Radiative corrections to the effective sextic couplings of Higgs self-interactions in the heavy supersymmetry

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Introduction

SM – low-energy effective theory. Supersymmetry.
MSSM Higgs sector: h (125) and H, A, H^+, H^-

$$\Phi_k = \begin{pmatrix} \phi_k^+(x) \\ \phi_k^0(x) \end{pmatrix} = \begin{pmatrix} -i\omega_k^+ \\ \frac{1}{\sqrt{2}}(v_k + \eta_k + i\chi_k) \end{pmatrix}, \qquad k = 1, 2 \tag{1}$$

$$v = \sqrt{v_1^2 + v_2^2} = 246 \text{ GeV}, \qquad v_2/v_1 = \tan\beta$$

$$\begin{aligned} U &= -\mu_1^2(\Phi_1^\dagger\Phi_1) - \mu_2^2(\Phi_2^\dagger\Phi_2) - [\mu_{12}^2(\Phi_1^\dagger\Phi_2) + h.c.] \\ &+ \lambda_1(\Phi_1^\dagger\Phi_1)^2 + \lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) \\ &+ [\lambda_5/2(\Phi_1^\dagger\Phi_2)(\Phi_1^\dagger\Phi_2) + \lambda_6(\Phi_1^\dagger\Phi_1)(\Phi_1^\dagger\Phi_2) + \lambda_7(\Phi_2^\dagger\Phi_2)(\Phi_1^\dagger\Phi_2) + h.c.] \end{aligned} \tag{2}$$

$$\begin{aligned} \lambda_{1,2}^{\texttt{tree}}(M_S) &= \frac{g_1^2 + g_2^2}{4}, \qquad \lambda_3^{\texttt{tree}}(M_S) = \frac{g_2^2 - g_1^2}{4}, \\ \lambda_4^{\texttt{tree}}(M_S) &= \frac{g_2^2}{2}, \qquad \lambda_{5,6,7}^{\texttt{tree}}(M_S) = 0, \end{aligned} \tag{3}$$

Radiative corrections to these tree-level expressions are parametrized using

$$
\begin{aligned}
\lambda_i(M) &= \lambda_i^{\texttt{tree}}(M_S) - \Delta\lambda_i(M)/2, \quad i = 1, 2, \\
\lambda_i(M) &= \lambda_i^{\texttt{tree}}(M_S) - \Delta\lambda_i(M), \quad i = 3, ...7,
\end{aligned} \tag{4}
$$

An example of one-loop threshold corrections[1]

$$
\begin{aligned}
\frac{\Delta\lambda_1}{2} = & -\frac{3}{32\pi^2}\Big[h_b^4 \frac{|A_b|^2}{M_{\rm SUSY}^2}\left(2 - \frac{|A_b|^2}{6M_{\rm SUSY}^2}\right) - h_t^4 \frac{|\,\mu|^4}{6M_{\rm SUSY}^4} + \\
& + 2h_b^4 l + \frac{g_2^2 + g_1^2}{4\,M_{\rm SUSY}^2}(h_t^2|\,\mu|^2 - h_b^2|A_b|^2)\Big] - \\
& - \frac{1}{768\pi^2}\left(11g_1^4 + 9g_2^4 - 36\,(g_1^2 + g_2^2)\,h_b^2\right) l,
\end{aligned} \tag{5}
$$

where $l = \log\left(\frac{M_S^2}{m_{top}^2}\right)$, $\qquad X_b = \frac{2A_b^2}{M_{\rm SUSY}^2}\left(1 - \frac{A_b^2}{12M_{\rm SUSY}^2}\right)$.

[1] Akhmetzyanova, Dolgopolov, Dubinin, Phys.Rev. D **71** (2005)

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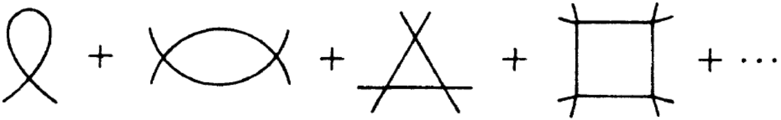
The one-loop approximation for the effective potential

The resummed potential at the one-loop[2]

$$U = U^{(2)} + U^{(4)} + U^{(6)} + U^{(8)} + ... \quad (6)$$

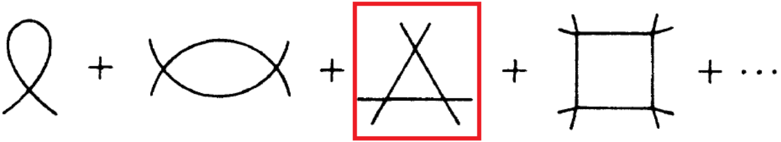


Two conditions of negligibly small contributions of higher order operators [3]

$$2|m_{\text{top}}A| < M_S^2, \qquad 2|m_{\text{top}}\mu| < M_S^2$$

[2] S. Coleman, E. Weinberg, Phys. Rev. D **7**(6) (1973) 1888

[3] Carena *et al.*, Phys. Lett. B 355, 1995



$$
\begin{aligned}
U^{(6)} \;=\;& \kappa_1(\Phi_1^\dagger\Phi_1)^3 + \kappa_2(\Phi_2^\dagger\Phi_2)^3 + \kappa_3(\Phi_1^\dagger\Phi_1)^2(\Phi_2^\dagger\Phi_2) + \kappa_4(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2)^2 + \\
+\;& \kappa_5(\Phi_1^\dagger\Phi_1)(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + \kappa_6(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \\
+\;& [\kappa_7(\Phi_1^\dagger\Phi_2)^3 + \kappa_8(\Phi_1^\dagger\Phi_1)^2(\Phi_1^\dagger\Phi_2) + \kappa_9(\Phi_1^\dagger\Phi_1)(\Phi_1^\dagger\Phi_2)^2 + \\
+\;& \kappa_{10}(\Phi_1^\dagger\Phi_2)^2(\Phi_2^\dagger\Phi_2) + \kappa_{11}(\Phi_1^\dagger\Phi_2)^2(\Phi_2^\dagger\Phi_1) + \kappa_{12}(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_2)^2 + \\
+\;& \kappa_{13}(\Phi_1^\dagger\Phi_1)(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_2) + h.c.].
\end{aligned}
$$

