The XXIV International Workshop
High Energy Physics and Quantum Field Theory

Sochi, Russia

Hyperfine structure of P-states in muonic ions of lithium, beryllium and boron

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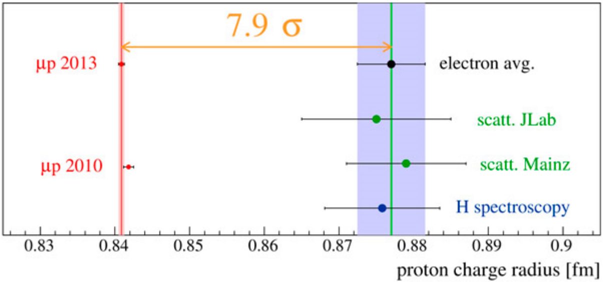
September 22 - September 29, 2019

- μLi, μBe, μB are two-particle bound states of muon and nucleus;
- The muon is two hundred times heavier than the electron. It leads to a lower Bohr radius of the muon. Thus, an influence of vacuum polarization and nuclear structure effects in hyperfine splitting increases;
- Muonic atoms play an important role in check of QED, theory of bound states and in precise measurement of fundamental constants;
- Measurement of the LS in light muonic atoms allows us to obtain more precise values of charge radii of corresponding atoms.

Proton radius puzzle

In the experiment carried out at PSI (Paul Scherrer Institute) transition frequency $2P_{3/2}^{F=2} - 2S_{1/2}^{F=1}$ in muonic hydrogen were measured with the following unexpected results:

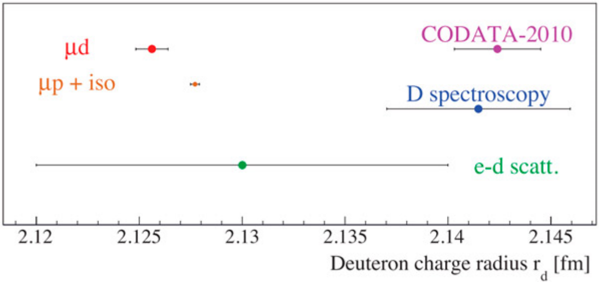
- New value of proton charge radius appeared to be 7.9 standard deviations smaller than the CODATA value $r_p = 0.8768(69) fm$.



A. Antognini et al., Science **339**, 417 (2013).

The experiment with muonic deuterium

Similar measurements for muonic deuterium also show noticeable discrepancy of 7.5σ with CODATA-2010 value. The discrepancy with CODATA-2014 value is slightly smaller - "only" 6σ.



R.Pohl et al. (the CREMA Collaboration). Laser spectroscopy of muonic deuterium // Science. 2016. V. 353. P. 669-637.

- One of the future scientific directions of CREMA collaboration is related with light muonic atoms of lithium, beryllium and boron.
- In our recent papers we calculated some corrections to the Lamb shift (2P-2S) and hyperfine splitting of S-states in muonic lithium, beryllium and boron and obtain more precise values of these energy intervals. This work continues our investigation to the case of P-wave part of the spectrum.

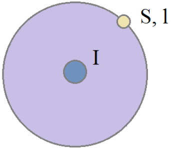
A. A. Krutov, A. P. Martynenko, F. A. Martynenko, O. S. Sukhorukova, Phys. Rev. A **94**, 062505 (2016).

A. E. Dorokhov et al., Phys. Rev. A **98**, 042501 (2018).

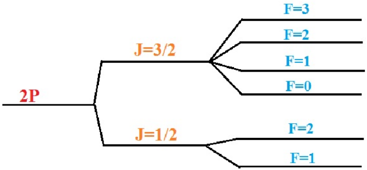
- The account of hyperfine structure of P-levels is also necessary because experimental transition frequencies are measured between different components of 2P and 2S states.

The aim of this work is to calculate hyperfine splitting intervals for P-states in muonic Li, Be, B with the account of corrections to vacuum polarization and nuclear structure.

HFS of spin-3/2 nuclei has the following structure:

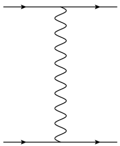


$$\vec{J} = \vec{l} + \vec{S}$$
$$\vec{F} = \vec{J} + \vec{I}$$



We use designation $n^{2F+1}L_J \to$ HFS consists of six states: $2^3P_{1/2}$, $2^5P_{1/2}$, $2^1P_{3/2}$, $2^3P_{3/2}$, $2^5P_{3/2}$, $2^7P_{3/2}$.

The contribution of the leading order α^4 to HFS of P-states is determined by the amplitude of one-photon interaction which is denoted $T_{1\gamma}$:



- In this work, we use both the momentum and coordinate representations to describe the interaction of particles.
- We begin with momentum representation of interaction amplitude in which two-particle bound state wave function of 2P-state can be written in the tensor form:

$$\psi_{2P}(\mathbf{p}) = (\varepsilon \cdot n_p)\, R_{21}(p),$$

where ε_δ is the polarization vector of orbital motion, $n_p = (0, \mathrm{p}/p)$ is the unit vector, $R_{21}(p)$ is the radial wave function in momentum space. Then the contribution to the energy spectrum is determined by the integral:

$$\Delta E^{hfs} = \int (\varepsilon^* \cdot n_q)\, R_{21}(q) \frac{d\mathbf{q}}{(2\pi)^{3/2}} \int (\varepsilon \cdot n_p)\, R_{21}(p) \frac{d\mathbf{p}}{(2\pi)^{3/2}} \Delta V^{hfs}(\mathbf{p}, \mathbf{q}).$$

The hyperfine potential ΔV^{hfs} can be constructed by means of one-photon interaction amplitude $T_{1\gamma}$ using the method of projection operators on states with definite quantum numbers. These projection operators can be written at the rest frame in covariant form.

$$T_{1\gamma}(\mathbf{p},\mathbf{q}) = 4\pi Z\alpha\left(\varepsilon^*\cdot n_q\right)\left[\overline{u}\left(q_1\right)\left(\left(p_1+q_1\right)_\mu 2m_1+\left(1+a_\mu\right)\sigma_{\mu\epsilon}\frac{k_\epsilon}{2m_1}\right)u\left(p_1\right)\right)\left(\varepsilon\cdot n_p\right)\times$$

$$\times D_{\mu\nu}(k)\overline{v}_\alpha\left(p_2\right)\left\{g_{\alpha\beta}\frac{\left(p_2+q_2\right)_\nu}{2m_2}F_1\left(k^2\right)+g_{\alpha\beta}\sigma_{\nu\lambda}\frac{k^\lambda}{2m_2}F_2\left(k^2\right)+\right.$$

$$\left.\frac{k_\alpha k_\beta}{4m_2^2}\frac{\left(p_2+q_2\right)_\nu}{2m_2}F_3\left(k^2\right)+\frac{k_\alpha k_\beta}{4m_2^2}\sigma_{\nu\lambda}\frac{k^\lambda}{2m_2}F_4\left(k^2\right)\right\}v_\beta\left(q_2\right),$$

