Non-perturbative Effects of the Electro-Weak Interaction at the LHC

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Introduction

The totality of data nowadays confirms main features of the Standard Model, which consists of QCD, describing strong interactions, and the EW theory, describing electroweak interactions. This confirmation is essentially based on numerous perturbative calculations which describe corresponding data. However in QCD the inevitable introduction of non-perturbative effects is also evident. First of all the low momenta region of the strong interaction definitely can not be described in the framework of the perturbative calculations.

Examples of non-perturbative quantities are well-known: vacuum averages the gluon condensate $< \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^a_{\mu\nu} >$, the quark condensate $< \bar{q}\, q >$ etc. One of the most powerful methods of dealing with the non-perturbative effects is provided in the framework of approaches using the so-called effective interactions. The eldest and the most popular such effective interaction is the famous Nambu–Jona-Lasinio interaction Y. Nambu and G. Jona-Lasinio, Phys. Rev., v. 122 p. 345 (1961); ibid v. 124 p. 246 (1961). With application to quark structure of hadrons this approach adequately describes the low momenta region.

The Nambu–Jona-Lasinio interaction deals with four-quark effective terms. However, the non-zero gluon condensate testifies for additional effective terms also in gluon interactions.

There were also proposals for such terms. In particular, the triple gluon interaction in the low momenta region of the following form

$$L_{eff} = -\frac{G}{3!} f_{abc} G^a_{\mu\nu} G^b_{\nu\rho} G^c_{\rho\mu}; \qquad (1)$$

where $G^a_{\mu\nu}$ is a gauge covariant gluon field and f_{abc} are structure constants of the color $SU(3)$
A. I. Alekseev, B. A. Arbuzov and V. A. Baikov, Theor. Math. Phys., 52 (1982), p. 739.

In the electro-weak interaction necessity of non-perturbative contributions nowadays is not so evident as in QCD. However the structure of gauge theories is the same for both cases.

One might expect similar features in three-boson interactions. In particular, the following three weak boson interaction was introduced
K. Hagiwara, R. D. Peccei, D. Zeppenfeld and K. Hikasa, Nucl. Phys. B, 282 (1987), p. 253.

$$L_{eff} = -\frac{G_W}{3!}\epsilon_{abc}W^a_{\mu\nu}W^b_{\nu\rho}W^c_{\rho\mu};\ G_W = \frac{g\,\lambda}{M_W^2};$$
$$W^3_{\mu\nu} = \cos\theta_W\,Z_{\mu\nu} + \sin\theta_W\,A_{\mu\nu}\,; \qquad (2)$$

where g is the electro-weak gauge coupling.

Interaction (2) would lead to effects *e.g.* ***in pair electro-weak bosons production and was studied in experiments. The best limitation for parameter*** λ ***is provided by recent data***
CMS Collaboration, arXiv: 1703.06095 (hep-ex)

$$-0.011 < \lambda < 0.011. \qquad (3)$$

Both the Nambu–Jona-Lasinio interaction and interaction (1) are supposed to act in low momenta region. This means, that in both cases form-factors are present, which guarantee decreasing of intensity of the interactions for large momenta. In the original NJL interaction a cut-off was introduced for the purpose.

Beginning with fundamental gauge theories of interactions of the Standard Model we have to understand the origin of such effective cut-off. This can be done under assumption of the effective interactions being spontaneously generated. The notion of a spontaneous generation is traced back to methods of the superconductivity theory. In application to superconductivity the conception of compensation principle was elaborated by N.N. Bogoliubov.

N.N. Bogoliubov, ZhETF, v. 34 pp. 58,66,73 (1958); N.N. Bogoliubov, Soviet Phys.-Uspekhi, v. 67 p. 236 (1959); N. N. Bogoliubov, Physica Suppl. (Amsterdam), v. 26, p. 1 (1960).

