Exotic polyquark states and their properties in QCD

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1. **Observations of exotic tetra- and pentaquark candidates**
2. **Non-resonant explanations of the observed distributions**
3. **Phenomenologocal models for polyquark resonances**
4. **Exotic bound states in QCD (lattice QCD, Green functions at large-N_c, QCD sum rules)**
5. **Conclusions**

• **For many decades only hadron states with quantum numbers of the $\bar{q}q$ and qqq systems have been observed, although one can construct many other color singlets.**

Why not $\bar{q}qqqq$ or $\bar{q}q\bar{q}q$? **[Large-N_c QCD no such states at the leading $1/N_c$]**

In fact we know many multiquark states (deuteron, nuclei) but these are of a bit different nature.

• **First attempt — the strange narrow pentaquark $\theta^+(1540)$ predicted in 1997 and "temporarily" observed in several experiments with large significance — was finally unsuccessful.**

• **In the 21th century many exotic candidates have been reported in the experiments:**

New charmonium states which do not fall in the usual $\bar{c}c$ picture, e.g.:

$Z^+(4430)$ $J^P = 1^+$ **seen in** $B^+ \to K(\psi'\pi+)$ **with** $\Gamma \sim 40^{+18+30}_{-13-13}$ **MeV** $\bar{c}c\bar{d}u$

$Z^+(3900)$ $J^P = 1^+$ **seen in** $\Upsilon(4260) \to \pi^-(J/\psi\pi+)$ **with** $\Gamma \sim 30$ **MeV** $\bar{c}c\bar{d}u$

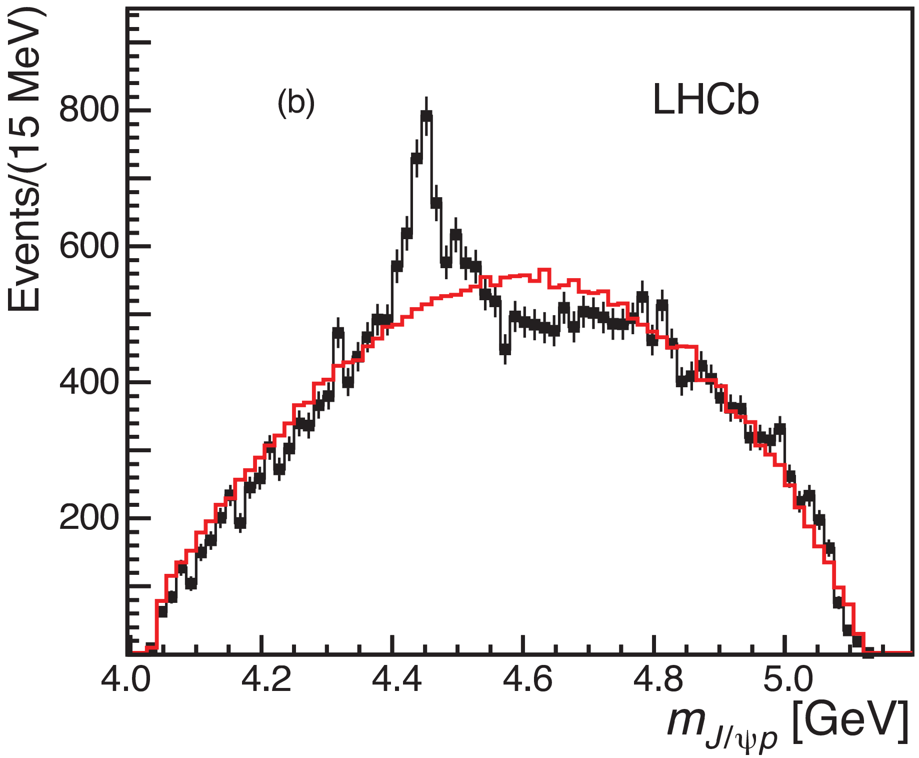
Similar charged states $\bar{b}b$ states $Z_b(10610)$ and $Z_b(10650)$.

Also many neutral states $\bar{c}c\bar{q}q$: e.g. $X(3872)$ a $J^{PC} = 1^{++}$ resonance $M = 3871.69 \pm 0.17$ MeV observed in $B^+ \to K^+(J/\psi\pi^+\pi^-)$ with the width $\Gamma < 1.2$ MeV very close to $D^0\bar{D}^{*0}$ threshold $M_{D^0} + M_{D^{*0}} - M_X = 0.11^{+0.6+0.1}_{-0.4-0.3}$ MeV.

Recently, also *pentaquark* candidates from LHCb in $\Lambda_b^0 \to K^-(J/\psi p)$

