{a\bar{b}c\bar{d}} \, \hat{v}^*_{\bar{c}} \hat{v}_d \right] = 0 \tag{7}$$

The corresponding conditions of minimum of the scalar potential are:

$\mathrm{Y}_1 = -\frac{1}{2} Z_1 v^2, \qquad Y_3 = -\frac{1}{2} Z_6 v^2$

Summary

Connection of the parameters of the Higgs potential in an arbitrary and the Higgs bases.

The considered $U(2)$ transformations are general rotations between different bases of the Higgs isodoublets in TDM. In particular, if expressions (7) are substituted into the conditions of minimization, expressions for the masses of the bosons h, H, A and constants of self-interaction of scalars written in terms of invariants, the formulas in an arbitrary ("generic") basis should be obtained.

In MSSM U(2) rotation in the space of isodoublets changes the form of Yukawa interaction and the Lagrangian for scalar quarks–Higgs bosons. The physical meaning of the general Lagrangian cannot and should not change.