In accordance to the Bogoliubov approach in application to QFT we look for a non-trivial solution of a compensation equation, which is formulated on the basis of the Bogoliubov procedure add – subtract. Let us show application of the method for spontaneous generation of interaction (2). Namely let us write down the initial Lagrangian in the form

$$L = L_0 + L_{int}\,;$$

$$L_0 = -\frac{1}{4}W^a_{\mu\nu}W^a_{\mu\nu} + \frac{G_W}{3!}F\cdot\epsilon_{abc}W^a_{\mu\nu}W^b_{\nu\rho}W^c_{\rho\mu}; \quad (4)$$

$$L_{int} = -\frac{G_W}{3!}F\cdot\epsilon_{abc}\,W^a_{\mu\nu}\,W^b_{\nu\rho}\,W^c_{\rho\mu}\,. \quad (5)$$

It is impotant, that in the anomalous interaction form-factor $F(p_i)$ is present.

Let us consider expression (4) as the new free *Lagrangian* L_0*, whereas expression (5) as the new* interaction *Lagrangian* L_{int}*. Then compensation conditions will consist in demand of full connected three-boson vertices of the structure (5) in the theory with Lagrangian* L_0 *to be zero. This demand gives an equation for form-factor* F*.*

Such equations according to terminology by N.N Bogoliubov are called compensation equations*. In a study of these equations it is always evident the existence of a perturbative trivial solution (in our case* $G = 0, F = 0$*), but, in general, a non-trivial non-perturbative solution may also exist.*

Most impressive effectiveness of the method was demonstrated in the light meson physics, where the spontaneously generated Nambu - Iona-Lazinio interaction was demonstrated. Application of the method leads to calculation of main light mesons' properties with good precision using only fundamental QCD parameters.

B. A. Arbuzov, M. K. Volkov and I. V. Zaitsev, Int. J. Mod.Phys. A, v. 21 p. 5721 (2006).

Application to the spontaneous generation of the wouldbe anomalous three-boson interaction to be discussed below. *B. A. Arbuzov, Eur. Phys. J., v. C61 p. 51 (2009), B. A. Arbuzov and I. V. Zaitsev, Phys. Rev., v. D85 : 093001 (2012).*

The analogous approach was applied to low energy gluon interaction: *B. A. Arbuzov and I.V. Zaitsev, Int. J. Mod. Phys., v. A28 : 1350127 (2013).*

For general review see book: *B. A. Arbuzov,* Non-perturbative Effective Interactions in the Standard Model, *De Gruyter, Berlin, 2014.*

Let us apply the method to wouldbe anomalous interactions of the Higgs.

Additional interactions of the Higgs

with electro-weak bosons

Three-boson vertex (2), indices a, b, c, Lorentz indices μ, ν, ρ, momenta p, q, k:

$$V(\mu, p; \nu, q; \rho, k) = -g\,\epsilon_{abc}\Big(g_{\mu\nu}(q_\rho - p_\rho) + g_{\nu\rho}(k_\mu - q_\mu) + g_{\rho\mu}(p_\nu - k_\nu) + \frac{\lambda}{M_W^2}F(p,q,k)(g_{\mu\nu}(q_\rho pk - p_\rho qk) + g_{\nu\rho}(k_\mu pq - q_\mu pk) + g_{\rho\mu}(p_\nu qk - k_\nu pq) + q_\mu k_\nu p_\rho - k_\mu p_\nu q_\rho)\Big). \tag{6}$$

Here $F(p,q,k)$ is a form-factor, which is defined in the framework of the spontaneous generation of effective interaction (2) in the compensation approach, which we have discussed in the Introduction.

Vertices for SM interaction of Higgs H with W^+W^- and $Z\,Z$:

$$\imath\, g M_W\, g_{\mu\nu}; \quad \imath\,\frac{g M_Z}{\cos\theta_W}\, g_{\mu\nu}. \tag{7}$$

We have additional contribution to $V V H$ vertex due to terms proportional to λ and to λ^2 according to diagrams presented in Fig. 1. We have additional contribution to vertices $V_{VV'H}$, where V and V' corresponds to electro-weak bosons W, Z, γ.

