Entanglement entropy in strongly correlated systems with confinement/deconfinement phase transition and anisotropy

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Based on work in progress with I.Ya.Aref'eva and A.Patrushev

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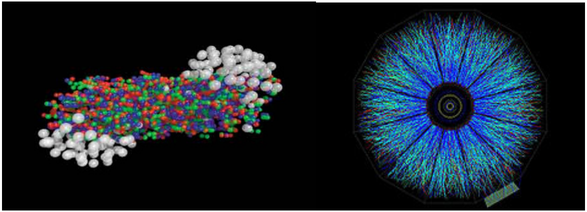
QFTHEP'2019

Heavy-ion collisions

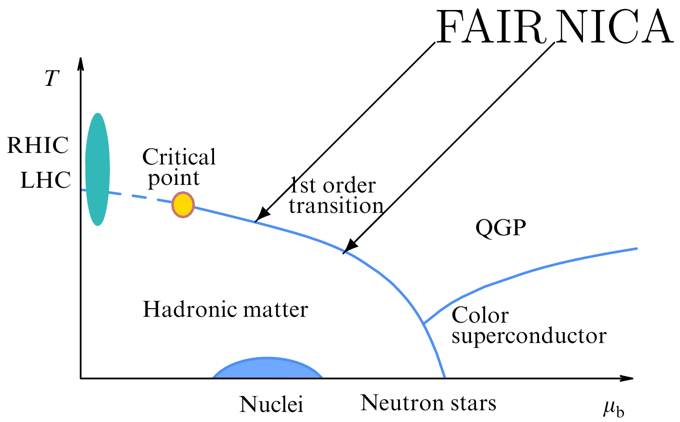


STAR at RHIC; LHC

- Quark gluon plasma formation $\tau \sim 0.1 fm/c$
- Spatial anisotropy with $\tau_{iso} \sim 2 fm/c$
- Strongly correlated system $\Longrightarrow$ Non-perturbative methods

M. Strickland, 1312.2285 [hep-ph]

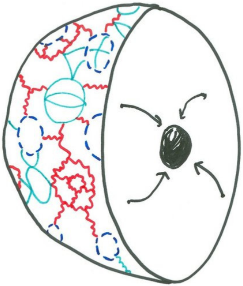
Phase diagram



$T_{cr} \sim 0.15$ GeV and $\mu_{cr} \sim 0.15$ GeV

AdS/CFT correspondence

J.Maldacena, D.Witten, S.Gubser and others



AdS/CFT correspondence $\Longrightarrow$ $\frac{\eta}{s} = \frac{1}{4\pi}$
G.Policastro, D.Son, A.Starinets, PRL '01

Motivation

Purpose: study of the QCD phase diagram as a function of temperature T and chemical potential μ in the anisotropic background

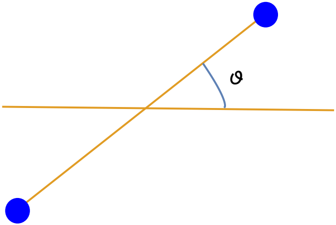
Multiplicity $\mathcal{M} \propto s_{AdS}^{0.33}$ vs $\mathcal{M} \propto s_{LHC}^{0.155}$

$\mathcal{M} \propto s^{\frac{2}{\nu+2}}$, $\nu = 4.5$ I.Aref'eva, A.Golubtsova, JHEP '14

Formulation of the problem

It is natural to expect, that the phase transition depends on the orientation of the quark pair relative to the anisotropy axis.

Anisotropy axis in the QGP created in the HIC is defined by the axes of ions collisions.
Confinement/deconfinement phase transition line depends on the angle θ between quarks' line and heavy ions collisions line.