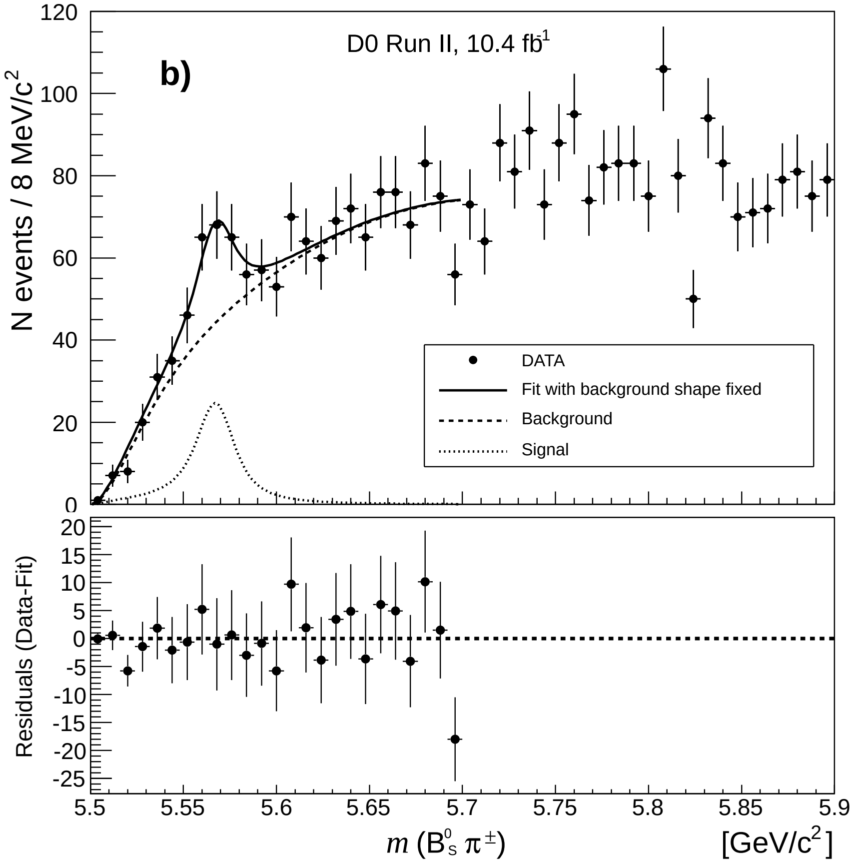
$P_c(4380)$ ($\Gamma \sim$ **200 MeV)**

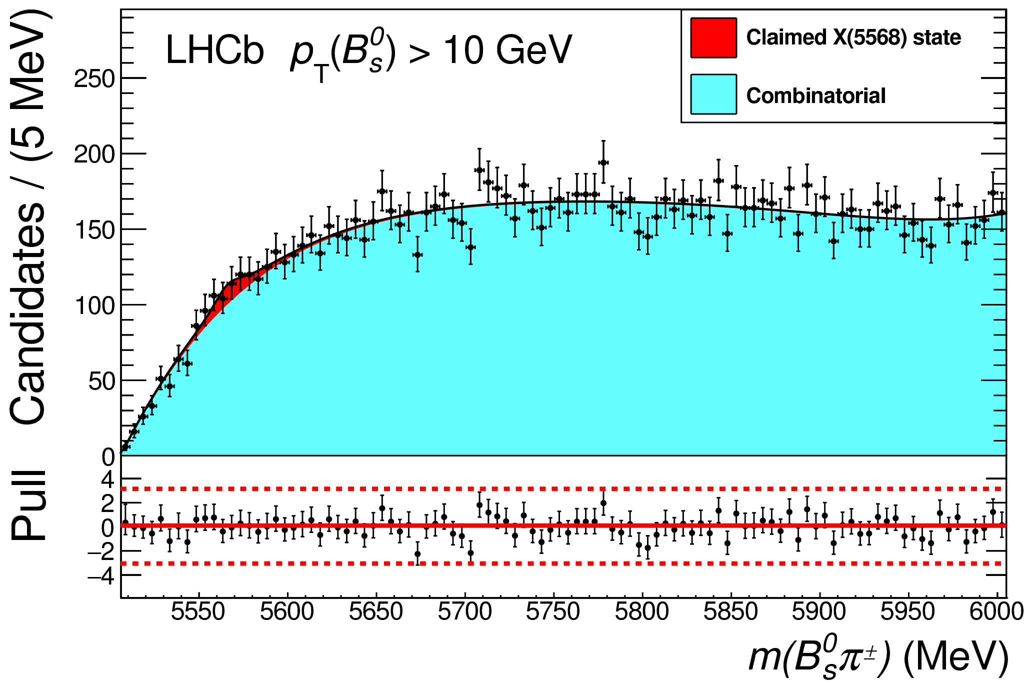
$P_c(4450)$ ($\Gamma \sim$ **30 MeV)**



LHCb, PRL(2015)

The only *flavour-exotic* candidate is $X(5568)$

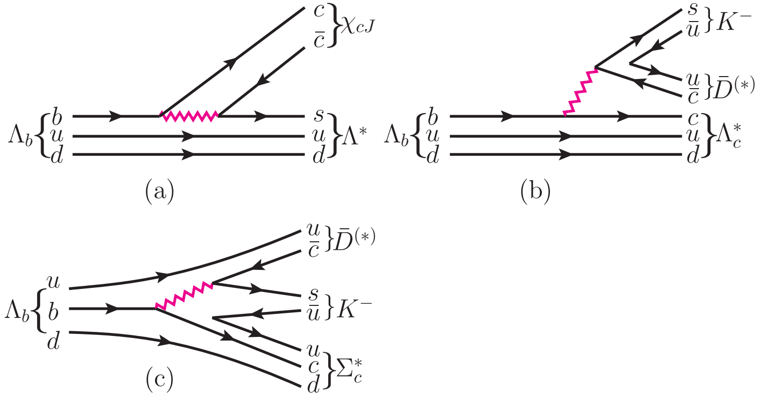


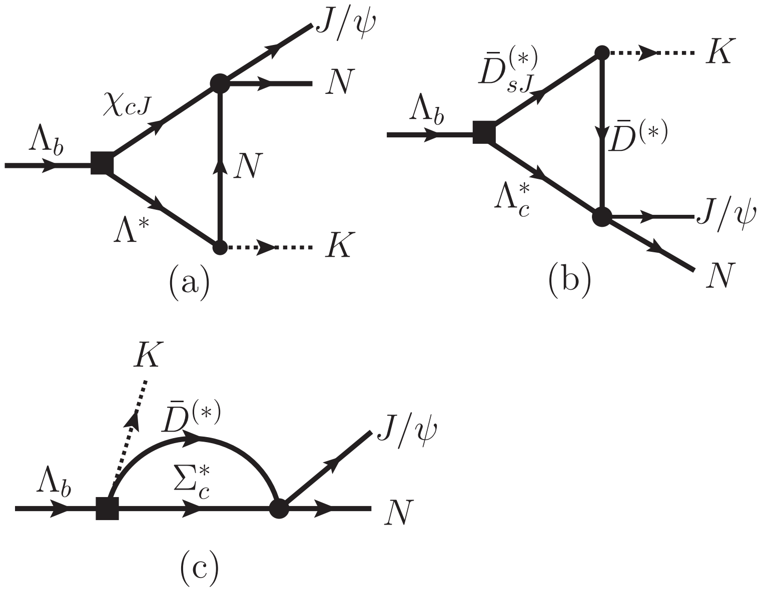


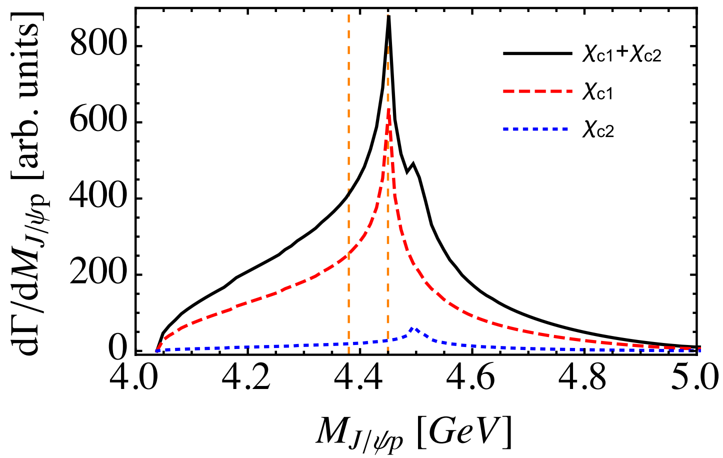
What are these structures — many scenarios.