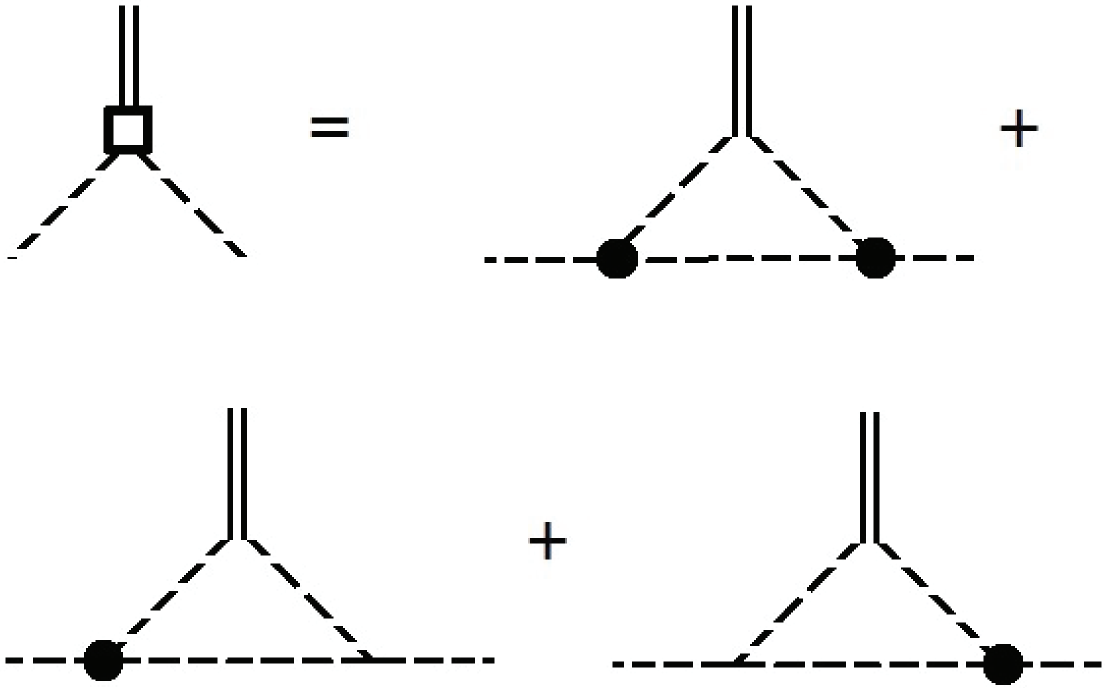

Figure 1: Diagrams for non-perturbative contributions to vertex VVH. Double lines - Higgs bosons, dotted lines - electro-weak bosons. Black spots - vertex (2, 6), simple points - SM interactions.

$$
\begin{aligned}
&V_{W^+W^-H} = \imath g M_W \left(g_{\mu\nu} + G_{WW}(g_{\mu\nu}pq - p_\nu q_\mu)\right);\\
&V_{ZZH} = \imath\,\frac{gM_Z}{\cos\theta_W}\left(g_{\mu\nu} + G_{ZZ}\left(g_{\mu\nu}pq - p_\nu q_\mu\right)\right);\\
&V_{ZAH} = \imath\,\frac{gM_Z}{\cos\theta_W}\,G_{Z\gamma}\left(g_{\mu\nu}pq - p_\nu q_\mu\right);\\
&V_{AAH} = \imath\,\frac{gM_Z}{\cos\theta_W}\,G_{\gamma\gamma}\left(g_{\mu\nu}pq - p_\nu q_\mu\right);\\
&G_{ZZ} = \cos^2\theta_W\,G_{W_0W_0};\;\; G_{\gamma\gamma} = \sin^2\theta_W\,G_{W_0W_0};\\
&G_{Z\gamma} = 2\cos\theta_W\sin\theta_W\,G_{W_0W_0};\\
&G_{W_0W_0} = \frac{2g}{M_W^2}\Big(\frac{3g\lambda}{8\pi^2}I_1 - \frac{\sqrt{2\lambda^2}}{\pi}I_2\Big);\\
&G_{WW} = \frac{g}{M_W^2}\Big(\frac{3g\lambda}{8\pi^2}I_{W1} - \frac{\sqrt{2\lambda^2}}{\pi}I_{W2} +\\
&\frac{3g\lambda}{8\pi^2}I_{Z1} - \frac{\sqrt{2\lambda^2}}{\pi}I_{Z2}\Big).
\end{aligned}
\tag{8}
$$

