Annihilation-Type Semileptonic B-Meson Decays

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Semileptonic Annihilation-Type B-Meson Decays

- Semileptonic decays with vector meson production under detail study $B \to K^{(*)} + \ell^- + \ell^+$ and $B_s^0 \to \phi + \ell^- + \ell^+$ $(\ell = e, \mu)$
- Pure annihilation-type decay modes (mixing in the $\omega - \phi$ sector is absent)
  - $B^0 \to \phi + \ell^- + \ell^+$
  - $B_s^0 \to \rho^0 + \ell^- + \ell^+$
  - $B_s^0 \to \omega + \ell^- + \ell^+$
- Sizable contribution from annihilation-type diagrams
  - $B^{+(0)} \to \rho^{+(0)} + \ell^- + \ell^+$
  - $B^0 \to \omega + \ell^- + \ell^+$
  - $B_s^0 \to K^0 + \ell^- + \ell^+$
- Important for the analysis of asymmetries

Effective Electroweak $b \to d$ Lagrangian

- An effective theory is derived from the Standard Model (SM) by integrating out heavy particles — the top quark, W- and Z-bosons, Higgs
- Lagrangian density includes all the other quark flavors $q = u, d, s, c, b$

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm QCD\times QED} + \mathcal{L}_{\rm weak}^{b\to d} + \mathcal{L}_{\rm weak}^{b\to s}$$

- Flavor-changing neutral current (FCNC) term $\mathcal{L}_{\rm weak}^{b\to d}$ describes $b \to d$ transition

$$\mathcal{L}_{\rm weak}^{b\to d} = -\frac{4G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p^{(d)} \sum_j C_j(\mu)\mathcal{P}_j(\mu) + \text{h. c.}$$

Operator Basis

- For most phenomenological applications, only operators $\mathcal{P}_j(\mu)$ of the dimension $d = 6$ are relevant
- Standard basis of four-fermion operators for $b \to d$ transition
  - Tree Operators $(p = u, c)$

$$\mathcal{P}_1^{(p)} = (\bar{d}\gamma_\mu L T^a p)(\bar{p}\gamma^\mu L T^a b) \qquad \mathcal{P}_2^{(p)} = (\bar{d}\gamma_\mu L p)(\bar{p}\gamma^\mu L b)$$

  - QCD Penguins

$$\mathcal{P}_3 = (\bar{d}\gamma_\mu L b)\sum_q(\bar{q}\gamma^\mu q) \quad \mathcal{P}_4 = (\bar{d}\gamma_\mu L T^a b)\sum_q(\bar{q}\gamma_\mu T^a q)$$

$$\mathcal{P}_5 = (\bar{d}\gamma_\mu\gamma_\nu\gamma_\rho L b)\sum_q(\bar{q}\gamma_\mu\gamma_\nu\gamma_\rho q)$$

$$\mathcal{P}_6 = (\bar{d}\gamma_\mu\gamma_\nu\gamma_\rho L T^a q)\sum_q(\bar{q}\gamma_\mu\gamma_\nu\gamma_\rho T^a q)$$

  - Electroweak Penguins
  - Electromagnetic and chromomagnetic dipole operators

$$\mathcal{P}_{7\gamma} = \frac{e}{16\pi^2}\left[\bar{d}\sigma^{\mu\nu}(m_b R + m_d L)b\right]F_{\mu\nu}$$

$$\mathcal{P}_{8g} = \frac{g_{st}}{16\pi^2}\left[\bar{d}\sigma^{\mu\nu}(m_b R + m_d L)T^a b\right]G^a_{\mu\nu}$$

  - Semileptonic operators

$$\mathcal{P}_{9\ell(10\ell)} = \frac{\alpha}{2\pi}(\bar{d}\gamma_\mu L b)\sum_\ell(\bar{\ell}\gamma^\mu(\gamma^5)\ell)$$

Wilson Coefficients

- At the matching scale μ_W, Wilson coefficients can be calculated as a perturbative expansion

