Complex- mass definition and the hypothesis of continuous mass

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Mass definitions
(Unstable particles)

OMS def.	Base	$(M,\,\Gamma)$
Pole-mass def.		
Complex-mass def.		$(M^2 - iM\Gamma) = M_p^2$

M and Γ def. $\Longleftrightarrow$ dressed propagator's structure (standard and model)

Dyson procedure $\Longrightarrow$ Renorm. propagator $\Longleftrightarrow$ $(M,\,\Gamma)$ - definition

$$\frac{1}{1-z} = \sum_{k=0}^{\infty} z^k = 1 + z + z^2 + \ldots, \qquad |z| < 1, \qquad z = \frac{\Pi_{(1)}(q)}{q^2 - M_0^2}$$

(d'Alambert convergence criterion)

Redefinition $z \longrightarrow \dfrac{\Im\Pi_{(1)}(q)}{q^2 - M^2} \approx \dfrac{M \cdot \Gamma}{q^2 - M^2}$ $\qquad |q^2 - M^2| < M\Gamma$ is excluded

Scheme of sequential fixed-order calculations $\longrightarrow$ GI violation

(Special methods, effective theories of UP etc.)

Normalized propagator $\Longrightarrow$ finite expression for calculations

$q_\mu q_\nu / f(q, M, \Gamma)$ (non-uniquence) | $f(q, M, \Gamma) = M^2, \;\; M^2 - iM\Gamma, \;\; q^2, \;\; (M - i\Gamma/2), \;\; \ldots$

Breit-Wigner approximation (~~GI~~)

$$D^V_{\mu\nu}(q^2) = \frac{-g_{\mu\nu} + q_\mu q_\nu / M_V^2}{q^2 - M_V^2 + iM_V\Gamma_V}; \quad D_F(\hat{q}) = \frac{\hat{q} + M_F}{q^2 - M_F^2 + iM_F\Gamma_F}$$

Electromagnetic Ward identity $\longrightarrow$ modified BW approximation **(GI)**

$$D^V_{\mu\nu}(q^2) = \frac{-g_{\mu\nu} + q_\mu q_\nu / (M_V^2 - iM_V\Gamma_V)}{q^2 - (M_V^2 - iM_V\Gamma_V)}; \quad D_F(\hat{q}) = \frac{\hat{q} + M_F - i\Gamma_F/2}{q^2 - (M_F - i\Gamma_F/2)^2}.$$

(Nowakowski and Pilaftsis, Z.Phys 1993)

$M^2 \longrightarrow M_P^2 = M^2 - i\Gamma M \qquad M \longrightarrow M_P = M - i\Gamma/2$ Complex-mass def.

Alternative approaches $\longrightarrow$ spectral representation of propagators (Lehman etc)

Matthews and Salam (PR 1956) m^2 -interpretation of spectral function

Model of UP with continuous (smeared) mass

Scalar UP: $D(q^2) = \int D_0(q^2, m^2)\rho(m^2)\, dm^2 \quad \longrightarrow$ spectral representation

$$D_0(q^2, m^2) = \frac{1}{q^2 - m^2 + i\epsilon}$$

SPA (fixed mass)

$\rho(m^2)$ - spectral function in m^2 -interpretation (Matthews and Salam, PR 1958)

$$D^M(q^2) = \int \frac{\rho(m^2)\, dm^2}{q^2 - m^2 + i\epsilon} = \frac{1}{q^2 - M^2 + iM\Gamma} = D^{st}(q^2) \implies \rho(m^2; M, \Gamma)$$

$$\int_a^b \frac{f(x)\, dx}{x \pm i\epsilon} = \mp i\pi f(0) + \mathcal{P}\int_a^b \frac{f(x)}{x} dx$$