$$
\begin{aligned}
I_1 &= \int_0^{t_0} \frac{F(t)\,dt}{2(\sqrt{t}+\mu)^2};\; I_2 = \int_0^{t_0} \frac{2tF^2(t)\,dt}{(\sqrt{t}+\mu)^3};\\
I_{W1} &= \int_0^{t_0} \frac{F(t)(\sqrt{t}+s\mu_Z)\,dt}{(\sqrt{t}+\mu_Z)^2(\sqrt{t}+\mu)};\\
I_{W2} &= \int_0^{t_0} \frac{F^2(t)\sqrt{t}\,(\sqrt{t}+s\mu_Z)\,dt}{(\sqrt{t}+\mu_Z)^2(\sqrt{t}+\mu)};\\
I_{Z1} &= \int_0^{t_0} \frac{F(t)\sqrt{t}\,dt}{(\sqrt{t}+\mu_Z)^2(\sqrt{t}+\mu)(1-s)};\\
I_{Z2} &= \int_0^{t_0} \frac{F^2(t)\,t\,dt}{(\sqrt{t}+\mu_Z)^2(\sqrt{t}+\mu)(1-s)};\\
\mu &= \frac{g|\lambda|}{16\sqrt{2}\pi};\quad \mu_Z = \frac{g|\lambda|}{16\sqrt{2}\pi\,(1-s)};\\
t &= \frac{G_W^2(p^2)^2}{512\pi^2};\; s = \sin^2\theta_W,\; t_0 = 9.6175.
\end{aligned}
\tag{9}
$$

$F(t) = F(p,-p,0)$ **B. A. A, I. V. Z, Phys. Rev., v. D85 : 093001 (2012) :**

$$F(t) = \frac{1}{2} G^{31}_{15}\left(t\,\middle|^{0}_{1,1/2,0,-1/2,-1}\right) - \frac{85\sqrt{2}g_0}{128\pi} \times$$
$$G^{31}_{15}\left(t\,\middle|^{1/2}_{1,1/2,1/2,-1/2,-1}\right) + C_1 G^{10}_{04}\left(t\,\middle|\frac{1}{2}, 1, -\frac{1}{2}, -1\right)$$
$$+ C_2 G^{10}_{04}\left(t\,\middle|1, \frac{1}{2}, -\frac{1}{2}, -1\right);\; t = \frac{G_W^2\,(p^2)^2}{512\,\pi^2}; \quad (10)$$
$$F(t) = 0,\, t \geq t_0; \quad t_0 = 9.6175,$$
$$g_0 = 0.6037,\, C_1 = -0.0351,\, C_2 = -0.0511.$$

Here $g_0 = g(t_0)$, $G^{mn}_{pq}\left(t\,\middle|\,^{a_1,\ldots a_p}_{b_1\ldots,b_q}\right)$ **- a Meijer function.**

Results for couplings in Table 1.

Table 1. Coupling constants $G_{VV'}$ GeV^{-1} of effective interactions HVV' in dependence on value of λ. All coupling values are multiplied by 10^7.

