Finite-temperature effective potentials in models with extended Higgs sector: typical scenarios

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Introduction

In the simple isoscalar model the standard-like Higgs potential

$$U(\varphi) = -\frac{1}{2}\mu^2\varphi^2 + \frac{1}{4}\lambda\varphi^4.$$

Two solutions

$$v(0) = 0 \text{ and } v^2(T) = \frac{\mu^2}{\lambda} - \frac{T^2}{4},$$

demonstrate the second order phase transition at the critical temperature

$$T_c = \frac{2\mu}{\sqrt{\lambda}} = 2v(0),$$

The thermal Higgs boson mass

$$m_h^2 = -\mu^2 + \lambda\frac{T^2}{4}.$$

Introduction

In a number of analyses the MSSM finite-temperature effective potential is taken in the representation

$$V_{eff}(v,T) = V_0(v_1,v_2,0) + V_1(m(v),0) + V_1(T) + V_{ring}(T), \quad (1)$$

- **V_0 is the tree-level MSSM two-doublet potential at the SUSY scale**
- **V_1 is the (non-temperature) one-loop resumed Coleman-Weinberg term, dominated by stop and sbottom contributions**
- **$V_1(T)$ is the one-loop temperature term**
- **V_{ring} is the correction of re-summed leading infrared contribution from multi-loop ring (or daisy) diagrams**

Finite temperature corrections of squarks

In the finite temperature field theory Feynman diagrams with boson propagators, containing Matsubara frequencies $\omega_n = 2\pi n T$ $(n = 0, \pm 1, \pm 2, ...)$, lead to structures of the form

$$I[m_1, m_2, ..., m_b] = T \sum_{n=-\infty}^{\infty} \int \frac{d\mathbf{k}}{(2\pi)^3} \prod_{i=1}^{b} \frac{(-1)^b}{(\mathbf{k}^2 + \omega_n^2 + m_j^2)}, \qquad (2)$$

$\mathbf{k}$ is the three-dimensional momentum in a system with the temperature T.

Finite temperature corrections of squarks

At $n \neq 0$ the result is

$$I[m_1, m_2, ..., m_b] = 2T\,(2\pi T)^{3-2b}\,\frac{(-1)^b \pi^{3/2}}{(2\pi)^3}\,\frac{\Gamma(b-3/2)}{\Gamma(b)}\,S(M, b{-}3/2), \quad (3)$$

where

$$S(M, b-3/2) = \int \{dx\} \sum_{n=1}^{\infty} \frac{1}{(n^2+M^2)^{b-3/2}}, \qquad M^2 \equiv \left(\frac{m}{2\pi T}\right)^2.$$

Finite temperature corrections of squarks

We calculate the integral

$$J_0[a_1, a_2] = \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{1}{(\mathbf{k}^2 + a_1^2)(\mathbf{k}^2 + a_2^2)} = \frac{1}{4\pi(a_1 + a_2)},$$

taking a residue in the spherical coordinate system.
$a_{1;2}^2$ are the sums of squared frequency and squared mass.
Derivatives of J_0 with respect to a_1 and a_2 can be used for calculation of integrals

$$J_1[a_1, a_2] = \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{1}{(\mathbf{k}^2 + a_1^2)^2(\mathbf{k}^2 + a_2^2)} = -\frac{1}{2a_1}\frac{\partial J_0}{\partial a_1} = \frac{1}{8\pi a_1(a_1 + a_2)^2},$$

$$J_2[a_1, a_2] = \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{1}{(\mathbf{k}^2 + a_1^2)^2(\mathbf{k}^2 + a_2^2)^2} = \frac{1}{4a_1 a_2}\frac{\partial^2 J_0}{\partial a_1 \partial a_2} = \frac{1}{8\pi a_1 a_2(a_1 + a_2)^3}.$$

Finite temperature corrections of squarks

Thus, the procedure of Feynman parametrization is not used. Substituting $a_1^2 = 4\pi^2 n^2 T^2 + m_1^2$ and $a_2^2 = 4\pi^2 n^2 T^2 + m_2^2$ to (??) and taking the sum over Matsubara frequencies after the integration we get

$$I_0[m_1, m_2] = \sum_{n=-\infty, n\neq 0}^{\infty} \frac{1}{4\pi(\sqrt{4\pi^2 n^2 T^2 + m_1^2} + \sqrt{4\pi^2 n^2 T^2 + m_2^2})}.$$

or, after redefinition of mass parameters $M_{1;2} = m_{1;2}/2\pi T$ the temperature corrections to effective potential are expressed by summed integrals.