Action and metric ansatz

$$S = \int \frac{d^5x}{16\pi G_5} \sqrt{-\det(g_{\mu\nu})} \left[R - \frac{f_1(\phi)}{4} F_{(1)}^2 - \frac{f_2(\phi)}{4} F_{(2)}^2 - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right]$$

$F_{\mu\nu}^{(1)} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $F_{\mu\nu}^{(2)} = q \; dy^1 \wedge dy^2$

$$ds^2 = G_{\mu\nu} dx^\mu dx^\nu = \frac{b(z)}{z^2} \left[-g(z) dt^2 + dx^2 + z^{2-\frac{2}{\nu}} (dy_1^2 + dy_2^2) + \frac{dz^2}{g(z)} \right]$$

ν – anisotropy parameter, $b(z) = e^{\frac{cz^2}{2}}$ – deformation factor, z_h – horizon

Blackening function $g(z)$:

$$g(z) = 1 - \frac{z^{2+\frac{2}{\nu}}}{z_h^{2+\frac{2}{\nu}}} \frac{\mathfrak{G}(\frac{3}{4}cz^2)}{\mathfrak{G}(\frac{3}{4}cz_h^2)} - \frac{\mu^2 c z^{2+\frac{2}{\nu}} e^{\frac{cz_h^2}{2}}}{4\left(1 - e^{\frac{cz_h^2}{4}}\right)^2} \mathfrak{G}(cz^2) + \frac{\mu^2 c z^{2+\frac{2}{\nu}} e^{\frac{cz_h^2}{2}}}{4\left(1 - e^{\frac{cz_h^2}{4}}\right)^2} \frac{\mathfrak{G}(\frac{3}{4}cz^2)}{\mathfrak{G}(\frac{3}{4}cz_h^2)}$$

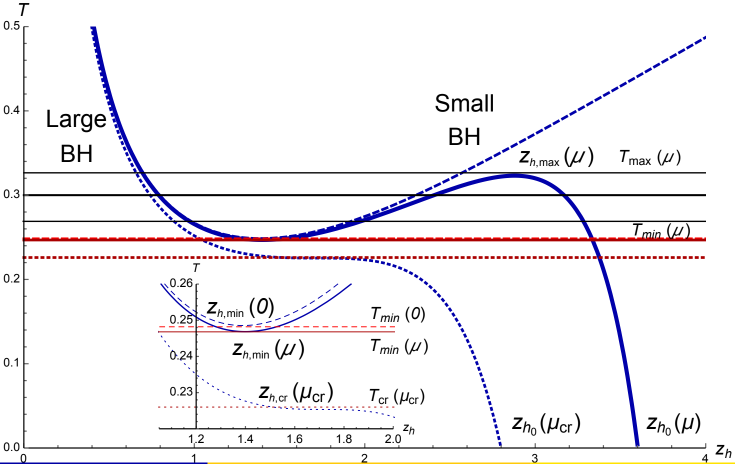
$$\mathfrak{G}(x) \equiv \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!(1+n+\frac{1}{\nu})}$$

I.Aref'eva, K.Rannu, JHEP '18

Thermodynamics (temperature)

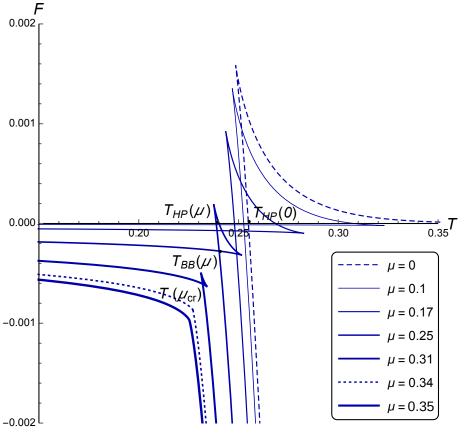
$$T = \frac{g'(z)}{4\pi}$$

I. Aref'eva, K.Rannu, JHEP '18



Thermodynamics (free energy)

$$F(z_h, c, \nu) = \int_{z_h}^{\infty} s(z_h, c, \nu)\, T'(z_h, c, \nu)\, dz_h$$



Temporal Wilson loop

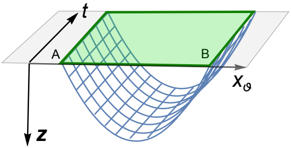
$$W[C_\vartheta] = e^{-S_{\vartheta,t}} \qquad \vec{n}: \qquad n_x = \cos\vartheta, \quad n_y = \sin\vartheta$$

