Semiclassical treatment of a photon decay in an external electromagnetic field at finite temperature

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Particle decays in an external electromagnetic field

- Standard approach based on exact solution of the Dirac equation in an external e/m field

review: Borisov et. al. 1997

book: Mikheev, Kuznetsov 2003-2014

...

- Plus: straightforward approach. Minus: complicated calculations
- Processes in thermal media $\rightarrow$ calculations even more complicated

Semiclassical approach

- Geometrically clear tunneling picture
- Significantly simpler calculations
- Valid only in the limit of exponential suppression

Idea of semiclassical "Worldline Instanton" approach

- Particle production in external field ϕ_{ext}

$$\Gamma \propto \text{Im} \int_{p.b.c} Dx_\mu e^{-S[x_\mu, \phi_{ext}]}.$$

- Path integral in saddle point approximation.
  E.o.m.: $\left.\frac{\delta S}{\delta x_\mu}\right|_{x_\mu^{cl}} = 0 +$ periodic b.c.
- Classical solution x_μ^{cl} — closed trajectory.
  $\Gamma \propto e^{-S[x_\mu^{cl}]}$ if semiclassical condition $S[x_\mu^{cl}] \gg 1$ is satisfied.
- Fluctuations near classical solution $\delta x_\mu = x_\mu - x_\mu^{cl}$.
  Integral over fluctuations $\rightarrow$ pre-exponential factor.
- Negative mode in 2nd variation $\delta^2 S[x_\mu] \rightarrow$ imaginary prefactor $\rightarrow$ particle production

Affleck, Alvarez Manton '82, Dunne Schubert '05'06, Monin '05, Monin, Voloshin '10 etc.

The Schwinger effect via "worldline instantons"

Affleck, Alvarez Manton 1982 (Nucl.Phys.B 197.3.509)

Scalar QED: $S_E = \int d^4x \left(-\frac{1}{4} F_{\mu\nu}^2 + |D_\mu \phi|^2 + m^2 |\phi|^2 \right).$

A_μ — classical external field. $Z[A_\mu] = \int D\phi^* D\phi e^{-S_E[A_\mu]} = e^{-W[A_\mu]}.$

$$\Gamma = \operatorname{Im} W[A_\mu]$$

Effective action in Schwinger *proper time* representation:

$$W[A_\mu] = -\frac{1}{4} F_{\mu\nu}^2 + \int_0^\infty \frac{ds}{s} e^{-m^2 s} \mathrm{Tr} \left(e^{D_\mu^2 s} \right).$$

Operator $\left(-D_\mu^2\right)$ can be interpreted as QM Hamiltonian.

$$\mathrm{Tr} \left(e^{s D_\mu^2} \right) = \int d^4x \langle x_\mu | e^{-s\left(-D_\mu^2\right)} | x_\mu \rangle = \int_{p.b.c.} Dx_\mu e^{-\int_0^s d\tau \left(\frac{\dot{x}_\mu^2}{4} + ieA_\mu \dot{x}_\mu \right)}.$$

The Schwinger effect via "worldline instantons"

$$\Gamma \propto \mathrm{Im} \int_0^\infty \frac{ds}{s} e^{-sm^2} \int_{p.b.c.} Dx_\mu e^{-\int_0^1 d\tau \left(\frac{\dot{x}_\mu^2}{4s} + ieA_\mu \dot{x}_\mu \right)}.$$

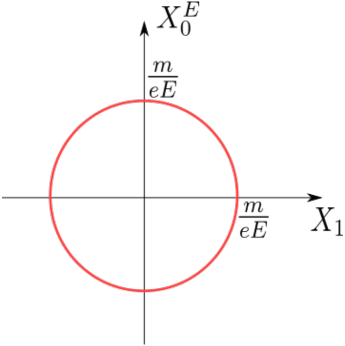
Solve integrals over x_μ and s in the saddle point approximation

Uniform constant electric field E.

