Effective potential in the conformal Standard Model

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Outline

Motivation

- Is the conformal symmetry fundamental?
- Yes, it is broken, but how?
- Can it be broken spontaneously?
- Conformal symmetry in GR?
- The hierarchy problem in the SM
- Is the SM an effective theory?
- Conformal symmetry breaking in QCD
- Application of the Coleman-Weinberg mechanism

Nonlinear realisations of affine and conformal symmetries

The Lorentz subgroup $SO(1,3)$ is chosen to be in linear realisation.

Nonlinear realisation of the affine group $\mathcal{A}(4)$ in the coset space over the Lorentz subgroup

$$\frac{\mathcal{A}(4)}{SO(1,3)} \sim \frac{P_m, L_{mn}, R_{mn}}{L_{mn}}$$

Nonlinear realisation of the conformal group in the coset with the same stability subgroup

$$\frac{SO(2,4)}{SO(1,3)} \sim \frac{P_m, L_{mn}, K_n, D}{L_{mn}}$$

Simultaneous covariance under both nonlinear realisations was constructed (see review: [E.A. Ivanov, PEPAN 2016])

[A.B. Borisov, V.I. Ogievetsky, Theor. Math. Phys. 1975]

GR as a nonlinear realisation

Einstein' gravity was obtained as a joint nonlinear realisation of the affine and conformal symmetries with the Lorentz symmetry as the stability subgroup. The minimal invariant action coincides with the Einstein–Hilbert action

$$-\frac{1}{16\pi G}\int d^4x\sqrt{-g}R,$$

where the dimensionful Newton constant G appeared after re-scaling of the dimensionless Goldstone field h_{mn}.

Thus, graviton is both a gauge boson of the diffeomorphism group and a Goldstone mode due to spontaneous symmetry breaking. Dilaton also appears as a Goldstone related to scale invariance breaking.

[A.B. Borisov, V.I. Ogievetsky, Theor. Math. Phys. 1975]
see also [A.B. Arbuzov, B.N. Latosh, arXiv:1904.06516 [gr-qc]]

The hierarchy problem in the SM

Quadratically divergent corrections to M_H:

$$M_H^2 = (M_H^0)^2 + \frac{3\Lambda^2}{8\pi^2 v^2}\left[M_H^2 + 2M_W^2 + M_Z^2 - 4m_t^2\right]$$

It looks unnatural to have $\Lambda \gg M_H$.

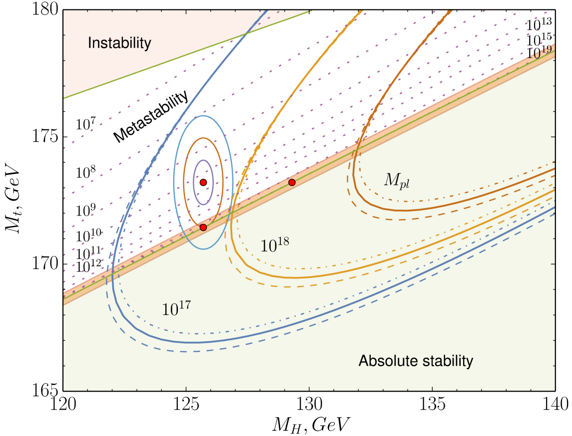
The most natural option would be $\Lambda \sim M_H$, i.e. everything is defined by the EW scale. But this is not the case of the SM and not found experimentally

Obviously, the problem is caused by the explicit breaking of the conformal symmetry in the SM

The best way out: to protect M_H by a (super)symmetry

W. Bardeen (1995): *"radiative stability of the Higgs boson mass, i.e. resolution of the naturalness problem, can be ensured by the classical scale invariance"*

The vacuum stability in SM



A relation between the EW and Planck scales?

Figure from: [A.V. Bednyakov, B.A. Kniehl, A.F. Pikelner, O.L. Veretin, Phys. Rev. Lett. '2015]

Conformal anomaly in QCD

The dimensional transmutation

$$\alpha_s(Q^2) = \frac{4\pi}{\beta_0 \ln(Q^2/\Lambda^2_{QCD})}$$

We do not know the origin of Λ_{QCD}, but we see

$$-\sqrt[3]{\langle \bar{q}\, q \rangle} \;\sim\; \sqrt[4]{G_{\mu\nu}G^{\mu\nu}} \;\sim\; M_q \;\sim\; \Lambda_{\rm QCD}$$

Very likely, the $\Lambda_{\rm QCD}$ scale comes from outside (massless!) QCD.
The QCD dynamics just helps it to propagate into M_q, $\langle \bar{q}\, q \rangle$, $\langle G_{\mu\nu}G^{\mu\nu} \rangle$, the scales of instantons and QCD vacuum domains