Mass basis for the case of effective potential with the dimension-six terms

$$\begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} = \mathcal{O}_\alpha \begin{pmatrix} H \\ h \end{pmatrix}, \qquad \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} = \mathcal{O}_\beta \begin{pmatrix} G^0 \\ A \end{pmatrix}, \qquad \begin{pmatrix} \omega_1^\pm \\ \omega_2^\pm \end{pmatrix} = \mathcal{O}_\beta \begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} \quad (7)$$

$$\mathcal{O}_X = \begin{pmatrix} \cos X & -\sin X \\ \sin X & \cos X \end{pmatrix}, \qquad X = \alpha, \beta \quad (8)$$

$$U = c_0 A + c_1 hA + c_2 HA + \frac{m_h^2}{2} h^2 + \frac{m_H^2}{2} H^2 + \frac{m_A^2}{2} A^2 + m_{H^\pm}^2 H^+ H^- + I_3 + I_4 + I_5 + I$$

Condition	Parameters
minimization	μ_1^2, μ_2^2
diagonalization	$\mathrm{Re}\mu_{12}^2$
$c_0 = 0$, $c_1 = 0$, $c_2 = 0$	$\mathrm{Im}\Delta\lambda$, $\mathrm{Im}\kappa$, $\mathrm{Im}\mu_{12}^2$

$$\frac{\partial U}{\partial \Phi_i} = 0, \qquad i = 1,2 \tag{9}$$

$$\begin{aligned}
\mu_1^2 &= -\mathrm{Re}\mu_{12}^2 t_\beta + \frac{v^2}{4}(4\lambda_1 c_\beta^2 + 3\mathrm{Re}\lambda_6 s_{2\beta} + 2s_\beta^2(\lambda_{345} + \mathrm{Re}\lambda_7 t_\beta)) + \qquad (10)\\
&+ \frac{v^4}{4}(3\kappa_1 c_\beta^4 + 5\mathrm{Re}\kappa_8 c_\beta^3 s_\beta + 3(\mathrm{Re}\kappa_7 + \mathrm{Re}\kappa_{11} + \mathrm{Re}\kappa_{13})c_\beta s_\beta^3 + \\
&+ (\mathrm{Re}\kappa_9 + (\kappa_3 + \kappa_5)/2)s_{2\beta}^2 + (\kappa_4 + \kappa_6 + 2\mathrm{Re}\kappa_{10} + \mathrm{Re}\kappa_{12} t_\beta)s_\beta^4),\\
\mu_2^2 &= -\mathrm{Re}\mu_{12}^2 \cot\beta + \frac{v^2}{4}(4\lambda_2 s_\beta^2 + 3\mathrm{Re}\lambda_7 s_{2\beta} + 2c_\beta^2(\lambda_{345} + \mathrm{Re}\lambda_6 \cot_\beta)) + \qquad (11)\\
&+ \frac{v^4}{4}(3\kappa_2 s_\beta^4 + 5\mathrm{Re}\kappa_{12} s_\beta^3 c_\beta + 3(\mathrm{Re}\kappa_7 + \mathrm{Re}\kappa_{11} + \mathrm{Re}\kappa_{13})s_\beta c_\beta^3 + \\
&+ (\mathrm{Re}\kappa_{10} + (\kappa_4 + \kappa_6)/2)s_{2\beta}^2 + (\kappa_3 + \kappa_5 + 2\mathrm{Re}\kappa_9 + \mathrm{Re}\kappa_8 \cot_\beta)c_\beta^4).
\end{aligned}$$

$$\mathcal{M}^2_Y = \begin{pmatrix} \mathcal{M}^2_{11} & \mathcal{M}^2_{12} \\ \mathcal{M}^2_{21} & \mathcal{M}^2_{22} \end{pmatrix}, \qquad \mathcal{M}^2_{ij} = \frac{\partial^2 U}{\partial Y_i \partial Y_j} \tag{12}$$

$$\mathcal{M}^2_{11} = m^2_A s^2_\beta + m^2_Z c^2_\beta, \qquad \mathcal{M}^2_{22} = m^2_A c^2_\beta + m^2_Z s^2_\beta, \qquad \mathcal{M}^2_{12} = -s_\beta c_\beta (m^2_A + m^2_Z$$

