On photon splitting in Lorentz-violating QED

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based on:
arXiv:1903.08464 [hep-ph] JCAP 1904 (2019) 054
arXiv:1906.08221 [astro-ph.HE]

23 September 2019

Motivation of Lorentz Invariance violation

- Different approaches to quantum gravity
  - Discrete spacetime, loop quantum gravity, non-commutative geometry e.t.c.
    Gambini, Pullin 1999
    Douglas, Nekrasov, 2001
    ...
  - Modifications of general relativity with large space derivatives (Hořava-Lifshitz e.t.c.)
    Hořava 2009
    Blas, Pujolas, Sibiryakov 2010
    ...
- Phenomenologically in non-gravity sector in the framework of EFT
  - Special type of LV (preserving other symmetries, motivations to concrete QG approaches)
    For example, $E^2 = m^2 + p^2(1+\delta) \pm \frac{p^4}{M_{LV}^2} \pm \dots$
  - The most general type — Standard Model Extension (SME)
    Kostelecky, Colladay 1998

Lorentz Invariance Violation: ways to constrain it

- Accurate measurements in the labs on the Earth
  Michelson-Morley-type experimiments, fine structure measurements..
- Observations in high-energy astrophysics:
  - Time-of-flight measurements (photons, neutrino, gravity waves..)
  - Modifications of cross-sections for some particle reactions, crutial to astrophysical processes (photon decay, modification of shower formation..)
- Accumulated effects in cosmology (structure grows e.t.c.)

Summary: *Data tables: Kostelecky, Russel, 2008-2018. arXiv: 0801.0287*

The model

LV: extra term quartic on spacial derivative, suppressed by LV mass scale M_{LV}.

$$\mathcal{L}_{QED}^{LV} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \mp \frac{1}{2M_{LV}^2}F_{ij}\Delta^2 F^{ij} + i\bar{\psi}\gamma^\mu D_\mu\psi - m\bar{\psi}\psi.$$

$$\Downarrow$$

Modified dispersion relation

$$E_\gamma^2 = p_\gamma^2 \pm \frac{p_\gamma^4}{M_{LV}^2}$$

- The similar dispersion relation may be considered for electrons. The constraint on LV mass scale $M_{LV,e} > 2 \times 10^{16}$ GeV is much better than for photons. *Liberati et.al., 2012*
- Dispersion relations like $E^2 = p^2 \pm \frac{p^3}{M_{LV,(1)}}$ are also studied. However, $M_{LV,(1)}$ is constrained at the level M_{pl} or higher; CPT is also violated.
- Quartic LV term may be generated as loop correction from possible quadratic LV in fermions *Cambiaso Lehnert Potting 2014, P.S. 2018*

Constraints on M_{LV} for photons

$$\mathcal{L}_{QED}^{LV} = \mathcal{L}_{QED}^{LI} \mp \frac{1}{2M_{LV}^2} F_{ij}\Delta^2 F^{ij} \qquad \leftrightarrow \qquad E^2 = p^2 \pm \frac{p^4}{M_{LV}^2}$$

Photon time of flight constraints (both signs $\pm$) (95% CL)

AGN: $M_{LV} > 7.3 \times 10^{10}$ GeV — *H.E.S.S. coll. 2019*

GRB: $M_{LV} > 1.3 \times 10^{11}$ GeV — *Fermi-LAT coll. 2013*

Subluminal LV — sign minus $E^2 = p^2 - \frac{p^4}{M_{LV}^2}$ (95% CL)

- Extragalactic photon absorption on EBL
  $M_{LV} > 7.8 \cdot 10^{11}$ GeV — *H.E.S.S. coll. 2019*
- No suppression of atmosphere shower formation
  (HEGRA: $E = 75$ TeV, 2.7σ) $M_{LV} > 2.1 \cdot 10^{11}$ GeV — *Rubtsov, P.S., Sibiryakov 2017*
  (Tibet: $E = 140$ TeV, 5σ) $M_{LV} > 5.7 \cdot 10^{12}$ GeV — *P.S. 2019*

Constraints on M_{LV} for photons: Superluminal LV

Superluminal LV — sign plus in disp. relation $E^2 = p^2 + \frac{p^4}{M_{LV}^2}$

Photon decay $\gamma \to e^+ e^-$

- Forbidden in LI, but allowed in LV if the photon energy exceed a certain threshold. *Coleman, Glashow 1997*
- If effective photon mass $m^2_{\gamma,eff} \equiv E^2 - p^2 \geq (2m_e)^2$ reaction is kinemnatically allowed!
- The constraint (photons from Crab nebula)

  (HEGRA: $E = 75$ TeV, 2.7σ) $M_{LV} > 2.8 \times 10^{12}$ GeV, 95% CL.

