Direct URCA-processes in neutron star quark core with strong magnetic field.

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Introduction

- Neutron stars: $\mathrm{R} = 10 \div 15$ km., $\quad \mathrm{M} = 1,4\ \mathrm{M}_\odot$
- There were discovered massive neutron stars ($\mathrm{M} \approx 2\ \mathrm{M}_\odot$), neutron stars with strong magnetic fields (magnetars).
- Cold electroneutral medium in β-equilibrium, consisting of u, d, s-quarks and electrons.
- These URCA-processes give leading contribution in neutrino emissivity:

$$\mathrm{u} + \mathrm{e}^- \to \mathrm{d} + \nu_\mathrm{e}\ \text{(crossing)},\ \ \mathrm{d} \to \mathrm{u} + \mathrm{e}^- + \bar{\nu}_\mathrm{e}\ \text{(decay)}.$$

- Lagrangian of four-fermion interaction:

$$\mathcal{L} = \frac{\mathrm{G_F} \cos\theta_\mathrm{c}}{\sqrt{2}} [\bar{\mathrm{d}}\gamma_\alpha(\mathrm{g_v} + \mathrm{g_a}\gamma^5)\mathrm{u}][\bar{\nu}\gamma^\alpha(1+\gamma^5)\mathrm{e}].$$

- Without magnetic field the neutrino emissivity is:

$$\mathrm{Q} \sim \mathrm{G_F^2}\alpha_\mathrm{c}\mu_\mathrm{u}\mu_\mathrm{d}\mu_\mathrm{e}\mathrm{T}^6 \quad \text{<Iwamoto, (1980)>}$$

Inclusion of strong magnetic field

- We solve the problem in approximations:

$$\mu_u^2, \mu_d^2 \gg m_q^2(\mu) > eB \gtrsim \mu_e^2 \gg T^2.$$

1) The magnetic field has small effect on QCD-dynamics and equation of state of quark medium. The quarks take a lot of Landau-levels.

2) The electrons take ground Landau-level.

- Substituting $\mu_{u,d} \approx 500$ MeV, $\mu_e \approx 30$ MeV, we obtain

$$B \sim 10^{17} \mathrm{Gs}.$$

- In the finite-temperature QCD:

$$m_q^2(\mu) = \frac{2\alpha_{st}}{3\pi}\mu_q^2.$$

- The quarks are ideal massive gas, electrons and neutrino are ideal massless gases in external magnetic field

$$m_e \to 0, \quad m_q = m_q(\mu_q).$$

Evaluation of neurtino emissivity

- The neutrino emissivity in crossing process:

$$\mathcal{Q}^{cr} = \frac{1}{V}\sum dn_u \; f_u \sum dn_e \; f_e \sum dn_d \; (1 - f_d) \sum dn_\nu \omega \frac{|S_{if}|^2}{\tau},$$

where sums mean summing under overall phase volumes of particles, f_i – are distribution function of particles, ω - is neutrino energy.
It is assumed in formula, that emitting medium is transparent for neutrinos.

- Sums for u, d, e are:

$$\sum dn = \sum_s \sum_n g_n L_y L_z \int \frac{dp_y dp_z}{(2\pi)^2}, \quad g_n = \begin{cases} 1, & \text{if } n = 0, \\ 2, & \text{if } n > 0, \end{cases}$$

and we use thise gauge: $\vec{B} = B \cdot \vec{n}_z, \; A^\mu = (0, 0, Bx, 0)$.

- Summation for neutrino is:

$$\sum dn = V \int \frac{d^3p}{(2\pi)^3}.$$

Evaluation of neurtino emissivity

- When we evaluate $|S_{if}|^2$, we use:

$$\sum_s \psi(x)\bar{\psi}(x') = \frac{e^{-i\Phi(x,x')}}{2E_n L_y L_z} \int \frac{dp_x}{2\pi} e^{-ip(x-x')} \hat{\rho}_n(p),$$

where $\Phi(x,x')$ - is phase, $E_n = \sqrt{p_z^2 + 2qeBn + m^2}$, $\hat{\rho}_n(p)$ - is density matrix of charged particle in magnetic field, which was summarized on spins.