$$\begin{aligned}
\Delta\mathcal{M}^2_{11} &= -v^2(\Delta\lambda_1 c^2_\beta + \mathtt{Re}\Delta\lambda_5 s^2_\beta + \mathtt{Re}\Delta\lambda_6 s_{2\beta}) + && (13)\\
&+ v^4[3\kappa_1 c^4_\beta + 4\mathrm{Re}\kappa_8 c^3_\beta s_\beta + (\kappa_3 + \kappa_5 + 3\mathrm{Re}\kappa_9) c^2_\beta s^2_\beta + \\
&+ (3\mathrm{Re}\kappa_7 + \mathrm{Re}\kappa_{11} + \mathrm{Re}\kappa_{13}) c_\beta s^3_\beta + \mathrm{Re}\kappa_{10} s^4_\beta], \\
\Delta\mathcal{M}^2_{22} &= -v^2(\Delta\lambda_2 s^2_\beta + \mathtt{Re}\Delta\lambda_5 c^2_\beta + \mathtt{Re}\Delta\lambda_7 s_{2\beta}) + && (14)\\
&+ v^4[\mathrm{Re}\kappa_9 c^4_\beta + (3\mathrm{Re}\kappa_7 + \mathrm{Re}\kappa_{11} + \mathrm{Re}\kappa_{13}) c^3_\beta s_\beta + \\
&+ (\kappa_4 + \kappa_6 + 3\mathrm{Re}\kappa_{10}) c^2_\beta s^2_\beta + 4\mathrm{Re}\kappa_{12} c_\beta s^3_\beta + 3\kappa_2 s^4_\beta], \\
\Delta\mathcal{M}^2_{12} &= -v^2(\Delta\lambda_{34} s_\beta c_\beta + \mathtt{Re}\Delta\lambda_6 c^2_\beta + \mathtt{Re}\Delta\lambda_7 s^2_\beta) + && (15)\\
&+ v^4[\mathrm{Re}\kappa_8 c^4_\beta + (\kappa_3 + \kappa_5 + \mathrm{Re}\kappa_9) c^3_\beta s_\beta + \\
&+ 2(\mathrm{Re}\kappa_{11} + \mathrm{Re}\kappa_{13}) c^2_\beta s^2_\beta + (\kappa_4 + \kappa_6 + \mathrm{Re}\kappa_{10}) c_\beta s^3_\beta + \mathrm{Re}\kappa_{12} s^4_\beta].
\end{aligned}$$

$$m^2_{H,h} = \frac{1}{2}(m^2_A + m^2_Z + \Delta\mathcal{M}^2_{11} + \Delta\mathcal{M}^2_{22} \pm \sqrt{m^4_A + m^4_Z - 2m^2_A m^2_Z c_{4\beta} + C}),$$

$$\begin{aligned} m^2_{H^\pm} &= m^2_W + m^2_A - \frac{v^2}{2}(\mathrm{Re}\Delta\lambda_5 - \Delta\lambda_4) + \qquad (16) \\ &+ \frac{v^4}{4}[c^2_\beta(2\mathrm{Re}\kappa_9 - \kappa_5) + s^2_\beta(2\mathrm{Re}\kappa_{10} - \kappa_6) - s_{2\beta}(\mathrm{Re}\kappa_{11} - 3\mathrm{Re}\kappa_7)] \end{aligned}$$

$$m^2_A = \frac{m^2_h(C_1 - m^2_h) + m^2_Z(C_2 - C_3) - \Delta\mathcal{M}^2_{11}\Delta\mathcal{M}^2_{22} + \Delta\mathcal{M}^4_{12}}{C_1 - C_2 - C_3 + m^2_Z c^2_{2\beta}}, \qquad (17)$$

$$\tan 2\alpha = \frac{2\Delta\mathcal{M}^2_{12} - (m^2_Z + m^2_A)s_{2\beta}}{(m^2_Z - m^2_A)c_{2\beta} + \Delta\mathcal{M}^2_{11} - \Delta\mathcal{M}^2_{22}} \qquad (18)$$

where

$$C = 4\Delta\mathcal{M}^4_{12} + (\Delta\mathcal{M}^2_{11} - \Delta\mathcal{M}^2_{22})^2 - 2(m^2_A - m^2_Z)(\Delta\mathcal{M}^2_{11} - \Delta\mathcal{M}^2_{22})c_{2\beta} - 4(m^2_A + m^2_Z)\Delta\mathcal{M}^2_{12}s_{2\beta},$$

$$C_1 = \Delta\mathcal{M}^2_{11} + \Delta\mathcal{M}^2_{22}, \qquad C_2 = m^2_h - \Delta\mathcal{M}^2_{12}s_{2\beta}, \qquad C_3 = \Delta\mathcal{M}^2_{11}s^2_\beta + \Delta\mathcal{M}^2_{22}c^2_\beta.$$

Two conditions restrict implicitly the MSSM parameter space

$$m^4_A + m^4_Z - 2m^2_A m^2_Z c_{4\beta} + C \geq 0, \quad m^4_A + m^4_Z + \Delta\mathcal{M}^2_{11} + \Delta\mathcal{M}^2_{22} - 2m^2_h \geq 0.$$

Effective potential method

The one-loop resummed MSSM potential at the renormalization scale m_{top}

$$U_{\rm eff} = U^0 + \frac{3}{32\pi^2}{\rm tr}\mathcal{M}^4\left(\ln\frac{\mathcal{M}^2}{m_{top}^2} - \frac{3}{2}\right), \tag{19}$$

where U^0 is a tree-level potential at the scale M_S,

$$\mathcal{M}^2_{ab} = \frac{\partial^2\mathcal{V}^0}{\partial\Psi_a\partial\Psi_b^*} \tag{20}$$

is the squark mass matrix squared,
$\Psi = (\tilde{Q}, \tilde{U}^*, \tilde{D}^*)$,
$\mathcal{V}^0$ is the most general scalar potential, including Higgs boson and one generation of squarks[4]

$$\mathcal{V}^0 = \mathcal{V}_M + \mathcal{V}_\Gamma + \mathcal{V}_\Lambda + \mathcal{V}_{\tilde{Q}} \tag{21}$$

