Asymptotic cosmological regimes in scalar-torsion gravity with a perfect fluid

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Introduction

- Teleparallel gravity (TG) is based on geometry of absolute parallelism (**Einstein, 1928**) – using of a field of orthonormal bases (tetrads) e^{μ}_{A} for tangent space-times at each point of space-time.
- Weitzenböck connection (**1923**) is applied in this theory instead Levi-Civita one, that leads to zero curvature ***R = 0*** and non-zero torsion ***T***.
- Equations of motion of TG coincide exactly with those of general relativity. However, modifications of these theories are not equivalents and their field equations differ from each other. Therefore, generalizations of TG (for example, *f(T)* gravity, scalar-torsion gravity) give rise to new cosmological dynamics and can describe **late-time cosmic acceleration of the Universe**

Introduction

Weitzenböck connection is used in teleparallel gravity

$$\overset{W}{\Gamma}{}^{\lambda}_{\nu\mu} = e^{\lambda}_{A}\partial_{\mu} e^{A}_{\nu} .$$

Then torsion tensor and torsion scalar are

$$T^{\lambda}_{\mu\nu} \equiv \overset{W}{\Gamma}{}^{\lambda}_{\nu\mu} - \overset{W}{\Gamma}{}^{\lambda}_{\mu\nu} = e^{\lambda}_{A}(\partial_{\mu} e^{A}_{\nu} - \partial_{\nu} e^{A}_{\mu}) ,$$

$$T \equiv \frac{1}{4} T^{\rho\nu\mu} T_{\rho\mu\nu} + \frac{1}{2} T^{\rho\nu\mu} T_{\nu\mu\rho} - T^{\rho}_{\rho\mu} T^{\nu\mu}_{\nu} .$$

The relation between of quantities, which are calculated applying Levi-Civita connection (L) and Weitzenböck one (W):

$$\overset{L}{R} = -\overset{W}{T} - 2\overset{L}{\nabla}{}^{\mu}\left(T^{\nu}_{\mu\nu}\right)$$

Purpose of the work

The purpose of this work was the investigation of cosmological dynamics in teleparallel gravity with nonminimal coupling with the Lagrangian of the form

$$L=\frac{1}{2}\sqrt{-g}\left(\frac{T}{8\pi G}+\partial_{\mu}\varphi\,\partial^{\mu}\varphi+\xi\,T\,B(\varphi)-2V(\varphi)\right)+L_{m},$$

where

$B(\varphi)=\varphi^{N}$, $N > 0$ and $V(\varphi)=V_{0}\varphi^{n}$,

$V_{0}\geq 0$, $n<0$

$p=\omega\rho$, $\omega\in[-1;1]$

The metrics and Planck units are used

$$ds^{2}=dt^{2}-a^{2}(t)dl^{2}$$

$$c=\hbar=1$$

Methods of the investigation

Methods of dynamical system theory,

a numerical integration,

an qualitative analysis of dynamics with
the effective potential,

algebraic methods

are applyed in this work.

6 Scheme of the basic method

- Intriduction of new variables $(a, \rho, \varphi, \ldots) \rightarrow (x, y, z, \ldots)$

$$\begin{cases} \dfrac{dx}{d(\ln(a))} = f_1(x, y, \ldots) \\ \dfrac{dy}{d(\ln(a))} = f_2(x, y, \ldots) \\ \ldots\ldots\ldots\ldots\ldots\ldots \end{cases}$$

- We find stationary points

$$\begin{cases} f_1 = 0 \\ f_2 = 0 \\ \ldots\ldots \end{cases} \Rightarrow \begin{matrix} (x_{cmaц1}, y_{cmaц1}, \ldots) \\ (x_{cmaц2}, y_{cmaц2}, \ldots) \\ \ldots\ldots\ldots\ldots \end{matrix}$$