Finite temperature corrections of squarks

The sum of integrals can be expressed by means of the generalized zeta-function.

$$I_0[M_a, M_b] = \frac{1}{16\pi^2 T} \int_0^1 dx\, \zeta(2, \frac{1}{2}, M^2),$$

where $\zeta(u, s, t)$ is the generalized Hurwitz zeta-function

$$\zeta(u, s, t) = \sum_{n=1}^{\infty} \frac{1}{(n^u + t)^s}.$$

Finite temperature corrections of squarks

So in the case under consideration the sums of integrals can be calculated by differentiation with respect to mass parameters participating in $M = M(M_a, M_b, x)$. Differentiation increases the power s in the denominator giving convergent integrals

$$I_1[M_a, M_b] = \frac{T}{2M_a}\frac{\partial}{\partial M_a} I_0 = -\frac{1}{64\pi^4 T^2}\int_0^1 dx\, x\, \zeta[2, \frac{3}{2}, M^2(x)],$$

$$I_2[M_a, M_b] = -\frac{1}{2M_b}\frac{\partial}{\partial M_b}(-I_1) =$$

$$= \frac{3}{256\pi^6 T^4}\int_0^1 dx\, x\,(1-x)\,\zeta[2, \frac{5}{2}, M^2(x)].$$

Effective potential of MSSM

In two-doublet model there are two identical $SU(2)$ doublets of complex scalar fields Φ_1 and Φ_2

$$\Phi_1 = \begin{pmatrix} \phi_1^+(x) \\ \phi_1^0(x) \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_2^+(x) \\ \phi_2^0(x) \end{pmatrix}$$

with nonzero vacuum expectation values

$$\langle\Phi_1\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \qquad \langle\Phi_2\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix}.$$

Neutral components of doublets

$$\phi_1^0(x) = \frac{1}{\sqrt{2}}(v_1 + \eta_1 + i\chi_1), \quad \phi_2^0(x) = \frac{1}{\sqrt{2}}(v_2 + \eta_2 + i\chi_2).$$

Effective potential of MSSM

The most general renormalizable hermitian $SU(2)\times U(1)$ invariant potential:

$$U(\Phi_1,\Phi_2) = -\mu_1^2(\Phi_1^\dagger\Phi_1) - \mu_2^2(\Phi_2^\dagger\Phi_2) - \mu_{12}^2(\Phi_1^\dagger\Phi_2) - \overset{*}{\mu_{12}^2}(\Phi_2^\dagger\Phi_1) +$$

$$+\frac{\lambda_1}{2}(\Phi_1^\dagger\Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) +$$

$$+\frac{\lambda_5}{2}(\Phi_1^\dagger\Phi_2)(\Phi_1^\dagger\Phi_2) + \frac{\overset{*}{\lambda_5}}{2}(\Phi_2^\dagger\Phi_1)(\Phi_2^\dagger\Phi_1) +$$

$$+\lambda_6(\Phi_1^\dagger\Phi_1)(\Phi_1^\dagger\Phi_2) + \overset{*}{\lambda_6}(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_1) +$$

$$+\lambda_7(\Phi_2^\dagger\Phi_2)(\Phi_1^\dagger\Phi_2) + \overset{*}{\lambda_7}(\Phi_2^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)$$

with effective real parameters μ_1^2, μ_2^2, λ_1,...,λ_4 and complex parameters μ_{12}^2, λ_5, λ_6, λ_7.