It is commonly assumed that radiatively induced dimensional transmutation is realized in QCD. It means a spontaneous breaking of conformal symmetry there contrary to the SM case

The Coleman-Weinberg mechanism (I)

S. Coleman & E. Weinberg, *"Radiative Corrections as the Origin of Spontaneous Symmetry Breaking"*, PRD 7 (1973) 1888

Semi-classical conformal-invariant $V = \lambda\phi^4/4!$ is transformed by quantum loop corrections into

$$V_{\text{eff}} = \frac{\lambda}{4!}\phi^4 + \frac{\lambda^2\phi^4}{256\pi^2}\left(\ln\frac{\phi^2}{M^2} - \frac{25}{6}\right)$$

where M is introduced to avoid infrared divergences

Conditions:

$$\left.\frac{\partial^2 V_{\text{eff}}(\phi)}{\partial\phi^2}\right|_{\phi=0} = m_\phi^2 \equiv 0, \qquad \left.\frac{\partial^4 V_{\text{eff}}(\phi)}{\partial\phi^4}\right|_{\phi=M} = \lambda$$

The quadratic hierarchy problem is removed by the scale-invariance condition $m_\phi = 0$. But we get a new hierarchy

The Coleman-Weinberg mechanism (II)

$$V_{\text{eff}} = \frac{\lambda}{4!}\phi^4 + \frac{\lambda^2\phi^4}{256\pi^2}\left(\ln\frac{\phi^2}{M^2} - \frac{25}{6}\right)$$

The minimum of the potential is not at zero:

$$\left.\frac{\partial V}{\partial \phi}\right|_{\phi=v} = 0 \qquad \Rightarrow \qquad \lambda \ln\frac{v^2}{M^2} = -\frac{32\pi^2}{3} + \frac{11}{3}\lambda$$

Non-perturbativity!?

Let's analyze the situation and construct an effective QFT model for $\phi \sim v$ and make the shift

$$\phi = \varphi + v$$

N.B. The Brout-Englert-Higgs mechanism gives then masses to gauge bosons and fermions

The Coleman-Weinberg mechanism (III)

In the vicinity of the minimum (in the mean field)

$$V_{\text{eff}}(\varphi) = \frac{\lambda}{4!}(\varphi + v)^4 + \frac{\lambda^2(\varphi+v)^4}{256\pi^2}\left(\ln\frac{(\varphi+v)^2}{M^2} - \frac{25}{6}\right) \quad \Rightarrow$$

$$V_{\text{eff}}(\varphi) \approx \frac{m_\varphi^2}{2}\varphi^2 + \frac{\kappa}{3!}\varphi^3 + \frac{\lambda_0}{4!}\varphi^4 + \mathcal{O}(\varphi^5)$$

$$m_\varphi^2 = \left.\frac{\partial^2 V_{\text{eff}}(\varphi)}{\partial\varphi^2}\right|_{\varphi=0} = \frac{\lambda^2}{32\pi^2}v^2 = \frac{\lambda_0}{11}v^2$$

$$\kappa = \left.\frac{\partial^3 V_{\text{eff}}(\varphi)}{\partial\varphi^3}\right|_{\varphi=0} = \frac{5\lambda^2}{32\pi^2}v = \frac{5\lambda_0}{11}v$$

$$\lambda_0 = \left.\frac{\partial^4 V_{\text{eff}}(\varphi)}{\partial\varphi^4}\right|_{\varphi=0} = \frac{11\lambda^2}{32\pi^2}$$

The Coleman-Weinberg mechanism (IV)

Open questions:

Is M an arbitrary renormalization scale or a physical one?
Remind Λ_{QCD}.

$\lambda_0 \sim \lambda^2 \qquad \leftrightarrow \qquad$ triviality?

A new hierarchy problem: $v \ll M$?

How do higher order corrections change the result?

Resummation of both legs and loops?

What happens in other QFT models?

COLEMAN-WEINBERG MECHANISM FOR A SUPERPOTENTIAL

[A.A. & D. Cirilo-Lombardo, *"Radiatively Induced Breaking of Conformal Symmetry in a Superpotential,"* Phys. Lett. B 758 (2016) 125]

Condensates were found in a simple SUSY model with one scalar and one fermion:

$$v^2 \equiv \langle\varphi\rangle^2 = M^2 \exp\left\{-\frac{196\pi^2}{\lambda}\right\}$$
$$\langle\bar{\psi}\psi\rangle = -v^3\frac{2\lambda}{7}$$

For $\lambda \lesssim 1$ and $v \sim 100$ GeV we get $M \gg M_{\text{Planck}}$

But what is the actual value of λ?