- After simplification, the crossing emissivity is:

$$\mathcal{Q}^{(cr)} = \frac{3G_F^2 \cos^2\theta_c g_v^2}{32(2\pi)^8} \sum_{n,n'} g_n g_{n'} \int \frac{d^3\mathcal{P}_1}{E_n} f_u \int \frac{d^3p}{\epsilon_{n''}} f_e \cdot$$

$$\cdot \int \frac{d^3\mathcal{P}_2}{E_{n'}} (1 - f_d) \int d^3k \delta^4(\mathcal{P}_1 + p - \mathcal{P}_2 - k) Sp[\hat{\rho}_d \gamma_\alpha \cdot$$

$$\cdot (1 + c\gamma^5)\hat{\rho}_u \gamma_\beta (1 + c\gamma^5)] Sp[\hat{\rho}_\nu \gamma^\alpha (1 + \gamma^5)\hat{\rho}_e \gamma^\beta (1 + \gamma^5)],$$

where $g = \frac{g_a}{g_v}$.

Evaluation of neurtino emissivity

The density matrixes for u, d, e, ν are <Gvozdev, Osokina, (2012)>:

$$\hat{\rho}_e(p) = 2e^{-\frac{u_e}{2}}\hat{p}_{||}\Pi_- \quad , \quad \hat{\rho}_\nu(k) = \frac{1}{2}\hat{k}(1-\gamma^5),$$

$$\hat{\rho}_u(\mathcal{P}_1) = 2(-1)^n e^{-\frac{u_u}{2}} \cdot$$
$$\cdot [(\hat{\mathcal{P}}_{1||} + m)(L_n(u_u)\Pi_+ - L_{n-1}(u_u)\Pi_- + 2\hat{\mathcal{P}}_{1\perp} L^1_{n-1}(u_u)],$$

$$\hat{\rho}_d(\mathcal{P}_2) = 2(-1)^{n'} e^{-\frac{u_d}{2}} \cdot$$
$$\cdot [(\hat{\mathcal{P}}_{2||} + m)(L_{n'}(u_d)\Pi_- - L_{n'-1}(u_d)\Pi_+) + 2\hat{\mathcal{P}}_{2\perp} L^1_{n'-1}(u_d)],$$

where $u = 2p_\perp^2/qeB$, qe - is module of particle charge, L_n, L^1_{n-1} - are Laguerre polynomial, $\hat{p}_{||} = p_\mu \gamma^\mu_{||}$, $\hat{p}_\perp = p_\mu \gamma^\mu_\perp$,

$$A^\mu = (A^0, A^1, A^2, A^3), \quad A^\mu = A^\mu_{||} - A^\mu_\perp,$$

$$A^\mu_{||} = (A^0, 0, 0, A^3), \quad A^\mu_\perp = (0, -A^1, -A^2, 0),$$

$\Pi_\sigma = \frac{1}{2}[1 + \sigma i \gamma^1 \gamma^2]$ - fermion projection operator.

Integrals on transversal components

It is necessary to evaluate the following integrals:

$$\mathcal{J}^{(n,m)} = \int d\mathcal{P}_{1x} d\mathcal{P}_{1y} \int d\mathcal{P}_{2x} d\mathcal{P}_{2y} \int dp_x dp_y \cdot$$
$$L_n(u_u) L_m(u_d) \exp\Big[-\frac{u_u + u_d + u_e}{2}\Big] \delta_\perp(\mathcal{P}_1 + p - \mathcal{P}_2 - k),$$