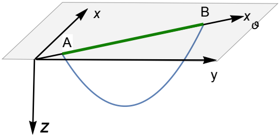
$$S = -\frac{\tau}{2\pi\alpha'}\int d\xi\ M(z(\xi))\sqrt{\mathcal{F}(z(\xi)) + (z'(\xi))^2}, \quad \tau = \int dt$$

$$M(z(\xi)) = \frac{b(z(\xi))}{z(\xi)^2}\, e^{\sqrt{\frac{2}{3}}\phi(z)},\ \mathcal{F}(z(\xi)) = g(z(\xi))\left(z(\xi)^{2-\frac{2}{\nu}}\sin^2(\theta) + \cos^2(\theta)\right)$$

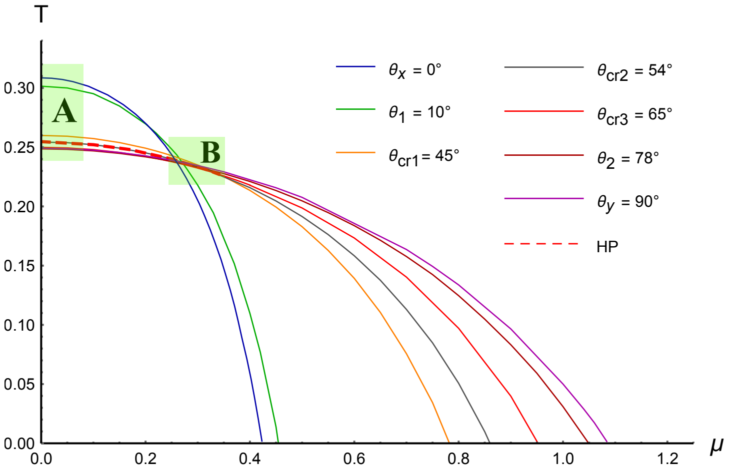
Let us introduce the effective potential:

$$\mathcal{V}(z) \equiv M(z)\sqrt{\mathcal{F}(z)}$$



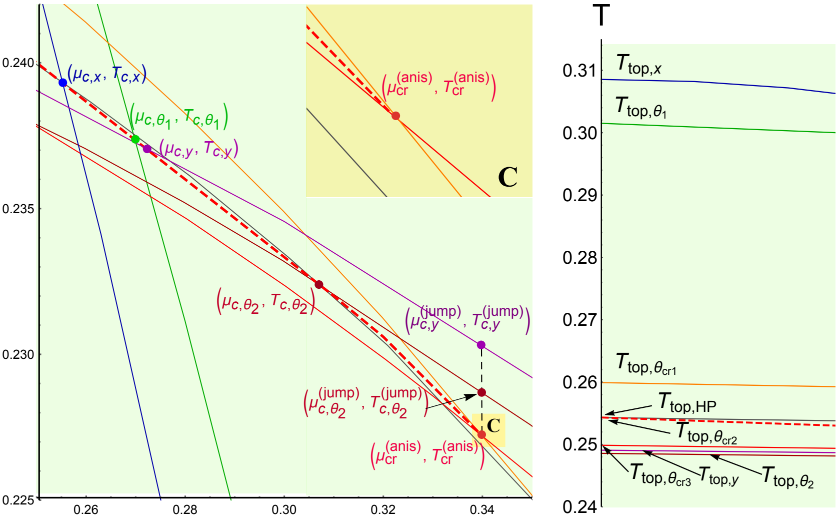


Phase diagram using Wilson loop angular parametrization



I.Aref'eva, K.Rannu, P.S., PLB '19

Phase diagram using Wilson loop angular parametrization



Entanglement Entropy

Quantum system is described by $|\Psi\rangle$ in a Hilbert space $\mathcal{H}$. For multiple degrees of freedom

$$\mathcal{H} = \mathcal{H}_{\mathcal{A}} \otimes \mathcal{H}_{\mathcal{B}}$$

$$\langle O \rangle = \langle \Psi | O | \Psi \rangle = \mathrm{Tr}[\rho \cdot O], \quad \rho = |\Psi\rangle\langle\Psi| - \text{density matrix}$$