Implications for the Standard Model

The one-loop effective potential in the SM (Λ^2 removed):

$$
\begin{aligned}
V^{(0)} &= \frac{m^2}{2}\varphi^2 + \frac{\lambda}{4!}\varphi^4 \\
V^{(1)} &= \frac{1}{64\pi^2}\Big\{H^2\left(\overline{\ln}H - \frac{3}{2}\right) + 3G^2\left(\overline{\ln}G - \frac{3}{2}\right) - 4N_C T^2\left(\overline{\ln}T - \frac{3}{2}\right) \\
&+ 6W^2\left(\overline{\ln}W - \frac{5}{6}\right) + 3Z^2\left(\overline{\ln}Z - \frac{5}{6}\right)\Big\} \\
H &= m^2 + \frac{\lambda^2}{2}\varphi^2, \quad T = \frac{y_t^2}{2}\varphi^2, \quad G = m^2 + \frac{\lambda^2}{6}\varphi^2, \quad W = \frac{g^2}{4}\varphi^2, \\
Z &= \frac{g^2 + g'^2}{4}\varphi^2, \quad \overline{\ln}X = \ln\frac{X}{\mu^2} + \gamma - \ln 4\pi
\end{aligned}
$$

People say that the C-W mechanism doesn't work in SM since y_t is big while λ, g, and g' are small. But what if λ is really big?

[C. Ford, I. Jack, D.R.T. Jones, NPB'1993, arXiv:hep-ph/0111190]

HIGGS COUPLING CONSTANTS

Collider	HL-LHC	ILC_{250}	$CLIC_{380}$	$CEPC_{240}$	FCC-$ee_{240\to365}$
$\mathcal{L}$ (ab^{-1})	3	2	1	5.6	5 + 0.2 + 1.5
Years		11.5 [5]	8	7	3 + 1 + 4
g_{HZZ} (%)	1.5 / 3.6	0.29 / 0.47	0.44 / 0.66	0.18 / 0.52	0.17 / 0.26
g_{HWW} (%)	1.7 / 3.2	1.1 / 0.48	0.75 / 0.65	0.95 / 0.51	0.41 / 0.27
g_{Hbb} (%)	3.7 / 5.1	1.2 / 0.83	1.2 / 1.0	0.92 / 0.67	0.64 / 0.56
g_{Hcc} (%)	SM / SM	2.0 / 1.8	4.1 / 4.0	2.0 / 1.9	1.3 / 1.3
g_{Hgg} (%)	2.5 / 2.2	1.4 / 1.1	1.5 / 1.3	1.1 / 0.79	0.89 / 0.82
$g_{H\tau\tau}$ (%)	1.9 / 3.5	1.1 / 0.85	1.4 / 1.3	1.0 / 0.70	0.66 / 0.57
$g_{H\mu\mu}$ (%)	4.3 / 5.5	4.2 / 4.1	4.4 / 4.3	3.9 / 3.8	3.9 / 3.8
$g_{H\gamma\gamma}$ (%)	1.8 / 3.7	1.3 / 1.3	1.5 / 1.4	1.2 / 1.2	1.2 / 1.2
$g_{HZ\gamma}$ (%)	11. / 11.	11. / 10.	11. / 9.8	6.3 / 6.3	10. / 9.4
g_{Htt} (%)	3.4 / 2.9	2.7 / 2.6	2.7 / 2.7	2.6 / 2.6	2.6 / 2.6
g_{HHH} (%)	50. / 52.	28. / 49.	45. / 50.	17. / 49.	19. / 34.
Γ_H (%)	SM	2.4	2.6	1.9	1.2

[J. de Blas et al. arXiv:1905.03764; A. Blondel et al. arXiv:1906.02693]

The crucial points

- It's worth to explore spontaneous breaking of conformal symmetry both in QFT and GR
- Conformal anomalies are natural for most QFT models
- The C-W formalism allows to evaluate them
- A modified interpretation of the C-W mechanism is suggested
- An effective field theory approximation is applied
- The assumption of the classical conformal invariance does solve the hierarchy problem in SM
- Instead of a linear hierarchy we get a logarithmic relation with a new (higher) scale

Open problems and questions

- The Dawn of the Post-Naturalness Era?
- Measurement of the Higgs self-couplings?
- Interpretation of the C-W mechanism?
- Origins of the M_{Pl}, EW, and QCD scales?
- Is there an IR/UV connection?
- Where is the new physics scale?