$$\mathcal{J}^{(n,m)\alpha} = \int d\mathcal{P}_{1x} d\mathcal{P}_{1y} \int d\mathcal{P}_{2x} d\mathcal{P}_{2y} \int dp_x dp_y \mathcal{P}_1^\alpha \cdot$$
$$L^1_{n-1}(u_u) L_m(u_d) \exp\Big[-\frac{u_u + u_d + u_e}{2}\Big] \delta_\perp(\mathcal{P}_1 + p - \mathcal{P}_2 - k),$$

$$\tilde{\mathcal{J}}^{(n,m)\alpha} = \int d\mathcal{P}_{1x} d\mathcal{P}_{1y} \int d\mathcal{P}_{2x} d\mathcal{P}_{2y} \int dp_x dp_y \mathcal{P}_2^\alpha \cdot$$
$$L_n(u_u) L^1_{m-1}(u_d) \exp\Big[-\frac{u_u + u_d + u_e}{2}\Big] \delta_\perp(\mathcal{P}_1 + p - \mathcal{P}_2 - k),$$

where

$$\delta_\perp(\mathcal{P}_1 + p - \mathcal{P}_2 - k) = \delta(\mathcal{P}_{1x} + p_x - \mathcal{P}_{2x} - k_x)\delta(\mathcal{P}_{1y} + p_y - \mathcal{P}_{2y} - k_y).$$

Integrals on transversal components

After evaluations:

$$\mathcal{J}^{(n,m)} = \frac{\pi^2}{3}(eB)^2(-1)^{n+m} I_0^{(n,m)}(v),$$

$$\mathcal{J}^{(n,m)\alpha} = k_\perp^\alpha \frac{\pi^2}{6}(eB)^2(-1)^{n+m+1} n \left(I_1^{(n-1,m)}(v) - I_1^{(n,m)}(v) \right),$$

$$\tilde{\mathcal{J}}^{(n,m)\alpha} = k_\perp^\alpha \frac{\pi^2}{6}(eB)^2(-1)^{n+m} m \left(I_1^{(n,m-1)}(v) - I_1^{(n,m)}(v) \right),$$

where $v^2 = \frac{k_\perp^2}{eB}$, $v \ll 1$, and

$$I_0^{(n,m)}(v) \equiv \int_0^\infty dx\, e^{-3x}\, L_n(2x) L_m(x) J_0(\sqrt{6} v \sqrt{x}),$$

$$I_1^{(n,m)}(v) \equiv \sqrt{\frac{2}{3}} \frac{1}{v} \int_0^\infty dx \frac{e^{-3x}}{\sqrt{x}}\, L_n(2x) L_m(x) J_1(\sqrt{6} v \sqrt{x}),$$

The special functions have maximum at $m = 2n - 1$.

Some approximations

- We denote:

$$E_n = \sqrt{p_z^2 + 2qeBn + m^2}, \quad 2qeBn \equiv p_\perp^2.$$

Accordingly for u-, d-quarks: $\mathcal{P}_{1\perp}^2 \equiv \frac{4}{3}eBn$, $\mathcal{P}_{2\perp}^2 \equiv \frac{2}{3}eBn'$, $\quad \mathcal{P}_{2\perp}^2 - \mathcal{P}_{1\perp}^2 \equiv {}_\Delta\mathcal{P}_\perp^2$.

- We have $\mu_e \ll \mu_u, \mu_d$, and $\mu_u < \mu_d$, that's why

$$f_u f_e (1 - f_d) = f_d (1 - f_u)(1 - f_e) e^{-\frac{\omega}{T}} e^{\frac{\mu_e - {}_\Delta\mu}{T}}.$$

$$f_d(1 - f_u) \to S \cdot \delta(E_{n'} - \mu_d), \quad {}_\Delta\mu \equiv \mu_d - \mu_u,$$

$$S = \int_0^\infty dE_n dE_{n'} f_d (1 - f_u) \delta(E_{n'} - E_n - q_0) \approx \frac{q_0 - {}_\Delta\mu}{\exp\left(\frac{q_0 - {}_\Delta\mu}{T}\right) - 1},$$

For crossing and decay accordingly: $q = p - k$, $q = p + k$.