  Martinez-Huerta, Perez-Lorenzana 2017

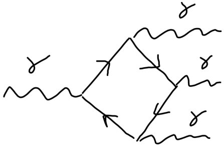
  (Tibet: $E = 140$ TeV, 5σ) $M_{LV} > 2 \cdot 10^{13}$ GeV *P.S. 2019*

If $m_{\gamma,eff} < 2m_e$ photon decay $\gamma \to e^+ e^-$ is forbidden but loop process of photon splitting $\gamma \to n\gamma$ may be allowed

The photon splitting

$$\mathcal{L}_{QED}^{LV} = \mathcal{L}_{QED}^{LI} - \frac{1}{2M_{LV}^2} F_{ij} \Delta^2 F^{ij} \qquad \leftrightarrow \qquad E^2 = p^2 + \frac{p^4}{M_{LV}^2}$$

- Photon splitting $\gamma \to n\gamma$ is kinematically allowed whenever the photon dispersion relation is superluminal (sign plus in dispersion relation)
- Splitting to two photons $\gamma \to 2\gamma$ do not occur due to the Furry theorem
- The main splitting process is $\gamma \to 3\gamma$



Far from pair production threshold Euler-Heisenberg effective Lagrangian may be used

$$\mathcal{L}_{E\text{-}H} = \frac{2\alpha^2}{45 m_e^4} \left[\left(\frac{1}{2} F_{\mu\nu} F^{\mu\nu} \right)^2 + 7 \left(\frac{1}{8} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} \right)^2 \right] .$$

The photon splitting

The width of of photon splitting in a similar model was estimated by

Gelmini Nussinov Yaguna 2005

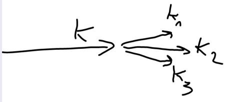
- Authors used notation of effective photon mass for dispersion relation $E^2 = p^2 + \frac{p^3}{M_{LV,(1)}}$
- They estimated the decay width of a "massive photon" in the rest frame, followed by subsequent boost to laboratory frame
- The estimation for the decay width (for quartic disp.relation):

$$\Gamma(\gamma \to 3\gamma) \sim 10^{-20} \frac{E_\gamma^{19}}{m_e^8 M_{LV}^{10}}.$$

- 50 TeV photons from Crab Nebula detected $\to$ estimated constraint $M_{LV} > 10^{13}$ GeV.
- More precise calculation seems to be necessary

Photon splitting calculation. The matrix element.

Kinematics



Angles between outgoing photons are assumed to be small.

$$\mathcal{L}_{E\text{-}H} = \frac{2\alpha^2}{45 m_e^4} \left[\left(\frac{1}{2} F_{\mu\nu} F^{\mu\nu} \right)^2 + 7 \left(\frac{1}{8} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} \right)^2 \right] .$$

We factorize polarization vectors in the matrix element (w/o factor $\frac{2\alpha^2}{45 m_e^4}$)

$$\mathcal{M} = \mathcal{M}_{\mu\nu\rho\lambda}(k_1, k_2, k_3, k) \ \times \ \varepsilon_\mu(k_1)\varepsilon_\nu(k_2)\varepsilon_\rho(k_3)\varepsilon^*_\lambda(k).$$

$$\mathcal{M}_{\lambda_1\lambda_2\lambda_3\lambda} = A_{\lambda_1\lambda_2\lambda_3\lambda} + \frac{7}{16}\tilde{A}_{\lambda_1\lambda_2\lambda_3\lambda}.$$

$$A_{\lambda_1\lambda_2\lambda_3\lambda} = 8(T_{\lambda_1\lambda_2}(k_1,k_2)T_{\lambda_3\lambda}(k_3,k) + T_{\lambda_1\lambda_3}(k_1,k_3)T_{\lambda_2\lambda}(k_2,k) + T_{\lambda_1\lambda}(k_1,k)T_{\lambda_3\lambda_2}(k_3,k_2)),$$

$$\tilde{A}_{\lambda_1\lambda_2\lambda_3\lambda} = 8(\tilde{T}_{\lambda_1\lambda_2}(k_1,k_2)\tilde{T}_{\lambda_3\lambda}(k_3,k) + \tilde{T}_{\lambda_1\lambda_3}(k_1,k_3)\tilde{T}_{\lambda_2\lambda}(k_2,k) + \tilde{T}_{\lambda_1\lambda}(k_1,k)\tilde{T}_{\lambda_3\lambda_2}(k_3,k_2)).$$

$$T_{\mu\nu}(k,p) = 2(pk)g_{\mu\nu} - 2p_\mu k_\nu, \qquad \tilde{T}_{\mu\nu}(k,p) = -4k^\rho p^\lambda \epsilon^{\mu\nu\rho\lambda}.$$