Are they real resonances or just structures in the cross-section (anomalous thresholds, cusps etc)?

Nonresonant nature of observed distributions







Amplitudes involving hadron rescatterings (triangle diagrams + two-point functions) may lead to the resonance structures in the distributions similar to the observed ones

Phenomenological approaches to exotic states

Exotic hadrons as quark – diquark confined bound states

Naive constituent-quark model:

Constituent quarks [$m_q = 250$ MeV ($q = u, d$), $m_s = 350$ MeV]

interacting via potential (confining at large distances + OGE at short distances)

are building blocks of ordinary hadrons: *quark-antiquark mesons* **and** *three-quark baryons***.**

Color-triplet–antitriplet interation via multigluon exchanges is confining at large distances

Concept of DIQUARKS: color-antitriplet $\bar{D}^a = \epsilon^{abc} q_b q_c$ made of two quarks.

Ordinary mesons: $\bar{q}^a q^a$ ($\bar{q}q$)

Ordinary baryons: $\epsilon_{abc} q^a q^b q^c = \bar{D}^a q^a$ (qqq)

Tetraquarks: $\bar{D}^a D^a$ ($\bar{q}q\bar{q}q$)

Pentaquarks: $\epsilon_{abc} \bar{D}^a \bar{D}^b \bar{q}^c$ ($\bar{q}qqqq$)

- **Why not $\epsilon_{abc} \bar{D}^a \bar{D}^b \bar{D}^c$ (confined 6-quark state)?**
- **Hierarchy of sizes: diquarks are not really compact objects, size similar to meson size**
- **Not easy to obtain narrow exotic states**

Exotic hadrons as molecular bound states

meson-meson long-distance interactions may produce a bound state (similar to the deuteron).

Mass below threshold; if relatively wide, may see the tail.

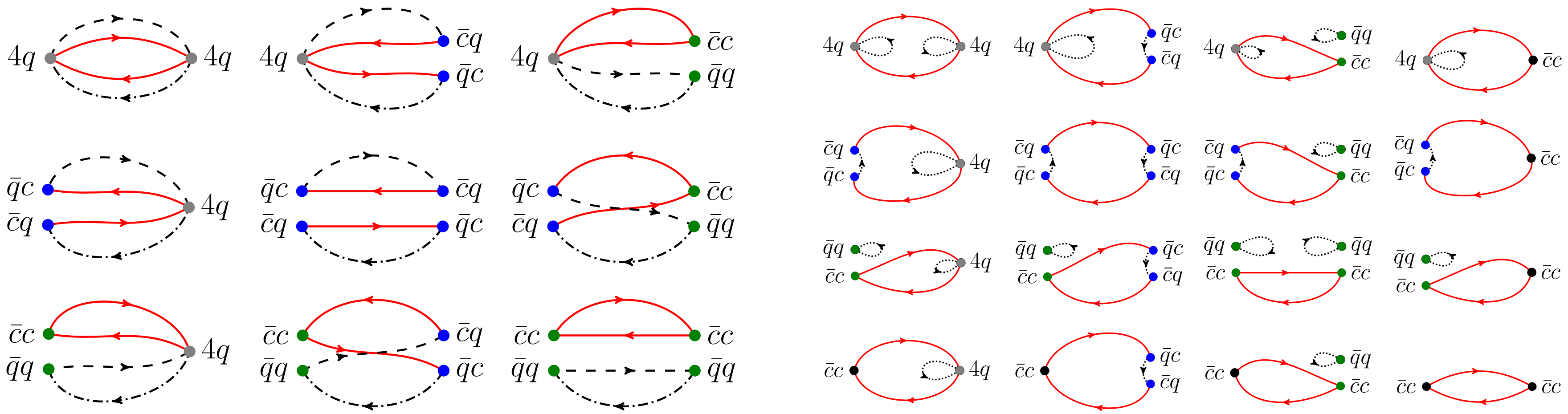
- **How to obtain from QCD the potential?**

How to systematically understand exotic states in QCD?

Lattice QCD

- **Put QCD on the lattice in a box (discrete spectrum of bound states)**
- **Consider two-point functions of a set of interpolating currents $\langle T\, J(x)\, j(0)\rangle$ in Eucledian space**
- **Identify those discrete states which yield meson-meson continuum states in the continuum limit**
- **By modifying the operator set identify the discrete continuum state and determine its properties**

Example of lattice study of $X(3872$ [Prelovsek et al, 2015: $\bar{c}c$, meson-meson, four-quark operators]



"A lattice candidate for $X(3872)$ with $I = 0$ is observed very close to the experimental state only if both $\bar{c}c$ and $\bar{D}D$ interpolators are included; the candidate is not found if diquark-antidiquark and $\bar{D}D$ are used in the absence of $\bar{c}c$. No candidate for neutral or charged $X(3872)$, or any other exotic candidates are found in the $I = 1$ channel. We also do not find signatures of exotic $\bar{c}c\bar{s}s$ candidates below 4.2 GeV". *No convincing signatures of tetraquark bound states.*

QCD at large Nc

$SU(N_c)$ gauge theory with $N_c \to \infty$ and $\alpha_s \sim 1/N_c$. At leading order, QCD Green functions have only non-interacting mesons as intermediate states; tetraquark bound states may emerge only in N_c-subleading diagrams. This fact was believed to provide the theoretical explanation of the non-existence of exotic tetraquarks.

However, even if the exotic tetraquark bound states appear only in subleading diagrams, the crucial question is their width: if narrow, they might be well observed in nature.

[W.Lucha, D.M., H.Sazdjian, 1706.06003]

We discuss four-point Green functions of bilinear quark currents, depend on 6 variables p_1^2, p_2^2, $p_1'^2$, $p_2'^2$, $p = p_1 + p_2 = p_1' + p_2'$, and the two Mandelstam variables $s = p^2$ and $t = (p_1 - p_1')^2$.