- Presenting $\delta(E_{n'} - \mu_d)$ in Lorentz-invariance form, we obtain

$$\sqrt{q_{||}^2} \approx s \frac{{}_\Delta\mathcal{P}_\perp^2}{2\mu_d}, \qquad \frac{\sqrt{q_{||}^2}}{\mu_d} \ll 1.$$

Integrals on parallel components

Next we evaluate integrals on parallel components:

$$\mathcal{I}_{\rm s} \equiv \int \frac{{\rm d}\mathcal{P}_{1z}}{{\rm E_n}} \int \frac{{\rm d}\mathcal{P}_{2z}}{{\rm E_{n'}}} \delta_{||}(\mathcal{P}_2 - \mathcal{P}_1 - {\rm q})\delta({\rm E}_2 - \mu_{\rm d})$$

$$\mathcal{I}_{\rm V}^{(1)\mu} \equiv \int \frac{{\rm d}\mathcal{P}_{1z}}{{\rm E_n}} \int \frac{{\rm d}\mathcal{P}_{2z}}{{\rm E_{n'}}} \delta_{||}(\mathcal{P}_2 - \mathcal{P}_1 - {\rm q})\delta({\rm E}_2 - \mu_{\rm d})\mathcal{P}^{\mu}_{1||}$$

$$\mathcal{I}_{\rm V}^{(2)\mu} \equiv \int \frac{{\rm d}\mathcal{P}_{1z}}{{\rm E_n}} \int \frac{{\rm d}\mathcal{P}_{2z}}{{\rm E_{n'}}} \delta_{||}(\mathcal{P}_2 - \mathcal{P}_1 - {\rm q})\delta({\rm E}_2 - \mu_{\rm d})\mathcal{P}^{\mu}_{2||}$$

$$\mathcal{I}_{\rm T}^{\alpha\beta} \equiv \int \frac{{\rm d}\mathcal{P}_{1z}}{{\rm E_n}} \int \frac{{\rm d}\mathcal{P}_{2z}}{{\rm E_{n'}}} \delta_{||}(\mathcal{P}_2 - \mathcal{P}_1 - {\rm q})\delta({\rm E}_2 - \mu_{\rm d})\mathcal{P}^{\alpha}_{1||}\mathcal{P}^{\beta}_{2||}$$

$$\mathcal{I}_{\phi} \equiv \int \frac{{\rm d}\mathcal{P}_{1z}}{{\rm E_n}} \int \frac{{\rm d}\mathcal{P}_{2z}}{{\rm E_{n'}}} \delta_{||}(\mathcal{P}_2 - \mathcal{P}_1 - {\rm q})\delta({\rm E}_2 - \mu_{\rm d})(\mathcal{P}_1\tilde{\phi}\mathcal{P}_2) = 0.$$

Integrals on parallel components

- Roughly we obtain:

$$\mathcal{I}_{\rm s} \approx \frac{2\delta(\cos\theta - \cos\theta^*)}{\mu_{\rm d}^2 {\rm v}_{\rm F}^2 \omega},$$

where ${\rm v}_{\rm F}^2 \equiv 1 - 2{\rm eBn}'/3\mu_{\rm d}^2$ - is square of Fermi velocity.

- Other integrals can be expressed through $\mathcal{I}_{\rm s}$:

$$\mathcal{I}_{\rm v}^{(1)\mu} = \frac{{}_\Delta\mathcal{P}_\perp^2 - {\rm q}_{||}^2}{2{\rm q}_{||}^2}\mathcal{I}_{\rm s}{\rm q}_{||}^\mu, \quad \mathcal{I}_{\rm v}^{(2)\mu} = \frac{{}_\Delta\mathcal{P}_\perp^2 + {\rm q}_{||}^2}{2{\rm q}_{||}^2}\mathcal{I}_{\rm s}{\rm q}_{||}^\mu,$$

