$\Delta - \Delta$ isovector axial form factors in QCD

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Outline

- Introduction
- Light-cone QCD sum rules (LCSR)
- $\Delta - \Delta$ Isovector axial vector transition in QCD
- Results
- Conclusions

Motivation

- Understanding the structure of the Δ resonance is great relevance to nuclear phenomenolgy.
- The Δ is a rather broad resonance close to the πN theroshold.
- it therefore couples strongly to nucleons and pions making it important ingredient in chiral expansions.
- The Δ baryon resists the experimental probing due to its short lifetime (10^{-23})

Introduction

- Form Factors ? Why are they important ?
- FFs describe how hadrons interact with each other and give information about the internal structure of the hadrons.
- FFs are well known that many fundamental properties hadrons, e.g., the distribution of their charge, the origin and strength of their magnetization, and their (possibly deformed) shape, can be studied on the basis of hadron form factors.
- Specifically, many of the classical hadron structure observables can be directly defined from form factors, including the charges or coupling constants, particularly the electromagnetic, axial and tensor charge of hadrons.

Light-cone QCD sum rules

- Form factors are non-perturbative objects.
- To study these processes, a reliable non perturbative method is needed.
- One of these methods is the LCSR* approach.
- LCSR method is one of the most powerful and applicable non-perturbative tools to hadron physics.
- In the LCSR, the hadronic parameters are expressed in terms of the properties of the vacuum and the distribution amplitudes.
- In the LCSR, OPE is carried out near light cone ($x^2 \simeq 0$).
- One starts with a correlation function of the form,

$$\Pi_{\mu\nu}(p,q) = i \int d^4 x e^{iqx} \langle 0 | T \left\{ J_\mu(x) J_\nu(0) \right\} | h(p,s) \rangle$$

*Braun et al. Z. Phys C 1989, Braun et al., Nucl.Phys. B 1989, Chernyak et al. Nucl.Phys. B 1990

Hadronic Representation

- The correlation functions can be expressed in terms of the properties of hadrons and also in terms of the properties of the vacuum.
- By inserting complete sets of hadronic states, the correlation function can be written as:

$$\Pi_{\mu\nu}(p,q) = \sum_{p'} \frac{\langle 0|J_\mu|h'(p')\rangle\langle h'(p')|J_\nu|h(p)\rangle}{m^2_{h'} - p'^2} + \ldots$$

- matrix elements

$$\langle 0|J_\mu(0)|h'(p')\rangle = \lambda_{h'} u^\mu(p')$$

where $u^\mu(p')$ spinor and $\lambda_{h'}$ residue.

- The matrix elements $\langle h'(p')|J_\nu|h(p)\rangle$ can be expressed in terms of coupling constants or form factors.

QCD Representation

- The correlation function can also be calculated in the deep Euclidean region using OPE:

$$\Pi = \sum_d C_d(x^2) O_d(x)$$

- In the case of mass sum rules or traditional sum rules, $O_d(x)$ are local operators. After Fourier transform, the correlation function becomes:

$$\Pi = \sum_d C_d(p^2) \langle h^{'}(p') | O_d | h(p) \rangle$$

- In case of light cone sum rules, matrix elements of the form $\langle h^{'}(p') | O_d | h(p) \rangle$ are needed.
- The matrix elements are expanded around $x^2 \simeq 0$ in terms of distribution amplitudes.

QCD sum rules

- Two expressions for the correlation function is matched using spectral representation.

$$\Pi(p^2) = \int ds \frac{\rho(s)}{s-p^2} + p^2(polynomial)$$

- To subtract the contributions of higher states and continuum, quark hadron duality is assumed:

$$\Pi^{hadron} = \int_0^{s_0} ds\rho(s)e^{-\frac{s}{M^2}}$$

- For $p^2 > 0$, the correlation function is calculated in terms of hadronic parameters. In the deep Euclidean region, $p^2 << 0$, the correlation function is calculated using the OPE in terms of QCD degrees of freedom.
- Sum rules are obtained by matching the two representation using spectral representation..