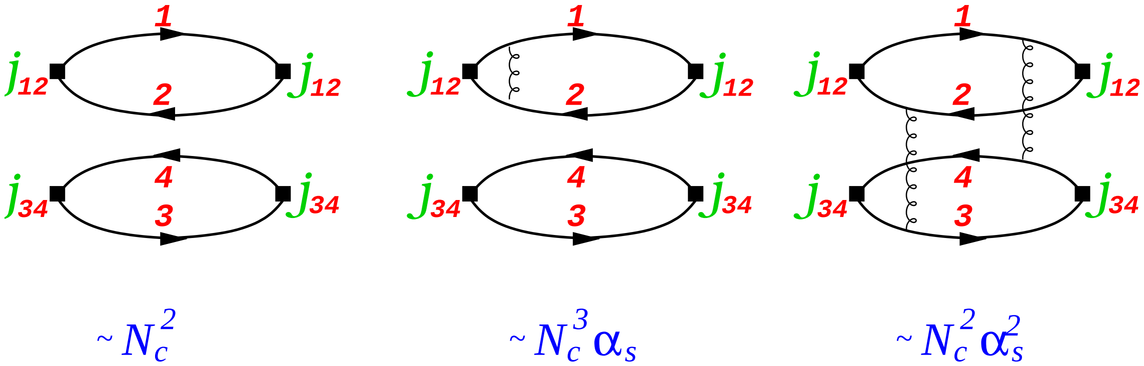
Criteria for selecting diagrams which potentially contribute to the tetraquark pole at $s = M_T^2$:

1. **The diagram should have a nontrivial (i.e., non-polynomial) dependence on the variable s.**
2. **The diagram should have a four-particle cut (i.e. threshold at $s = (m_1 + m_2 + m_3 + m_4)^2$), where m_i are the masses of the quarks forming the tetraquark bound state. The presence or absence of this cut is established by solving the Landau equations for the corresponding diagram.**

Flavour-exotic tetraquarks

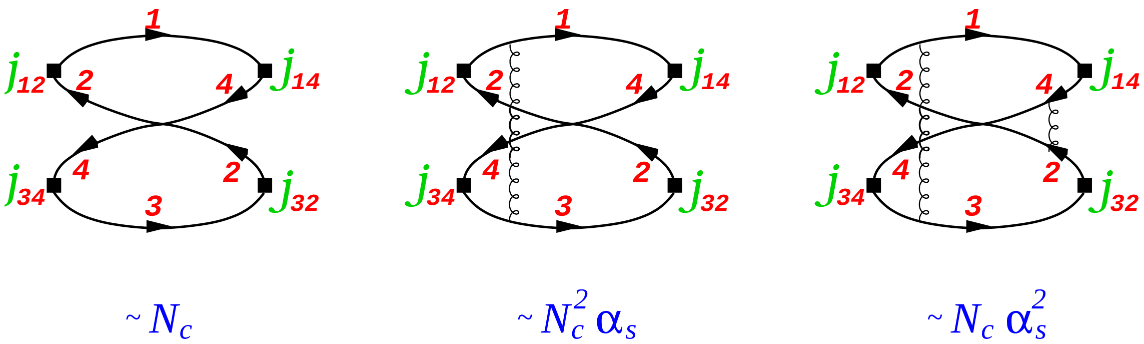
Bilinear quark currents $j_{ij} = \bar{q}_i q_j$ producing M_{ij} from the vacuum, $\langle 0|j_{ij}|M_{ij}\rangle = f_{M_{ij}}$, $f_M \sim \sqrt{N_c}$.

"Direct" 4-point functions $\Gamma_{\rm I}^{\rm (dir)} = \langle j_{12}^\dagger j_{34}^\dagger j_{12} j_{34}\rangle$ and $\Gamma_{\rm II}^{\rm (dir)} = \langle j_{14}^\dagger j_{32}^\dagger j_{14} j_{32}\rangle$:

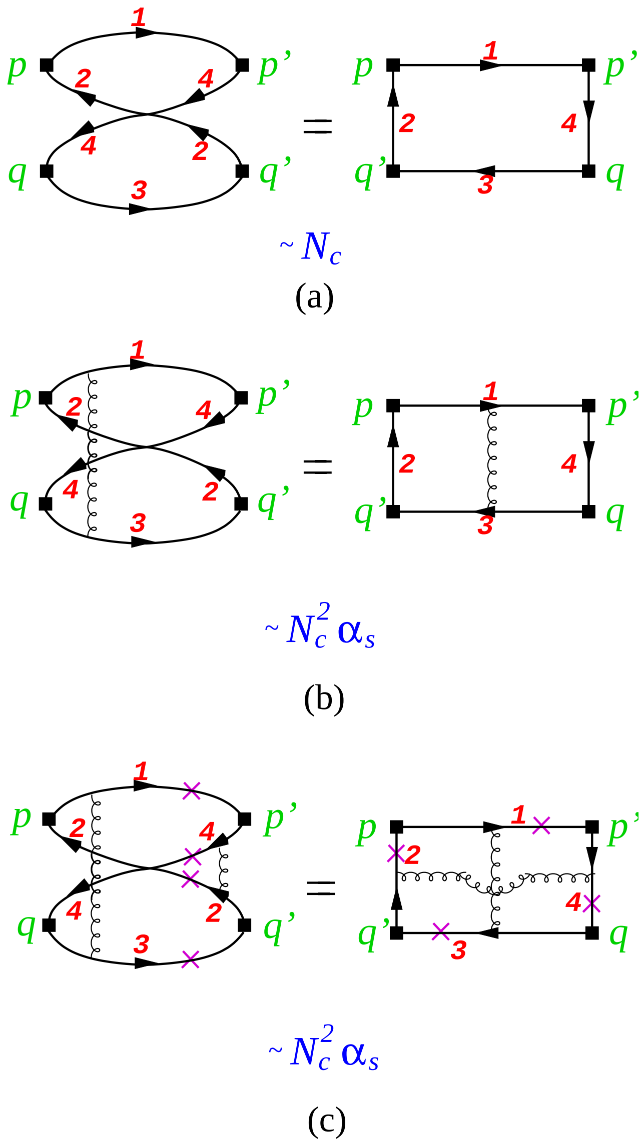


(a) (b) (c)

"Recombination" functions $\Gamma^{\rm (rec)} = \langle j_{12}^\dagger j_{34}^\dagger j_{14} j_{32}\rangle$ and $\Gamma^{\rm (rec)\dagger}$:



(a) (b) (c)


p
p'
q
q'
1
2
3
4
=
~ N_c
(a)
~ $N_c^2 \alpha_s$
(b)
~ $N_c^2 \alpha_s$
(c)

$$\Gamma^{(\mathrm{dir})}_{\mathrm{I},T} = \langle j^\dagger_{12} j^\dagger_{34} j_{12} j_{34} \rangle = O(N_c^0), \quad \Gamma^{(\mathrm{dir})}_{\mathrm{II},T} = \langle j^\dagger_{14} j^\dagger_{32} j_{14} j_{32} \rangle = O(N_c^0), \quad \Gamma^{(\mathrm{rec})}_{T} = \langle j^\dagger_{12} j^\dagger_{34} j_{14} j_{32} \rangle = O(N_c^{-1}).$$