[4]Gunion, Haber, Nucl. Phys. B272, 1986; Haber, Hempfling, Phys. Rev. D 48, 1993

$$
\begin{aligned}
\mathcal{V}_M &= -\mu_{ij}^2 \Phi_i^\dagger \Phi_j + M_{\tilde{Q}}^2 (\tilde{Q}^\dagger \tilde{Q}) + M_{\tilde{U}}^2 (\tilde{U}^* \tilde{U}) + M_{\tilde{D}}^2 (\tilde{D}^* \tilde{D}), && (22)\\
\mathcal{V}_\Gamma &= \Gamma_i^D (\Phi_i^\dagger \tilde{Q}) \tilde{D} + \Gamma_i^U (i \Phi_i^T \sigma_2 \tilde{Q}) \tilde{U} + h.c., && (23)\\
\mathcal{V}_\Lambda &= \Lambda_{ik}^{jl} (\Phi_i^\dagger \Phi_j)(\Phi_k^\dagger \Phi_l) + (\Phi_i^\dagger \Phi_j)[\Lambda_{ij}^Q (\tilde{Q}^\dagger \tilde{Q}) + \Lambda_{ij}^U (\tilde{U}^* \tilde{U}) + \Lambda_{ij}^D (\tilde{D}^* \tilde{D})] \\
&+ \overline{\Lambda}_{ij}^Q (\Phi_i^\dagger \tilde{Q})(\tilde{Q}^\dagger \Phi_j) + \frac{1}{2}[\Lambda \epsilon_{ij} (i \Phi_i^T \sigma_2 \Phi_j) \tilde{D}^* \tilde{U} + h.c.] && (24)
\end{aligned}
$$

$$M_S = M_{\tilde{Q},\tilde{U},\tilde{D}}$$

$$
\begin{aligned}
&\Lambda^Q = \mathrm{diag}[\frac{1}{4}(g_2^2 - g_1^2 Y_Q), h_U^2 - \frac{1}{4}(g_2^2 - g_1^2 Y_Q)], && (25)\\
&\overline{\Lambda}^Q = \mathrm{diag}(h_D^2 - \frac{1}{2} g_2^2, \frac{1}{2} g_2^2 - h_U^2), && (26)\\
&\Lambda^U = \mathrm{diag}(-\frac{1}{4} g_1^2 Y_U, h_U^2 + \frac{1}{4} g_1^2 Y_U), && (27)\\
&\Lambda^D = \mathrm{diag}(h_D^2 - \frac{1}{4} g_1^2 Y_D, \frac{1}{4} g_1^2 Y_D), && (28)\\
&\Lambda = -h_U h_D, && (29)\\
&\Gamma_{1,2}^U = h_U(-\mu, A_U), \qquad \Gamma_{1,2}^D = h_D(A_D, -\mu), && (30)
\end{aligned}
$$

$g_{1,2}$ are couplings of $SU(2)_L \times U(1)_Y$, $Y_{Q,U,D} = \{\frac{1}{3}(-1), \frac{2}{3}(2), -\frac{4}{3}\}$ – squark (slepton) hypercharges, $h_U = \frac{g_2 m_U}{\sqrt{2} m_W s_\beta}$, $h_D = \frac{g_2 m_D}{\sqrt{2} m_W c_\beta}$ – Yukawa couplings, $A_{U,D}$ – trilinear couplings, μ – Higgs superfield mass parameter.

Symbolic expressions for κ_i and numerical results

Effective potential terms of the dimension-six in the decomposition are

$$U_{\text{eff}}^{(6)} = \frac{3}{32M_S^2\pi^2}\Big(\frac{1}{3}\text{tr}(\mathcal{M}_\Lambda^2)^3 - \frac{1}{2M_S^2}\text{tr}[(\mathcal{M}_\Gamma^2)^2(\mathcal{M}_\Lambda^2)^2] + \frac{1}{6M_S^4}\text{tr}[(\mathcal{M}_\Gamma^2)^4\mathcal{M}_\Lambda^2] - \frac{1}{60M_S^6}\text{tr}(\mathcal{M}_\Gamma^2)^6\Big). \quad (31)$$

For instance,

Fields	$U^{(6)}$	$U_{\text{eff}}^{(6)}$
$(\phi_1^+\phi_1^-)^3$	κ_1	$\frac{h_D^6}{32M_S^2\pi^2}\left(2 - \frac{3\lvert A_D\rvert^2}{M_S^2} + \frac{\lvert A_D\rvert^4}{M_S^4} - \frac{\lvert A_D\rvert^6}{10M_S^6}\right)$ $-h_D^4\frac{g_1^2+g_2^2}{128M_S^2\pi^2}\left(3 - 3\frac{\lvert A_D\rvert^2}{M_S^2} + \frac{\lvert A_D\rvert^4}{2M_S^4}\right)$ $+\frac{h_D^2}{512M_S^2\pi^2}\left(\frac{5}{3}g_1^4 + 2g_1^2g_2^2 + 3g_2^4\right)\left(1 - \frac{\lvert A_D\rvert^2}{2M_S^2}\right)$ $-h_U^6\frac{\lvert\mu\rvert^6}{320M_S^8\pi^2} + h_U^4\frac{(g_1^2+g_2^2)\lvert\mu\rvert^4}{256M_S^6\pi^2} - h_U^2\frac{(17g_1^4-6g_1^2g_2^2+9g_2^4)\lvert\mu\rvert^2}{3072M_S^4\pi^2}$ $+\frac{g_1^2}{1024M_S^2\pi^2}(g_1^4 - g_2^4)$