(Sokhotski-Plemelj formula)

$$\Im D(q) = -\pi\rho(q^2) = \frac{-M\Gamma}{[q^2 - M^2]^2 + M^2\Gamma^2};$$

$$\Re D(q) = \mathcal{P}\int \frac{\rho(m^2)\, dm^2}{q^2 - m^2} = \frac{q^2 - M^2}{[q^2 - M^2]^2 + M^2\Gamma^2}.$$

$$\rho(m^2) = \frac{1}{\pi}\frac{M\Gamma}{[m^2 - M^2]^2 + M^2\Gamma^2}.$$

$(-\infty < m^2 < \infty)$

Negative component of spectral representation

$$P(m^2 < 0) = \int_{-\infty}^{0} \rho(m^2; M, \Gamma)\, dm^2 \approx \frac{\Gamma}{\pi M}, \quad (\frac{\Gamma}{M} << 1)$$

$$\epsilon = \frac{\delta D}{D}, \quad \delta D = \int_{-\infty}^{0} D_0(q^2, m^2)\rho(m^2)dm^2$$

$$\epsilon(q^2; M, \Gamma) = \frac{1}{\pi}\frac{\Gamma M}{q^2 - M^2 - i\Gamma M}[\frac{1}{2}\ln\frac{q^4}{M^2(M^2+\Gamma^2)} + \pi\frac{q^2 - M^2}{\Gamma M}]$$

$q^2 = M^2$ $\qquad$ $\epsilon \approx \frac{-i}{2\pi}\frac{\Gamma^2}{M^2} \quad (\frac{\Gamma}{M} << 1)$

$q^2 \to \infty$ $\qquad$ $\epsilon \to 1$ (asymptotic)

$q^2 << M^2$ $\qquad$ $\epsilon \approx \frac{\Gamma}{\pi M}(\pi\frac{M}{\Gamma} + \frac{1}{2}\ln\frac{M^4}{q^4})$

Considerable q^2 -dependence of $\epsilon(q^2; \Gamma, M) \longleftarrow$ integration rule (SP formula)

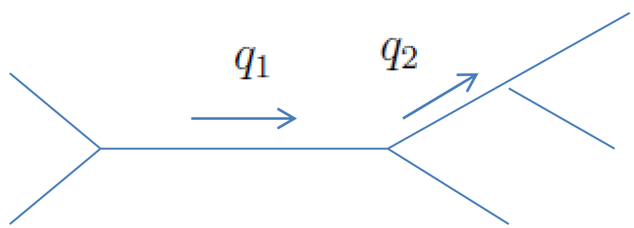
Conclusion: we can not cut off the negative component and interpret it as the error of BW approximation

Problem with negative component $m^2 < 0$ (tachyon?)

$$\phi(x) = \frac{1}{(2\pi)^{3/2}} \int \int \phi(\mathbf{p}, m^2) e^{ipx} d\mathbf{p}\, \omega(m^2)\, dm^2,$$

$p = (\mathbf{p}, p^0)$, $\phi(\mathbf{p}, m^2)$ is defined in standard way at fixed mass $p^2 = m^2$

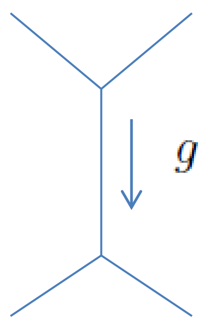
1. The model of UP: $p^2 > 0$ time-like region (s-channel processes)



$q_1^2 > 0$ | q^2 - Inv. mass

$q_2^2 > 0$ | $\Gamma(q^2)$

2. Generalization: $p^2 < 0$ space-like region (t, u-channel processes)



$g^2 < 0$

The model of UP is effective theory

($m^2 < 0$ -unobserved value)

tachyon?

3. BW, MBW, ... approximations $\longleftrightarrow$ full two-point function ?

PROPAGATOR OF VECTOR UNSTABLE PARTICLES

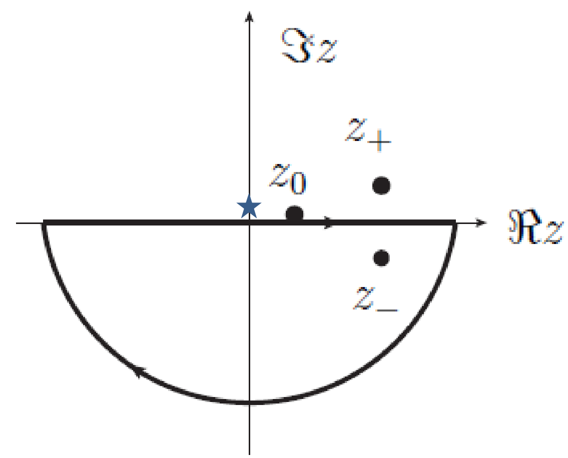
$$D_{\mu\nu}(q) = \frac{1}{\pi}\int_{-\infty}^{+\infty} \frac{-g_{\mu\nu} + q_\mu q_\nu/(m^2 - i\epsilon)}{q^2 - m^2 + i\epsilon} \frac{M\Gamma\, dm^2}{[m^2 - M^2]^2 + M^2\Gamma^2}.$$