Parameters of Effective Potential of MSSM

In the tree approximation on the energy scale M_{SUSY}, the parameters λ_{1-7} are real and are expressed using the coupling constants g_1 and g_2 of electroweak group of the gauge symmetry $SU(2) \otimes U(1)$ as follows:

$$\lambda_1(M_{SUSY}) = \lambda_2(M_{SUSY}) = \frac{1}{4}\left(g_2^2(M_{SUSY}) + g_1^2(M_{SUSY})\right),$$

$$\lambda_3(M_{SUSY}) = \frac{1}{4}\left(g_2^2(M_{SUSY}) - g_1^2(M_{SUSY})\right),$$

$$\lambda_4(M_{SUSY}) = -\frac{1}{2}g_2^2(M_{SUSY}),$$

$$\lambda_5(M_{SUSY}) = \lambda_6(M_{SUSY}) = \lambda_7(M_{SUSY}) = 0.$$

Parameters of Effective Potential of MSSM

The supersymmetric scalar potential of interaction of Higgs bosons with the third generation quark superpartners on the tree level has the form

$$\mathcal{V}^0 = \mathcal{V}_M + \mathcal{V}_\Gamma + \mathcal{V}_\Lambda + \mathcal{V}_{\widetilde{Q}}\,,$$

$$\mathcal{V}_M = (-1)^{i+j} m_{ij}^2 \Phi_i^\dagger \Phi_j + M_{\widetilde{Q}}^2 \left(\widetilde{Q}^\dagger \widetilde{Q}\right) + M_{\widetilde{U}}^2 \widetilde{U}^* \widetilde{U} + M_{\widetilde{D}}^2 \widetilde{D}^* \widetilde{D}\,,$$

$$\mathcal{V}_\Gamma = \Gamma_i^D \left(\Phi_i^\dagger \widetilde{Q}\right) \widetilde{D} + \Gamma_i^U \left(i\Phi_i^T \sigma_2 \widetilde{Q}\right) \widetilde{U} + \overset{*}{\Gamma}{}_i^D \left(\widetilde{Q}^\dagger \Phi_i\right) \widetilde{D}^* - \overset{*}{\Gamma}{}_i^U \left(i\widetilde{Q}^\dagger \sigma_2 \Phi_i^*\right) \widetilde{U}^*\,,$$

$$\mathcal{V}_\Lambda = \Lambda_{ik}^{jl} \left(\Phi_i^\dagger \Phi_j\right) \left(\Phi_k^\dagger \Phi_l\right) + \left(\Phi_i^\dagger \Phi_j\right) \left[\Lambda_{ij}^Q \left(\widetilde{Q}^\dagger \widetilde{Q}\right) + \Lambda_{ij}^U \widetilde{U}^* \widetilde{U} + \Lambda_{ij}^D \widetilde{D}^* \widetilde{D}\right] +$$

$$+ \overline{\Lambda}_{ij}^Q \left(\Phi_i^\dagger \widetilde{Q}\right) \left(\widetilde{Q}^\dagger \Phi_j\right) + \frac{1}{2} \left[\Lambda \epsilon_{ij} \left(i\Phi_i^T \sigma_2 \Phi_j\right) \widetilde{D}^* \widetilde{U} + \text{э.с.}\right]\,, i, j, k, l = 1, 2\,,$$

$\mathcal{V}_{\widetilde{Q}}$ denotes the terms of interaction of four scalar quarks.

Parameters of Effective Potential of MSSM

Calculation of the finite-temperature diagrams for the general case of complex-valued μ and $A_{t,b}$ gives the result

$$\Delta\lambda_1^{thr} = 3h_t^4|\mu|^4 I_2[m_Q, m_U] + 3h_b^4|A_b|^4 I_2[m_Q, m_D] +$$

$$+h_t^2|\mu|^2(-\frac{g_1^2 - 3g_2^2}{2} I_1[m_Q, m_U] + 2g_1^2 I_1[m_U, m_Q])$$

$$+h_b^2|A_b|^2(\frac{12h_b^2 - g_1^2 - 3g_2^2}{2} I_1[m_Q, m_D] + (6h_b^2 - g_1^2) I_1[m_D, m_Q])$$

