Neutrino-electron scattering in a dense magnetized plasma

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Motivation

Neutrino physics plays a decisive role in astrophysical cataclysms such as supernova explosions. The studies of neutrino interactions and in particular neutrino-electron processes in an external active medium are of considerable interest.

In the SN explosion, as in any astrophysical applications, the most interesting are the mean values of the neutrino energy and momentum losses.

- Bezchastnov & Haensel (1996) performed numerical calculations of the differential cross-section of the neutrino-electron scattering in dense magnetized plasma in the limit of rather weak magnetic field B, $eB < \mu E$ (μ is the plasma chemical potential, E is the typical neutrino energy).
- Kuznetsov & Mikheev (1999, 2000) evaluated the probability of the total sum of νe processes ($\nu \to \nu e^- e^+$, $\nu e^\mp \to \nu e^\mp$, $\nu e^- e^+ \to \nu$) and the volume density of the neutrino energy and momentum losses, integrated over the momenta of the plasma electrons, in a strong magnetic field, $eB \gg (\mu^2, T^2, E^2) \gg m_e^2$, ($T$ is the plasma temperature); electrons and positrons occupied only the ground Landau levels.

- Mikheev & Narynskaya (2000, 2003) evaluated the probability of the $\nu e \to \nu e$ process and the volume density of the neutrino energy and momentum losses, summarized over all initial states of the plasma electrons, in a moderate magnetic field, while the density of plasma is large:
  $\mu^2 > eB \gg (T^2, E^2) \gg m_e^2$, and $eB \gg \mu E$.
  It was concluded that transitions were dominating when both initial and final electrons occupied the same Landau levels.

The purpose of the present research is to calculate analytically the probability of the $\nu e \to \nu e$ process and the volume density of the neutrino energy and momentum losses, for a more general case when the initial and final electrons could occupy any physically allowed Landau levels.

Solutions of the Dirac equation in a magnetic field

There exist several descriptions of solving the Dirac equation in an external magnetic field, e.g.:

- M. H. Johnson & B. A. Lippmann (1949)
- A. I. Akhiezer & V. B. Berestetskii, *Quantum Electrodynamics* (1965)
- A. A. Sokolov & I. M. Ternov, *Synchrotron Radiation* (1968)
- D. B. Melrose & A. J. Parle (1983)
- K. Bhattacharya & P. B. Pal (2004)
- I. A. Balantsev, Yu. V. Popov & A. I. Studenikin (2011)
- A. V. Kuznetsov & N. V. Mikheev, *Electroweak Processes in External Active Media* (2013)

Solutions of the Dirac equation in a magnetic field

We use the solutions which are the eigenstates of the covariant operator of a magnetic polarization (Sokolov, 1968):

$$\hat{\mu}_z = m_e \Sigma_z - \mathrm{i}\gamma_0\gamma_5 [\boldsymbol{\Sigma} \times \hat{\boldsymbol{P}}]_z \qquad \hat{\mu}_z \, \Psi^{\pm}_{p,n}(X) = \pm \, M_n \, \Psi^{\pm}_{p,n}(X)$$

where $\hat{\boldsymbol{P}} = -\mathrm{i}\boldsymbol{\nabla} + e\boldsymbol{A}$. Landau gauge: $A^{\mu} = (0, 0, xB, 0)$.

$$\Psi^{s}_{p,n}(X) = \frac{\mathrm{e}^{-\mathrm{i}(\varepsilon_n t - p_y y - p_z z)}}{\sqrt{4\varepsilon_n M_n (\varepsilon_n + M_n)(M_n + m_e) L_y L_z}} \, u^{s}_{n}(\xi)$$

where $\varepsilon_n = \sqrt{M_n^2 + p_z^2}$, $\quad M_n = \sqrt{m_e^2 + 2\beta n}$, $\quad \beta = eB$.

Solutions of the Dirac equation in a magnetic field

$$u_n^-(\xi) = \begin{pmatrix} -\mathrm{i}\sqrt{2\beta n}\, p_z V_{n-1}(\xi) \\ (\varepsilon_n + M_n)(M_n + m_e) V_n(\xi) \\ -\mathrm{i}\sqrt{2\beta n}(\varepsilon_n + M_n) V_{n-1}(\xi) \\ -p_z (M_n + m_e) V_n(\xi) \end{pmatrix}, \quad u_n^+(\xi) = \dots$$

The harmonic oscillator functions $V_n(\xi)\,(n = 0, 1, 2, \dots)$ are expressed via Hermite polynomials $H_n(\xi)$:

$$V_n(\xi) = \frac{\beta^{1/4} \mathrm{e}^{-\xi^2/2}}{\sqrt{2^n n! \sqrt{\pi}}} H_n(\xi)\,, \quad \xi = \sqrt{\beta}\left(x + \frac{p_y}{\beta}\right)$$

Polarization amplitudes are written in manifestly relativistic invariant form.