$$D_{\mu\nu}(q) = -\frac{M\Gamma}{\pi}\oint_{C_-} \frac{(g_{\mu\nu} - q_\mu q_\nu/(z - i\epsilon))\, dz}{(z - z_-)(z - z_+)(z - z_0)}$$

$$= -2iM\Gamma \frac{g_{\mu\nu} - q_\mu q_\nu/(z_-)}{(z_- - z_+)(z_- - z_0)} = \frac{-g_{\mu\nu} + q_\mu q_\nu/(M^2 - iM\Gamma)}{q^2 - M^2 + iM^2\Gamma^2}$$

$$z_0 = q^2 + i\epsilon$$

$$z_\pm = M^2 \pm iM\Gamma$$



Complex-mass structure

$$D(q) = \frac{1}{q^2 - M_P^2}; \qquad D_{\mu\nu}(q) = \frac{-g_{\mu\nu} + q_\mu q_\nu/M_P^2}{q^2 - M_P^2} \qquad M_P^2 = M^2 - iM\Gamma$$

PROPAGATOR OF SPINOR UNSTABLE PARTICLES

$$\hat{D}(q) = \frac{1}{\hat{q} - m + i\epsilon} = \frac{\hat{q} + m - i\epsilon}{q^2 - (m - i\epsilon)^2}.$$

$$\hat{D}(q) = \int \frac{\hat{q} + m}{q^2 - (m - i\epsilon)^2}\, \rho(m)\, dm$$

$$\rho(m) = \frac{1}{\pi}\, \frac{\Gamma/2}{[m - M]^2 + \Gamma^2/4}$$

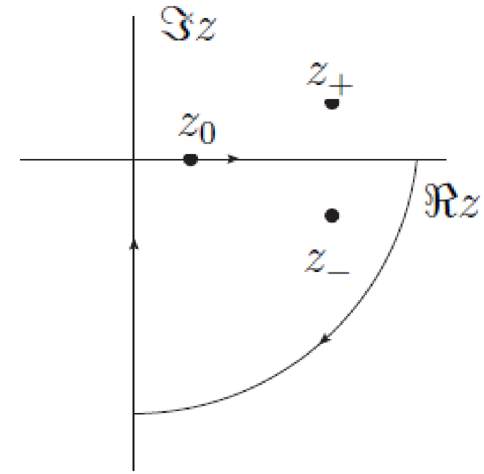
$$M(q) = M_0 + \Re\Sigma(q)$$

$$\Gamma(q) = \Im\Sigma(q)$$

$$m - i\epsilon \to M - i\Gamma/2$$

$$\hat{D}_-(q) = -\frac{\Gamma}{2\pi} \int_{C_-} \frac{dz}{z - z_-}\, \frac{\hat{q} + z}{(z^2 - z_0^2)(z - z_+)}$$
$$= -\, i\Gamma(q) \frac{\hat{q} + z_-}{(z_-^2 - z_0^2)(z_- - z_+)} = \frac{\hat{q} + M - i\Gamma/2}{q^2 - (M - i\Gamma/2)^2}.$$

$$z_0^2 = q^2 + i\epsilon,\; z_\pm = M \pm i\Gamma/2$$



Complex-mass structure

$$M_P = M_\rho - i\Gamma_\rho/2$$

CONCLUSIONS

1. There are some problems with dressed propagators construction and mass definition for UP.
2. Dyson procedure contains the problems of convergence criterium at peak region and scheme of sequantial fixed-order calculations.
3. Spectral representation leads to the "negative component" in the spectral expansion which nature is not clear.
4. The model of UP with continuous mass should be modified at $m^2 < 0$.

Thank You for attention