Where:

- $p_{1,2}=\frac{m_{1,2}}{(m_1+m_2)}P\pm p$ are four-momenta of initial muon and nuclear, $q_{1,2}=\frac{m_{1,2}}{(m_1+m_2)}Q\pm q$ are four-momenta of final muon and nuclear (expressed in terms of total two-particle momenta P, Q and relative momenta p, q);
- a_μ is the muon anomalous magnetic moment;
- $D_{\mu\nu}(k)$ is the photon propagator which is taken to be in the Coulomb gauge;
- Four form factors which parameterise the nucleus electromagnetic current can be expressed through multipole form factors measured in experiments: charge G_{E0}, electroquadrupole G_{E2}, magnetic-dipole G_{M1} and magnetic-octupole G_{M3} form factors.

To describe the hyperfine structure of state $2P_{1/2}$ we introduce:

- The projection operator on the muon state with $j=1/2$:

$$\hat{\Pi}_{j=1/2}=[u(0)\varepsilon_\omega(0)]_{j=1/2}=\frac{1}{\sqrt{3}}\gamma_5\left(\gamma_\omega-v_\omega\right)\psi(0),$$

  where $\psi(0)$ is the Dirac spinor describing the muon state with $j=1/2$, $v=(1,0,0,0)=P/\left(m_1+m_2\right)$ is the auxiliary four vector.

- To project muon-nucleus pair on state with total momentum $F=2$ we use the projection operator:

$$\hat{\Pi}_{j=1/2}(F=2)=[\psi(0)\overline{v}_\alpha(0)]_{F=2}=\frac{1+\hat{v}}{2\sqrt{2}}\gamma_\tau\varepsilon_{\alpha\tau},$$

  where the tensor $\varepsilon_{\alpha\tau}$ describes the state $F=2$.

We make the summation over projections of the total momentum F using the equation

$$\sum_{M_F=-2}^{2}\varepsilon^*_{\beta\lambda}\varepsilon_{\alpha\rho}=\left[\frac{1}{2}X_{\beta\alpha}X_{\lambda\rho}+\frac{1}{2}X_{\beta\rho}X_{\lambda\alpha}-\frac{1}{3}X_{\beta\lambda}X_{\alpha\rho}\right],\quad X_{\beta\alpha}=\left(g_{\alpha\beta}-v_\beta v_\alpha\right)$$

Then the averaged over the projections M_F amplitude takes the form:

$$\overline{T_1\gamma(\mathbf{p},\mathbf{q})}^{j=1/2}_{F=2} = \frac{Z\alpha}{5} n_q^\delta n_p^\omega Tr\left\{\gamma_\sigma \frac{1+\hat{v}}{2\sqrt{2}}(\gamma_\delta - v_\delta)\gamma_5 \frac{(\hat{q}_1+m_1)}{2m_1}\Gamma_\mu \frac{(\hat{p}_1+m_1)}{2m_1}\gamma_5(\gamma_\omega - v_\omega)\times\right.$$

$$\left.\frac{1+\hat{v}}{2\sqrt{2}}\gamma_\rho \frac{(\hat{p}_2-m_2)}{2m_2}\Gamma^\nu_{\alpha\beta}\frac{(\hat{q}_2-m_2)}{2m_2}\right\} D_{\mu\nu}(k)\hat{\Pi}_{\beta_1\sigma,\alpha_1\rho}L_{\alpha\alpha_1}L_{\beta\beta_1},$$

where we introduce for the convenience short designations of nucleus vertex function

$$\Gamma^\nu_{\alpha\beta} = \left[g_{\alpha\beta}\frac{(p_2+q_2)_\nu}{2m_2}F_1\left(k^2\right) + g_{\alpha\beta}\sigma_{\nu\lambda}\frac{k^\lambda}{2m_2}F_2\left(k^2\right) + \frac{k_\alpha k_\beta}{4m_2^2}\frac{(p_2+q_2)_\nu}{2m_2}F_3\left(k^2\right) + \frac{k_\alpha k_\beta}{4m_2^2}\sigma_{\nu\lambda}\frac{k^\lambda}{2m_2}F_4\left(k^2\right)\right]$$

the lepton vertex function

$$\Gamma_\mu = \frac{p_{1,\mu}+q_{1,\mu}}{2m_1} + (1+a_\mu)\,\sigma_{\mu\epsilon}\frac{k_\epsilon}{2m_1},$$

and the Lorentz factors of vector fields

$$L_{\alpha\alpha_1}L_{\beta\beta_1} = \left[g_{\alpha\alpha_1} - \left(v_\alpha - \frac{p_\alpha}{2m_2}\right)\left(v_{\alpha_1} - \frac{p_{\alpha_1}}{m_2}\right)\right]\left[g_{\beta\beta_1} - \left(v_\beta - \frac{p_\beta}{2m_2}\right)\left(v_{\beta_1} - \frac{p_{\beta_1}}{m_2}\right)\right].$$

Using Form, we obtain the muon-nucleus interaction operator for the state $2^5P_{1/2}$:

$$V_{1\gamma}(\mathbf{p},\mathbf{q})^{j=1/2}_{F=2}=\frac{2\alpha\mu_N}{27m_1m_p(\mathbf{p}-\mathbf{q})^2}\left\{\frac{9}{2}pq+\frac{9m_1}{4m_2}pq-\frac{9}{4}(\mathbf{pq})\left(\frac{p}{q}+\frac{q}{p}\right)+\frac{27m_1}{8m_2}(\mathbf{pq})\left(\frac{p}{q}+\frac{q}{p}\right)\right.$$

$$-\frac{9m_1}{m_2}\frac{(\mathbf{pq})^2}{pq}+a_\mu\left[\frac{9}{4}pq-\frac{9}{4}(\mathbf{pq})\left(\frac{p}{q}+\frac{q}{p}\right)+\frac{9}{4}\frac{(\mathbf{pq})^2}{pq}\right]+\frac{a_\mu}{F_2(0)}\left[-\frac{27}{4}pq\left(1+\frac{m_2}{m_1}\right)+\frac{27}{4}\frac{(\mathbf{pq})^2}{pq}+\right.$$

$$\left.\frac{27m_2}{8m_1}(\mathbf{pq})\left(\frac{p}{q}+\frac{q}{p}\right)\right]+\frac{27(\mathbf{pq})\left(\mathbf{p}^2-\mathbf{q}^2\right)^2}{8(\mathbf{p}-\mathbf{q})^2F_2(0)pq}-\frac{27}{8F_2(0)}\left[pq\left(2+\frac{m_1}{m_2}+\frac{m_2}{m_1}\right)-\frac{m_2}{m_1}(\mathbf{pq})\left(\frac{p}{q}+\frac{q}{p}\right)\right.$$

$$\left.\left.+(\mathbf{pq})\left(\frac{p}{q}+\frac{q}{p}\right)+4m_1m_2\frac{(\mathbf{pq})}{pq}-\frac{2m_1}{m_2}\frac{(\mathbf{pq})^2}{pq}\right]\right\}.$$