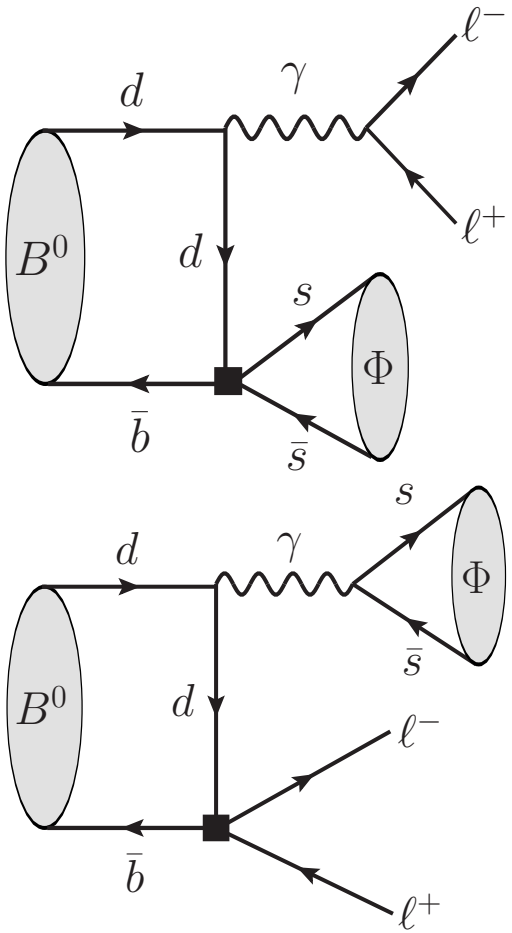
$$C_j(\mu_W) = \sum_{k=0}^{\infty} \left[\frac{\alpha_s(\mu_W)}{4\pi}\right]^k C_j^{(k)}(\mu_W)$$

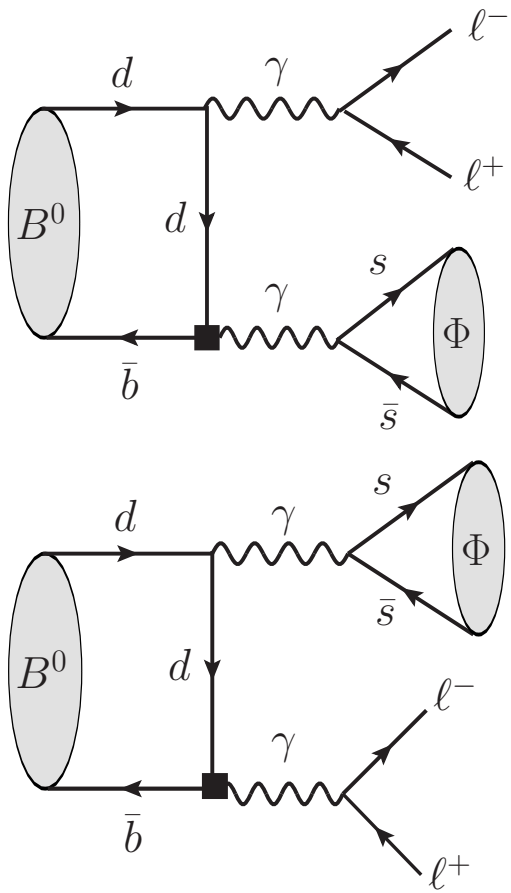
- Mixed under the operator renormalization
- Evolved to the b-quark scale by RGE
- Hierarchy in the Wilson coefficients exists; in the naive dimentional regularization scheme at the next-to-leading logarithmic (NLL) order