The fact that "dir" and "rec" amplitudes have different behaviors in N_c requires at least two exotic poles:
T_A couples stronger to $M_{12}M_{34}$ channel, T_B couples stronger to $M_{14}M_{32}$ channel.

$$\begin{aligned}
\Gamma^{(\mathrm{dir})}_{\mathrm{I},T} &= O(N_c^0) = f_M^4 \left(\frac{|A(M_{12}M_{34} \to T_A)|^2}{p^2 - M_{T_A}^2} + \frac{|A(M_{12}M_{34} \to T_B)|^2}{p^2 - M_{T_B}^2} \right) + \cdots, \\
\Gamma^{(\mathrm{dir})}_{\mathrm{II},T} &= O(N_c^0) = f_M^4 \left(\frac{|A(M_{14}M_{32} \to T_A)|^2}{p^2 - M_{T_A}^2} + \frac{|A(M_{14}M_{32} \to T_B)|^2}{p^2 - M_{T_B}^2} \right) + \cdots, \\
\Gamma^{(\mathrm{rec})}_{T} &= O(N_c^{-1}) = f_M^4 \left(\frac{A(M_{12}M_{34} \to T_A) A(T_A \to M_{14}M_{32})}{p^2 - M_{T_A}^2} + \frac{A(M_{12}M_{34} \to T_B) A(T_B \to M_{14}M_{32})}{p^2 - M_{T_B}^2} \right) + \cdots.
\end{aligned}$$

We seek tetraquarks with finite mass at large N_c:

$$\begin{aligned}
A(T_A \to M_{12}M_{34}) &= O(N_c^{-1}), \qquad A(T_A \to M_{14}M_{32}) = O(N_c^{-2}), \\
A(T_B \to M_{12}M_{34}) &= O(N_c^{-2}), \qquad A(T_B \to M_{14}M_{32}) = O(N_c^{-1}).
\end{aligned}$$

The widths $\Gamma(T_{A,B}) = O(N_c^{-2})$.

Mixing between T_A and T_B:

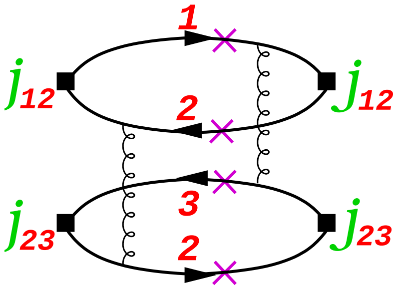
Introducing mixing parameter g_{AB}, we get additional contributions to the Green functions. Most restrictive for g_{AB} is the recombination function, for which mixing provides the additional contribution

$$\Gamma_T^{(\mathrm{rec})} = O(N_c^{-1}) = f_M^4 \left(\frac{A(M_{12}M_{34} \to T_A)}{p^2 - M_{T_A}^2} g_{AB} \frac{A(T_B \to M_{14}M_{32})}{p^2 - M_{T_B}^2} \right) + \cdots .$$

The mixing parameter $g_{AB} \leq O(N_c^{-1})$: the two flavor-exotic tetraquarks of the same flavor content do not mix at large N_c.

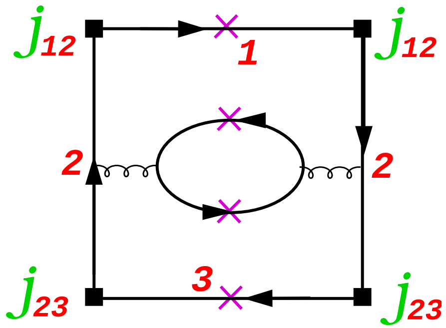
Diagrams of new topologies emerge.

For direct functions $\Gamma^{(\mathrm{dir})}_{(\mathrm{I,II}),T}$, new diagrams do not change leading large-N_c behavior:



$\sim N_c^2\,\alpha_s^2$

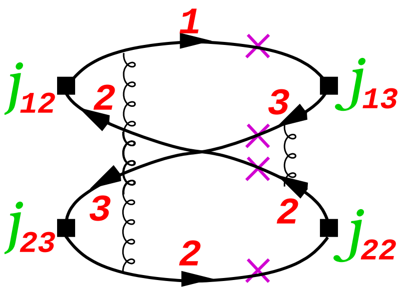
(a)



$\sim N_c^2\,\alpha_s^2$

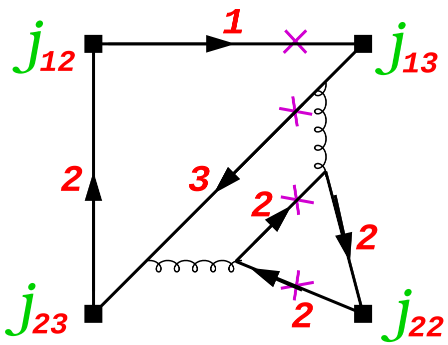
(b)

For recombination functions, the new diagram modifies leading large-N_c behavior



$\sim N_c\,\alpha_s^2$

(a)



$\sim N_c^2\,\alpha_s^2$

(b)

The new diagram modifies the leading large-N_c behavior:

$$\Gamma^{(\mathrm{dir})}_{\mathrm{I},T} = \langle j^\dagger_{12} j^\dagger_{23} j_{12} j_{23}\rangle = O(N_c^0), \quad \Gamma^{(\mathrm{dir})}_{\mathrm{II},T} = \langle j^\dagger_{13} j^\dagger_{22} j_{13} j_{22}\rangle = O(N_c^0), \quad \Gamma^{(\mathrm{rec})}_{T} = \langle j^\dagger_{12} j^\dagger_{23} j_{13} j_{22}\rangle = O(N_c^0).$$

"dir" and "rec" functions have the same behavior, and one exotic state T is enough:

$$A(T \to M_{12}M_{23}) = O(N_c^{-1}), \qquad A(T \to M_{13}M_{22}) = O(N_c^{-1}).$$

Its width is $\Gamma(T) = O(N_c^{-2})$.