The expression contains typical momentum integrals:

$$J_1=\int R_{21}(q)\frac{d\mathbf{q}}{(2\pi)^{3/2}}\int R_{21}(p)\frac{d\mathbf{p}}{(2\pi)^{3/2}}\frac{pq}{(\mathbf{p}-\mathbf{q})^2}=\left\langle\frac{pq}{(\mathbf{p}-\mathbf{q})^2}\right\rangle=\frac{3}{16},$$

$$J_2=\left\langle\frac{(\mathbf{pq})^2}{pq(\mathbf{p}-\mathbf{q})^2}\right\rangle=\frac{5}{48},\quad J_3=\left\langle\frac{(\mathbf{pq})\left(p^2+q^2\right)}{pq(\mathbf{p}-\mathbf{q})^2}\right\rangle=\frac{5}{24},\quad J_4=\left\langle\frac{(\mathbf{pq})\left(\mathbf{p}^2-\mathbf{q}^2\right)^2}{pq(\mathbf{p}-\mathbf{q})^4}\right\rangle=\frac{1}{6}.$$

It is important to note that when constructing potentials in this way, we obtain not only the hyperfine part of the potentials, but also the Coulomb contributions and contributions to the fine structure, which are further reduced when considering hyperfine splitting.

Let us consider also the construction of the potential in the case of $2^3P_{1/2}$ state. We represent the state with $s_2=3/2$ as the sum of two moments $\tilde{s}_2=1/2$ and $l_2=1$:

$$\Psi_{s_2=3/2,F=1,M_F}=\sqrt{\frac{2}{3}}\Psi_{\tilde{S}=0,F=1,M_F}+\sqrt{\frac{1}{3}}\Psi_{\tilde{S}=1,F=1,M_F},$$

The projection operators on these states:

$$\hat{\Pi}_\alpha(\tilde{S}=0,F=1)=\frac{1+\hat{v}}{2\sqrt{2}}\gamma_5\varepsilon_\alpha,\hat{\Pi}_\alpha(\tilde{S}=1,F=1)=\frac{1+\hat{v}}{4}\gamma_\sigma\varepsilon_{\alpha\sigma\rho\omega}v^\rho\varepsilon^\omega.$$

$$\overline{T_{1\gamma}(\mathbf{p},\mathbf{q})}^{j=1/2}_{F=1}(\tilde{S}=0)=\frac{Z\alpha}{3}n_q^\delta n_p^\omega\, Tr\left\{\gamma_5\frac{1+\hat{v}}{2\sqrt{2}}\left(\gamma_\delta-v_\delta\right)\gamma_5\frac{(\hat{q}_1+m_1)}{2m_1}\Gamma_\mu\frac{(\hat{p}_1+m_1)}{2m_1}\gamma_5\left(\gamma_\omega-v_\omega\right)\times\right.$$
$$\frac{1+\hat{v}}{2\sqrt{2}}\gamma_5\frac{(\hat{p}_2-m_2)}{2m_2}\left[g_{\alpha\beta}\frac{(p_2+q_2)_\nu}{2m_2}F_1\left(k^2\right)+g_{\alpha\beta}\sigma_{\nu\lambda}\frac{k^\lambda}{2m_2}F_2\left(k^2\right)+\frac{k_\alpha k_\beta}{4m_2^2}\frac{(p_2+q_2)_\nu}{2m_2}F_3\left(k^2\right)+\right.$$
$$\left.\left.\frac{k_\alpha k_\beta}{4m_2^2}\sigma_{\nu\lambda}\frac{k^\lambda}{2m_2}F_4\left(k^2\right)\right]\frac{(\hat{q}_2-m_2)}{2m_2}\right\}D_{\mu\nu}(k)\hat{\Pi}_{\beta_1\sigma\alpha_1\rho}L_{\alpha\alpha_1}L_{\beta\beta_1}\left(g_{\alpha_1\beta_1}-v_{\alpha_1}v_{\beta_1}\right).$$

$$\overline{T_{1\gamma}(\mathbf{p},\mathbf{q})}^{j=1/2}_{F=1}(\tilde{S}=1)=\frac{Z\alpha}{3}n_q^\delta n_p^\omega\, Tr\left\{\gamma_\rho\frac{1+\hat{v}}{4}\left(\gamma_\delta-v_\delta\right)\gamma_5\frac{(\hat{q}_1+m_1)}{2m_1}\Gamma_\mu\frac{(\hat{p}_1+m_1)}{2m_1}\gamma_5\left(\gamma_\omega-v_\omega\right)\times\right.$$
$$\frac{1+\hat{v}}{2\sqrt{2}}\gamma_5\frac{(\hat{p}_2-m_2)}{2m_2}\left[g_{\alpha\beta}\frac{(p_2+q_2)_\nu}{2m_2}F_1\left(k^2\right)+g_{\alpha\beta}\sigma_{\nu\lambda}\frac{k^\lambda}{2m_2}F_2\left(k^2\right)+\frac{k_\alpha k_\beta}{4m_2^2}\frac{(p_2+q_2)_\nu}{2m_2}F_3\left(k^2\right)+\right.$$
$$\left.\left.\frac{k_\alpha k_\beta}{4m_2^2}\sigma_{\nu\lambda}\frac{k^\lambda}{2m_2}F_4\left(k^2\right)\right]\frac{(\hat{q}_2-m_2)}{2m_2}\right\}D_{\mu\nu}(k)L_{\alpha\alpha_3}L_{\beta\beta_3}\epsilon_{\rho\beta_3\alpha_1\beta_1}\epsilon_{\tau\alpha_3\rho_1\omega_1}\left(g_{\omega_1\beta_1}-v_{\omega_1}v_{\beta_1}\right).$$

Omitting other details of the calculation we obtain the hyperfine splitting of $2P_{1/2}$ state as follows:

$$\Delta E^{hfs}\left(2^5P_{1/2}-2^3P_{1/2}\right)=\frac{2\alpha(Z\alpha)^3\mu^3\mu_N}{27m_1m_p}\left[1+\frac{1}{2}a_\mu+\frac{m_1}{2m_2}-\frac{3m_1}{4m_2F_2(0)}\right]=$$

$$=\begin{cases} {}^7_3Li: 210.8960\mathrm{meV}\\ {}^9_4Be: -183.2929\mathrm{meV}\\ {}^{11}_5B: 818.1086\mathrm{meV}\end{cases}$$