$C_1(m_b)$	1.080	$C_3(m_b)$	0.011	$C_7(m_b)$	4.9×10^{-4}
$C_2(m_b)$	-0.177	$C_4(m_b)$	-0.033	$C_8(m_b)$	4.6×10^{-4}
$C_{7\gamma}(m_b)$	-0.317	$C_5(m_b)$	0.010	$C_9(m_b)$	-9.8×10^{-3}
$C_{8g}(m_b)$	0.149	$C_6(m_b)$	-0.040	$C_{10}(m_b)$	1.9×10^{-3}

$B^0 \to \phi \ell^+ \ell^-$ Decay: Leading Order

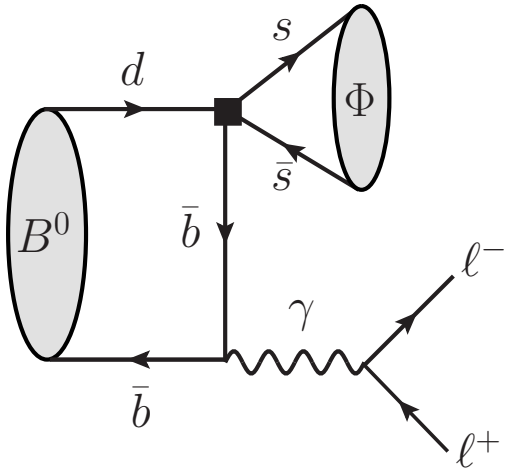
- Leading contribution in the $\Lambda_{\rm QCD}/M_B$ expansion

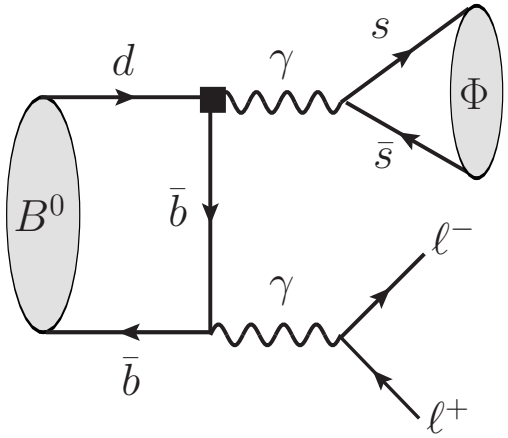


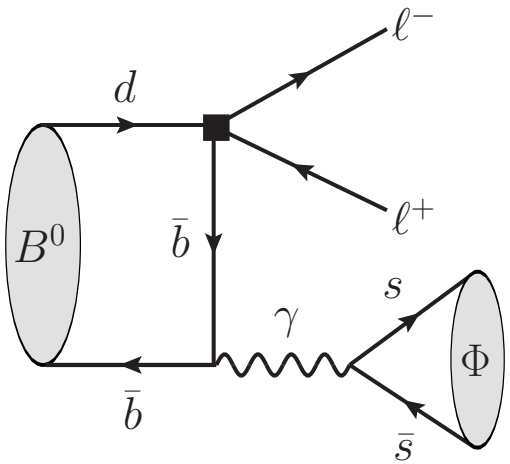


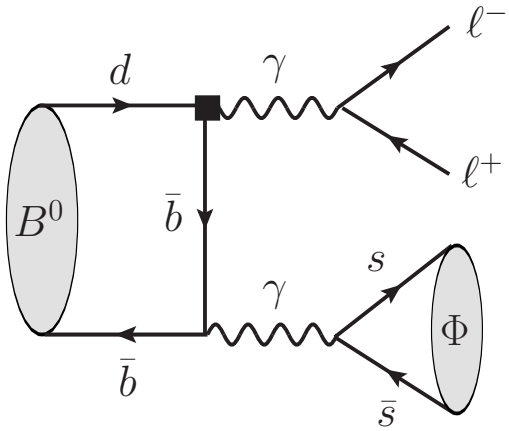
$B^0 \to \phi \ell^+ \ell^-$ Decay: Leading Order