The reduced density matrix ρ_A for the subsystem A by tracing out with respect to $\mathcal{H}_B$ by

$$\rho_A = \mathrm{Tr}_B[\rho]$$

The entanglement entropy is defined as the von-Neumann entropy for ρ_A

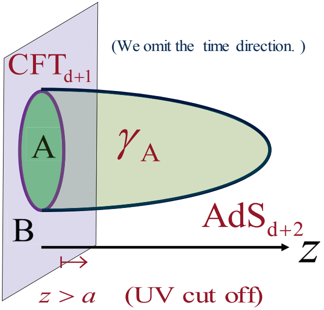
$$S_A = -\mathrm{Tr}[\rho_A \log \rho_A]$$

Holographic Entanglement Entropy (HEE)

Area law proposition S.Ryu,T.Takayanagi, PRL '06

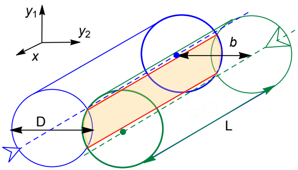
$$S_A = \frac{\textit{Area of } \gamma_A}{4G_N^{(d+2)}}$$

where γ_A is the d dimensional static minimal surface in AdS_{d+2} whose boundary is given by ∂A.

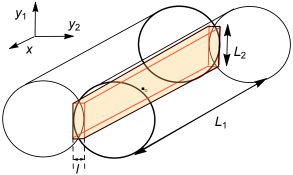


HEE and phase transition

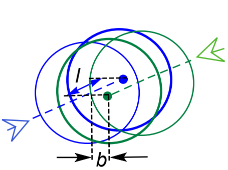
- Entanglement as a Probe of Confinement
  I.Klebanov, D.Kutasov, A.Murugan 0709.2140,
  A.Lewkowycz 1204.0588
- Isotropic model
  S.He, S-Y.Wu, Y.Yang, P-H.Yuan 1301.0385,
  D.Dudal and S.Mahapatra 1708.0699
- Wilson loops in anisotropic model
  D.Ageev, I.Aref'eva, A.Golubtsova, E.Gourgoulhon, NPB '17

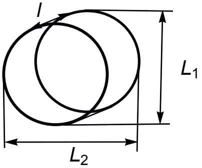


A



B





Subsystem A_{xYY} cut out along x-direction:

$$x \in [0, |l_x| << L_x], \quad y_1 \in [0, L_{y_1}], \quad y_2 \in [0, L_{y_2}]$$

$$\mathcal{S}_{xYY} = \frac{\mathcal{A}_{xYY}}{L_{y_1} L_{y_2}} = \int_0^{z_*} \frac{b_s^{3/2}(z)}{z^{1+2/\nu}} \sqrt{1 + \frac{z'^2}{g(z)}} dx$$

,

$$b_s(z, \nu) \equiv e^{cz^2/2 + \sqrt{\frac{2}{3}}\phi(z, z_h, \nu)}$$

Subsystem A_{yXY} cut out along y_1-direction (which is equivalent to y_2):

$$x \in [0, L_x], \quad y_1 \in [0, |l_{y_1}| << L_{y_1}], \quad y_2 \in [0, L_{y_2}]$$

$$\mathcal{S}_{yXY} = \int_0^{z*} \frac{b_s^{3/2}(z)}{z^{1+2/\nu}} \sqrt{1 + \frac{z'^2}{g(z)\, z^{2-2/\nu}}} dx$$

BI-action:

$$\mathcal{S} \;=\; \frac{T}{2\pi\alpha} \int_{-\ell}^{\ell} M(z) \sqrt{\mathcal{F}(z) + z'^2} dx, \quad \mathcal{V}(z) = M(z)\sqrt{\mathcal{F}(z)}$$

Renormalization

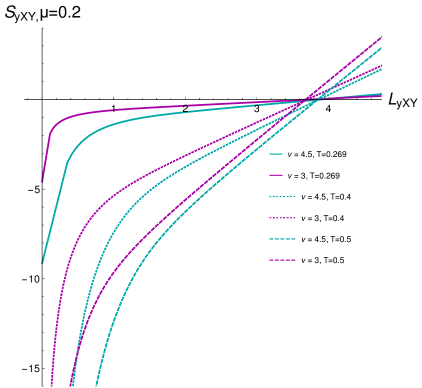
For xYY case and $1 \leq \nu \leq 1.67$ we have to perform renormalization (just one substraction):