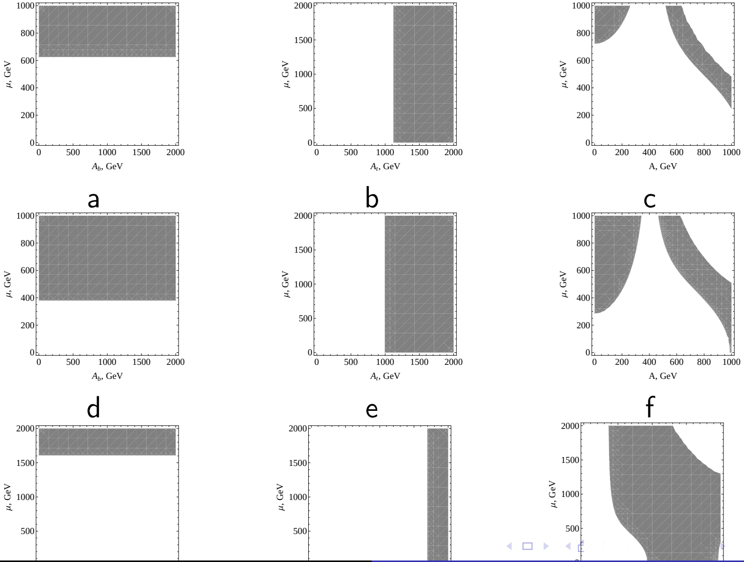
$$\Delta\lambda_2^{thr} = 3h_t^4|A_t|^4 I_2[m_Q, m_U] + 3h_b^4|\mu|^4 I_2[m_Q, m_D] +$$

$$+h_b^2|\mu|^2(\frac{g_1^2 + 3g_2^2}{2} I_1[m_Q, m_D] + g_1^2 I_1[m_D, m_Q]) +$$

$$+h_t^2|A_t|^2(\frac{12h_t^2 + g_1^2 - 3g_2^2}{2} I_1[m_Q, m_U] + (6h_t^2 - 2g_1^2) I_1[m_U, m_Q])$$

Bifurcation sets

N	Solutions	Hessian $H(\overline{v}_1, \overline{v}_2)$	local minimum conditions
1	$\overline{v}_1 = 0, \quad \overline{v}_2 = 0$	$-\begin{pmatrix} \overline{\mu}_1^2 & 0 \\ 0 & \overline{\mu}_2^2 \end{pmatrix}$	$\overline{\mu}_1^2 + \overline{\mu}_2^2 < 0, \quad \overline{\mu}_1^2 \cdot \overline{\mu}_2^2 \geq 0$
2	$\overline{v}_1 = 0, \quad \lambda_2 \overline{v}_2^2 - \overline{\mu}_2^2 = 0$	$\begin{pmatrix} -\overline{\mu}_1^2 + \frac{\lambda_{345}}{2} \overline{v}_2^2 & 0 \\ 0 & 2\lambda_2 \overline{v}_2^2 \end{pmatrix}$	$-\overline{\mu}_1^2 + \overline{v}_2^2(2\lambda_2 + \frac{1}{2}\lambda_{345}) > 0$ $(-\overline{\mu}_1^2 + \frac{1}{2}\lambda_{345}\overline{v}_2^2)\lambda_2 \overline{v}_2^2 \geq 0$
3	$\overline{v}_2 = 0, \quad \lambda_1 \overline{v}_2^2 - \overline{\mu}_1^2 = 0$	$\begin{pmatrix} 2\lambda_1 \overline{v}_1^2 & 0 \\ 0 & -\overline{\mu}_2^2 + \frac{\lambda_{345}}{2} \overline{v}_1^2 \end{pmatrix}$	$-\overline{\mu}_2^2 + \overline{v}_1^2(2\lambda_1 + \frac{1}{2}\lambda_{345}) > 0$ $(-\overline{\mu}_2^2 + \frac{1}{2}\lambda_{345}\overline{v}_1^2)\lambda_1 \overline{v}_1^2 \geq 0$
4	$\lambda_1 \overline{v}_1^2 + \frac{\lambda_{435}}{2} \overline{v}_2^2 - \overline{\mu}_1^2 = 0,$ $\lambda_2 \overline{v}_2^2 + \frac{\lambda_{435}}{2} \overline{v}_1^2 - \overline{\mu}_2^2 = 0$	$\begin{pmatrix} 2\lambda_1 \overline{v}_1^2 & \lambda_{345}\overline{v}_1\overline{v}_2 \\ \lambda_{345}\overline{v}_1\overline{v}_2 & 2\lambda_2 \overline{v}_2^2 \end{pmatrix}$	$\lambda_1 \overline{v}_1^2 + \lambda_2 \overline{v}_2^2 > 0$ $\overline{v}_1^2 \overline{v}_2^2 (4\lambda_1\lambda_2 - \lambda_{345}^2) \geq 0$