T can mix with the ordinary meson M_{13}. The restriction on the mixing parameter $g_{TM_{13}}$:

$$\Gamma^{(\mathrm{dir})}_{\mathrm{I},T} = O(N_c^0) = f_M^4 \left(\frac{A(M_{12}M_{23} \to T)}{p^2 - M_T^2} g_{TM_{13}} \frac{A(M_{13} \to M_{12}M_{23})}{p^2 - M^2_{M_{13}}} \right) + \cdots .$$

$A(M_{13} \to M_{12}M_{23}) \sim 1/\sqrt{N_c}$, **so** $g_{TM_{13}} \leq O(1/\sqrt{N_c})$.

The analysis of Green functions in large-N_c QCD allows one to restrict some properties of the possible exotic states.

QCD sum rules for exotic states

• **The basic object**

T**-product of a number of the interpolating currents** $j(x)$**:**

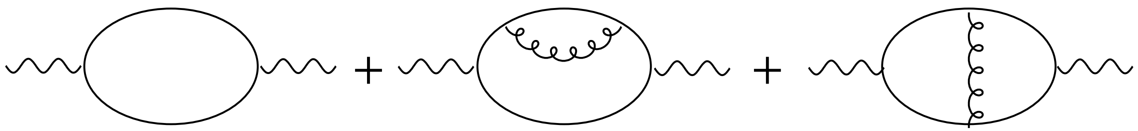
$$\langle\Omega|j(0)|M\rangle = f_M \neq 0.$$

(E.g. $j(x) = \bar{q}_1(x)Oq_2(x)$ **for "normal" mesons, 4-quark currents for exotic mesons).**

The simplest object—2-point function

$$\Pi(p^2) = i\int d^4x\, e^{ipx}\left\langle\Omega\left|T\left(j(x)j^\dagger(0)\right)\right|\Omega\right\rangle$$

Perturbative expansion of the two-point function of 2 bilinear currents:



Perturbation theory works with Feynman propagators, whereas in the soft region due to the confinement the exact non-perturbative propagators differ strongly from Feynman propagators. This should be "corrected."

Regions of soft momenta in Feynman integrals lead to power-suppressed terms in correlators.

$$\int dk \frac{1}{k^2}\frac{1}{(p-k)^2}, \qquad\qquad k \sim \Lambda \to \frac{\Lambda^2}{p^2}$$

Thus, confinement effects should lead to power-suppressed terms in p^2.

• **Wilsonian OPE - separation of distances:**

$$T\left(j(x)j^\dagger(0)\right) = C_0(x^2,\mu)\hat{1} + \sum_n C_n(x^2,\mu) : \hat{O}_n(x=0,\mu) :$$

$$\Pi(p^2) = \Pi_{\rm pert}(p^2,\mu) + \sum_n \frac{C_n}{(p^2)^n}\langle\Omega| : \hat{O}_n(x=0,\mu) : |\Omega\rangle$$

• **Physical QCD vacuum $|\Omega\rangle$ is complicated and differs from perturbative QCD vacuum $|0\rangle$.**

Condensates – nonzero expectation values of gauge-invariant operators over physical vacuum:

$$\boxed{\langle\Omega| : \hat{O}(0,\mu) : |\Omega\rangle \neq 0}$$

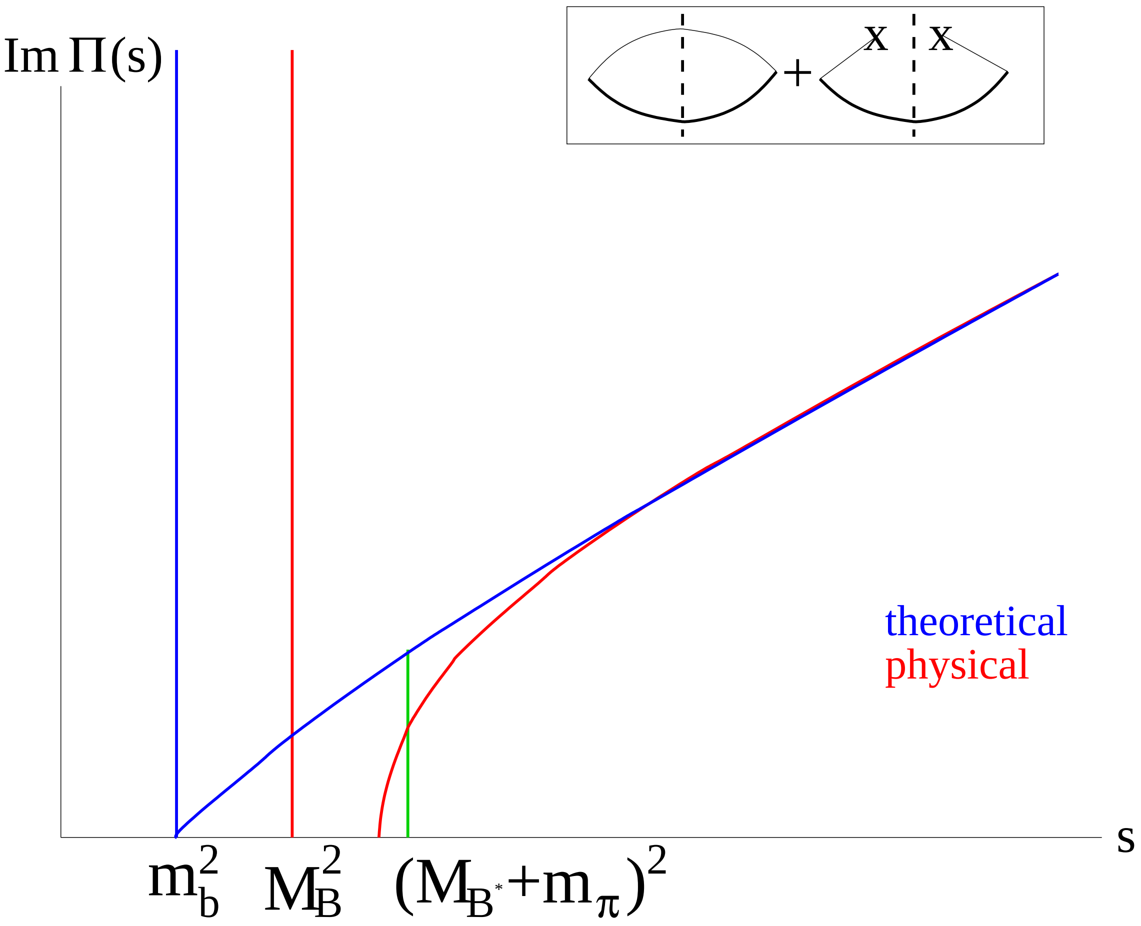
$$\langle\Omega|\bar{q}q(2\text{ GeV}|\Omega\rangle = (271 \pm 3\text{ MeV})^3, \qquad \langle\Omega|\alpha_s/\pi GG|\Omega\rangle = 0.012 \pm 0.006\text{ GeV}.$$

2-point function is analytic function of p^2

$$\Pi(p^2) = \int \frac{ds}{s - p^2}\rho(s),$$

One calculates the spectral densities using OPE and using hadron states

$$\rho_{\rm theor}(s) = \left[\rho_{\rm pert}(s,\mu) + \sum_n C_n \delta^{(n)}(s)\langle\Omega|O_n(\mu)|\Omega\rangle\right], \qquad \rho_{\rm hadr}(s) = f^2\delta(s - M^2) + \rho_{\rm cont}(s)$$



How to relate to each other truncated $\Pi_{\rm OPE}(p^2)$ and $\Pi_{\rm hadron}(p^2)$?