- $1/M_B$-suppressed contributions are ignored









- NLO corrections can be numerically important but neglected

$B^0 \to \phi \ell^+ \ell^-$ Decay: Differential Branching Fraction

- Factorization approach is applicable
- ϕ-meson assumed to be energetic
- Small q^2 range is considered $\quad (q^2 \lesssim M_{J/\phi}^2)$
- Masses of leptons ($\ell = e, \mu$) are neglected

$$\frac{d\mathcal{B}}{dq^2} = \tau_B \, \frac{G_F^2 |V_{td}^* V_{tb}|^2 \alpha^2}{216\pi} \, M_B f_B^2 f_\phi^2 Q_d^2 \lambda^3(1, m_\phi/M_B, \sqrt{q^2}/M_B)$$

$$\times |C_3 + 4C_5|^2 \left[\left| \frac{1}{\lambda_-^B(q^2)} \right|^2 + \frac{m_\phi^2}{q^2 \left(1 - q^2/M_B^2\right)^2} \left| \frac{1}{\lambda_+^B(q^2)} \right|^2 \right]$$

- First inverse moments $[\lambda_\pm^B(q^2)]^{-1}$ of the B-meson distribution amplitudes (DAs) $\phi_\pm^B(\omega)$ are non-perturbative quantities; dependent on the DAs models

Heavy-Meson Distribution Amplitudes

- Meson-to-vacuum transition at $z^2 = 0$ (scalar heavy quark) [Grozin & Neubert (1996)]

$$\langle 0|Q^*(0)\,E(0,z)\,q(z)|M(v)\rangle = f_M \left\{\tilde{\varphi}_+(t) + [\tilde{\varphi}_-(t) - \tilde{\varphi}_+(t)]\,\frac{\hat{z}}{2t}\right\} U(v)$$

- f_M is a constant with dimension of a mass
- $t = (vz)$ is the time in heavy-meson rest frame
- Four-velocity of heavy meson at rest $v^\mu = (1,0,0,0)$
- $U(v)$ is non-relativistic wave-function of spinor-like meson
- $\tilde{\varphi}_\pm(t)$ are distribution amplitudes (DAs) of heavy meson
- Fourier transformations of DAs are required in constructing matrix elements

$$\tilde{\varphi}_\pm(t) = \int_0^\infty d\omega\, \mathrm{e}^{-i\omega t}\phi_\pm(\omega)$$

EoM Relations Among B-Meson LCDAs

- Heavy B- and B^*-mesons are degenerate in this approach
- Meson-to-vacuum transition at $z^2 = 0$ in real world [Grozin & Neubert (1996); Beneke & Feldmann (2001)]

$$\langle 0|q_\alpha(z)\,E(0,z)\,h_{v,\beta}(0)|\bar{B}(v)\rangle = -\frac{if_B m_B}{4}$$
$$\times\left[(1+\hat{v})\left\{\tilde{\varphi}_+^B(t) - \left[\tilde{\varphi}_+^B(t) - \tilde{\varphi}_-^B(t)\right]\frac{\hat{z}}{2t}\right\}\gamma_5\right]_{\beta\alpha}$$

- Heavy- and light-quark equation of motion relates DAs [Kawamura, Kodaira, Qiao & Tanaka (2001)]

$$\phi_+^B(\omega) + \omega\,\frac{d\phi_-^B(\omega)}{d\omega} = I(\omega), \quad (\omega - 2\bar{\Lambda})\,\phi_+^B(\omega) + \omega\,\phi_-^B(\omega) = J(\omega)$$

- $I(\omega)$ and $J(\omega)$ are determined by quark-antiquark-gluon DAs
- Wandzura-Wilczek relation follows when $I(\omega) = 0$

$$\phi_-^B(\omega) = \int_\omega^\infty \frac{\phi_+^B(\omega')}{\omega'}\,d\omega'$$