μ, GeV
A_b, GeV
A_t, GeV
A, GeV
a
b
c
d
e
f

Effective potential of NMSSM

In the NMSSM two identical scalar $SU(2)$ doublets of the complex scalar fields Φ_1 and Φ_2 are introduced

$$\Phi_1 = \begin{pmatrix} \phi_1^+(x) \\ \phi_1^0(x) \end{pmatrix} = \begin{pmatrix} -i\omega_1^+ \\ \frac{1}{\sqrt{2}}(v_1 + \eta_1 + i\chi_1) \end{pmatrix},$$

$$\Phi_2 = \begin{pmatrix} \phi_2^+(x) \\ \phi_2^0(x) \end{pmatrix} = \begin{pmatrix} -i\omega_2^+ \\ \frac{1}{\sqrt{2}}(v_2 + \eta_2 + i\chi_2) \end{pmatrix}$$

Singlet superfield S:

$$S = \frac{1}{\sqrt{2}}(v_3 + s_1 + is_2).$$

Effective potential of NMSSM

The most general Hermitian form of the renormalized $SU(2) \times U(1)$ invariant potential for system of fields has the form:

$$U(\Phi_1, \Phi_2, S) = -\mu_1^2(\Phi_1^\dagger\Phi_1) - \mu_2^2(\Phi_2^\dagger\Phi_2) - \mu_3^2 S^* S - \mu_{12}^2(\Phi_1^\dagger\Phi_2) - \overset{*}{\mu}{}_{12}^2 (\Phi_2^\dagger\Phi_1)$$

$$+\frac{\lambda_1}{2}(\Phi_1^\dagger\Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) +$$

$$+\frac{\lambda_5}{2}(\Phi_1^\dagger\Phi_2)(\Phi_1^\dagger\Phi_2) + \frac{\overset{*}{\lambda}_5}{2}(\Phi_2^\dagger\Phi_1)(\Phi_2^\dagger\Phi_1) +$$

$$+\lambda_6(\Phi_1^\dagger\Phi_1)(\Phi_1^\dagger\Phi_2) + \overset{*}{\lambda}_6 (\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_1) + \lambda_7(\Phi_2^\dagger\Phi_2)(\Phi_1^\dagger\Phi_2) + \overset{*}{\lambda}_7 (\Phi_2^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)$$

$$+k_1(\Phi_1^\dagger\Phi_1)S^* S + k_2(\Phi_2^\dagger\Phi_2)S^* S + k_3(\Phi_1^\dagger\Phi_2)S^* S + \overset{*}{k}_3 (\Phi_2^\dagger\Phi_1)S^* S + k_4(S^* S)^2 +$$

$$+k_5(\Phi_1^\dagger\Phi_1)S + k_6(\Phi_2^\dagger\Phi_2)S + k_7(\Phi_1^\dagger\Phi_2)S + \overset{*}{k}_7 (\Phi_2^\dagger\Phi_1)S^* + k_8 S^3.$$

Parameters of Effective Potential of NMSSM

In the tree approximation on the energy scale M_{SUSY}, the parameters λ_i, κ_j expressed as:

$$\lambda_1 = \lambda_2 = \frac{g_1^2 + g_2^2}{8},\ \lambda_3 = \frac{g_2^2 - g_1^2}{4},\ \lambda_4 = -\frac{g_2^2}{2}, \qquad \lambda_5 = \lambda_6 = \lambda_7 = 0 \quad (4)$$

$$k_1 = |\lambda|^2,\ \ k_2 = |\lambda|^2,\ \ k_3 = \lambda k^*,\ \ k_4 = |k|^2,\ \ k_5 = \lambda A_\lambda,\ \ k_6 = \frac{1}{3} k A_k, \quad (5)$$

The free parameters of the model are chosen in the range possible values:

$$1.0 < tg\beta \leq 60, \quad M_1 = M_2, \quad 100\,\text{GeV} \leq M_2 \leq 2000\,\text{GeV},$$

$$0.0001 \leq \lambda \leq 0.7, \quad 0 \leq \kappa \leq 0.65.$$

$$0\,\text{GeV} \leq A_\lambda \leq 1000\,\text{GeV}, \quad -100\,\text{GeV} \leq A_\kappa \leq -10\,\text{GeV}$$