λ	G_{WW}	G_{ZZ}	$G_{Z\gamma}$	$G_{\gamma\gamma}$
0.01	3.1	4.40	4.71	1.26
0.006	2.20	3.10	3.32	0.89
0.003	1.33	1.86	2.00	0.54
0	0	0	0	0
−0.003	−3.51	−4.83	−5.17	−2.58
−0.006	−6.54	−9.01	−9.65	−2.58
−0.01	−10.3	−14.2	−15.2	−4.08

Then we calculate cross sections of pair weak boson production accompanied by the Higgs. In doing this we apply the CompHEP package and come to results for cross sections of processes

$p + p \longrightarrow W^+W^-H + X$, ($\sigma(+-)$)

$p + p \longrightarrow W^+Z\,H + X$, ($\sigma(+0)$)

$p + p \longrightarrow W^-Z\,H + X$ ($\sigma(-0)$)

in dependence on value of λ in admissible interval (3), that is for values

$|\lambda| = 0, 0.003, 0.006, 0.01.$

Table 2. Production LO cross sections of $VV'H$ for $\sqrt{s} = 8\,TeV$ at the LHC.

λ	$\sigma(+-)\,fb$	$\sigma(+0)\,fb$	$\sigma(-0)\,fb$
-0.01	5.18	1.21	0.50
-0.006	4.62	1.09	0.45
-0.003	4.22	1.02	0.43
0	3.86	0.98	0.42
0.003	3.75	0.98	0.42
0.006	3.69	0.98	0.42
0.01	3.62	0.98	0.42

Table 3. Production LO cross sections of $VV'H$ for $\sqrt{s} = 13\,TeV$ at the LHC.

λ	$\sigma(+-)\,fb$	$\sigma(+0)\,fb$	$\sigma(-0)\,fb$
−0.01	21.16	3.60	1.47
−0.006	17.06	2.71	1.20
−0.003	14.36	2.27	1.07
0	11.90	2.08	1.00
0.003	11.14	2.09	1.00
0.006	10.70	2.12	1.02
0.01	10.35	2.19	1.03

From Table 3 we see, that for $\sqrt{s} = 13\,TeV$ with negative λ the effect is noticeable, especially for process $p + p \to W^+W^-H + X$, and e.g for $\lambda = -0.01$ the cross-section is almost two times more than the SM one. However the cross-section itself presumably is not sufficiently high.

Let us note also, that $VV'H$ additional interaction with $\lambda \neq 0$ might give effect for VBF Higgs production. However calculations show, that with couplings from Table 1 effects even for $\sqrt{s} = 13\,TeV$ are insignificant, as well as effects for branching ratios of the Higgs decays.

Possible manifestations of vertices (8) were studied in decays $H \to W^+W^-$, $H \to Z\,Z$

V. Khachatryan et al. (CMS Collaboration) Phys. Rev. D, 92 (2015) Article 012004.

Results of this work give limitations, which definitely do not contradict values for couplings presented in Table 1.

In the next section we consider additional contributions of interactions (8) to interaction of the Higgs with quarks, especially with the heavy ones, which can lead to essentially more significant effects at the LHC.

Additional t and b quarks interaction with the Higgs

We use vertices (8)

$$V_{W^+W^-H},\; V_{ZZH},\; V_{ZAH},\; V_{AAH}$$

to define additional contributions for interactions of quarks with the Higgs. For the beginning we shall be interested in interactions of the most heavy top quarks.
We consider following diagrams.