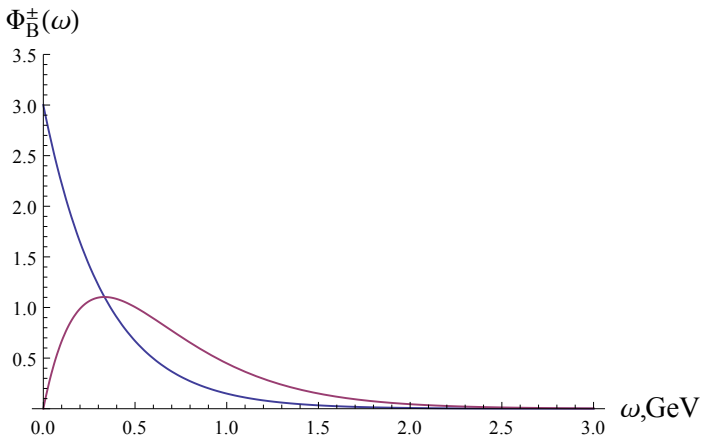
Models of the B-Meson Distribution Amplitudes

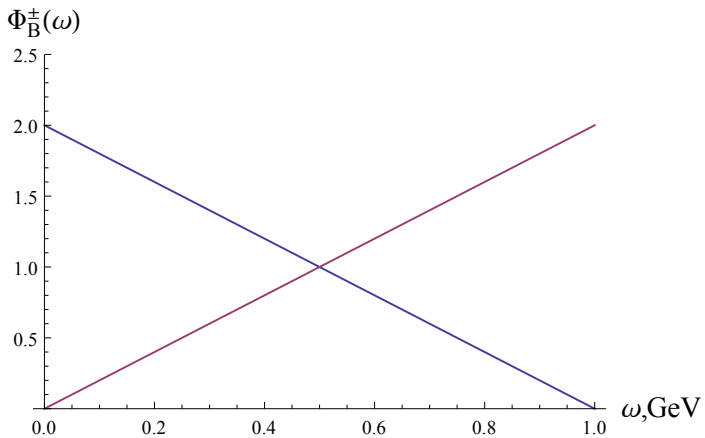
- Exponential model [A. Grozin & M. Neubert, PRD (1996)]

$$\phi_B^+(\omega) = \frac{\omega}{\omega_0^2}\, e^{-\omega/\omega_0}, \qquad \phi_B^-(\omega) = \frac{1}{\omega_0}\, e^{-\omega/\omega_0}, \qquad \omega_0 = \frac{2}{3}\,\bar{\Lambda}$$

- Linear model [H. Kawamura et al., PLB (2001)]
  Motivated by asymptotic form of light-meson LCDAs

$$\phi_B^+(\omega) = \frac{\omega}{2\bar{\Lambda}^2}\,\theta(2\bar{\Lambda}-\omega), \qquad \phi_B^-(\omega) = \frac{2\bar{\Lambda}-\omega}{2\bar{\Lambda}^2}\,\theta(2\bar{\Lambda}-\omega)$$





Distribution Amplitude Moments

- LCDAs are involved as convolutions into B-meson decay amplitudes
- First inverse moments are of great importance

$$\lambda_B^{-1}(\mu) = \int_0^\infty \frac{d\omega}{\omega}\,\phi_+^B(\omega;\mu), \qquad \frac{\sigma_{B,n}(\mu)}{\lambda_B(\mu)} = \int_0^\infty \frac{d\omega}{\omega}\,\phi_+^B(\omega;\mu)\ln^n\frac{\mu}{\omega}$$

- In particular, B-meson-to-photon transition to leading order in $1/m_B$ and $1/(2E_\gamma)$ expansion

$$F^{(0)}_{B\to\gamma^*_\perp} = \frac{Q_u f_B m_B}{2E_\gamma}\,\lambda^{-1}_{B,+}(q^2), \qquad F^{(0)}_{B\to\gamma^*_\parallel} = \frac{Q_u f_B m_B}{2E_\gamma}\,\lambda^{-1}_{B,-}(q^2)$$