Squared Matrix element:

$$\overline{|\mathcal{M}|^2} = \frac{1}{2}\sum_{pols}\mathcal{M}^*\mathcal{M} = \mathcal{M}^*_{\alpha\beta\gamma\delta}\mathcal{M}_{\mu\nu\rho\lambda}\sum_{s_1}\varepsilon^{*(s_1)}_{\alpha}(k_1)\varepsilon^{(s_1)}_{\mu}(k_1)\sum_{s_2}\varepsilon^{*(s_2)}_{\beta}(k_2)\varepsilon^{(s_2)}_{\nu}(k_2)\cdot$$

$$\cdot\sum_{s_3}\varepsilon^{*(s_3)}_{\gamma}(k_3)\varepsilon^{(s_3)}_{\rho}(k_3)\,\frac{1}{2}\sum_{s}\varepsilon^{(s)}_{\delta}(k)\varepsilon^{*(s)}_{\lambda}(k).$$

Polarization sums in our model:

$$\sum_{s=1,2}\varepsilon^{*(s)}_{\mu}(k)\varepsilon^{(s)}_{\nu}(k) = -g_{\mu\nu} - \frac{k_0^2}{M_{LV}^2}u_\mu u_\nu,$$

$u_\mu = (1,0,0,0)$.

Calculations in FeynCalc plugin for Wolfram Mathematica:

A very large output was generated. Here is a sample of it:

$$16384\ aa1^2\ aa2^2\ aa3^2\ k1\cdot k4^2\ k2\cdot k3^2\ u^{24} + 16384\ aa1^2\ aa2^2\ aa3^2\ k1\cdot k3^2\ k2\cdot k4^2\ u^{24} + 16384\ aa1^2\ aa2^2\ aa3^2\ k1\cdot k2^2\ k3\cdot k4^2\ u^{24} +$$

$$32768\ aa1^2\ aa2^2\ aa3^2\ k1\cdot k3\ k1\cdot k4\ k2\cdot k3\ k2\cdot k4\ u^{24} + 32768\ aa1^2\ aa2^2\ aa3^2\ k1\cdot k2\ k1\cdot k4\ k2\cdot k3\ k3\cdot k4\ u^{24} + 32768\ aa1^2\ aa2^2\ aa3^2\ k1\cdot k2\ k1\cdot k3\ k2\cdot k4\ k3\cdot k4\ u^{24} +$$

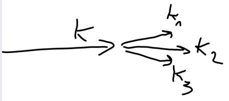
$$\ll 1822 \gg + 58880\ aa1^2\ k1\cdot k2\ k1\cdot u\ k2\cdot u\ k3\cdot k4\ k3\cdot u\ k4\cdot u + 69632\ aa1^2\ aa2^2\ k1\cdot k2\ k1\cdot u\ k2\cdot u\ k3\cdot k4\ k3\cdot u\ k4\cdot u + 58880\ aa2^2\ k1\cdot k2\ k1\cdot u\ k2\cdot u\ k3\cdot k4\ k3\cdot u\ k4\cdot u +$$

$$58880\ aa1^2\ aa3^2\ k1\cdot k2\ k1\cdot u\ k2\cdot u\ k3\cdot k4\ k3\cdot u\ k4\cdot u + 58880\ aa2^2\ aa3^2\ k1\cdot k2\ k1\cdot u\ k2\cdot u\ k3\cdot k4\ k3\cdot u\ k4\cdot u + 69632\ aa3^2\ k1\cdot k2\ k1\cdot u\ k2\cdot u\ k3\cdot k4\ k3\cdot u\ k4\cdot u$$

Show Less | Show More | Show Full Output | Set Size Limit...

Photon splitting calculation. Phase volume.

Kinematics



Angles between outgoing photons are assumed to be small.

