Supersymmetric models with broken Lorentz invariance.

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The XXIII International Workshop "High Energy Physics and Quantum Field Theory", 2017

Motivation

Motivation for the Lorentz breaking:

- Extended models in theoretical physics
- New approaches to quantization of gravity (Horava, 2009; Blas, Pujolas, Sibiryakov, 2010)
- Phenomenology (Rubakov, 2006; Blas, Sibiryakov, 2011)

Gravity with broken Lorentz invariance

Action for the Einstein-aether gravity (Jacobson, Mattingly, 2001):

$$S = S_{GR} + S_{æ},$$

Action for the aether:

$$S_{æ} = -\frac{1}{2}\int d^4x\sqrt{-g}\{c_1(\nabla_n u_m)^2 + c_2(\nabla_m u_m)^2 + \\ + c_3\nabla_n u_m \nabla^m u^n - c_4 u^r u^s \nabla_r u_m \nabla_s u^m\}$$

Aether norm:

$$u_m u^m = -1$$

Constraints on the theory parameters

PPN-parameters:

$$\alpha_1 = -8\frac{{c_3}^2 + c_1 c_4}{2c_1 - {c_1}^2 + {c_3}^2}$$

$$\alpha_2 = \frac{\alpha_1}{2} - \frac{(c_1 + 2c_3 - c_4)(2c_1 + 3c_2 + c_3 + c_4)}{(c_1 + c_2 + c_3)(2 - c_1 - c_4)}$$

Experimental constraints:

$$|\alpha_1| \lesssim 10^{-4}$$

$$|\alpha_2| \lesssim 4 \times 10^{-7}$$

From gravity to supergravity

- Ordinary gravity is compatible with Lorentz breaking, the formulation is parametrized by four constants.
- What about supergravity?
- The first step: supersymmetric models with Lorentz breaking.

Supersymmetry and Lorentz breaking

$$\Phi^i = \left(A^i, \Psi^i_\alpha\right)$$

Commutators of SUSY transformations on-shell

$$[\delta_\xi \delta_\eta]\Phi^i = 2i\left(\eta\sigma_m\bar{\xi} - \xi\sigma_m\bar{\eta}\right)\partial_m\Phi^i =>$$

$$\delta_\xi A^i = \xi\Psi^i$$

$$\delta_\xi \Psi^i_\alpha = 2i\left(\sigma_m\bar{\xi}\right)_\alpha \partial_m A^i + C_1^{ij} i u_m \xi_\alpha \partial_m \bar{A}_j + C_2^{ij} i u_m \left(\sigma_{mn}\xi\right)_\alpha \partial_n \bar{A}_j$$

Supersymmetry and Lorentz breaking

From the Lagrangian, invariant under transformations,

$$\delta_\xi L = 0 =>$$

$$C_1^{ij} = -C_1^{ji}$$

$$C_2^{ij} = 0$$

$$L = i\Psi^i \sigma_m \partial_m \bar{\Psi}^i + \bar{A}^i \Box A^i -$$
$$- \frac{i}{4} C_1^{ij} u_m \left(\Psi^i \partial_m \Psi^j + \text{h. c.} \right) - \frac{1}{2} \left(C_1^2 \right)^{ij} u_m u_n \partial_m A^i \partial_n \bar{A}^j$$

Supersymmetry and Lorentz breaking

- There is no non-trivial Lorentz-violating supersymmetric models for the vector supermultiplet.
- The existence of the models for the scalar supermultiplet depends on the number of scalars and spinors
- These results agree with the superfield formalism (Bolokhov, Groot Nibbelink, Pospelov, 2005).
- Do non-trivial theories for the gravitational multiplet exist?

Supersymmetry and broken Lorentz invariance

Supersymmetric aether (Pujolas, Sibiryakov, 2011):

$$U^m = u^m(x_L) + \sqrt{2}\theta\eta^m(x_L) + \theta^2 G^m(x_L)$$

Super-aether norm:

$$U_m U^m = -1$$

Super-aether action:

$$S = \int d^8 z f\left(U_m \bar{U}^m\right) + \int d^6 z \Lambda\left(U_m U^m + 1\right)$$

leads to the constraints:

$$c_2 + c_3 = 0,$$

$$c_4 = 0.$$

Non-minimal supergravity (e. g., Buchbinder, Kuzenko, 1998):

$$S_{SG} = \frac{1}{\varkappa^2}\int d^8z\Big[\frac{1}{4}\Big((\partial_k H_m)^2 - (\Delta_k H_m)^2\Big) + \frac{n+1}{2n}(\partial_m H^m)^2 + \\ + \frac{n+1}{2}(\Delta_m H^m)^2 - i\frac{3n+1}{2n}\partial_m H^m\left(\Gamma - \bar{\Gamma}\right) + \\ + \frac{3n+1}{2}\Delta_m H^m\left(\Gamma + \bar{\Gamma}\right) + \frac{9n^2-1}{8n}(\Gamma^2 + \bar{\Gamma}^2) + \frac{(3n+1)^2}{4n}\Gamma\bar{\Gamma}\Big].$$