Parameters of Effective Potential of NMSSM

The supersymmetric scalar potential of interaction of Higgs bosons with the third generation quark superpartners on the tree level has the form

$$V = |y_u(\widetilde{Q}\epsilon H_u)|^2+|y_d(\widetilde{Q}\epsilon H_d)|^2+|y_u\widetilde{u}_R^* H_u^0-y_d\widetilde{d}_R^* H_d^-|^2+|y_d\widetilde{d}_R^* H_d^0-y_d\widetilde{u}_R^* H_u^+|^2-$$

$$-y_u(\widetilde{u}_R\widetilde{u}_L^*\lambda S H_d^0+\widetilde{u}_R\widetilde{d}_L^*\lambda S H_d^-+c.c.)-y_d(\widetilde{d}_R\widetilde{d}_L^*\lambda S H_u^0+\widetilde{d}_R\widetilde{d}_L^*\lambda S H_u^++c.c.)+$$

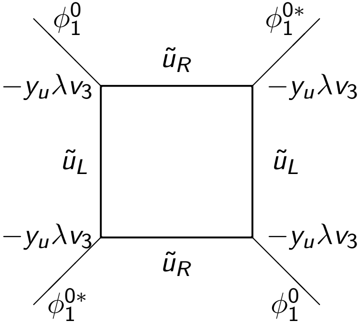
$$+\frac{g_2^2}{8}(4|H_d^\dagger\widetilde{Q}|^2-2(H_d^\dagger H_d)(\widetilde{Q}^\dagger\widetilde{Q})+4|H_u^\dagger\widetilde{Q}|^2-2(H_u^\dagger H_u)(\widetilde{Q}^\dagger\widetilde{Q}))+$$

$$+\frac{g_1^2}{2}(\frac{1}{6}(\widetilde{Q}^\dagger\widetilde{Q})-\frac{2}{3}\widetilde{u}_R^*\widetilde{u}_R+\frac{1}{3}\widetilde{d}_R^*\widetilde{d}_R+\frac{1}{2}(H_u^\dagger H_u)-\frac{1}{2}(H_d^\dagger H_d))^2+$$

$$+(\widetilde{u}_R^* y_u A_u(\widetilde{Q}^T\epsilon H_u)-\widetilde{d}_R y_d A_d(\widetilde{Q}^T\epsilon H_d)+c.c.)$$

Parameters of Effective Potential of NMSSM

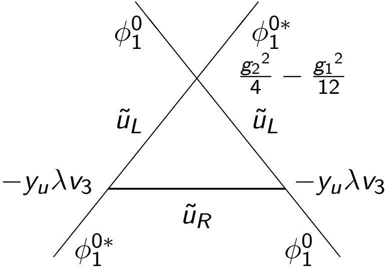
The oneloop corrections to the parameters of effective potential



$$(y_u\lambda v_3)^4 I_2[m_Q, m_U]$$

Parameters of Effective Potential of NMSSM

The oneloop corrections to the parameters of effective potential



$$(-y_u\lambda v_3)^2\left(\frac{{g_2}^2}{4} - \frac{{g_1}^2}{12}\right)I_1[m_Q, m_U]$$

Parameters of Effective Potential of NMSSM

The oneloop corrections to the parameters of effective potential

$$\Delta\lambda_1 = h_u^4\lambda^4 v_3^4 I_2[m_Q, m_U] + h_d^4 A_d^4 I_2[m_Q, m_D] +$$

$$+h_u^2\lambda^2 v_3^2 \left(\left(\frac{g_2^2}{4} - \frac{g_1^2}{12} \right) I_1[m_Q, m_U] + \frac{1}{3} g_1^2 I_1[m_U, m_Q] \right) +$$

$$+h_d^2 A_d^2 \left(\left(h_d^2 - \frac{g_1^2}{12} - \frac{g_2^2}{4} \right) I_1[m_Q, m_D] + \left(h_d^2 - \frac{g_1^2}{6} \right) I_1[m_D, m_Q] \right)$$

$$\Delta\lambda_2 = h_u^4 A_u^4 I_2[m_Q, m_U] + h_d^4\lambda^4 v_3^4 I_2[m_Q, m_D] +$$