Hadronic Representation

- $\Pi_{\mu\nu}(p,q) = i\int d^4x\, e^{iqx}\, \langle 0|T\left\{J^{\Delta}_{\mu}(0)J^{A}_{\nu}(x)\right\}|\Delta(p)\rangle$

- In this part, we show how to correlation function is related to the physically observed hadrons. Inserting a complete set of intermediate hadronic states with the same quantum number as interpolating currents,

- $\Pi_{\mu\nu}(p,q) = \sum_{s'} \frac{\langle 0|J_\mu|\Delta(p',s')\rangle\langle\Delta(p',s')|J_\nu|\Delta(p,s)\rangle}{m^2_\Delta - p'^2} + \ldots$

  where;

- $\langle 0|J_\mu(0)|\Delta\rangle = \lambda_\Delta \Delta^\mu(s',p')$

- $$\langle\Delta(p',s')|A_\nu(x)|\Delta(p,s)\rangle = \frac{-i}{2}\overline{\Delta}^{\alpha}(p',s')\Big[g_{\alpha\beta}\Big(g_1^A(q^2)\gamma_\nu\gamma_5 + g_3^A(q^2)\frac{q_\nu\gamma_5}{2M_\Delta}\Big) + \frac{q^\alpha q^\beta}{4M_\Delta^2}\Big(h_1^A(q^2)\gamma_\nu\gamma_5 + h_3^A(q^2)\frac{q_\nu\gamma_5}{2M_\Delta}\Big)\Big]\Delta^\beta(p,s)$$

Hadronic Representation

- Hadronic side of the correlation function;

$$\Pi_{\mu\nu} = i\frac{\lambda_\Delta}{2(M_\Delta^2 - p'^2)}\Bigg[\left\{-\frac{(3M_\Delta^2+4q^2)}{24M_\Delta^5}h_3^A(q^2) - \frac{g_3^A(q^2)}{3M_\Delta^3}\right\} q_\mu q_\nu \not{q}\gamma_5 q^\beta \Delta_\beta$$
$$+ g_1^A(q^2)\left\{-\frac{4}{3}q_\mu\gamma_5\Delta_\nu + p_\nu\gamma_5\Delta_\mu + \gamma_\nu\not{q}\gamma_5\Delta_\mu - \frac{2}{3M_\Delta}q_\mu\not{q}\gamma_5\Delta_\nu\right\}$$
$$-\left\{g_3^A(q^2)+2g_1^A(q^2)\right\} q_\nu\gamma_5\Delta_\mu + \left\{\frac{(M_\Delta^2+2q^2)}{6M_\Delta^4}h_1^A(q^2) + \frac{2g_1^A(q^2)}{3M_\Delta^2}\right\} q_\mu\gamma_\nu\not{q}\gamma_5 q^\beta$$
$$+\left\{\frac{(3M_\Delta^2-4q^2)}{6M_\Delta^4}h_1^A(q^2) + \frac{2g_1^A(q^2)}{3M_\Delta^2}\right\} q_\mu p_\nu\gamma_5 q^\beta\Delta_\beta - \frac{h_1^A(q^2)}{M_\Delta^3}q_\mu p_\nu\not{q}\gamma_5 q^\beta\Delta_\beta$$
$$-\frac{g_3^A(q^2)}{2M_\Delta}q_\nu\not{q}\gamma_5\Delta_\mu + \left\{-\frac{(3M_\Delta^2+2q^2)}{12M_\Delta^3}h_1^A(q^2)\right\} q_\mu\gamma_\nu\gamma_5 q^\beta\Delta_\beta$$
$$+\left\{-\frac{(-3M_\Delta^2+5q^2)}{12M_\Delta^4}h_3^A(q^2) - \frac{(5M_\Delta^2+2q^2)}{6M_\Delta^4}h_1^A(q^2) - \frac{4g_1^A(q^2)}{3M_\Delta^2}\right\} q_\mu q_\nu\gamma_5 q^\beta\Delta_\beta$$
$$-\frac{2g_3(q^2)}{3M_\Delta^2}q_\mu q_\nu\gamma_5 q^\beta\Delta_\beta\Bigg]$$