$$\kappa_1 = h_D^6 C_9^D - h_D^4 G_4 C_8^D + h_D^2 G_2 B_1^D + h_U^6 A_1 + h_U^4 G_4 A_2 - h_U^2 G_3 A_3 + G_1, \quad (32)$$

$$\kappa_2 = h_D^6 A_1 + h_D^4 G_4 A_2 - h_D^2 G_2 A_3 + h_U^6 C_9^U - h_U^4 G_4 C_8^U + h_U^2 G_3 B_1^U - G_1, \quad (33)$$

$$\kappa_3 = h_D^6 C_7^D + h_D^4 G_4 B_3^D - h_D^2 G_2 (2B_1^D + A_3) \quad (34)$$

$$+ h_U^6 C_1^U - h_U^4 G_4 B_4^U |\mu|^2 + h_U^2 G_3 (B_1^U + 2A_3) - 3G_1,$$

$$\kappa_4 = h_D^6 C_1^D - h_D^4 G_4 B_4^D |\mu|^2 + h_D^2 G_2 (B_1^D + 2A_3) \quad (35)$$

$$+ h_U^6 C_7^U + h_U^4 G_4 B_3^U - h_U^2 G_3 (2B_1^U + A_3) + 3G_1,$$

$$\kappa_5 = h_D^6 C_7^D + h_D^4 G_4 B_3^D - h_D^2 G_2 (2B_1^D + A_3) \quad (36)$$

$$+ h_U^6 C_1^U - h_U^4 G_4 B_4^U |\mu|^2 + h_U^2 G_3 (B_1^U + 2A_3) - 3G_1,$$

$$\kappa_6 = h_D^6 C_1^D - h_D^4 G_4 B_4^D |\mu|^2 + h_D^2 G_2 (B_1^D + 2A_3) \quad (37)$$

$$+ h_U^6 C_7^U + h_U^4 G_4 B_3^U - h_U^2 G_3 (2B_1^U + A_3) + 3G_1,$$

$$\kappa_7 = \frac{\mu^3}{320 M_S^8 \pi^2} (A_D^3 h_D^6 + A_U^3 h_U^6), \quad (38)$$

$$\kappa_8 = h_D^6 C_6^D + 2h_D^4 G_4 C_4^D + h_D^2 G_2 A_7^D + h_U^6 A_2^U + h_U^4 G_4 A_5^U + h_U^2 G_3 A_7^U, \quad (39)$$

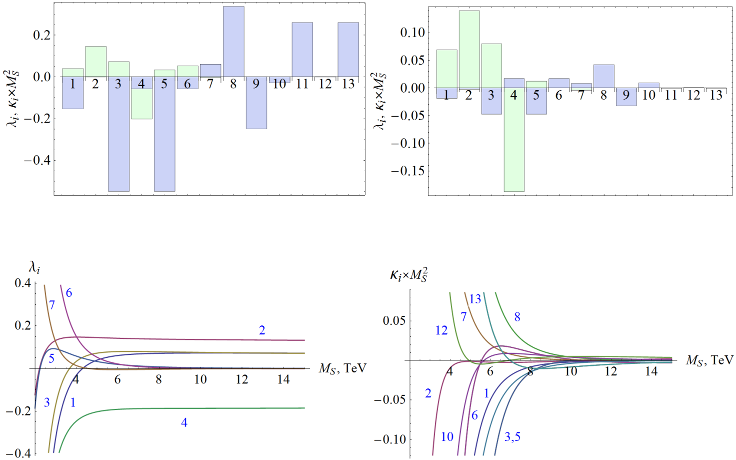
$$\kappa_9 = h_D^6 C_2^D - h_D^4 G_4 A_6^D + h_U^6 A_4^U + h_U^4 G_4 A_6^U, \quad (40)$$

$$\kappa_{10} = h_D^6 A_4^D + h_D^4 G_4 A_6^D + h_U^6 C_2^U - h_U^4 G_4 A_6^U, \quad (41)$$

$$\kappa_{11} = h_D^6 C_3^D + h_D^4 G_4 C_5^D - 2h_D^2 G_2 A_7^D + h_U^6 C_3^U + h_U^4 G_4 C_5^U - 2h_U^2 G_3 A_7^U, \quad (42)$$

$$\kappa_{12} = h_D^6 A_2^D + h_D^4 G_4 A_5^D + h_D^2 G_2 A_7^D + h_U^6 C_6^U + 2h_U^4 G_4 C_4^U + h_U^2 G_3 A_7^U, \quad (43)$$

$$\kappa_{13} = h_D^6 C_3^D + h_D^4 G_4 C_5^D - 2h_D^2 G_2 A_7^D + h_U^6 C_3^U + h_U^4 G_4 C_5^U - 2h_U^2 G_3 A_7^U, \quad (44)$$



The dimensionless parameters λ_i and $\kappa_i \cdot M_S^2$ for $A_t = A_b = 10$ TeV, $\mu = 14$ TeV, $\tan\beta = 5$, M_S=5,7 TeV (top row). λ_i are evaluated using analytical formulae[5]

[5] E. Akhmetzyanova, M. Dolgopolov, and M. Dubinin, Phys. Rev. D **71**, 075008 (2005); Phys. Part. Nucl. **37**(5), 677 (2006).