$$+h_u^2 A_u^2 \left(\left(\frac{g_1^2}{12} - \frac{g_2^2}{4} \right) I_1[m_Q, m_U] + (h_u^2 - \frac{1}{3} g_1^2 I_1[m_U, m_Q] \right) +$$

$$+h_d^2\lambda^2 v_3^2 \left(\left(\frac{g_1^2}{12} + \frac{g_2^2}{4} \right) I_1[m_Q, m_D] + \frac{g_1^2}{6} I_1[m_D, m_Q] \right)$$

solutions	Hessian $H(v_1, v_2, v_3)$	local minimum conditions
$v_1 = 0$ $v_2 = 0$ $v_3 = 0$	$\begin{pmatrix} -\mu_1^2 & 0 & 0 \\ 0 & -\mu_2^2 & 0 \\ 0 & 0 & -2\mu_3^2 \end{pmatrix}$	$-\mu_1^2 - \mu_2^2 - 2\mu_3^2 > 0,$ $\mu_1^2 \cdot \mu_2^2 \cdot \mu_3^2 < 0.$
$v_1 \neq 0$ $v_2 = 0$ $v_3 = 0$	$\begin{pmatrix} v_1^2\lambda_1 & 0 & 0 \\ 0 & \frac{1}{2}v_1^2\lambda_{34} - \mu_2^2 & k_5 v_1 \\ 0 & k_5 v_1 & k_1 v_1^2 - 2\mu_3^2 \end{pmatrix}$	$k_1 v_1^2 - \mu_2^2 - 2\mu_3^2 + \lambda_1 v_1^2 +$ $\frac{1}{2}(\lambda_3 + \lambda_4)v_1^2 > 0,$ $\lambda_1 v_1^2\{(\left(k_1 v_1^2 - 2\mu_3^2\right)$ $\left(\frac{1}{2}\lambda_{34} v_1^2 - \mu_2^2\right) - k_5^2 v_1^2)\} > 0.$
$v_1 = 0$ $v_2 \neq 0$ $v_3 = 0$	$\begin{pmatrix} \frac{1}{2}v_2^2\lambda_{34} - \mu_1^2 & 0 & k_5 v_2 \\ 0 & v_2^2\lambda_2 & 0 \\ k_5 v_2 & 0 & k_2 v_2^2 - 2\mu_3^2 \end{pmatrix}$	$k_2 v_2^2 - \mu_1^2 - 2\mu_3^2 + \lambda_2 v_2^2 +$ $\frac{1}{2}(\lambda_3 + \lambda_4)v_2^2 > 0,$ $\lambda_2 v_2^2\{\left(k_2 v_2^2 - 2\mu_3^2\right)$ $\left(\frac{1}{2}\lambda_{34} v_2^2 - \mu_1^2\right) - k_5^2 v_2^2\} > 0.$

Bifurcation sets

Solutions	Hessian $H(v_1, v_2, v_3)$	Local minimum conditions
$v_1 \neq 0$ $v_2 \neq 0$ $v_3 = 0$	$\begin{pmatrix} v_1^2\lambda_1 & v_1 v_2 \lambda_{34} & 0 \\ v_1 v_2(\lambda_3 + \lambda_4) & v_2^2\lambda_2 & 0 \\ 0 & 0 & k_1 v_1^2 + 2k_3 v_2 v_1 + k_2 v_2^2 - 2\mu_3^2 \end{pmatrix}$	$Det > 0, \;\; Tr > 0$
$v_1 \neq 0$ $v_2 = 0$ $v_3 \neq 0$	$\begin{pmatrix} v_1^2\lambda_1 & 0 & 2k_1 v_1 v_3 \\ 0 & \frac{1}{2}\left(\lambda_{34} v_1^2 + 2k_2 v_3^2 - 2\mu_2^2\right) & k_3 v_1 v_3 \\ 2k_1 v_1 v_3 & k_3 v_1 v_3 & 2v_3(3k_6 + 4k_4 v_3) \end{pmatrix}$	$Det > 0, \;\; Tr > 0$
$v_1 = 0$ $v_2 \neq 0$ $v_3 \neq 0$	$\begin{pmatrix} \frac{1}{2}\left(\lambda_{34} v_2^2 + 2k_1 v_3^2 - 2\mu_1^2\right) & 0 & k_3 v_2 v_3 \\ 0 & v_2^2\lambda_2 & 2k_2 v_2 v_3 \\ k_3 v_2 v_3 & 2k_2 v_2 v_3 & 2v_3(3k_6 + 4k_4 v_3) \end{pmatrix}$	$Det > 0, \;\; Tr > 0$
$v_1 \neq 0$ $v_2 \neq 0$ $v_3 \neq 0$	$\begin{pmatrix} H_{11} & H_{12} & H_{13} \\ H_{21} & H_{22} & H_{23} \\ H_{31} & H_{32} & H_{33} \end{pmatrix}$ $H_{11} = v_1^2\lambda_1 - v_2 v_3(k_5 + k_3 v_3)/v_1,$ $H_{12} = H_{21} = v_3(k_5 + k_3 v_3) + v_1 v_2(\lambda_3 + \lambda_4),$ $H_{13} = H_{31} = k_5 v_2 + 2(k_1 v_1 + k_3 v_2)v_3,$	$* * *$