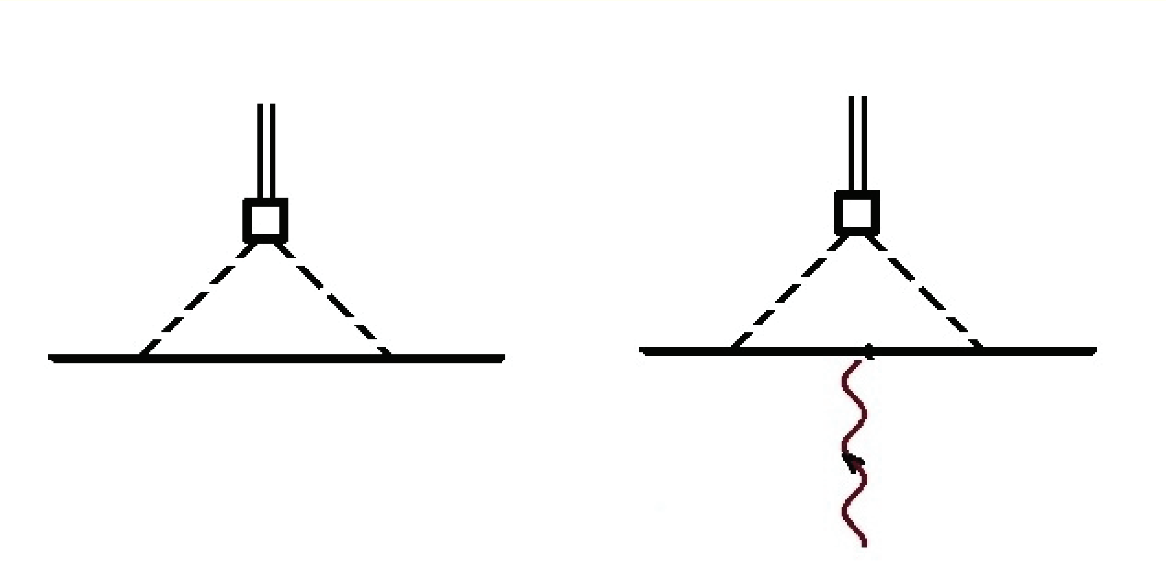

Figure 2: The diagram representation of non-perturbative contributions to $\bar{t}tH$ and $\bar{t}tHG$ vertices. Double lines - Higgs bosons, dotted lines - electro-weak bosons. Thick lines - the t quark, the wavy line - a gluon.

Taking into account these vertices we calculate loop diagrams presented in Fig. 2 to obtain the following expression for $\bar{t}tH$ vertex, which corresponds to the first diagram in Fig.2

$$V_{\bar{t}tH} = -\frac{g}{2M_W}\,\bar{t}\left(M_t + 9\cos\theta_W M_Z M_W \times G_{WW} I_1\,(\hat{p}_1 - \hat{p}_2)(1+\gamma_5)\right) tH; \quad (11)$$
$$\hat{a} = a_\mu \gamma^\mu;$$

where G_{WW} is already defined in (8) and calculated in Table 1, p_1 and p_2 are respectively outcoming momenta of t and $\bar{t}$ quarks. Integral I_1 is defined in (9). For calculation of the integral we here use the same form-factor $F(t)$ (10).

Due to QCD gauge invariance we have to take into account also vertex for fourfold interaction involving also a gluon: $\bar{t}tHG_\mu$, which actually corresponds to the second diagram in Fig.2

$$V_{\bar{t}tHG} = 9gg_s\cos\theta_W M_Z G_{WW} I_1\, \bar{t}\hat{G}(1+\gamma_5)tH; \quad (12)$$

where g_s is the QCD gauge coupling constant and of course usual structure of the QCD is used. Then we perform calculations for cross sections of process $p+p\longrightarrow \bar{t}\,t\,H+X$ for two energies of the LHC: $\sqrt{s}=8\,TeV$ and $\sqrt{s}=13\,TeV$

Let us define for the same values of $\sqrt{s}$ ratios of cross-sections with nonzero λ in admissible interval (3) and its SM value for $\lambda = 0$

$$\mu_{\sqrt{s}} = \frac{\sigma_\lambda(pp \to \bar{t}tH)}{\sigma_0(pp \to \bar{t}tH)}; \quad (13)$$

where σ_0 is actually the SM value for the cross section. Results of calculations with application of CompHEP package are shown in Table 4. We use current value for the strong coupling

$$\alpha_s(M_Z) = 0.1181 \pm 0.0011. \quad (14)$$

Table 4. Production LO cross sections fb of $\bar{t}tH$ for $\sqrt{s} = 8\,TeV$, $13\,TeV$ and ratio $\mu_{\sqrt{s}}$ (13) at the LHC.