We work in terms of longitudial and transverse momenta, $k_i^{\perp}$ is assumed to be of the order of k^2/M_{LV}

We introduce dimensionless variables α_1, α_2, α_3, β_1, β_2:

$$k_i^{\parallel} = k\,\alpha_i, \qquad \alpha_1 + \alpha_2 + \alpha_3 = 1, \qquad k_i^{\perp} = \frac{k^2}{M_{LV}} \cdot \beta_i.$$

$$(k_3^{\perp})^2 = (k_1^{\perp})^2 + (k_2^{\perp})^2 + 2k_1^{\perp} k_2^{\perp} \cos\varphi_2.$$

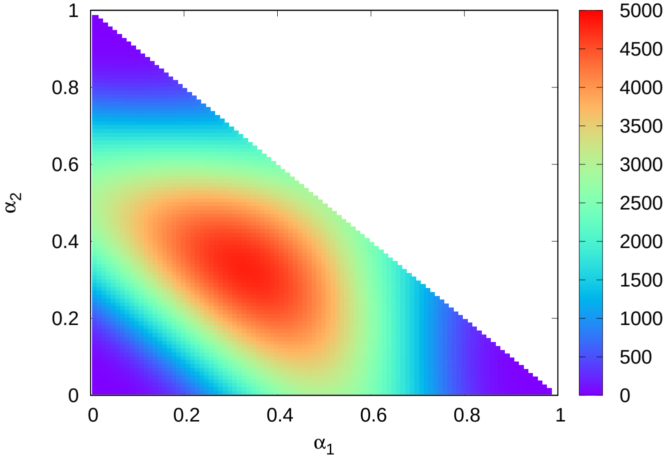
Decay width

$$\Gamma_{\gamma\to 3\gamma} = \frac{1}{2^7\, 3!\, \pi^4} \frac{k^4}{M_{LV}^2} \int \frac{d\alpha_1\, d\alpha_2\, d\beta_1 d\beta_2}{\alpha_1\alpha_2} \frac{\overline{|\mathcal{M}|^2}}{\sin\varphi_2|_{\varphi_2=\varphi_2(\alpha_1,\alpha_2,\beta_1,\beta_2)}}.$$

First integrate numerically over transverse momenta β_1, β_2. Area of integration is determined by $|\sin\varphi_2| \leq 1$.

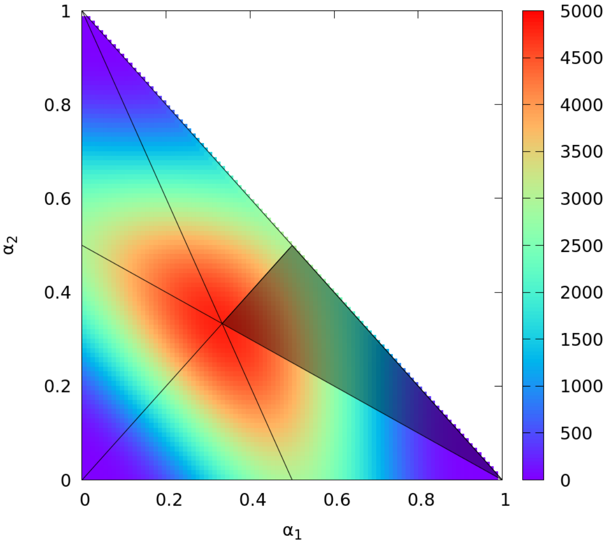
The decay width density $d^2\Gamma/d\alpha_1 d\alpha_2$

Integrate over transverse momenta β_1, β_2. $\alpha_i = k_i/k$, $\alpha_3 = 1 - \alpha_1 - \alpha_2$.



Maximum at $\alpha_1 = \alpha_2 = \alpha_3 = 1/3$ but $\alpha_1 \sim \alpha_2 \sim 0.5, \alpha_3 \sim 0$ is not really suppressed.

The decay width density. Permutations symmetry.

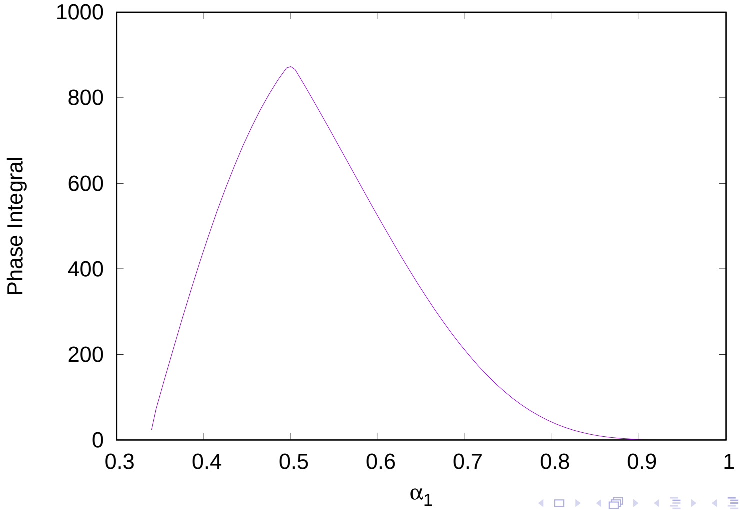


Symmetry of final photons permutations $\alpha_i \leftrightarrow \alpha_j,\ i,j = 1,2,3,\ i \neq j,\ \alpha_3 = 1 - \alpha_1 - \alpha_2,$ $\alpha_i = k_i/k.$