- Expressed through q^2-dependent first inverse moments

$$\lambda^{-1}_{B,\pm}(q^2) = \int_0^\infty \frac{\phi_\pm^B(\omega)\,d\omega}{\omega - q^2/M_B - i\epsilon}$$

- $\lambda^{-1}_{B,-}(q^2)$ is logarithmically divergent in the limit $q^2 \to 0$

First Inverse Moments in Exponential Model

- Simple and logarithmic first inverse moments

$$\lambda_B(\mu) = \omega_0(\mu) = \frac{2}{3}\,\bar{\Lambda}; \quad \sigma_B(\mu) = \ln\frac{\mu}{\lambda_B(\mu)} + \gamma_E$$

- $\gamma_E \simeq 0.577$ is the Euler constant
- Numerically (b-quark mass is in the $\overline{\rm MS}$-scheme):
  $\lambda_B(1.0\ \text{GeV}) \simeq 0.33\ \text{GeV}$ & $\sigma_B(1.0\ \text{GeV}) \simeq 1.67$
- q^2-dependent first inverse moments

$$\lambda_{B,+}^{-1}(q^2) = \lambda_B^{-1} + \zeta\,\lambda_{B,-}(q^2), \quad \lambda_{B,-}^{-1}(q^2) = \lambda_B^{-1}\,e^{-\zeta}\,[-\text{Ei}(\zeta) + i\pi]$$

- $\zeta = q^2/(m_B\lambda_B)$ — reduced momentum squared
- $\text{Ei}(x)$ is the exponential integral function

First Inverse Moments in Linear Model

- Simple and logarithmic first inverse moments

$$\lambda_B(\mu) = \bar{\Lambda} = m_B - m_b; \quad \sigma_B(\mu) = \ln \frac{\mu}{2\lambda_B(\mu)} + 1$$

- Numerically (b-quark mass is in the $\overline{\rm MS}$-scheme):
  $\lambda_B(1.0\ \text{GeV}) \simeq 0.5\ \text{GeV}$ & $\sigma_B(1.0\ \text{GeV}) \simeq 1$
- q^2-dependent first inverse moments

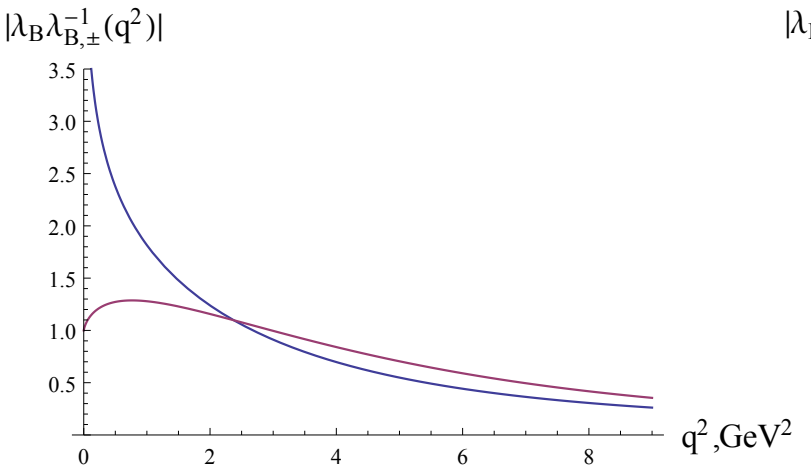
$$\lambda_{B,+}^{-1}(q^2) = \lambda_B^{-1}\left[\xi \ln|1/\xi - 1| + 1 + i\pi\xi\,\Theta(1-\xi)\right]$$

$$\lambda_{B,-}^{-1}(q^2) = \lambda_B^{-1}\left[(1-\xi)\ln|1/\xi - 1| - 1 + i\pi\,(1-\xi)\,\Theta(1-\xi)\right]$$