The leading solution is a circle:

$$x_0 = \frac{m}{eE}\sin(2\pi\tau), \qquad x_1 = \frac{m}{eE}\cos(2\pi\tau),$$

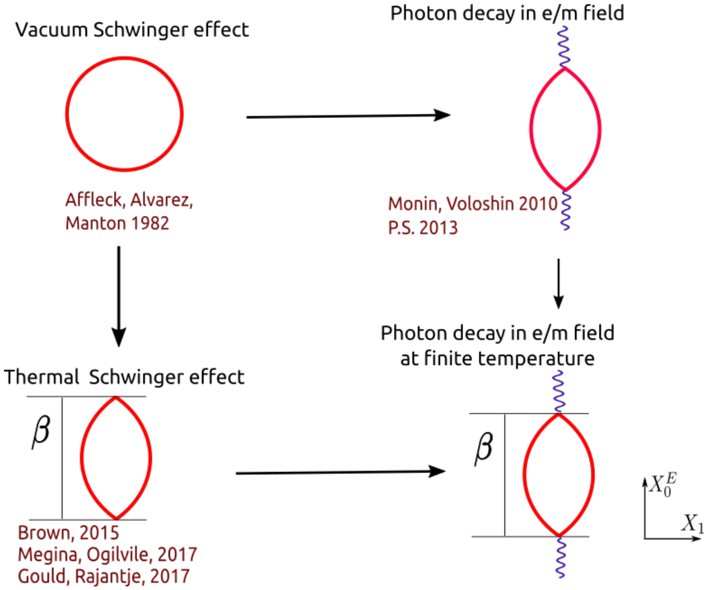
$$x_2 = x_3 = 0, \qquad s = \frac{2\pi}{eE}.$$



The action on the solution x_μ is $S = \frac{\pi m^2}{eE}$.

$$\Gamma = \frac{(eE)^2}{(2\pi)^3} e^{-\frac{\pi m^2}{eE}}.$$

Generalizations to external photon and finite temperature



Photon decay in electric/magnetic field

Monin, Voloshin 2010 (arXiv:1001.3354)

P.S. 2013 (arXiv:1301.5707)

Ext. photon $k_\mu = (\omega, 0, \omega, 0)$

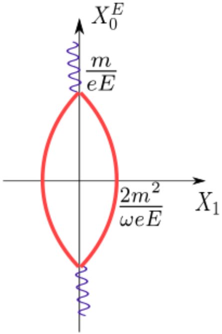
Optical theorem:
$\Gamma \propto \mathrm{Im}\left(\Pi_{\mu\nu}(k)\varepsilon^*_\mu(k)\varepsilon_\nu(k)\right)$.

Photon: insertion
$\oint d\tau \dot{x}_\mu(\tau)\, e^{-ik_\mu x_\mu(\tau)}$
into the path integral for Γ.

Classical solution —
two arcs of circle (electric)
two hyperbolas (magnetic)

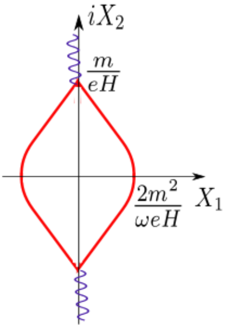
$$\Gamma \propto e^{-S}.$$

Electric field



$$S = \frac{8m^3}{3\omega eE}$$

Magnetic field



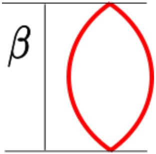
$$S = \frac{8m^3}{3\omega eH}$$

in the limit $\omega \gg 2m$

The Schwinger effect at finite temperature T

Brown 2015 (arXiv:1512.05716)

- QFT at finite temperature — euclidean time x_0^E is periodic with period $\beta = \frac{1}{T}$. New semiclassical condition: $T \ll m$.
- Small temperatures — the same instanton as at zero temperature until the size of instanton $\frac{2m}{eE}$ is less than β.
- At critical temperature $T_c = \frac{eE}{2m}$ the size of instanton $=$ the length of compact dimension



- Larger temperatures — a new solution: two arc of circle.

$$\Gamma_T \propto \exp\left(-\frac{2m^2}{eE}\arcsin\left(\frac{T_c}{T}\right) - \frac{m}{T}\sqrt{1-\frac{T_c^2}{T^2}}\right)$$

The limit $T \gg T_c$: $\Gamma \propto e^{-\frac{2m}{T}}$ — Boltzmann exponent. $2m$ – energy of a pair.

Inclusive rate at fixed energy $\mathcal{E}$

Photon decay in th. bath is non-equilibrium process. Thermal approach does not work.