QCD Representation

- Correlation Function

$$\Pi_{\mu\nu}(p,q) = i\int d^4x\, e^{iqx}\, \langle 0|\, T\left\{J_\mu^\Delta(0) J_\nu^A(x)\right\} |\Delta(p)\rangle$$

where;

$$J_\mu^\Delta(0) = \frac{1}{\sqrt{3}}\varepsilon^{abc}[2(u^{aT}(0)C\gamma_\mu d^b(0))u^c(x) + (u^{aT}(0)C\gamma_\mu u^b(0))d^c(0)]$$

$$J_\nu^A(x) = \frac{1}{2}\left\{\bar{u}^d(x)\gamma_\nu\gamma_5 u^d(x) - \bar{d}^e(x)\gamma_\nu\gamma_5 d^e(x)\right\}$$

QCD Representation

- QCD side of the correlation function:

$$\Pi_{\mu\nu} = \frac{i}{16\sqrt{3}} \int d^4 x e^{iqx} \, (C\gamma_\mu)_{\alpha\beta}(\gamma_\nu\gamma_5)_{\rho\sigma} \Big\{ 4\epsilon^{abc}\langle 0|u^a_\sigma(x1)u^b_\theta(x2)d^c_\phi(x3)|\Delta(p,s)\rangle$$
$$\Big[2\delta^\eta_\alpha\delta^\theta_\sigma\delta^\phi_\beta S(-x)_{\lambda\rho} + 2\delta^\eta_\lambda\delta^\theta_\sigma\delta^\phi_\beta S(-x)_{\alpha\rho} + \delta^\eta_\alpha\delta_{\sigma\theta}\delta^\phi_\lambda S(-x)_{\beta\rho} + \delta^\eta_\beta\delta^\theta_\sigma\delta^\lambda_\phi S(-x)_{\alpha\rho}\Big]$$
$$- 4\epsilon^{abc}\langle 0|u^a_\sigma(x1)u^b_\theta(x2)d^c_\phi(x3)|\Delta(p,s)\rangle \Big[2\delta^\eta_\alpha\delta^\theta_\lambda\delta^\phi_\sigma S(-x)_{\beta\rho} + \delta^\eta_\alpha\delta^\theta_\beta\delta^\phi_\sigma S(-x)_{\lambda\rho}\Big]\Big\}$$

Where;

$$S(x) = \frac{i\not{x}}{2\pi^2 x^4} - \frac{\langle q\bar{q}\rangle}{12}\left(1 + \frac{m_0^2 x^2}{16}\right) - ig_s \int_0^1 d\upsilon \left[\frac{\not{x}}{16\pi^2 x^4} G_{\mu\nu}\sigma^{\mu\nu} - \upsilon x^\mu G_{\mu\nu}\gamma^\nu \frac{i}{4\pi^2 x^2}\right].$$

$$\langle 0|\varepsilon^{abc} u^a(x1)u^b(x2)d^c(x3)|\Delta(p)\rangle = \frac{f_\Delta}{4}\Big[V(x_i)M_\Delta(\gamma_\mu C)\Delta^\mu + A(x_i)M_\Delta(\gamma_\mu\gamma_5 C)(\gamma_5\Delta^\mu)$$
$$+ T(x_i)(i\sigma_{\mu\nu}p^\nu C)\Delta^\mu\Big]^*$$

*C.E. Carlson and J. L. Poor., PRD 38, 1988

Results

The QCD sum rules are obtained by matching the short-distance (QCD side) of the correlation function with the long-distance (phenomenological side) calculation and taking the fourier transformations, we obtain;