The process of the $\nu e \to \nu e$ scattering

The effective local Lagrangian:

$$\mathcal{L} = -\frac{G_{\rm F}}{\sqrt{2}} \left[\bar{e}\gamma_\alpha (C_V - C_A\gamma_5) e\right] \left[\bar{\nu}\gamma^\alpha (1 - \gamma_5)\nu\right]$$

The constants depend on the neutrino flavor:

$$C_V^{(e)} = +\frac{1}{2} + 2\sin^2\theta_{\rm W} \,, \qquad C_A^{(e)} = +\frac{1}{2}$$

$$C_V^{(\mu,\tau)} = -\frac{1}{2} + 2\sin^2\theta_{\rm W} \,, \qquad C_A^{(\mu,\tau)} = -\frac{1}{2}$$

The process of the $\nu e \to \nu e$ scattering

The S matrix element of the subprocess $\nu e^-_{(\ell)} \to \nu e^-_{(n)}$

$$S = \mathrm{i}\, \frac{G_{\mathrm{F}}}{\sqrt{2}}\, \frac{(2\pi)^3\, \delta(\varepsilon'_n - \varepsilon_\ell - q_0)\, \delta(p'_y - p_y - q_y)\, \delta(p'_z - p_z - q_z)}{\sqrt{2EV\, 2E'V\, 2\varepsilon_\ell L_y L_z\, 2\varepsilon'_n L_y L_z}} \times$$

$$\times \mathrm{e}^{-q_\perp^2/4eB - \mathrm{i} q_x (p_y + p'_y)/2eB} [\bar{u}(p'_\parallel)\, \hat{\jmath}(C_V - C_A \gamma_5)\, u(p_\parallel)]$$

where $q = P - P' = p' - p$, ε_ℓ and ε'_n are the energies of the initial and final electrons, $q_\perp$ is the projection of the vector $\boldsymbol{q}$ on the plane perpendicular to the vector $\boldsymbol{B} = (0, 0, B)$, $q_\perp^2 = q_x^2 + q_y^2$, and $j_\alpha = \bar{\nu}(P')\gamma_\alpha(1 - \gamma_5)\nu(P)$ is the Fourier tranform of the current of the left-handed neutrinos.

The process of the $\nu e \to \nu e$ scattering

The probability of the subprocess $\nu e^-_{(\ell)} \to \nu e^-_{(n)}$ is obtained by integration over the initial and final electron momentum states and by summation over the polarization states

$$W_{\ell n} = \sum_{s,s'} W_{\ell n}^{ss'} = W_{\ell n}^{--} + W_{\ell n}^{-+} + W_{\ell n}^{+-} + W_{\ell n}^{++}$$

The process of the $\nu e \to \nu e$ scattering

$$W_{(\nu e^- \to \nu e^-)} = \frac{1}{\mathcal{T}} \sum_{\ell} \sum_{n} \sum_{s,s'} \int |\, S \,|^2 \, \mathrm{d}n_{e^-} \, \mathrm{d}n'_{e^-} \, \frac{\mathrm{d}^3 P' \, V}{(2\pi)^3} \, (1 - f(E'))$$

$$\mathrm{d}n_{e^-} = \frac{\mathrm{d}p_y \, \mathrm{d}p_z \, L_y \, L_z}{(2\pi)^2} \, f(\varepsilon_\ell), \quad \mathrm{d}n'_{e^-} = \frac{\mathrm{d}p'_y \, \mathrm{d}p'_z \, L_y \, L_z}{(2\pi)^2} \, (1 - f(\varepsilon'_n))$$

where p_z is the electron momentum along the magnetic field, p_y is the generalized momentum which defines the position of the center of a Gaussian packet along the x axis, $x_0 = -p_y/eB$, while $\varepsilon_\ell = \sqrt{p_z^2 + 2eB\ell + m_e^2}$ is the energy of the plasma electron occupying the ℓ-th Landau level, $f(\varepsilon_\ell)$ is a distribution function of electrons: $f(\varepsilon_\ell) = [\mathrm{e}^{(\varepsilon_\ell - \mu)/T} + 1]^{-1}$.