In case of $j=3/2$ we can perform calculations in the similar way. We obtain the hyperfine splitting of $2P_{3/2}$ state as follows:

$$\Delta E^{hfs}\left(2^7P_{3/2}-2^5P_{3/2}\right)=\frac{\alpha(Z\alpha)^3\mu^3\mu_N}{45m_1m_p}\left[1-\frac{1}{4}a_\mu+\frac{5m_1}{4m_2}-\frac{15m_1}{8m_2F_2(0)}\right]=\begin{cases} {}^7_3Li: 63.8246\mathrm{meV}\\ {}^9_4Be: -55.7466\mathrm{meV}\\ {}^{11}_5B: 246.6252\mathrm{meV}\end{cases}$$

$$\Delta E^{hfs}\left(2^5P_{3/2}-2^3P_{3/2}\right)=\frac{2\alpha(Z\alpha)^3\mu^3\mu_N}{135m_1m_p}\left[1-\frac{1}{4}a_\mu+\frac{5m_1}{4m_2}-\frac{15m_1}{8m_2F_2(0)}\right]=\begin{cases} {}^7_3Li: 42.5497meV\\ {}^9_4Be: -37.1644meV\\ {}^{11}_5B: 164.4168meV\end{cases}$$

$$\Delta E^{hfs}\left(2^3P_{3/2}-2^1P_{3/2}\right)=\frac{\alpha(Z\alpha)^3\mu^3\mu_N}{135m_1m_p}\left[1-\frac{1}{4}a_\mu+\frac{5m_1}{4m_2}-\frac{15m_1}{8m_2F_2(0)}\right]=\begin{cases} {}^7_3Li: 21.2932\mathrm{meV}\\ {}^9_4Be: -18.5822\mathrm{meV}\\ {}^{11}_5B: 82.2084\mathrm{meV}\end{cases}$$

The numerical values of these contributions are large. Therefore, it makes sense to consider a number of corrections to these formulas.

However, firstly we want to use another approach to solve this problem in the coordinate representation. To calculate the HFS of the spectrum of P-levels, it is necessary to use the following term from the Hamiltonian:

$$\Delta H^{hfs} = \frac{Z\alpha g_N}{2m_1 m_2 r^3}\left[1+\frac{m_1}{m_2}-\frac{m_1}{m_2 g_N}\right](\mathbf{L}\mathbf{s}_2) - \frac{Z\alpha\,(1+a_\mu)\,g_N}{2m_1 m_2 r^3}\left[\mathbf{s}_1\mathbf{s}_2 - 3\,(\mathbf{s}_1\mathbf{r})\,(\mathbf{s}_2\mathbf{r})\right]$$

A fine part of the Hamiltonian:

$$\Delta H^{fs} = \frac{Z\alpha}{m_1 m_2 r^3}\left[1+\frac{m_2}{2m_1}+a_\mu\left(1+\frac{m_2}{m_1}\right)\right](\mathbf{L}\mathbf{s}_1)$$

Averaging this formula over the wave functions of the 2P state, we obtain the main contribution to the fine splitting:

$$\Delta E^{fs} = \frac{(Z\alpha)^4\mu^3}{16 m_1 m_2}\left[1+\frac{m_2}{2m_1}+a_\mu\left(1+\frac{m_2}{m_1}\right)\right] = \begin{cases} {}^7_3Li : 747.8581\,\mathrm{meV} \\ {}^9_4Be : 2372.2215\,\mathrm{meV} \\ {}^{11}_5B : 5804.9674\,\mathrm{meV}\end{cases}$$

The hyperfine part includes operators:

$$T_1 = \mathbf{L}\mathbf{s}_2, \quad T_2 = \mathbf{s}_1\mathbf{s}_2 - 3\,(\mathbf{s}_1\mathbf{n})\,(\mathbf{s}_2\mathbf{n})$$

$$E\left(2^{2F+1}P_j\right) = \frac{\alpha(Z\alpha)^3\mu^3\mu_N}{72 m_1 m_p}\left[\overline{T}_1+\frac{m_1}{m_2}\overline{T}_1-\frac{3m_1}{2m_2F_2(0)}\overline{T}_1-(1+a_\mu)\,\overline{T}_2\right]$$

$$E\left(2^7P_{3/2}\right)=E^{fs}+\frac{\alpha(Z\alpha)^3\mu^3\mu_N}{60m_1m_p}\left[1-\frac{a_\mu}{4}+\frac{5m_1}{4m_2}-\frac{15m_1}{8m_2F_2(0)}\right]=\begin{cases}{}^7_3Li: 795.7265\,\mathrm{meV}\\ {}^9_4Be: 2330.4116\,\mathrm{meV}\\ {}^{11}_5B: 5989.9363\,\mathrm{meV}\end{cases}$$

$$E\left(2^5P_{3/2}\right)=E^{fs}-\frac{\alpha(Z\alpha)^3\mu^3\mu_N}{180m_1m_p}\left[1-\frac{a_\mu}{4m}+\frac{5m_1}{4m_2}-\frac{15m_1}{8m_2F_2(0)}\right]=\begin{cases}{}^7_3Li: 731.9019\,\mathrm{meV}\\ {}^9_4Be: 2386.1582\,\mathrm{meV}\\ {}^{11}_5B: 5743.3111\,\mathrm{meV}\end{cases}$$

$$E\left(2^3P_{3/2}\right)=E^{fs}-\frac{11\alpha(Z\alpha)^3\mu^3\mu_N}{540m_1m_p}\left[1-\frac{a_\mu}{4}+\frac{5m_1}{4m_2}-\frac{15m_1}{8m_2F_2(0)}\right]=\begin{cases}{}^7_3Li: 689.3522\,\mathrm{meV}\\ {}^9_4Be: 423.325\,\mathrm{meV}\\ {}^{11}_5B: 5578.8943\,\mathrm{meV}\end{cases}$$

$$E\left(2^1P_{3/2}\right)=E^{fs}-\frac{\alpha(Z\alpha)^3\mu^3\mu_N}{36m_1m_p}\left[1-\frac{a_\mu}{4}+\frac{5m_1}{4m_2}-\frac{15m_1}{8m_2F_2(0)}\right]=\begin{cases}{}^7_3Li: 668.0773\,\mathrm{meV}\\ {}^9_4Be: 2441.9047\,\mathrm{meV}\\ {}^{11}_5B: 5496.6859\,\mathrm{meV}\end{cases}$$