Borel transform $p^2 \to \tau$ [$\frac{1}{s-p^2} \to \exp(-\tau p^2)$]

$$\Pi(\tau) = \int ds \exp(-s\tau)\rho(s) = f^2 e^{-M_B^2\tau} + \int\limits_{s_{\rm phys}}^{\infty} ds\, e^{-s\tau}\rho_{\rm hadr}(s) = \int\limits_{(m_b+m)^2}^{\infty} ds\, e^{-s\tau}\rho_{\rm pert}(s,\mu) + \Pi_{\rm power}(\tau,\mu).$$

Here $s_{\rm phys}$ is the physical threshold, and f is the decay constant defined by

$$\langle 0|\bar{q}Ob|B\rangle = f.$$

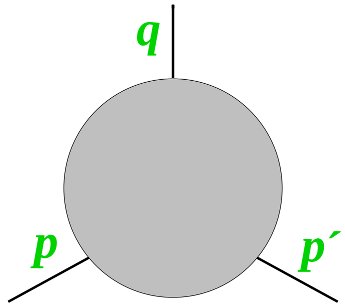
To get rid of the excited-state contributions, one adopts the *duality Ansatz*: all contributions of excited states are counterbalanced by the perturbative contribution above an *effective continuum threshold*, $s_{\rm eff}(\tau)$ which differs from the physical continuum threshold.

Applying the duality assumption for the bilinear quark currents yields:

$$f^2 e^{-M_B^2\tau} = \int\limits_{(m_b+m)^2}^{s_{\rm eff}(\tau)} ds\, e^{-s\tau}\rho_{\rm pert}(s,\mu) + \Pi_{\rm power}(\tau,\mu).$$

Strong decays from 3 – point vertex functions

- **The basic object:**



$$\Gamma(p, p', q) = \int \langle\Omega|T(J(x)j(0)j'(x')|\Omega\rangle \exp(ipx - ip'\, x')dxdx'$$

This correlator contains the triple-pole in the Minkowski region: namely

$$\Gamma(p, p', q) = \frac{ff'}{(p^2 - M^2)(p'^2 - M'^2)} F(q^2) + \cdots$$

where the form factor $F(q^2)$ contains pole at $q^2 = M_q^2$:

$$F(q^2) = \frac{f_q g_{MM'M_q}}{(q^2 - M_q^2)} + \cdots$$

$g_{MM'M_q}$ **describes the $M \to M_1 M_2$ strong transition;**

f**,** f'**, and** f_{M_q} **are the decay constants of the mesons $\langle 0|j(0)|M\rangle = f_M$.**

The three-point function satisfies the double spectral representation

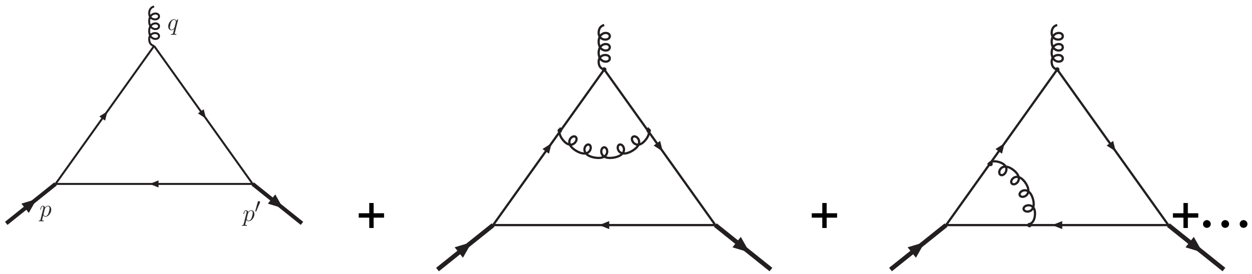
$$\Gamma(p, p', q) = \int \frac{ds}{s - p^2} \frac{ds'}{s' - p'^2} \Delta(s, s', q^2)$$

Perform double Borel transform $p^2 \to \tau$, $p'^2 \to \tau'$ **and applying duality we obtain**

$$\exp(-M^2\tau)\exp(-M'^2\tau') f f' F(q^2) = \int^{s_{\rm eff}} ds \exp(-s\tau) \int^{s'_{\rm eff}} ds' \exp(-s'\tau') \Delta_{\rm OPE}(s, s', q^2)$$

• **Normal hadrons:**

$$\Gamma_{\rm OPE}(p^2, p'^2, q^2) = \Gamma_0(p^2, p'^2, q^2) + \alpha_s \Gamma_1(p^2, p'^2, q^2) + \dots$$

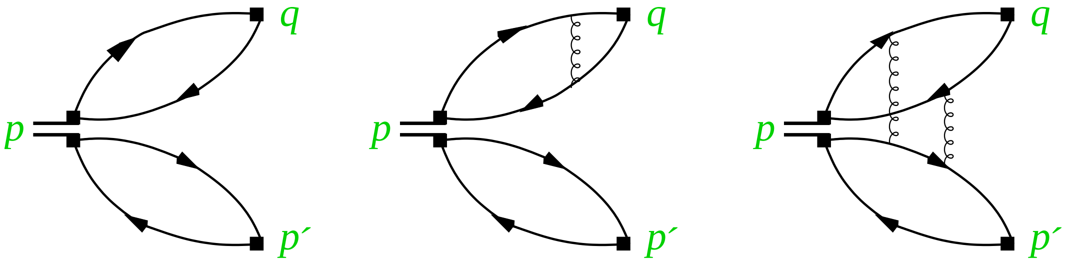


Already one-loop zero-order diagram has a nonzero double-spectral density. And therefore provides a nonzero contribution to the form factor at small and intermediate momentum transfers (and to the coupling). Radiative corrections and are crucial for large q^2 **and improve the result.**