λ	$\sigma(13\,TeV)$	$\sigma(8\,TeV)$	μ_{13}	μ_8
-0.01	1628.2	460.6	3.14	3.15
-0.006	1212.9	342.6	2.34	2.34
-0.003	853.3	241.0	1.65	1.65
0	517.8	146.1	1.00	1.00
0.003	401.4	113.6	0.78	0.78
0.006	348.7	98.4	0.67	0.67
0.01	304.4	85.9	0.59	0.59

The uncertainty in (14) means 2% accuracy for calculated cross-sections. With taking into account of other sources of uncertainties we estimate overall accuracy to be around 10%.

The combination of the ATLAS and the CMS data, collected with $\sqrt{s} = 7\,TeV$ and $8\,TeV$,
G. Aad et al. (ATLAS and CMS Collaborations) JHEP 1608 (2016) 045. $\longrightarrow \mu_8$

$$\mu_8 \;=\; 2.3^{+0.7}_{-0.6}\,. \tag{15}$$

With numbers from Table 4 we have from (15)

$$\lambda \;=\; -\,0.0057^{+0.0028}_{-0.0039}\,. \tag{16}$$

The result is safely inside limitation (3).

Of course we have here again only two standard deviations effect, which undoubtedly needs further studies. We see that values of μ, practically, do not depend on $\sqrt{s}$ but with $\sqrt{s} = 13\,TeV$ cross sections are more than three times as much as those for conditions of result (15). One might hope to check the predictions in forthcoming experimental studies at the LHC with increased statistics. Emphasize, that in case of this study would give result $\lambda \neq 0$, we would come to the fundamental conclusion of non-perturbative effects in the electro-weak interaction to be necessarily present.

Let us consider also associated production of the Higgs with other $\bar{q}\, q$ pairs. Indeed, interactions (11,12) in our consideration also exist for other quarks. All the difference is connected with a value of the quark mass in (11). In particular, it is advisable to consider also process of b quark associated pair production with Higgs

$$p + p \longrightarrow \bar{b} b H \, + \, X. \qquad (17)$$

Results of calculations are shown in Table 5 with the same level of an accuracy as in Table 4.

Table 5. Production LO cross sections fb of $\bar{b}bH$ for $\sqrt{s} = 8\,TeV$, $13\,TeV$ and ratio $\mu_{b\sqrt{s}}$ at the LHC.

λ	$\sigma(13\,TeV)$	$\sigma(8\,TeV)$	μ_{b13}	μ_{b8}
−0.01	1612.5	572.7	2.93	2.77
−0.006	1204.4	433.6	2.19	2.10
−0.003	903.0	329.1	1.64	1.59
0	550.8	206.7	1.00	1.00
0.003	446.0	169.0	0.81	0.82
0.006	396.4	151.8	0.72	0.73
0.01	354.6	137.1	0.64	0.66

As a matter of fact, results of Table 5 are are also valid for light quarks u, d, c, s as well.

Conclusion

The problem of an existence of non-perturbative contributions in the electro-weak interaction is without doubt a fundamental one. Anomalous three-boson interaction (2) provides the crucial test for this problem. We have shown above, that there are promising processes

$$p + p \longrightarrow \bar{t}tH + X, \quad p + p \longrightarrow \bar{b}bH + X; \qquad (18)$$

for investigation of the problem at the LHC, and we can hope, that future results for these processes at $\sqrt{s} = 13\,TeV$ *will confirm the existence of non-perturbative effects in the electro-weak interaction.*

The important problem is, if predictions of the present work could in any way contradict the present knowledge. We have already mentioned, that contributions of the additional interactions to branching ratios of the Higgs are negligible. The effects in process $p+p\to\bar{q}qH+X$, where we have to take into account all six flavors of quarks would lead to an additional contribution to the total Higgs production cross section. For example, for $\lambda=-0.006$, which actually is quite close to the central value in estimate (16), we have from Tables 4, 5

$$\Delta\,\sigma(8TeV)=1.36\,pb;\;\sigma_{SM}(8TeV)=22.3\,pb;$$
$$\Delta\,\sigma(13TeV)=4.03\,pb;\;\sigma_{SM}(13TeV)=50.6\,pb.\quad(19)$$