$$E\left(2^5P_{1/2}\right)=\frac{\alpha(Z\alpha)^3\mu^3\mu_N}{36m_1m_p}\left[1+\frac{a_\mu}{2}+\frac{m_1}{2m_2}-\frac{3m_1}{4m_2F_2(0)}\right]=\begin{cases}{}^7_3Li: 79.0860\,\mathrm{meV}\\ {}^9_4Be: -68.7348\,\mathrm{meV}\\ {}^{11}_5B: 306.7907\,\mathrm{meV}\end{cases}$$

$$E\left(2^3P_{1/2}\right)=-\frac{5\alpha(Z\alpha)^3\mu^3\mu_N}{108m_1m_p}\left[1+\frac{a_\mu}{2}+\frac{m_1}{2m_2F_2(0)}\right]=\begin{cases}{}^7_3Li: -131.8100\,meV\\ {}^9_4Be: 114.5581\,\mathrm{meV}\\ {}^{11}_5B: -511.3179\,\mathrm{meV}\end{cases}$$

The contribution of quadruple interaction

In the leading order α^4 in the energy spectrum of muonic ions Li, Be, B there is important contribution of the quadrupole interaction.
The muon and nucleus interaction operator (where $\rho_n(r')$ is charge density distribution):

$$V_{\mu N} = -e \int \frac{\rho_n(r') d^3 r'}{|r - r'|},$$

using the multipole expansion, where θ is the angle between r and r':

$$\frac{1}{|r - r'|} = \sum_{l=0}^{\infty} \frac{r^l}{r'^{l+1}} P_l(\cos\theta),$$

$$V_{\mu N} = -e \int \sum_{l=0}^{\infty} \frac{|r^l|}{|r'^{l+1}|} \rho_n(r') d^3 r' P_l(\cos\theta),$$

According to the addition theorem for spherical harmonics:

$$P(\cos\theta) = \sum_{q=-l}^{l} (-1)^q C_q^l(\theta'\phi')C_{-q}^l(\theta''\phi''),$$

where

$$C_q^l(\theta'\phi') = \sqrt{\frac{4\pi}{2l+1}}\, Y_{lq}(\theta,\phi).$$

Then for $l = 2$, separating muon and nuclear coordinates, we have:

$$V_{\mu N} = -e \sum_{q=-2}^{2} (-1)^q \int |r'^2| \rho_n(r') C_q^2(\theta'\phi') d^3 r' \,\frac{1}{|r|^3}\, C_{-q}^2(\theta''\phi'').$$

In terms of irreducible tensor operators:

$$V^Q = -eQ_q^2(n)T_q^2(\mu),$$

where $Q_q^2(n)$, $T_q^2(\mu)$ are irreducible tensor operators of rank 2 for nucleus and muon cloud quadrupole moments respectively.

For irreducible tensor operators we have the following explicit expressions:

$$Q_q^2(n) = \sqrt{\frac{4\pi}{5}}\, r'^2 \rho_n(r') Y_{2q}(\theta''\phi'').$$

$$T_q^2(\mu) = \sqrt{\frac{4\pi}{5}}\, \frac{1}{r^3}\, Y_{2q}(\theta'\phi').$$

To calculate the contribution of quadrupole interaction to HFS, we have to average V^Q over the muon-nucleus wave functions:

$$\psi_F^m = |jIF>=\sum_{\mu} C(IjF; m-\mu, \mu)\Phi_I^{m-\mu}\Psi_j^{\mu},$$

where m is the projection of total angular momentum $F = L + S + I$ on z-axis,
μ - is the projection $J = L + S$ on z-axis,
$C(IjF; m-\mu, \mu)$ is Clebsch-Gordan coefficient,
$\Phi_I^{m-\mu} = |Im-\mu>$ is the nuclear wave function,
$\Psi_j^{\mu} = |j\mu>$ is the muon wave function.
In the first order of perturbation theory, we need to calculate the following matrix element:

$$\Delta E^Q = \left\langle j' IF \left| V^Q \right| jIF \right\rangle = -e\sum_{\mu,\mu'} C(IjF; m-\mu, \mu)C(IjF; m-\mu', \mu')\times$$

$$\times \left\langle Im-\mu \left| Q_q^2(n) \right| Im-\mu' \right\rangle \left\langle j\mu \left| T_q^2(\mu) \right| j\mu' \right\rangle$$

For the next step we need to use Wigner-Eckart theorem:

$$\left\langle j'\mu' \left| T_q^{\kappa} \right| j\mu \right\rangle = (-1)^{\kappa} \frac{\langle j' \| T^{\kappa} \| j\rangle}{\sqrt{2j'+1}} C(j\kappa j'; -\mu, q, \mu')$$

where $\langle j' \| T^{\kappa} \| j\rangle$ is reduced matrix element, ${T_q}^{\kappa}$ is irreducible tensor operator, κ is tensor rank.

И.И. Собельман, "Введение в теорию атомных спектров"

Let us apply Wigner-Eckart theorem to our expression:

$$\Delta E^Q = -e(-1)^q \sum_{q,\mu,\mu'} C(IjF; m-\mu, \mu) C(IjF; m-\mu', \mu') \times$$

$$\times \underline{\left\langle Im-\mu \,\middle|\, Q^2_q(n) \,\middle|\, Im-\mu' \right\rangle \left\langle j\mu \,\middle|\, T^2_q(\mu) \,\middle|\, j\mu' \right\rangle} =$$

$$= -e \sum_{q,\mu,\mu'} C(IjF; m-\mu, \mu) C(IjF; m-\mu', \mu') \times$$

$$\times \frac{C(j2j'; \mu, q, \mu')}{\sqrt{(2J'+1)}} \frac{C(I2I; m-\mu, -q, m-\mu')}{\sqrt{(2I+1)}} \left\langle j' \,\middle\|\, T^2 \,\middle\|\, j \right\rangle \left\langle I \,\middle\|\, Q^2 \,\middle\|\, I \right\rangle.$$

Then we have to take into account the following relation for Clebsch-Gordan coefficients:

$$C(j2j'; \mu, \mu'-\mu, \mu) C(j'IF; \mu', m-\mu') = \sqrt{2f+1}\sqrt{2J'+1} \times$$
$$\times \sum_f W(j2FI; j'f) C(2If; \mu'-\mu, m-\mu') C(jfF; \mu, m-\mu).$$

Assuming that $q = \mu' - \mu$:

$$\Delta E^Q = -e \sum_{\mu\mu' f} (-1)^{\mu'-\mu} \frac{\sqrt{2f+1}}{\sqrt{2I+1}} C(jIF; \mu, m-\mu) C(I2I; m-\mu, \mu-\mu') \times$$

$$\times C(2If; \mu'-\mu, m-\mu') C(jfF; \mu, m-\mu) W(j2FI; j'f) \left\langle j' \,\middle\|\, T^2 \,\middle\|\, j \right\rangle \left\langle I \,\middle\|\, Q^2 \,\middle\|\, I \right\rangle.$$