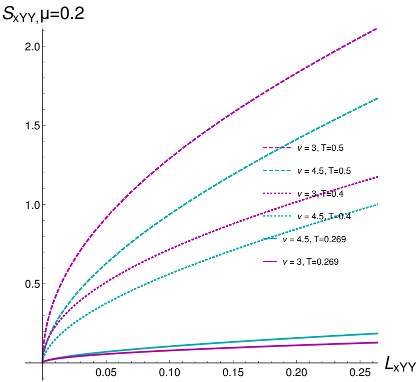
$$\frac{1}{2}\mathcal{S}_{xYY,ren} = \int_{\epsilon}^{z_*} dz \left[\frac{b_s^{3/2}(z)}{z^{1+2/\nu}} \frac{1}{\sqrt{g(z)(1-\frac{\mathcal{V}^2_{xYY}(z_*)}{\mathcal{V}^2_{xYY}(z)})}} - \frac{b_{s,as}^{3/2}(z)}{z^{1+2/\nu}} \right] + \int^{z_*} \frac{b_{s,as}^{3/2}(z)}{z^{1+2/\nu}} dz$$

For xYY case and $\nu > 1.67$ we have an integrable singularity
For yXY case and $\nu \geq 1$ we have nonintegrable singularity and have to perform renormalization (just one substraction):

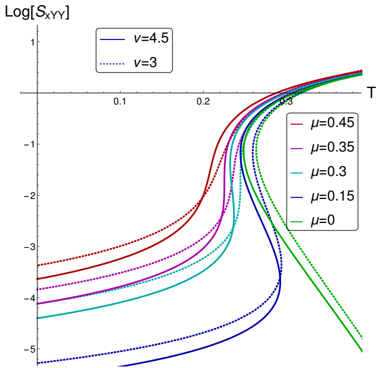
$$\frac{1}{2}\mathcal{S}_{yXY,ren} = \int_{\epsilon}^{z_*} dz \left[\frac{b_s^{3/2}(z)}{z^{2+1/\nu}} \frac{1}{\sqrt{g(z)(1-\frac{\mathcal{V}^2_{yXY}(z_*)}{\mathcal{V}^2_{yXY}(z)})}} - \frac{b_{s,as}^{3/2}(z)}{z^{2+1/\nu}} \right] + \int^{z_*} \frac{b_{s,as}^{3/2}(z)}{z^{2+1/\nu}} dz$$

HEE dependence on length

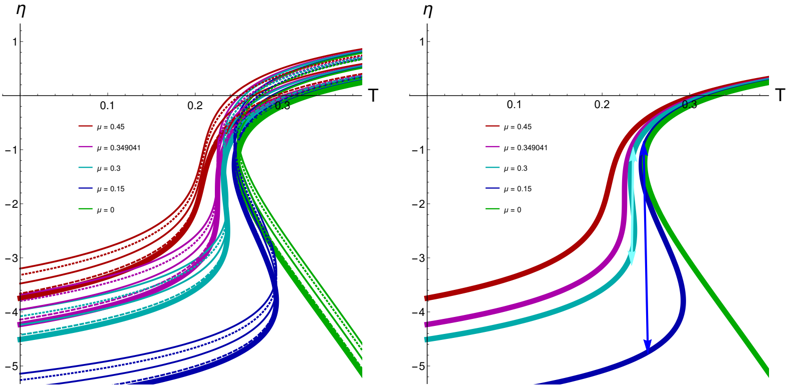




HEE dependence on temperature



HEE density for different angles ($\theta = 0^0, 30^0, 45^0, 60^0, 90^0$)



$$\eta = \frac{dS(\ell)}{d\ell} = \frac{b_s^{3/2}(z_*)}{z_*^{1+2/\nu}} \qquad \eta = V_{xYY}(z_*) = V_{yXY}(z_*) = V_{\theta Y}(z_*)$$

Results

- Holographic entanglement entropy (HEE) was investigated for subsystems with arbitrary spatial orientation
- The expressions of HEE were renormalized
- Density of HEE was calculated
- The dependence of HEE and its density on temperature and characteristic length were studied

- *MAIN OPEN QUESTION: how to measure the HEE?*

Thank you for your attention!