Super-gauge transformations:

$$\delta H_{\alpha\dot{\alpha}} = \bar{D}_{\dot{\alpha}} L_{\alpha} - D_{\alpha}\bar{L}_{\dot{\alpha}}$$

$$\delta\Gamma = -\frac{n+1}{4(3n+1)}\bar{D}^2 D^{\alpha} L_{\alpha} + \frac{1}{4}\bar{D}^{\dot{\alpha}} D^2 \bar{L}_{\dot{\alpha}}$$

Super-aether transformations

$$U^a = w^a + V^a$$

Super-gauge transformations:

$$\delta V^a = w^b M_b{}^a$$

$$M_{ab} = \frac{1}{4}(\sigma_{ab})_\beta{}^\alpha D_\alpha \bar{D}^2 L^\beta + \frac{1}{4}(\bar{\sigma}_{ab})^{\dot{\alpha}}{}_{\dot{\beta}} \bar{D}^{\dot{\beta}} D^2 \bar{L}_{\dot{\alpha}}$$

Chirality constraint:

$$\bar{D}_{\dot{\alpha}} V^c = -w^b \Phi_{\dot{\alpha} b}{}^c \ .$$

Superconnection:

$$\Phi_{\dot{\alpha} bc} = -\frac{1}{4}(\sigma_{bc})_\alpha{}^\beta \bar{D}^2 D^\alpha H_{\beta\dot{\alpha}} - (\bar{\sigma}_{bc})^{\dot{\beta}}{}_{\dot{\alpha}} \bar{D}_{\dot{\beta}} \Gamma$$

Supergravity with broken Lorentz invariance

Lorentz-violating supergravity:

$$S = S_{SG} + \frac{C}{2\kappa^2}\int d^8z \Big[V_a \bar{V}^a + i w^a w^b \partial_a H_b (\Gamma - \bar{\Gamma}) + w^a w^b \Delta_a H_b (\Gamma + \bar{\Gamma}) +$$
$$+ \frac{1}{4}\left(\Delta_k H_m \Delta^k H^m - \partial_k H_m \partial^k H^m - (\Delta_m H^m)^2 + (\partial_m H^m)^2\right) +$$
$$+ \frac{i}{4}\partial_m H^m (\Gamma - \bar{\Gamma}) + \frac{1}{4}\Delta_m H^m (\Gamma + \bar{\Gamma}) + \frac{3}{8}(\Gamma^2 + \bar{\Gamma}^2) \Big] ,$$

(Marakulin, Sibiryakov, 2016)

Super-aether expansion:

$$V_b|_{\text{bos}} = v_b(x_L) + \theta^2 G_b(x_L) + \theta\sigma^k\bar{\theta} f_{bk}(x_L).$$

Bosonic part of the theory:

$$L = \frac{1}{2\varkappa^2}\Bigg\{\frac{1}{4}h_{km}\Box h^{km} + \frac{1}{2}\partial^k h_{km}\partial_l h^{lm} - \frac{1}{2}\partial_k h^{km}\partial_m h +$$
$$+\frac{1}{4}\partial_m h\partial^m h - \partial_m \hat{v}^R_a \partial^m \hat{v}^{R,a} - \partial_m \hat{v}^I_a \partial^m \hat{v}^{I,a} + \sqrt{C}\,\hat{v}^{R,a} w^b\left(\partial_b\partial^k h_{ka} - \partial_a\partial^k h_{kb}\right) -$$
$$-\frac{C}{4}w^a w^b\left(\partial_a h_{mn} - \partial_m h_{na}\right)\left(\partial_b h^{mn} - \partial^m h^n{}_b\right) - \frac{C}{2}w^a w^b \partial_a \hat{v}^{I,m}\partial_b \hat{v}^I_m +$$
$$+\frac{C}{2}(\partial_a \hat{v}^{I,a})^2 - C w^b w^c \epsilon_{bkam}\partial^k \hat{v}^{R,a}\partial_c \hat{v}^{I,m} + O(C^{3/2})\Bigg\}.$$

(Marakulin, Sibiryakov, 2016)

Results and outlook

- Supersymmetric models with broken Lorentz invariance are constructed and classified at the component level
- Lorentz violating linearized supergravity lagrangian is constructed in terms of superfields
- Bosonic part of this theory is discussed
- The theory parameters are:

$$c_1 = C; \; c_2 = c_3 = c_4 = 0.$$

- Dispersion relations for the small perturbations are obtained.
- Dispersion relation of the gravitational waves is

$$E^2 = (1 + C)\, p^2$$

- The next step: non-linear extension (Grim, Müller, Wess, 1984)