Considering normalization and symmetry properties of Clebsch-Gordan coefficients, we have the next expression:

$$\sum_{\mu'}(-1)^{\mu'-\mu}C(I2I;m-\mu,\mu-\mu')C(2If;\mu'-\mu,m-\mu')=(-1)^2\delta_{If}.$$

Delta-function δ_{If} removes the sum over f in the initial equation and the remaining sum is equal to 1 because of the orthogonality of Clebsch-Gordan coefficients. Racah coefficient can be expressed in the following way:

$$W(j2FI;j'I)=(-1)^{-2-F+I+J'}W(jIj'I;F2).$$

Thus we obtain the following general expression:

$$\underline{\Delta E^Q=-e(-1)^{I+J'-F}W(jIj'I;F2)\left\langle j'\left\| T^2\right\| j\right\rangle\left\langle I\left\| Q^2\right\| I\right\rangle.}$$

In our calculation we use the relation between Racah coefficient and 6j-symblos:

$$W(j_1j_2j_5j_4;j_3j_6)=(-1)^{-j_1-j_2-j_4-j_5}\left\{\begin{array}{ccc} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{array}\right\}. \qquad (1)$$

By definition:

$$eQ = 2 \left\langle II \,\middle|\, Q_0^2(n) \,\middle|\, II \right\rangle .$$

and according to Wigner-Eckart theorem when $q = 0$, the matrix element has the form:

$$\left\langle II \,\middle|\, Q_0^2(n) \,\middle|\, II \right\rangle = (-1)^{I-I} \left\langle I \,\|\, Q(n) \,\|\, I \right\rangle \begin{pmatrix} I & 2 & I \\ -I & 0 & I \end{pmatrix}.$$

$$eQ = 2 \left\langle I \,\|\, Q(n) \,\|\, I \right\rangle \begin{pmatrix} I & 2 & I \\ -I & 0 & I \end{pmatrix}.$$

Then we have:

$$\left\langle I \,\|\, Q(n) \,\|\, I \right\rangle = \frac{eQ}{2} \left[\begin{pmatrix} I & 2 & I \\ -I & 0 & I \end{pmatrix} \right]^{-1}.$$

In the same way:

$$\left\langle j' \,\|\, T(\mu) \,\|\, j \right\rangle = -\sqrt{2j+1}\sqrt{2j'+1}(-1)^{j'+\frac{1}{2}} \begin{pmatrix} j' & 2 & j \\ \frac{1}{2} & 0 & -\frac{1}{2} \end{pmatrix} < \frac{1}{r^3} >,$$

$$< \frac{1}{r^3} > = \int_0^\infty \Psi_{2P}^*(r) \frac{1}{r} \Psi_{2P}(r) dr.$$

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Therefore, for matrix elements of quadrupole correction we have the following relation:

$$\Delta E^{Q,VP} = (-1)^{j'+\frac{1}{2}-F-j+1-I} \left\{ \begin{array}{ccc} j & I & F \\ I & j' & 2 \end{array} \right\} \frac{\alpha Q}{2} \left[\left(\begin{array}{ccc} I & 2 & I \\ -I & 0 & I \end{array} \right) \right]^{-1} \times$$
$$\times \sqrt{2j+1}\sqrt{2j'+1} \left(\begin{array}{ccc} j' & 2 & j \\ \frac{1}{2} & 0 & -\frac{1}{2} \end{array} \right) < \frac{1}{r^3} > .$$

$$\Delta E^{Q} = \frac{\alpha^4 \mu^3 Z^3 Q}{240} (5\delta_{F,0} + \delta_{F,1} - 3\delta_{F,2} + \delta_{F,3}).$$

$$\Delta E^{Q} = \begin{cases} {}^{7}_{3}Li : -299.136 \text{ meV}, \\ {}^{9}_{4}Be : 933.724 \text{ meV}, \\ {}^{11}_{5}B : 1412.630 \text{ meV}. \end{cases}$$

Drake G.W.F., Byer L.L. Lamb shifts and fine-structure splittings for the muonic ions μLi, μBe, and μB: A proposed experiment //Phys. Rev. A. 1985. V. 32. P. 713-719.

In order to obtain vacuum polarization correction in quadrupole interaction, we use modified Coulomb potential:

$$V_v^C p(r) = -\frac{Z\alpha^2}{3\pi}\int_1^\infty \rho(\xi)d\xi \int \frac{\rho(r')d^3r'}{|r-r'|}e^{-2m_e\xi|r-r'|}.$$

The general expression for VP-correction has the form:

$$\Delta E^{Q,VP} = (-1)^{j'+\frac{1}{2}-F-j+1-I}\left\{\begin{array}{ccc} j & I & F \\ I & j' & 2 \end{array}\right\}\frac{\alpha Q}{2}\left[\left(\begin{array}{ccc} I & 2 & I \\ -I & 0 & I \end{array}\right)\right]^{-1}\times$$
$$\times\sqrt{2j+1}\sqrt{2j'+1}\left(\begin{array}{ccc} j' & 2 & j \\ \frac{1}{2} & 0 & -\frac{1}{2} \end{array}\right)[\frac{\alpha}{3\pi}\frac{e^{-2m_e\xi r}}{r^3}(1+2m_e\xi r+\frac{4m_e^2\xi^2 r^2}{3})].$$

After numerical integration, we have:

$$\Delta E_V^Q P = \begin{cases} {}^7_3Li : -0.510 \text{ meV}, \\ {}^9_4Be : 1.954 \text{ meV}, \\ {}^{11}_5B : 3.403 \text{ meV}. \end{cases}$$