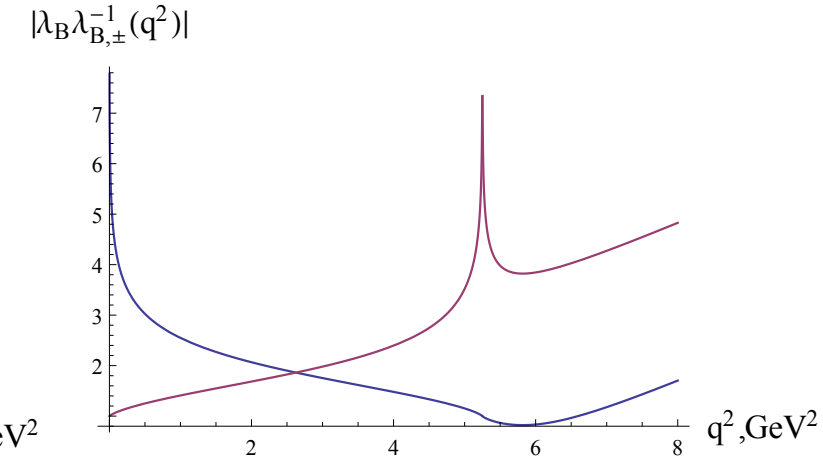
- $\xi = q^2/(2m_B\lambda_B)$ — reduced momentum squared
- $\Theta(x)$ is the unit-step function

First Inverse Moments of DAs Models

Exponential model



Linear model



$B^0 \to \phi \ell^+ \ell^-$ Decay: Numerical Results

- Partially integrated branching fraction

$$\Delta\mathcal{B}(q^2_{\rm min} < q^2 < q^2_{\rm max}) = \int_{q^2_{\rm min}}^{q^2_{\rm max}} \frac{d\mathcal{B}}{dq^2}\, dq^2$$

- Estimated in two models:

  GN model: $\Delta\mathcal{B}(1\ \text{GeV}^2 < q^2 < 8\ \text{GeV}^2) = 5.70 \times 10^{-13}$

  KKQT model: $\Delta\mathcal{B}(1\ \text{GeV}^2 < q^2 < 8\ \text{GeV}^2) = 5.25 \times 10^{-13}$
- Reduction is around 10%
- Relatively large error due to the choice of a model
- Comparable with uncertainty originated by the scale variation
- Total branching fraction

$$\mathcal{B}(B^0 \to \phi \ell^+ \ell^-) \sim 10^{-12}$$

- Too small, to be observed experimentally at present

Comments about Other Decay Modes

- In $B_s \to \rho^0(\omega)\ell^-\ell^+$ decays, B_s-meson DAs are necessary
- For the first inverse moment, a simple rescaling is used

$$\frac{M_{B_s}}{\lambda_{B_s}}\,(\lambda_{\rm QCD}+m_s)=\frac{M_B}{\lambda_B}\,\lambda_{\rm QCD}$$

- $SU(3)_F$-breaking correction in DAs are not workred out
- In semileptonic decays with sizable annihilation contributions, the choice of B-meson model can result the uncertainty of order 10% into the annihilation contribution
- Can affect the uncertainties of CP- and isospin-asymmetries; requires careful study
- Similar effects one can expected in the semileptonic decays with pseudoscalar production

Conclusions

- Annihilation-type semileptonic decay $B \to \phi \ell^- \ell^+$ is considered in the factorization approach
- Branching fraction is sensitive to the choice of the B-meson distribution amplitude in the form of momentum-dependent first inverse moments
- Two models for the B-meson distribution amplitudes are considered and their first inverse moments are calculated
- The partially integrated branching fractions are calculated in the range $1\ \mathrm{GeV}^2 < q^2 < 8\ \mathrm{GeV}^2$ and dependence on the DAs choice results uncertainty of order 10%
- The branching fraction of $B \to \phi \ell^- \ell^+$ decays is too small to be observed experimentally at present
- This uncertainty in rare semileptonic FCNC-induced decays, being small in total and differential branching fractions, can be substantial in CP- and isospin-asymmetries which are sensitive to annihilation contributions