$$\mathcal{I}_{\rm T}^{\alpha\beta} = \left({\rm a}\tilde{\Lambda}^{\alpha\beta} + {\rm b}\frac{{\rm q}_{||}^\alpha {\rm q}_{||}^\beta}{{\rm q}_{||}^2}\right)\mathcal{I}_{\rm s},$$

$${\rm a} = -\frac{{}_\Delta\mathcal{P}_\perp^2 + {\rm q}_{||}^2}{2} + \mathcal{P}_{2\perp}^2 + {\rm m}^2 - \frac{({}_\Delta\mathcal{P}_\perp^2)^2 - ({\rm q}_{||}^2)^2}{4{\rm q}_{||}^2},$$

$${\rm b} = -\frac{({}_\Delta\mathcal{P}_\perp^2)^2 - ({\rm q}_{||}^2)^2}{2{\rm q}_{||}^2} + \frac{{}_\Delta\mathcal{P}_\perp^2 + {\rm q}_{||}^2}{2} - \mathcal{P}_{2\perp}^2 - {\rm m}^2.$$

Terms

In our gauge:

$$F_{\mu\nu} = B \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad \phi_{\mu\nu} \equiv \frac{F_{\mu\nu}}{B},$$

$$\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma} = B \begin{pmatrix} 0 & 0 & 0 & +1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}, \quad \tilde{\phi}_{\mu\nu} \equiv \frac{\tilde{F}_{\mu\nu}}{B}.$$

The tensors $\Lambda_{\mu\nu}$,$\tilde{\Lambda}_{\mu\nu}$ are:

$$\Lambda_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad \tilde{\Lambda}_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix},$$

$$g_{\mu\nu} = \tilde{\Lambda}_{\mu\nu} - \Lambda_{\mu\nu}.$$

Convolutions

- Evaluating of emissivity, we can see, that

$$(\mathrm{p}_1\tilde{\phi}\mathrm{p}_2) \to 0.$$

- We express other convolutions in terms $\mathrm{q}_{||}^2, \mathrm{k}_{||}^2$. For crossing:

$$(\mathrm{p}\tilde{\Lambda}\mathrm{k}) = \frac{\mathrm{k}_{||}^2 - \mathrm{q}_{||}^2}{2}, \quad (\mathrm{k}\tilde{\Lambda}\mathrm{q}) = -\frac{\mathrm{k}_{||}^2 + \mathrm{q}_{||}^2}{2}, \quad (\mathrm{p}\tilde{\Lambda}\mathrm{q}) = \frac{\mathrm{q}_{||}^2 - \mathrm{k}_{||}^2}{2}.$$

  For decay:

$$(\mathrm{p}\tilde{\Lambda}\mathrm{k}) = \frac{\mathrm{q}_{||}^2 - \mathrm{k}_{||}^2}{2}, \quad (\mathrm{k}\tilde{\Lambda}\mathrm{q}) = \frac{\mathrm{k}_{||}^2 + \mathrm{q}_{||}^2}{2}, \quad (\mathrm{p}\tilde{\Lambda}\mathrm{q}) = \frac{\mathrm{q}_{||}^2 - \mathrm{k}_{||}^2}{2}.$$

- We take leading contribution in S-matrix element:

$$\mu_{\mathrm{d}}^2, \mathcal{P}_{2\perp}^2 \gg \mathrm{q}_{||}^2, \mathrm{m}^2, {}_{\Delta}\mathcal{P}_{\perp}^2.$$

- We neglect contribution from $\mathrm{I}_1^{(\mathrm{n,m})}$.

Summs

- From the conservation law of square components of impulses, for crossing and decay accordingly:

$$2n - n' > 0, \quad n' - 2n > 0.$$

- Therefore we introduce new variables for crossing and decay, and replace summs:

$$k = 2n - n', \quad k = n' - 2n,$$

$$\sum_{n,n'} \to \sum_{n=1}^{N_1} \sum_{k=1}^{2n}, \qquad \sum_{n,n'} \to \sum_{n'=0}^{N_2} \sum_{k=1}^{n'}.$$

where $N_1 = 3\mu_u^2/4eB$, $N_2 = 3\mu_d^2/2eB$.