6 physically equivalent regions.
Shaded region – hierarchy $k_1 > k_2 > k_3$

The decay width density $d\Gamma/d\alpha_1$

Hierarchy $k_1 > k_2 > k_3$ is fixed.



The total splitting width and mean free path for photons.

$$\Gamma_{\gamma\to 3\gamma} \simeq 1.2\cdot 10^3 \left(\frac{2\alpha^2}{45}\right)^2 \frac{E_\gamma^{19}}{2^7\, 3!\, \pi^4 m_e^8 M_{LV}^{10}} \simeq 9\cdot 10^{-14} \frac{E_\gamma^{19}}{m_e^8 M_{LV}^{10}}.$$

The same parametric dependence as in *Gelmini Nussinov Yaguna 2005*
but 5 orders of magnitude larger. Very sharp dependence on E_γ!

Mean free path

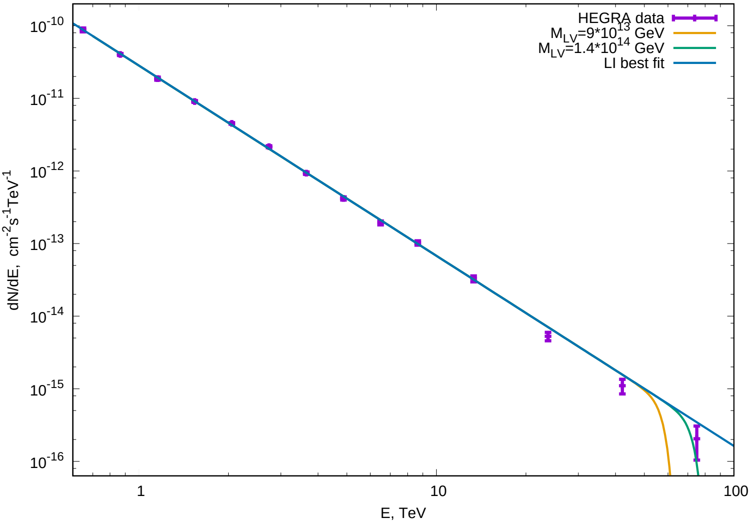
$$\langle L\rangle_{\gamma\to 3\gamma} \simeq 8\times \left(\frac{M_{LV}}{10^{14}\,\text{GeV}}\right)^{10} \left(\frac{E_\gamma}{40\,\text{TeV}}\right)^{-19} \text{Mpc}.$$

Estimated constraint on M_{LV} dependent on L_{source}

$$M_{LV} > \left(\frac{E_\gamma}{40\,\text{TeV}}\right)^{1.9} \left(\frac{L_{source}}{8\,\text{Mpc}}\right)^{0.1} \times 10^{14}\ \text{GeV}.$$

Crab Nebula spectrum (HEGRA coll.)

best LI fit, splitting with fixed M_{LV}.



Splitting bound from the last bin of Crab spectrum

- HEGRA (2004): $E = 75$ TeV, significance 2.7σ

$$M_{LV} > 1.3 \times 10^{14} \text{ GeV}, \quad 95\% \text{ CL}.$$

- HAWC (2019): $E = 102(118)$ TeV, significance $4.5(5.4)\sigma$

$$M_{LV} > 2.2(3.0) \times 10^{14} \text{ GeV}, \quad 95\% \text{ CL}.$$

- Tibet (2019): $E = 140$ TeV, significance 5σ

$$M_{LV} > 4.1 \times 10^{14} \text{ GeV}, \quad 95\% \text{ CL}.$$

Conclusions

- Direct calculations of the splitting process support estimations of *Gelmini Nussinov Yaguna 2005*
- Photon indeed lose energy in the splitting process: the configuration of two soft photons in the final state is suppressed.
- The bound on M_{LV} from the absence of the splitting process is an order of magnitude better than from the photon decay
- New observational data from HAWC and Tibet (photon energy more than 100 TeV) significantly improve the bound

Thank you for your attention!