- Type of stability is determined by **Lyapunov (1892)** in linear approach

$$\begin{pmatrix} (\delta x)' \\ (\delta y)' \\ \ldots \end{pmatrix} = \begin{pmatrix} \dfrac{\partial f_1}{\partial x} & \dfrac{\partial f_1}{\partial y} & ,,, \\ \dfrac{\partial f_2}{\partial x} & \dfrac{\partial f_2}{\partial y} & ,,, \\ \ldots & \ldots & \ldots \end{pmatrix} \begin{pmatrix} \delta x \\ \delta y \\ \ldots \end{pmatrix} \Rightarrow \lambda_1, \lambda_2, \lambda_3$$

eigenvalues

Main equations

Equations of gravitation and scalar fields are derived after varying the action with the considered Lagrangian

$$3H^2 = K\left(\frac{1}{2}\dot{\varphi}^2 + V(\varphi) - 3\xi H^2 B(\varphi) + \rho\right), \qquad (1)$$

$$2\dot{H} = -K\left(\dot{\varphi}^2 + 2\xi H\dot{\varphi}B'(\varphi) + 2\xi\dot{H}B(\varphi) + \rho(1+\omega)\right), \qquad (2)$$

$$\ddot{\varphi} + 3H\dot{\varphi} + 3\xi H^2 B'(\varphi) + V'(\varphi) = 0, \qquad (3)$$

here $K = 8\pi G$, $' = \frac{d}{d\varphi}$.

The equation for $\ddot{\varphi}$, when $\rho=0$

The equation for $\ddot{\varphi}$ is obtained from the system (1)-(3)

$$\ddot{\varphi}=-3H\dot{\varphi}-\frac{K\xi B'(\varphi)\dot{\varphi}^2}{2(1+K\xi B(\varphi))}-\frac{K\xi B'(\varphi)V(\varphi)+V'(\varphi)(1+K\xi B(\varphi))}{1+K\xi B(\varphi)} \quad (4)$$

where *H* is excluded using (5), which follows from (1)

$$H=\pm\sqrt{\frac{K\left(\frac{1}{2}\dot{\varphi}^2+V(\varphi)\right)}{3(1+K\xi B(\varphi))}} \quad (5)$$

We choose sign «+» for the consideration of the expansion case.

The effective potential

In scalar-torsion gravity Einstein frame **does not exist**, but we show in this work that using the effective potential of the form

$$V_{eff}(\varphi)=V(\varphi)(1+K\xi B(\varphi)) \quad ,$$

which has derivative with respect to φ

$$V'_{eff}(\varphi)=K\xi B'(\varphi)V(\varphi)+V'(\varphi)(1+K\xi B(\varphi)) \quad ,$$

allows to analyze of cosmological dynamics qualitatively.

Stability of de Sitter solution

1. For $\rho = 0$ de Sitter solution is **stable**, when

$$-N < n < 0, \quad \text{where } N > 0.$$

It is obtained in this work by means of the analyse of the effective potential and it coincides with the result for partial cases $N = 2$, $N = 4$, studied in paper of **M.A.Skugoreva et al.**, Phys. Rev. D **91** (2015) 044023.

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Asymptotic solutions ($\varphi \to \infty$) for $N = 2$, $V(\varphi) = V_0 \varphi^n$, $n < 0$, $\rho = 0$

2. $$a(t) = a_0 |t - t_0|^{\frac{1}{2+3\sqrt{6\xi}}}, \quad \varphi(t) = \varphi_0 |t - t_0|^{\frac{\sqrt{6}\xi}{2+3\sqrt{6\xi}}}$$

$$n > -2, \quad \xi > \frac{6}{(n+2)^2}, \quad t \to \infty$$

3. $$a(t) = a_0 |t - t_0|^{\frac{1}{2-3\sqrt{6\xi}}}, \quad \varphi(t) = \varphi_0 |t - t_0|^{\frac{\sqrt{6}\xi}{3\sqrt{6\xi}-2}}$$

$$-2 < n < 0, \quad 1).\ 0 < \xi < \frac{3}{8}, \quad t \to t_0$$

$$2).\ \frac{3}{8} < \xi < \frac{6}{(n+2)^2}, \quad t \to \infty$$

3a. $$a(t) = a_0 e^{H_0(t-t_0)}, \quad \varphi(t) = \varphi_0 |t - t_0|^{-\sqrt{6}\xi H_0 (t-t_0)}$$

$$\xi = \frac{3}{8}$$

Asymptotic solutions ($\varphi \to \infty$) for $N = 2$, $V(\varphi) = V_0 \varphi^n$, $n < 0$, $\rho = 0$