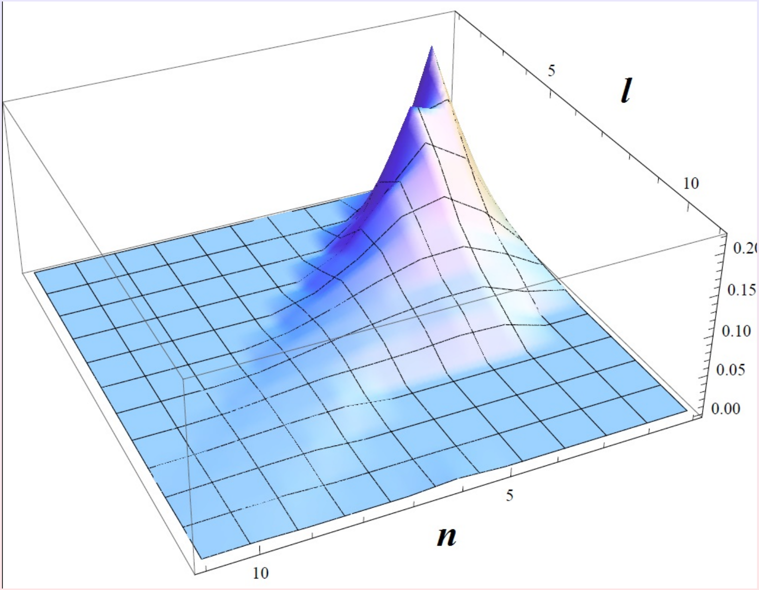
The probability of the subprocess $\nu e^-_{(\ell)} \to \nu e^-_{(n)}$

$$W_{\ell n} = \frac{eB}{(2\pi)^4 16E} \int \frac{\mathrm{d}^3 P'}{E'} \left(1 - f(E')\right)$$

$$\times \int \frac{\mathrm{d}p_z}{\varepsilon'_n \, \varepsilon_\ell} \, \delta(\varepsilon'_n - \varepsilon_\ell - q_0) \, f(\varepsilon_\ell) \left(1 - f(\varepsilon'_n)\right) \sum_{s,s'} |\mathcal{M}^{ss'}_{\ell n}|^2 \, ,$$

$\mathcal{M}^{ss'}_{\ell n}$ $(s, s' = \pm 1)$ are the invariant polarization amplitudes.
It should be noted, that no upper limits arise from kinematics both on the Landau level numbers ℓ, n and the neutrino momenta, and the convergence of both summation and integration is provided by the distribution functions of initial and final electrons.

The process of the $\nu e \to \nu e$ scattering



The process of the $\nu e \to \nu e$ scattering

As the analysis shows, the subprocesses with $\ell = n$, where both initial and final electrons occupy the same Landau levels, are dominating, but the subprocesses with $\ell \neq n$ can give an essential contribution also. This means that the results for the probability limited to the case $\ell = n$, were underestimated.

Neutrino force

The four-vector of the mean values of the neutrino energy and momentum losses:

$$Q^\alpha = E \int (P - P')^\alpha \, \mathrm{d}W = -E \, (\mathcal{I}, \boldsymbol{F}) \, ,$$

where $\mathrm{d}W$ is the total differential probability of the process.

The zeroth component of Q^α is connected with the mean energy lost by a neutrino per unit time due to the process considered, $\mathcal{I} = \mathrm{d}E/\mathrm{d}t$.

The space components are connected with the mean neutrino momentum loss per unit time, $\boldsymbol{F} = \mathrm{d}\boldsymbol{P}/\mathrm{d}t$.

Neutrino force

An analysis of the four-vector Q^α in a general case for the magnetic field of arbitrary strength, where electrons can occupy the states corresponding to excited Landau levels, now is in progress.

The force density $\boldsymbol{F}$ could lead to a very interesting consequences if a strong toroidal magnetic field is generated in the supernova envelope, providing an asymmetry of the supernova explosion and, in particular, it can explain the phenomenon of high pulsar kick-velocities.

Conclusions

- The probability of the $\nu e \to \nu e$ process in a dense magnetized plasma is calculated analytically, for a general case when the initial and final electrons could occupy any physically allowed Landau levels.
- The subprocesses dominate where both initial and final electrons occupy exactly the same Landau levels, as it was mentioned in previous calculations, but the subprocesses where the Landau levels are not equal but close, are also essential.
- An analysis of the four-vector Q^α describing the mean values of the neutrino energy and momentum losses, in a general case for the magnetic field of arbitrary strength, where electrons can occupy the states corresponding to excited Landau levels, now is in progress.

Thank you for your attention!