Photon decay rate at fixed energy $\mathcal{E}$:

$$\Gamma_T = \int_0^\infty d\mathcal{E}\, e^{-\mathcal{E}/T}\, \Gamma_{\mathcal{E}}$$

$\Gamma_{\mathcal{E}}$ — decay rate of off-shell photon $\gamma_{\mathcal{E}}$ with 4-momentum $(\mathcal{E},0,0,0)$.

Thermal Schwinger effect via integral over fixed energy

$$\Gamma_T = \int_0^\infty d\mathcal{E} \int_0^\infty \frac{ds}{s} \int_{p.b.c.} Dx_\mu\, e^{-m^2 s - \int_0^s d\tau \left(\frac{\dot{x}_\mu^2}{4} + ieA_\mu \dot{x}_\mu\right) - \mathcal{E}(x_0(1/2) - x_0(0)) - \mathcal{E}/T}.$$

Photon decay in ext. field $\quad = \quad \gamma\gamma_{\mathcal{E}} \to e^+e^-$

effective external 4-momentum $(\omega + \mathcal{E},0,\omega,0)$.

Photon decay in electric field at finite temperature

$T < T_c \;\rightarrow\;$ the same solution as for $T = 0$. $\quad T > T_c \;\rightarrow\;$ new solution.

$$\Gamma \propto e^{-S}.$$

Action on the classical solution, exactly on arbitrary $\frac{2m}{\omega}$ and $\frac{T_c}{T} \leq 1$.

$$S = \frac{4m^2}{eE} \cdot \arctan\left(\frac{2mT_c}{\omega T}\right)\left[1 - \frac{1}{2}\left(\left(\frac{2m}{\omega}\right)^{-2} + \frac{T_c^2}{T^2}\right) + \left(\frac{2m}{\omega}\right)^{-2}\left[1 - \left(\frac{2m}{\omega}\right)^2 \theta^2\right]\right] -$$

$$- \frac{2m^2}{eE} \cdot \left(\frac{2m}{\omega}\right)^{-1} \cdot \frac{T_c}{T} + \frac{4m^2}{eE}\left(\frac{2m}{\omega}\right)^{-1}\frac{T_c}{T}\left[1 - \sqrt{1 - \left(\frac{2m}{\omega}\right)^2 \theta^2}\right], \qquad \theta^2 = 1 - \frac{T_c^2}{T^2}.$$

In the limit $\omega \gg 2m$

$$S = \frac{4m^3}{\omega eE} \cdot \frac{T_c}{T}\left(1 - \frac{T_c^2}{T^2}\right) + \frac{8m^3}{3\omega eE} \cdot \left(\frac{T_c}{T}\right)^3.$$

In the limit $T \gg T_c \quad \rightarrow \quad S = \frac{2m^2}{\omega T}$,
in the limit $T = T_c \quad \rightarrow \quad S = \frac{8m^3}{3\omega eE}$, the same as for $T = 0$.

Photon decay in **magnetic** field at finite temperature

Critical temperature $T_c = \frac{eH}{2m}$. Semiclassics: $T \ll m$

$T < T_c \;\rightarrow\;$ the same solution as for $T = 0$. $T > T_c \;\rightarrow\;$ new solution.

In the limit $\omega \gg 2m$:

$$\Gamma \propto e^{-S}, \qquad S = \frac{4m^3}{\omega e H} \cdot \frac{T_c}{T}\left(1 - \frac{T_c^2}{T^2}\right) + \frac{8m^3}{3\omega e H} \cdot \left(\frac{T_c}{T}\right)^3 .$$

In the limit $T \gg T_c \quad \rightarrow \quad S = \frac{2m^2}{\omega T}$,
in the limit $T = T_c \quad \rightarrow \quad S = \frac{8m^3}{3\omega e H}$, the same as for $T = 0$.

Semiclassics: $T \ll m \;\rightarrow$ only sub-Schwinger magnetic field
$H \ll m/e \sim 10^{13}\,\mathrm{G}$.

Conclusions

- Worldline instanton method may be applied to photon decay in external electromagnetic field at zero and nonzero temperature in the regime of exponential suppression.
- Astrophysical applications of photon decay in thermal magnetized plasma for sub-Schwinger magnetic field?
- Possible generalization to nontrivial chemical potential?

Thank you for your attention!