$* * *$ The last case $(v_1 \neq 0,\ v_2 \neq 0,\ v_3 \neq 0)$:

$$-\frac{k_5 v_1 v_2}{v_3} + 8k_4 v_3^2 + 6k_6 v_3 - v_3(k_3 v_3 + k_5)\frac{v_1^2 + v_2^2}{v_1 v_2} + \lambda_1 v_1^2 + \lambda_2 v_2^2 > 0,$$

$$\frac{1}{v_1 v_2 v_3} \cdot \Bigg(v_3 \Big(k_5 v_2 + 2k_1 v_1 v_3 + 2k_3 v_2 v_3 \Big) \Big(v_1 v_2 \Big(k_5 v_1 + 2k_3 v_1 v_3 + 2k_2 v_2 v_3 \Big) \times$$

$$\times \Big(k_3 v_3^2 + k_5 v_3 + v_1 v_2(\lambda_3 + \lambda_4) \Big) - v_1 \Big(k_5 v_2 + 2k_1 v_1 v_3 + 2k_3 v_2 v_3 \Big) \Big(- k_3 v_1 v_3^2 -$$

$$- k_5 v_1 v_3 + \lambda_2 v_2^3 \Big) \Big) - v_3 \Big(k_5 v_1 + 2k_3 v_1 v_3 + 2k_2 v_2 v_3 \Big) \Big(v_2 \Big(k_5 v_1 + 2k_3 v_1 v_3 +$$

$$+ 2k_2 v_2 v_3 \Big) \Big(- k_3 v_2 v_3^2 - k_5 v_2 v_3 + \lambda_1 v_1^3 \Big) - v_1 v_2 \Big(k_5 v_2 + 2k_1 v_1 v_3 + 2k_3 v_2 v_3 \Big) \times$$

$$\times \Big(k_3 v_3^2 + k_5 v_3 + v_1 v_2(\lambda_3 + \lambda_4) \Big) \Big) + \Big(8k_4 v_3^3 + 6k_6 v_3^2 - k_5 v_1 v_2 \Big) \Big(\Big(- k_3 v_2 v_3^2 -$$

$$- k_5 v_2 v_3 + \lambda_1 v_1^3 \Big) \Big(- k_3 v_1 v_3^2 - k_5 v_1 v_3 + \lambda_2 v_2^3 \Big) - v_1 v_2 \Big(k_3 v_3^2 + k_5 v_3 +$$

$$+ v_1 v_2(\lambda_3 + \lambda_4) \Big)^2 \Big) \Bigg) > 0.$$

Conclusion

- Our analysis of the effective MSSM and NMSSM finite-temperature potentials is based on a calculation of various one-loop temperature corrections from the squark-Higgs boson sector for the case of nonzero trilinear parameters A_t, A_b and Higgs superfield parameter μ.
- Quantum corrections are incorporated in control parameters $\lambda_{1,...7...}(T)$ of the effective two-doublet (+singlet) potential, which is then explicitly rewritten in terms of Higgs boson mass eigenstates.
- Bifurcation sets types for the two-Higgs-doublet(+singlet) potential $U_{eff}(v_1, v_2)$ are determined.