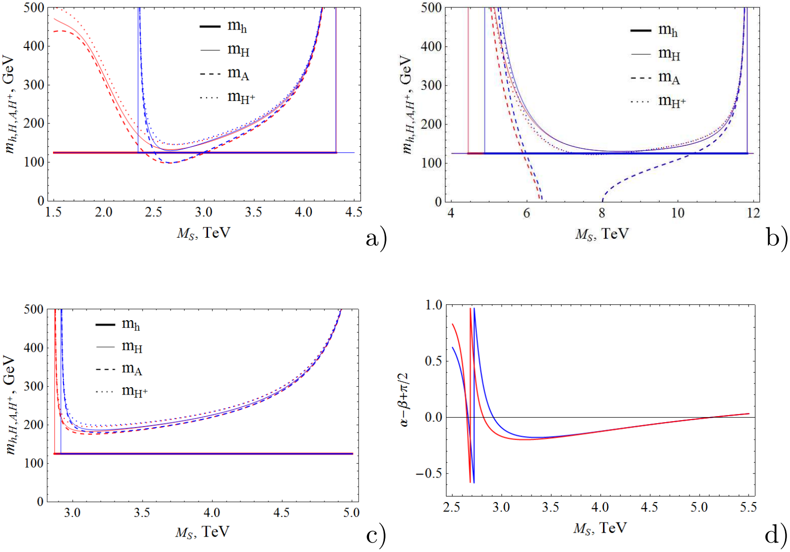


Figure: (a) $\tan\beta = 4$, $A = 10$ TeV, $\mu = 8$ TeV; (b) $\tan\beta = 8$, $A = 25$ TeV, $\mu = 30$ TeV, (c),(d) $\tan\beta = 5$, $A = 10$ TeV and $\mu = 5$ TeV.

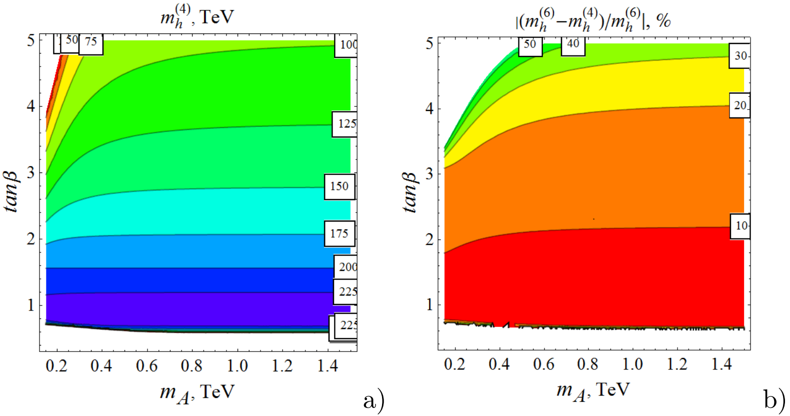


Figure: (a) contours for the Higgs boson mass $m_h^{(4)}$ calculated with the dimension-four potential terms; (b) the relative difference in percent between $m_h^{(6)}$ and $m_h^{(4)}$ masses; the parameter set $A = 10$ TeV, $\mu = 8.3$ TeV, $M_S = 2$ TeV.

The vacuum stability conditions

The necessary condition

$$\frac{\partial U}{\partial \phi_1} = 0, \qquad \frac{\partial U}{\partial \phi_2} = 0 \tag{45}$$

and sufficient condition

$$\Delta = \left| \begin{array}{cc} U''_{\phi_1\phi_1} & U''_{\phi_1\phi_2} \\ U''_{\phi_2\phi_1} & U''_{\phi_2\phi_2} \end{array} \right|_v > 0 \tag{46}$$

of extremum existence.
The minimum condition

$$U''_{\phi_1\phi_1} > 0. \tag{47}$$

$\lambda_{6,7} = \kappa_i = 0$, $\mathrm{Re}\mu_{12}^2 = 0$:
$\lambda_1 > 0, \qquad \lambda_2 > 0, \qquad \sqrt{\lambda_1 \lambda_2} > |\lambda_{345}|.$

Conditions of positivly definite Higgs potential

1. $\lambda_{6,7} = \kappa_i = 0$, $\mathrm{Re}\mu_{12}^2 = 0$: $\lambda_1 \geq 0, \qquad \lambda_2 \geq 0, \qquad \lambda_{345} \geq -2\sqrt{\lambda_1 \lambda_2}.$

2. $\kappa_i = 0$:

$$\lambda_1 \geq 0, \qquad \lambda_2 \geq 0, \qquad \lambda_{345} \geq 6\sqrt{\lambda_1 \lambda_2},$$
$$\mathrm{Re}\lambda_6 \geq -2\sqrt[4]{\lambda_1^3 \lambda_2}, \qquad \mathrm{Re}\lambda_7 \geq -2\sqrt[4]{\lambda_1 \lambda_2^3}.$$

3. The general case

$$\kappa_1 \geq 0, \qquad \kappa_2 \geq 0, \qquad \mathrm{Re}\kappa_8 \geq -3\sqrt[6]{\kappa_1^5 \kappa_2}, \qquad \mathrm{Re}\kappa_{12} \geq -3\sqrt[6]{\kappa_1 \kappa_2^5}, \quad (48)$$
$$\kappa_4 + \kappa_6 + 2\mathrm{Re}\kappa_{10} \geq 15\sqrt[3]{\kappa_1 \kappa_2^2}, \qquad \kappa_3 + \kappa_5 + 2\mathrm{Re}\kappa_9 \geq 15\sqrt[3]{\kappa_1^2 \kappa_2},$$
$$\mathrm{Re}\kappa_7 + \mathrm{Re}\kappa_{11} + \mathrm{Re}\kappa_{13} \geq -10\sqrt{\kappa_1 \kappa_2}.$$

Summary

In the model-dependent case of the MSSM when the resummed effective potential is expanded up to dimension-six operators induced by the soft supersymmetry breaking terms, we

1. calculated symbolically corrections to the effective sextic couplings
2. and used them to determine the post-Higgs discovery mass spectrum of the heavy MSSM Higgs bosons.