Using the method of projection operators formulated above, we can distinguish in the amplitude of the one-photon interaction a part proportional to the quadrupole form factor $G_{E2}\left(k^2\right)$. Its value at zero $G_{E2}(0)=m_2^2Q/Z$, and the magnitude of the quadrupole moment of the nucleus Q sets the numerical value of this correction. The averaged amplitudes of quadrupole interaction for different states have the form:

$$\overline{T_{\mathbf{1}\gamma,Q}(\mathbf{p},\mathbf{q}))}^{j=\mathbf{3}/\mathbf{2}}_{F=\mathbf{3}}=\frac{\alpha Q}{20(\mathbf{p}-\mathbf{q})^{\mathbf{2}}}\left[pq-4(\mathbf{pq})\left(\frac{p}{q}+\frac{q}{p}\right)+7\frac{(\mathbf{pq})^{\mathbf{2}}}{pq}\right],$$

$$\overline{T_{\mathbf{1}\gamma,Q}(\mathbf{p},\mathbf{q}))}^{j=\mathbf{3}/\mathbf{2}}_{F=\mathbf{2}}=\frac{\alpha Q}{60(\mathbf{p}-\mathbf{q})^{\mathbf{2}}}\left[9pq+4(\mathbf{pq})\left(\frac{p}{q}+\frac{q}{p}\right)-17\frac{(\mathbf{pq})^{\mathbf{2}}}{pq}\right],$$

$$\overline{T_{\mathbf{1}\gamma,Q}(\mathbf{p},\mathbf{q})}^{j=\mathbf{3}/\mathbf{2}}_{F=\mathbf{1}}=\frac{\alpha Q}{20(\mathbf{p}-\mathbf{q})^{\mathbf{2}}}\left[pq-4(\mathbf{pq})\left(\frac{p}{q}+\frac{q}{p}\right)+7\frac{(\mathbf{pq})^{\mathbf{2}}}{pq}\right],$$

$$\overline{T_{\mathbf{1}\gamma,Q}(\mathbf{p},\mathbf{q})}^{j=\mathbf{3}/\mathbf{2}}_{F=\mathbf{0}}=\frac{\alpha Q}{12(\mathbf{p}-\mathbf{q})^{\mathbf{2}}}\left[-3pq+4(\mathbf{pq})\left(\frac{p}{q}+\frac{q}{p}\right)-5\frac{(\mathbf{pq})^{\mathbf{2}}}{pq}\right].$$

Making momentum integration we obtain contributions to the energy levels $2^{2F+1}P_{3/2}$:

$$\Delta E_Q^{hfs}=\frac{\alpha Q(\mu Z\alpha)^3}{240}\left[\delta_{F3}-3\delta_{F2}+\delta_{F1}+5\delta_{F0}\right].$$

This result coincides exactly with calculations made in coordinate representation.

To calculate VP-correction in momentum representation we should use the following replacement in the photon propagator $\overline{T_{1\gamma,Q}(\mathbf{p},\mathbf{q})}^{j=3/2}_{F=0} - \overline{T_{1\gamma,Q}(\mathbf{p},\mathbf{q})}^{j=3/2}_{F=3}$:

$$\frac{1}{(\mathbf{p}-\mathbf{q})^2} \to \frac{\alpha}{3\pi}\int_1^\infty \frac{\rho(\xi)d\xi}{(\mathbf{p}-\mathbf{q})^2+4m_e^2\xi^2}, \quad \rho(\xi)=\sqrt{\xi^2-1}\left(2\xi^2+1\right)/\xi^4.$$

Then the correction to vacuum polarization in the quadrupole interaction can be expressed in terms of three momentum integrals which are calculated analytically:

$$I_1 = \int R_{21}(q)\frac{d\mathbf{q}}{(2\pi)^{3/2}}\int R_{21}(p)\frac{d\mathbf{p}}{(2\pi)^{3/2}}\frac{pq}{[(\mathbf{p}-\mathbf{q})^2+4m_e^2\xi^2]} =$$

$$= \left\langle \frac{pq}{[(\mathbf{p}-\mathbf{q})^2+4m_e^2\xi^2]}\right\rangle = \frac{a(3a+8)+6}{2(a+2)^4}, \quad a=\frac{4m_e\xi}{\mu Z\alpha}.$$

$$I_2 = \left\langle \frac{(\mathbf{pq})^2}{pq[(\mathbf{p}-\mathbf{q})^2+4m_e^2\xi^2]}\right\rangle = \frac{a(3a+8)+10}{6(a+2)^4}, \quad I_3 = \left\langle \frac{(\mathbf{pq})(p^2+q^2)}{pq[(\mathbf{p}-\mathbf{q})^2+4m_e^2\xi^2]}\right\rangle = \frac{2(4a+5)}{3(a+2)^4}.$$

As a result we obtain following formula for the energy correction:

$$\Delta E^Q_{vp}(2^{(2F+1)}P_{3/2}) = \frac{\alpha^2(Z\alpha)^3 Q}{90\pi\,(4-a_1{}^2)^{5/2}}\Bigg\{2\sqrt{4-a_1^2}\left(a_1{}^2-1\right)+$$

$$+\left(5a_1{}^2-8\right)\ln\Bigg[\frac{\left(2-\sqrt{4-a_1{}^2}\right)}{a_1}\Bigg]\Bigg\}\left[5\delta_{F0}+\delta_{F1}-3\delta_{F2}+\delta_{F3}\right], \; a_1=\frac{4m_e}{\mu Z\alpha}.$$

Corresponding numerical results for the states $2^{(2F+1)}P_{3/2}$ are included in the Table.

Corrections to the vacuum polarization and nucleus structure

The main contribution of the effects of vacuum polarization in the hyperfine structure of the energy spectrum of the P-states is related with a modification of the Hamiltonian, which in turn is determined by the replacement in the photon propagator.

$$\Delta V^{hfs}_{vp}(2^7P_{3/2}-2^5P_{3/2})=\frac{\alpha}{135\pi}\int_1^\infty\frac{\rho(\xi)d\xi}{(\mathbf{p}-\mathbf{q})^2+4m_e^2\xi^2}\Big\{12pq+15\frac{m_1}{m_2}pq+12(\mathbf{pq})\left(\frac{p}{q}+\frac{q}{p}\right)-$$

$$36\frac{(\mathbf{pq})^2}{pq}-15\frac{m_1}{m_2}\frac{(\mathbf{pq})^2}{pq}+a_\mu\left[-3pq+12(\mathbf{pq})\left(\frac{p}{q}+\frac{q}{p}\right)-21\frac{(\mathbf{pq})^2}{pq}\right]-\frac{45m_1}{2m_2F_2(0)}\left[pq-\frac{\mathbf{pq})^2}{pq}\right],$$

$$\Delta V^{hfs}_{vp}(2^5P_{3/2}-2^3P_{3/2})=\frac{2}{3}\Delta V^{hfs}_{vp}(2^7P_{3/2}-2^5P_{3/2})=2\Delta V^{hfs}_{vp}(2^3P_{3/2}-2^1P_{3/2}).$$

$$\Delta V^{hfs}_{vp}(2^5P_{1/2}-2^3P_{1/2})=\frac{2\alpha^2}{81\pi}\int_1^\infty\frac{\rho(\xi)d\xi}{(\mathbf{p}-\mathbf{q})^2+4m_e^2\xi^2}\Big\{12pq+6\frac{m_1}{m_2}pq-6(\mathbf{pq})\left(\frac{p}{q}+\frac{q}{p}\right)-$$

$$6\frac{m_1}{m_2}\frac{(\mathbf{pq})^2}{pq}+a_\mu\left[6pq-6(\mathbf{pq})\left(\frac{p}{q}+\frac{q}{p}\right)+6\frac{(\mathbf{pq})^2}{pq}\right]-\frac{9m_1}{m_2F_2(0)}\left[pq-\frac{\mathbf{pq})^2}{pq}\right].$$