These additional contributions lead to a change in the global signal strength, which currently reads

$$\mu = 1.09 \pm 0.07 \pm 0.04 \pm 0.03 \pm 0.07. \quad (20)$$

We easily see, that additional contributions (19) give the following changes for theoretical predictions for effective μ instead of the unity

$$\mu\,(8\,TeV) = 1.061\,; \quad \mu\,(13\,TeV) = 1.080. \quad (21)$$

The results evidently do not contradict (20), which is based mostly on data collected with $\sqrt{s} = 7$ and $8\,TeV$. The results being discussed see also in B. A. Arbuzov and I. V. Zaitsev, arXiv: 1704.05293 (2017) (hep-ph), Phys.Lett. B to be published.

In case of an existence of triple interaction (2),** e.g. **in processes (18), being confirmed, extensive studies of other possible non-perturbative effects will be desirable.

For example, effects in the top pair production in an association with an electro-weak boson** $W^{\pm}, Z$ **were discussed in work: B. A. A. and I. V. Z., Prog. Theor. Exp. Phys. 85 (2016), 093001.

Process of** $\bar{t}tH$ **production without doubt will be studied at the LHC. What if the result would give significant effect,** e.g. **value** $\mu_{13} \simeq 2.3$ **would be established with a sufficient reliability? Of course, the deviation from the SM would need explanation.

From the point of view of the present talk this wouldbe effect is to be connected with non-perturbative effects, and first of all it is defined by coupling λ of the three-boson anomalous interaction. We have shown above (see Table 4), that just value (15)

$$\mu_{13} = 2.3; \tag{22}$$

corresponds to value (16)

$$\lambda = -0.0057; \tag{23}$$

For confirmation of the non-perturbative origin of the woudbe effect the study of the anomalous three-boson interaction (2) is necessary.

That is, the double boson production is to be studied. Let us show estimates for process

$$p + p \to W^+ + \gamma + X. \qquad (24)$$

Table 6. Cross sections for double production $\sigma(W^+\gamma)\, fb$ at LHC, $\sqrt{s} = 8\, TeV$ with $p_{\gamma\, t} > p_t^0$.

$\lvert\lambda\rvert\ /\ p_t^0\ GeV$	100	200	300	400	500	600
0.010	411.6	41.96	10.61	4.29	2.26	1.36
0.006	407.6	39.56	8.76	2.92	1.27	0.66
0.003	406.4	38.42	7.97	2.35	0.85	0.36
0.000	406.2	38.1	7.71	2.15	0.72	0.26

Table 7. Cross sections for double production $\sigma(W^+\gamma)\, fb$ at LHC, $\sqrt{s} = 13\, TeV$ with $p_{\gamma t} > p_t^0$.

$\lvert\lambda\rvert$ / $p_t^0\, GeV$	100	200	300	400	500	600
0.010	790.2	97.1	30.8	15.7	10.2	7.45
0.006	781.6	89.1	23.9	9.93	5.38	3.44
0.003	777.0	85.5	21.1	7.49	3.36	1.76
0.000	775.0	84.4	20.1	6.68	2.67	1.20

We see, that for $\sqrt{s} = 13\, TeV$ conditions are more preferable for studying of the effect for $\lvert\lambda\rvert \simeq 0.006$ in comparison with $\sqrt{s} = 8\, TeV$.

In case there would be the predicted correspondence of effects in $\bar{t}tH$ production and λ, the non-perturbative origin of the effect would be proved. This might be the first manifestation of a non-perturbative effect of the electro-weak interaction. If so, we shall encounter the very interesting and productive field of a work both on experimental and theoretical problems in forthcoming studies.

Thanks for the attention