- For the $\Delta - \Delta$ axial transition

$$g_1^A(q^2)\frac{\lambda_\Delta}{M_\Delta - p'^2} = -\frac{f_\Delta M_\Delta}{\sqrt{3}}\Bigg[\int_0^1 dx_2 \frac{1}{(q-px_2)^2}\int_0^{1-x_2} d_{x_1}\, 4V(x_1, x_2, 1-x_1-x_2) - \int_0^1 dx_3 \frac{1}{(q-px_3)^2}\int_0^{1-x_3} d_{x_1}\,[-T + A - 2V](x_1, 1-x_1-x_3, x_3)\Bigg]$$

$$g_3^A(q^2)\frac{\lambda_\Delta}{M_\Delta - p'^2} = -\frac{f_\Delta M_\Delta}{\sqrt{3}}\Bigg[\int_0^1 dx_2 \frac{1}{(q-px_2)^2}\int_0^{1-x_2} d_{x_1}\,[2T + 4A + 8V](x_1, x_2, 1-x_1-x_2) + \int_0^1 dx_3 \frac{1}{(q-px_3)^2}\int_0^{1-x_3} d_{x_1}\,[-3T + 3A + 2V](x_1, 1-x_1-x_3, x_3)\Bigg],$$

Results

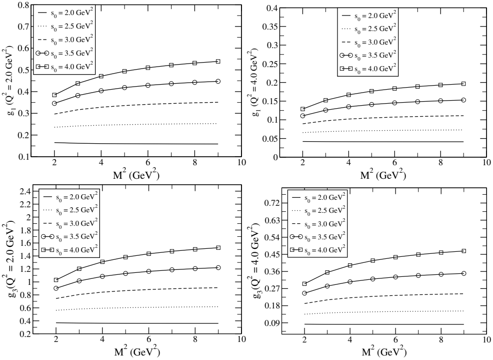

Figure: The dependence of the form factors; on the Borel parameter squared M_B^2 for the values of the continuum threshold $s_0 = 2.0\ GeV^2$, $s_0 = 2.5\ GeV^2$ $s_0 = 3.0\ GeV^2$, $s_0 = 3.5\ GeV^2$ and $s_0 = 4.0\ GeV^2$ and $Q^2 = 2.0$ and $4.0\ GeV^2$, for g_1^A and g_3^A axial form factors.

Results

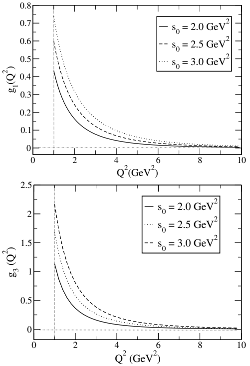

Figure: The dependence of the form factors for the values of the continuum threshold $s_0 = 2.0\ GeV^2$, $s_0 = 2.5\ GeV^2$, $s_0 = 3.0\ GeV^2$ and $M^2 = 3.0$ for g_1^A and g_3^A axial form factors.

Results

- Axial Charge;
  Fit function; $g_A(q^2) = g_A(0)\exp[-Q^2/M_A^2]$

Fit Region (GeV^2)	$g_A(0)$	M_A (GeV)
[1.0 – 10]	3.48	1.15
[1.5 – 10]	2.64	1.24
[2.0 – 10]	2.10	1.32

- Table: The values of exponential fit parameters, g_A and M_A for axial form factors.

Results

-

	[1]	[2]	[3]	[4]	[5]	This Work
g_A	-1.9 ± 0.1	-4.50	-4.47	-4.48	-4.30	-2.70 ± 0.6

- Table: Different results from theoretical models which are Lattice QCD, ChPT, quark models and also our model.

1- Alexandrou et al., PRD 87, 2013.
2- Jiang et al.,PRD 78, 2008.
3- Theussl et al., EPJC 12, 2001.
4- Glozman et al., PRD 58, 1998.
5- Glantshnig et al., EPJA 23, 2005.

Conclusion

- In this study, we have calculated the isovector axial vector form factors for Δ baryon within the LCSR method.
- We extract axial g_1^A, g_3^A form factors. The form factors h_1^A, h_3^A could not obtain from our approach since the necessary DAs have not been known yet.
- We compared our fit results with the lattice QCD, ChPT and quark models.
- Unfortunately, there is no experimental data yet to compare our results within this region.

Thank you......