5. $$a(t) = a_0 |t - t_0|^{\frac{2}{\xi(n^2-4)}}, \quad \varphi(t) = \varphi_0 |t - t_0|^{\frac{2}{2-n}}$$

$-2 < n < 0, \; 0 < \xi < \frac{6}{(n+2)^2}, \; t \to \infty$ unstable,

$n < -2, \; 0 < \xi < \frac{6}{(n+2)^2}, \; t \to \infty$ **stable.**

13 Asymptotic solutions ($\varphi \to \infty$) for $N = 2$, $V(\varphi) = V_0 \varphi^n$, $n < 0$, $\rho \neq 0$

3. $a(t) = a_0 |t - t_0|^{\frac{1}{2 - 3\sqrt{6\xi}}}$, $\quad \varphi(t) = \varphi_0 |t - t_0|^{\frac{\sqrt{6\xi}}{3\sqrt{6\xi} - 2}}$

$n < 0$, **1).** $\omega \in [-1; 0)$, $0 < \xi < \frac{3(1-\omega)^2}{8}$, $t \to t_0$

2). $\omega \in [0; 1)$, $\frac{3}{8} < \xi < \frac{6}{(n+2)^2}$, $t \to \infty$

4. $a(t) = a_0 |t - t_0|^{\frac{2(\omega - 1)}{3\omega^2 - 3 + 8\xi}}$, $\varphi(t) = \varphi_0 |t - t_0|^{\frac{8\xi}{3\omega^2 - 3 + 8\xi}}$, $\rho(t) = \rho_0 |t - t_0|^{\frac{6(1-\omega^2)}{3\omega^2 - 3 + 8\xi}}$

$n < 0$, $\quad 0 < \xi < \frac{3(1-\omega^2)}{8}$, $\omega \in (-1; 1)$, $\quad t \to t_0$

$n < 0$, $\quad \xi = \frac{3}{8}(1 - \omega^2)$, $\omega \in (-1; 0)$, $\quad t \to -\infty$