An improved precision can be reached using such procedure, especially at the low EFT cut-off scale.

For moderately heavy supersymmetry ($M_S \sim 2$–3 TeV) additional corrections induced by higher-order terms in the expansion of the effective potential should be taken into account.

Thank you for your attention

$$
\begin{aligned}
\mathrm{Re}\mu_{12}^2 &= s_\beta c_\beta \left(m_A^2 + \frac{v^2}{2}(2\mathtt{Re}\lambda_5 + \mathtt{Re}\lambda_6 \cot\beta + \mathtt{Re}\lambda_7 \tan\beta) \right) \qquad (49) \\
&+ v^4 \{ \mathrm{Re}\kappa_9 c_\beta^3 s_\beta + \mathrm{Re}\kappa_{10} c_\beta s_\beta^3 + \frac{1}{4}[\mathrm{Re}\kappa_8 c_\beta^4 \\
&+ \mathrm{Re}\kappa_{12} s_\beta^4 + (9\mathrm{Re}\kappa_7 + \mathrm{Re}\kappa_{11} + \mathrm{Re}\kappa_{13}) s_\beta^2 c_\beta^2] \}
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{Im}\mu_{12}^2 &= \frac{v^2}{2}(s_\beta c_\beta \mathrm{Im}\lambda_5 + c_\beta^2 \mathrm{Im}\lambda_6 + s_\beta^2 \mathrm{Im}\lambda_7) \qquad (50) \\
&+ \frac{v^4}{4}\{ \mathrm{Im}\kappa_8 c_\beta^4 + 2\mathrm{Im}\kappa_9 c_\beta^3 s_\beta \\
&+ (3\mathrm{Im}\kappa_7 + \mathrm{Im}\kappa_{11} + \mathrm{Im}\kappa_{13}) c_\beta^2 s_\beta^2 + 2\mathrm{Im}\kappa_{10} c_\beta s_\beta^3 + \mathrm{Im}\kappa_{12} s_\beta^4 \}
\end{aligned}
$$

$$
\begin{aligned}
c_1 &= v^2(-1/2 \cdot \mathrm{Im}\lambda_5 c_{\alpha+\beta} + \mathrm{Im}\lambda_6 s_\alpha c_\beta - \mathrm{Im}\lambda_7 c_\alpha s_\beta) \qquad (51)\\
&+ \frac{v^4}{4}(-c_{\alpha+\beta} s_{2\beta}(3\mathrm{Im}\kappa_7 + \mathrm{Im}\kappa_{11} + \mathrm{Im}\kappa_{13}) + 4(s_\alpha c_\beta^3 \mathrm{Im}\kappa_8 - c_\alpha s_\beta^3 \mathrm{Im}\kappa_{12})\\
&+ 2(s_\beta^2(-3c_\alpha c_\beta + s_\alpha s_\beta)\mathrm{Im}\kappa_{10} - c_\beta^2(c_\alpha c_\beta - 3 s_\alpha s_\beta)\mathrm{Im}\kappa_9),\\
c_2 &= -\frac{v^2}{2}\{\mathrm{Im}\lambda_5 s_{\alpha+\beta} + 2(\mathrm{Im}\lambda_6 c_\beta c_\alpha + \mathrm{Im}\lambda_7 s_\beta s_\alpha) \qquad (52)\\
&+ v^2[2\mathrm{Im}\kappa_8 c_\beta^3 c_\alpha + \mathrm{Im}\kappa_9 c_\beta^2(s_{\alpha+\beta} + 2c_\alpha s_\beta) + \mathrm{Im}\kappa_{10} s_\beta^2(s_{\alpha+\beta} + 2c_\beta s_\alpha)\\
&+ 2\mathrm{Im}\kappa_{12} s_\beta^3 s_\alpha + \frac{1}{2}(3\mathrm{Im}\kappa_7 + \mathrm{Im}\kappa_{11} + \mathrm{Im}\kappa_{13}) s_{2\beta} s_{\alpha+\beta}]\}
\end{aligned}
$$