Further integration over the momentum variables and spectral parameter ξ can be performed analytically. But the answer for hyperfine splitting in the energy spectrum is more conveniently written in the integral form over ξ:

$$\Delta E^{hfs}_{vp}(2^7P_{3/2}-2^5P_{3/2})=\frac{\alpha^2(Z\alpha)^3\mu^3\mu_N}{135\pi m_1 m_p}\int_1^\infty\frac{\rho(\xi)d\xi}{(a+2)^4}\Big[16+20\frac{m_1}{m_2}+a(32+40\frac{m_1}{m_2})+$$

$$15a^2\frac{m_1}{m_2}-a_\mu(4+8a+15a^2)-\frac{15m_1}{2m_2F_2(0)}(4+8a+3a^2)\Big],$$

$$\Delta E^{hfs}_{vp}(2^5P_{3/2}-2^3P_{3/2})=\frac{2}{3}\Delta E^{hfs}_{vp}(2^7P_{3/2}-2^5P_{3/2})=2\Delta E^{hfs}_{vp}(2^3P_{3/2}-2^1P_{3/2}),$$

$$\Delta E^{hfs}_{vp}(2^5P_{1/2}-2^3P_{1/2})=\frac{2\alpha^2(Z\alpha)^3\mu^3\mu_N}{81\pi m_1 m_p}\int_1^\infty\frac{\rho(\xi)d\xi}{(a+2)^4}\Big[16+32a+18a^2+$$

$$2\frac{m_1}{m_2}(4+8a+3a^2)+a_\mu(8+16a+12a^2)-\frac{3m_1}{m_2F_2(0)}(4+8a+3a^2)\Big].$$

The numerical results are presented in the Table for separate energy levels.

Nucleus of Li, Be and B have sufficiently large size, so the structure effects can be significant. For their estimation in order α^6 we use an expansion of charge, magnetic dipole and electric quadrupole form factors:

$$F_1(k^2) \approx 1 - \frac{1}{6} r_{E0}^2 \mathbf{k}^2,\ F_2(k^2) \approx F_2(0)[1 - \frac{1}{6} r_M^2 \mathbf{k}^2],\ F_3(k^2) \approx 2[1 - \frac{1}{6} r_{E0}^2 \mathbf{k}^2] - 2G_{E2}(0)[1 - \frac{1}{6} r_{E2}^2 \mathbf{k}^2],$$

and take into account terms proportional to charge r_E, magnetic dipole r_{M1} and electric quadrupole r_{E2} radii. For example, the potential for $j = 1/2$ hyperfine splitting in momentum representation is the following:

$$\Delta V_{str}(2^5P_{1/2} - 2^3P_{1/2}) = \frac{2Z\alpha}{27 m_1 m_2}\Big\{ -r_{E0}^2 \frac{3m_1}{2m_2}[pq - \frac{(\mathbf{pq})^2}{pq}] +$$

$$+ F_2(0) r_{M1}^2 \big[2pq + \frac{m_1}{m_2} pq - \frac{m_1}{m_2} \frac{(\mathbf{pq})^2}{pq} + a_\mu \big(pq + \frac{(\mathbf{pq})^2}{pq} \big) \big] \Big\}.$$

The calculation of remaining momentum integrals gives $< pq >= 3/8$, $< \frac{(\mathbf{pq})^2}{pq} >= 1/8$ and shifts of the energy levels $2^{2F+1}P_J$. To obtain corresponding numerical results we set approximately $r_{E0} = r_{M1}$ and omit quadruple radius r_{E2}.

HFS of $2P$-states in muonic ions Li, Be, B.

The contribution	$2^3P_{1/2}$, meV	$2^5P_{1/2}$, meV	$2^1P_{3/2}$, meV	$2^3P_{3/2}$, meV	$2^5P_{3/2}$, meV	$2^7P_{3/2}$, meV
Leading order α^4 correction	-131.8100 114.5581 -511.3179	79.0860 -68.7348 306.7907	668.0773 2441.9047 5496.6859	689.3522 2423.3225 5578.8943	731.9019 2386.1582 5743.3111	795.7265 2330.4116 5989.9363
Quadrupole correction of order α^4	0 0 0	0 0 0	-186.9598 583.5774 882.8935	-37.3920 116.7155 176.5787	112.1759 -350.1465 -529.7361	-37.3920 116.7155 176.5787
VP correction of order α^5	-0.2122 0.2275 -1.1746	0.1273 -0.1365 0.7048	-0.0618 0.0743 -0.4038	-0.0453 0.0545 -0.2961	-0.0124 0.0149 -0.0808	0.0371 -0.0446 0.2423
Quadrupole and VP correction of order α^5	0 0 0	0 0 0	-0.3189 1.2214 2.1266	-0.0638 0.2443 0.4253	0.1913 -0.7328 -1.2760	-0.0638 0.2443 0.4253
Relativistic correction of order α^6	0.7320 1.8588 -12.9534	-1.2200 -1.1153 7.7721	-0.1090 0.1661 -1.1575	-0.0799 0.1218 -0.8489	-0.0218 0.0332 -0.2315	0.0654 -0.0997 0.6945
VP correction of order α^6	-0.0011 0.0011 -0.0054	-0.0007 -0.0007 0.0032	-0.0004 0.0005 -0.0023	-0.0003 0.0003 -0.0017	-0.0001 0.0001 -0.0005	0.0002 -0.0003 0.0014
Structure correction of order α^6	-0.0784 0.1295 -0.8292	0.0471 -0.0777 0.4975	-0.0008 0.0018 -0.0050	-0.0007 0.0015 -0.0043	-0.0004 0.0008 -0.0028	-0.0001 -0.0001 -0.0007
Summary contribution	-131.3697 116.7750 -526.2805	78.0397 -70.0650 315.7683	480.6266 3026.9462 6380.9450	651.7702 2540.4604 5755.3395	843.8518 2035.3279 5212.1450	758.3733 2447.2267 6167.2528

Summary and discussion

- The account of hyperfine structure of P-levels is necessary because experimental transition frequencies are measured between different components of 2P and 2S states.
- HFS of P-states in muonic ions of Li, Be and B was calculated previously in

G. W. F. Drake and L. L. Byer, Phys. Rev. A **32**, 713 (1985).

However, this work contains only general formula of hyperfine structure in leading order.

Thank you