14 Asymptotic solutions ($\varphi \to \infty$) for $N = 2$, $V(\varphi) = V_0 \varphi^n$, $n < 0$, $\rho \neq 0$

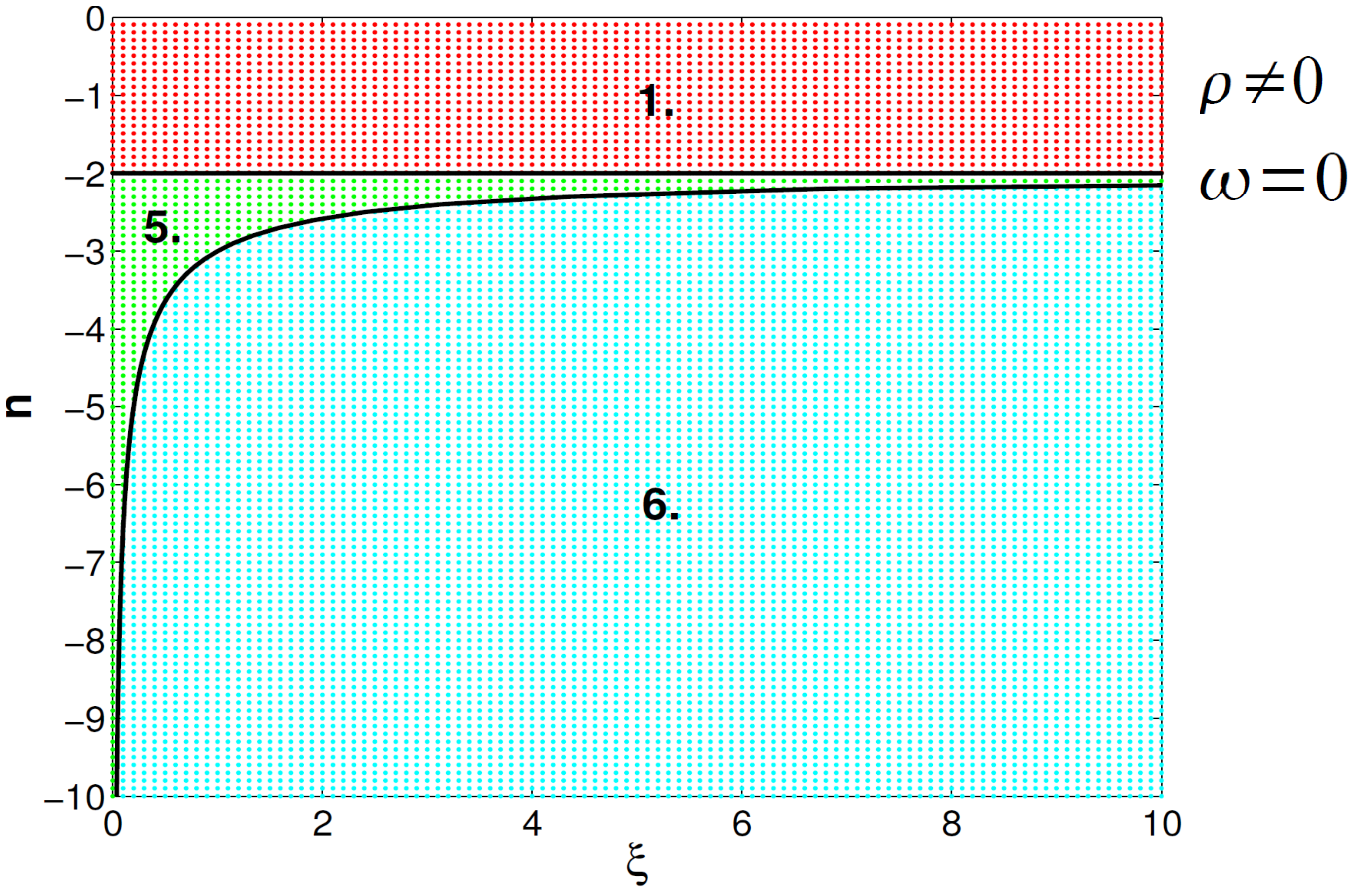
5. $$a(t) = a_0 |t - t_0|^{\frac{2}{\xi(n^2-4)}} \qquad \varphi(t) = \varphi_0 |t - t_0|^{\frac{2}{2-n}}$$

$$n < -2, \quad 0 < \xi < \frac{3(1+\omega)}{n(n+2)},$$
$$\omega \in (-1; 1], \quad t \to \infty$$

6. $$a(t) = a_0 |t - t_0|^{\frac{2n}{3(1+\omega)(n-2)}}, \ \varphi(t) = \varphi_0 |t - t_0|^{\frac{2}{2-n}}, \ \rho(t) = \rho_0 |t - t_0|^{\frac{2n}{2-n}}$$

$$n < -2, \quad \xi > \frac{3(1+\omega)}{n(n+2)},$$
$$\omega \in (-1; 1], \quad t \to \infty$$

Fig. 1. Regions of a stability of solutions 1, 5, 6 for *N* = 2, $V(\varphi)=V_0\varphi^n$, $n<0$



Conclusion

In the model of teleparallel gravity with nonminimal coupling function $B(\varphi)=\varphi^N$, *N* > 0 and the potential $V(\varphi)=V_0\varphi^n$, $n<0$

1. we have shown, that it is possible to use the effective potential (as in the standard scalar-curvature coupling) in vacuum case for a qualitative analysis of cosmological dynamics, and we have found **the stability condition of de Sitter solution: *n* > – *N***.

2. For ***n* < – *N*, *N* = 2**, $\rho=0$ the evolution of the Universe ends at one of two **power-law stage** (depending on *n*, ξ).

3. For ***n* < – *N*, *N* = 2**, $\rho\neq0$ the final of cosmological evolution is either the **power-law regime** or the **tracker soluiton** (depending on *n*, ξ).