$$\begin{aligned}
\kappa_1 &= h_D^6 C_9^D - h_D^4 G_4 C_8^D + h_D^2 G_2 B_1^D + h_U^6 A_1 + h_U^4 G_4 A_2 - h_U^2 G_3 A_3 + G_1, && (53)\\
\kappa_2 &= h_D^6 A_1 + h_D^4 G_4 A_2 - h_D^2 G_2 A_3 + h_U^6 C_9^U - h_U^4 G_4 C_8^U + h_U^2 G_3 B_1^U - G_1, && (54)\\
\kappa_3 &= h_D^6 C_7^D + h_D^4 G_4 B_3^D - h_D^2 G_2 (2B_1^D + A_3) && (55)\\
&+ h_U^6 C_1^U - h_U^4 G_4 B_4^U |\mu|^2 + h_U^2 G_3 (B_1^U + 2A_3) - 3G_1, &&\\
\kappa_4 &= h_D^6 C_1^D - h_D^4 G_4 B_4^D |\mu|^2 + h_D^2 G_2 (B_1^D + 2A_3) && (56)\\
&+ h_U^6 C_7^U + h_U^4 G_4 B_3^U - h_U^2 G_3 (2B_1^U + A_3) + 3G_1, &&\\
\kappa_5 &= h_D^6 C_7^D + h_D^4 G_4 B_3^D - h_D^2 G_2 (2B_1^D + A_3) && (57)\\
&+ h_U^6 C_1^U - h_U^4 G_4 B_4^U |\mu|^2 + h_U^2 G_3 (B_1^U + 2A_3) - 3G_1, &&\\
\kappa_6 &= h_D^6 C_1^D - h_D^4 G_4 B_4^D |\mu|^2 + h_D^2 G_2 (B_1^D + 2A_3) && (58)\\
&+ h_U^6 C_7^U + h_U^4 G_4 B_3^U - h_U^2 G_3 (2B_1^U + A_3) + 3G_1, &&\\
\kappa_7 &= \frac{\mu^3}{320 M_S^8 \pi^2} (A_D^3 h_D^6 + A_U^3 h_U^6), && (59)\\
\kappa_8 &= h_D^6 C_6^D + 2h_D^4 G_4 C_4^D + h_D^2 G_2 A_7^D + h_U^6 A_2^U + h_U^4 G_4 A_5^U + h_U^2 G_3 A_7^U, && (60)\\
\kappa_9 &= h_D^6 C_2^D - h_D^4 G_4 A_6^D + h_U^6 A_4^U + h_U^4 G_4 A_6^U, && (61)\\
\kappa_{10} &= h_D^6 A_4^D + h_D^4 G_4 A_6^D + h_U^6 C_2^U - h_U^4 G_4 A_6^U, && (62)\\
\kappa_{11} &= h_D^6 C_3^D + h_D^4 G_4 C_5^D - 2h_D^2 G_2 A_7^D + h_U^6 C_3^U + h_U^4 G_4 C_5^U - 2h_U^2 G_3 A_7^U, && (63)\\
\kappa_{12} &= h_D^6 A_2^D + h_D^4 G_4 A_5^D + h_D^2 G_2 A_7^D + h_U^6 C_6^U + 2h_U^4 G_4 C_4^U + h_U^2 G_3 A_7^U, && (64)\\
\kappa_{13} &= h_D^6 C_3^D + h_D^4 G_4 C_5^D - 2h_D^2 G_2 A_7^D + h_U^6 C_3^U + h_U^4 G_4 C_5^U - 2h_U^2 G_3 A_7^U, && (65)
\end{aligned}$$

$$G_1 = \frac{1}{M_S^2}\frac{g_1^2(g_1^4-g_2^4)}{1024\pi^2}, \qquad G_2 = \frac{5g_1^4+6g_1^2g_2^2+9g_2^4}{3072\pi^2}, \tag{66}$$

$$G_3 = \frac{17g_1^4-6g_1^2g_2^2+9g_2^4}{3072\pi^2}, \qquad G_4 = \frac{g_1^2+g_2^2}{256\pi^2},$$

$$A_1 = -\frac{|\mu|^6}{320M_S^8\pi^2}, \qquad A_2 = \frac{|\mu|^4}{M_S^6}, \qquad A_3 = \frac{|\mu|^2}{M_S^4}, \qquad A_2^X = \frac{3A_X\mu|\mu|^4}{320M_S^8\pi^2}, \tag{67}$$

$$A_4^X = -\frac{3A_X^2\mu^2|\mu|^2}{320M_S^8\pi^2}, \qquad A_5^X = -\frac{2A_X\mu|\mu|^2}{M_S^6}, \qquad A_6^X = \frac{A_X^2\mu^2}{M_S^6}, \qquad A_7^X = \frac{\mu A_X}{M_S^4},$$

$$B_1^X = -\frac{|A_X|^2}{M_S^4} + \frac{2}{M_S^2}, \qquad B_2^X = -\frac{4|A_X|^2}{M_S^6} + \frac{6}{M_S^4}, \tag{68}$$

$$B_3^X = C_8^X + |\mu|^2 B_2^X, \qquad B_4^X = \frac{|\mu|^2}{M_S^6} + B_2^X,$$

$$C_1^X = \frac{|\mu|^4}{320\pi^2}\left(-\frac{9|A_X|^2}{M_S^8} + \frac{10}{M_S^6}\right), \qquad C_2^X = \frac{A_X^2\mu^2}{320\pi^2}\left(-\frac{3|A_X|^2}{M_S^8} + \frac{10}{M_S^6}\right), \tag{69}$$

$$C_3^X = \frac{A_X\mu|\mu|^2}{320\pi^2}\left(\frac{9|A_X|^2}{M_S^8} - \frac{20}{M_S^6}\right), \qquad C_4^X = A_X\mu\left(\frac{|A_X|^2}{M_S^6} - \frac{3}{M_S^4}\right),$$

$$C_5^X = -2A_X\mu\left(\frac{|A_X|^2-|\mu|^2}{M_S^6} - \frac{3}{M_S^4}\right), \qquad C_6^X = \frac{A_X\mu}{320\pi^2}\left(\frac{3|A_X|^4}{M_S^8} - \frac{20|A_X|^2}{M_S^6} + \frac{30}{M_S^4}\right),$$

$$C_7^X = -\frac{|\mu|^2}{320\pi^2}\left(\frac{9|A_X|^4}{M_S^8} - \frac{40|A_X|^2}{M_S^6} + \frac{30}{M_S^4}\right), \qquad C_8^X = \frac{|A_X|^4}{M_S^6} - \frac{6|A_X|^2}{M_S^4} + \frac{6}{M_S^2},$$

$$C_9^X = -\frac{1}{320\pi^2}\left(\frac{|A_X|^6}{M_S^8} - \frac{10|A_X^4|}{M_S^6} + \frac{30|A_X|^2}{M_S^4} - \frac{20}{M_S^2}\right).$$