Results

The leading contributions in neutrino emissivity are:

$$\mathcal{Q}^{(cr)} \approx \frac{2G_F^2 \cos^2\theta_c(g_v^2+g_a^2)}{(\pi)^5}(eB)^2\mu_e^2\,{}_{\Delta}\mu T^2 \cdot$$

$$\cdot \exp\left(-\frac{\Delta\mu}{T}\right)\exp\left(-\frac{eB}{3\mu_d T}\right)\sum_n I_0^{(n,2n-1)}\frac{1+v_F^2}{v_F^2}.$$

$$\mathcal{Q}^{(dec)} \leqslant \frac{9\sqrt{\pi}G_F^2\cos^2\theta_c(g_v^2+g_a^2)}{4\sqrt{2}(\pi)^5}(eB)\mu_d\sqrt{\mu_e}(\mu_e - {}_{\Delta}\mu + \frac{5}{2}T)T^{\frac{9}{2}} \cdot$$

$$\cdot \exp\left(-\frac{\mu_e - {}_{\Delta}\mu}{T}\right)\sum_{n'} I_0^{(\frac{n'}{2},n')}\frac{1-v_F^2}{v_F^2},$$

where for crossing and decay accordingly: $v_F^2 = 1 - 4eBn/3\mu_d^2$, $v_F^2 = 1 - 2eBn'/3\mu_d^2$.

Conclusions

- The direct URCA-processes in our approximations

$$\mu_u^2, \mu_d^2 \gg m_q^2(\mu) > eB \gtrsim \mu_e^2 \gg T^2$$

$$\mu_d^2, \mathcal{P}_{2\perp}^2 \gg q_{||}^2, m^2, {}_\Delta\mathcal{P}_\perp^2$$

are exponentially suppressed
1) The decay suppression is a consequence of small region of overlap of statistical factors u- and d- quarks

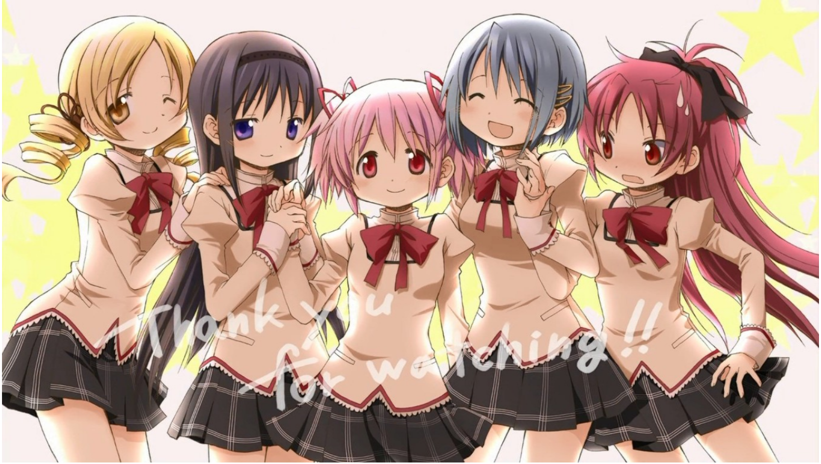
$$S = \int_0^\infty dE_n dE_{n'} f_d (1 - f_u) \delta(E_{n'} - E_n - q_0) \approx \frac{q_0 -_\Delta \mu}{\exp\left(\frac{q_0 -_\Delta \mu}{T}\right) - 1},$$

2) The crossing suppression is a consequence of one-dimensional kinematics:

$$\omega > \epsilon, \quad \epsilon \geqslant \mu_e.$$

- The main question is how do URCA-processes go on at gradual decrease of magnetic field?

Thank you for watching!!



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