• **Exotic hadrons:** $\langle T(\theta(x) j_1(0) j_2(y))\rangle$.
Many possibilities to write interpolating current for X, $\langle 0|\theta|X\rangle = f_X, \qquad f_X \neq 0.$

$$\theta = M(x)M(x), \qquad M(x) = \bar{q}(x)O\bar{q}(x)$$
$$\theta = M^A(x)M^A(x), \qquad M^A(x) = \bar{q}(x)\lambda^A O\bar{q}(x) \quad [A = 1\ldots, 8]$$
$$\theta = \bar{D}^a(x)D^a(x), \qquad D^a(x) = \epsilon^{abc}(q_c(x))^T C O q_b(x), \quad [a, b, c = 1, \ldots, 3]$$

Color Singlet - color singlet:



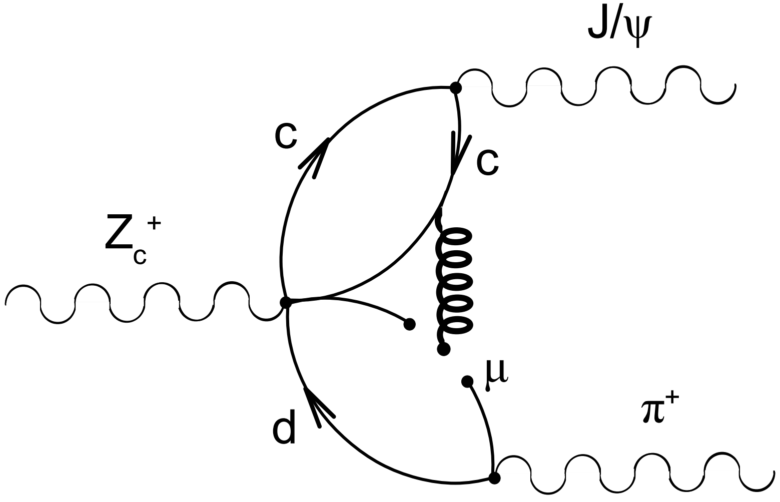
$$\Gamma_{\rm OPE}(p^2, p'^2, q^2) = \Pi(p'^2)\Pi(q^2) + (\alpha_s)^2\Gamma_{\rm connected}(p^2, p'^2, q^2)$$

After the Borel transform $p^2 \to \tau$, the disconnected leading-order contribution vanishes.

Thus the LO contribution is not related to the exotic-state decay. **Clear from the factorization property** $\Gamma(p, p', q) = \Pi(p'^2)\Pi(q^2)$ **and from the large-N_c behaviour of the QCD diagrams.**

The "fall-apart" decay mechanism of exotic hadrons differs from the decay mechanism of the ordinary hadrons and requires the appropriate treatment within QCD sum rules. The calculation of the radiative corrections is mandatory for a reliable analysis of the properties of the exotic states.

Do power corrections contribute to the decay rate?



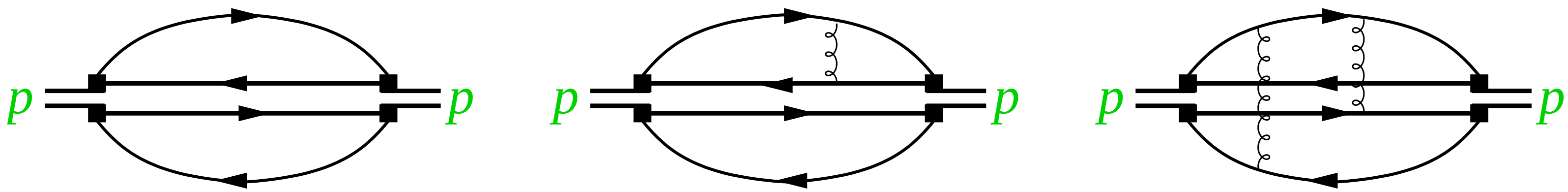
[from M.Nielsen, arXiv:1611.03300]

Power corrections are given in terms of the local condensates, i.e. there is no momentum flow from one part of the diagram to the other. Therefore again factorizes in the function of $F(q^2)F(p'^2)$ and vanishes after the Borel transform in p^2.

No contibution of the gluon condensate at leading order in α_s to the $X \to M_1 M_2$ amplitude.

2 – point function of exotic currents

Singlet-singlet:



How to write down the duality relation?

$$f_X^2 \exp(-M_X^2\tau) + \Pi^{\rm cont}_{meson-meson}(\tau) = \int ds\,\exp(-s\tau)\rho_{XX}(s) + \Pi_{\rm power}(\tau)$$

The right prescription:

Do not include the LO (i.e. start with order $O(\alpha_s^2)$ in the singlet-singlet case: $f_X^{(1)} \sim O(\alpha_s)$.

Outlook

• **Around 20 candidates for exotic resonances:** $\bar{c}c\bar{q}q$ **or** $\bar{b}b\bar{q}q$ **+** $\bar{c}cqqq$**.**
$X(5568)$ **in** $B_s\pi^\pm$ **(?) is doubtful. No light-quark candidates** $\bar{q}q\bar{q}q$**. No convincing interpretation. Various possibilities.**

• *Non-resonance explanations*:
the observed structures are due to hadron low-energy diagrams (triangles and loops). The amplitudes contains many unknown couplings. Difficult to obtain a certain conclusion.

• *Phenomenological models*:
Based on diquarks in a confining potential predict spectrum of exotic states
Based on hadron-hadron potentials predict molecular states (depending on specific potentials)

• *Lattice QCD*:
Very difficult setup; no convincing results for exotic states

• *QCD at large* N_c:
Two exotic $\bar{q}_1q_2\bar{q}_3q_4$ **narrow states** $\Gamma \sim O(1/N_c^2)$**, each decaying into one meson-meson channel.**
One cryptoexotic state $\bar{q}_1q_2\bar{q}_2q_3$ $\Gamma \sim O(1/N_c^2)$ **decaying into various meson-meson channels with similar probabilities.**

• *QCD sum rules*
Dozens of papers, many confirming the resonance interpretation of exotic states. However, not all conceptual issues of the method for exotic states have been fully settled.

• **Dynamics of fall-apart decays of exotic resonances has fundamental difference from dynamics of ordinary-meson decays: the appropriate contributions to Green functions describing decays of exotic states emerge only at subleading α_s orders; the leading order disconnected diagrams are** *not related to strong decays of exotic hadrons*. **This makes the calculation of α_s-corrections mandatory.**

Many efforts for otaining reliable predictions are necessary!