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Higgs field as the Goldberger-Wise field

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The question of the stabilization of the size of the extra dimension with help of the Higgs field was raised earlier in the papers:

- L. Vecchi, “A Natural Hierarchy and a low New Physics scale from a Bulk Higgs,”(2011);
- M. Geller, S. Bar-Shalom and A. Soni, “Higgs-radion unification: Radius stabilization by an SU(2) bulk doublet and the 126 GeV scalar,” (2014).

There was considered a perturbative solution. We attempt to find an exact one.

The Randall-Sundrum model

We consider two branes with tension interacting with gravity in a five-dimensional space-time $E = M_4 \times S^1 / Z_2$

In this report the interbrane separation is assumed to be stabilized by a two-component complex scalar field. On "our" brane it will implement the Higgs mechanism of spontaneous symmetry breaking.

The background solution

Let us consider a scalar field $\phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$.

The action of the model can be written as

$$S = S_g + S_{SM+\phi},$$

where the gravitational action S_g is given by

$$S_g = 2M^3 \int d^4x \int_{-L}^{L} dy R \sqrt{-g},$$

and $S_{SM+\phi}$ is an action of the scalar field, branes and the Standard model:

$$S_{SM+\phi} = -M \int d^4x \int_{-L}^{L} dy \left(\partial_M \bar{\phi} \partial^M \phi + V(\bar{\phi}\phi) \right) \sqrt{-g} -$$

$$- \int_{y=L} \lambda_1(\bar{\phi}\phi) \sqrt{-\tilde{g}}\, d^4x + \int_{y=L} \left(-\lambda_2(\bar{\phi}\phi) + L_{SM-HP}(\phi, \bar{\phi}) \right) \sqrt{-\tilde{g}}\, d^4x.$$

A solution for the metric, which preserves the Poincare invariance in any four-dimensional subspace $y = const$, is sought in the form:

$$ds^2 = \gamma_{MN} dx^M dx^N = e^{-2A(y)} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2,$$

and for the multidimensional Higgs field in the form:

$$\phi(x, y) = \phi(y).$$

By the variation of the action we get the equations of motion:

$$\frac{1}{2}\left(\bar{\phi}'\phi+V+\frac{\lambda_1}{M}\delta(y)+\frac{\lambda_2}{M}\delta(y-L)\right)=2M^2\left(3A''-6\left(A'\right)^2\right),$$

$$12M^2\left(A'\right)^2+\frac{1}{2}\left(V-\bar{\phi}'\phi'\right)=0,$$

$$\frac{dV}{d\phi}+\frac{1}{M}\frac{d\lambda_1}{d\phi}\delta(y)+\frac{1}{M}\frac{d\lambda_2}{d\phi}\delta(y-L)=\bar{\phi}''-4A'\bar{\phi}',$$

$$\frac{dV}{d\bar{\phi}}+\frac{1}{M}\frac{d\lambda_1}{d\bar{\phi}}\delta(y)+\frac{1}{M}\frac{d\lambda_2}{d\bar{\phi}}\delta(y-L)=\phi''-4A'\phi'.$$

An ansatz looks like:

$$V = \frac{1}{4}\frac{dW}{d\phi}\frac{dW}{d\bar{\phi}} - \frac{1}{24M^2}\left(W(\bar{\phi}\phi)\right)^2,$$

$$\phi'(y) = sign(y)\frac{1}{2}\frac{dW}{d\bar{\phi}}, \qquad \bar{\phi}'(y) = sign(y)\frac{1}{2}\frac{dW}{d\phi},$$

$$A'(y) = sign(y)\frac{1}{24M^2}W(\bar{\phi}\phi).$$

The equations of motion are valid everywhere, except for the branes.

We choose the function $W(\bar{\phi}\phi)$ in the form:

$$W = 24M^2k - 2u\bar{\phi}\phi.$$

Then the brane potentials should be defined as follows:

$$\lambda_1(\bar{\phi}\phi) = MW(\bar{\phi}\phi) + \beta_1\left(\bar{\phi}\phi - \frac{\upsilon_1^2}{2}\right)^2,$$

A Higgs-like potential

$$\lambda_2(\bar{\phi}\phi) = -MW(\bar{\phi}\phi) + \beta_2\left(\bar{\phi}\phi - \frac{\upsilon_2^2}{2}\right)^2.$$

We finally get:

$$\phi(y) = \begin{pmatrix} 0 \\ \frac{\upsilon}{\sqrt{2}} e^{-u(|y|-L)} \end{pmatrix}, \quad \upsilon = 246\, GeV,$$

$$A(y) = k(|y| - L) + \frac{\upsilon^2}{96M^2}\left(e^{-2u(|y|-L)} - 1\right).$$

The interbrane distance is defined by the boundary conditions for the scalar field and is expressed in terms of the parameters of the model by the relation:

$$L = \frac{1}{u} \ln\left(\frac{\upsilon_1}{\upsilon}\right),$$

so we have the size of the extra dimension stabilized.

The equation for the fluctuations of the scalar field

In order to build the linearized theory we represent the metric and the five-dimensional Higgs field in the unitary gauge as:

$$g_{MN}(x,y) = \gamma_{MN}(y) + \frac{1}{\sqrt{2M^3}} h_{MN}(x,y),$$

$$\phi(x,y) = \phi(y) + \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2M^2}} f(x,y) \end{pmatrix}.$$

Let us define a new function $g = e^{-2A(y)} h_{44}(x, y)$.

After the mode decomposition of g we get the equation in the Sturm-Liouville form:

$$\frac{d}{dy}\left(\frac{e^{2A}}{(\phi_2')^2} g_n'\right) - \frac{e^{2A}}{6M^2} g_n = -\mu_n^2 g_n \frac{e^{4A}}{(\phi_2')^2},$$

$y \in (0, L)$, and the boundary conditions on the branes:

$$\left(\frac{1}{4M}\frac{d^2\lambda_1}{d\phi_2^2} - \frac{\phi_2''}{\phi_2'}\right) g_n' + \mu_n^2 e^{2A} g_n \Big|_{y=+0} = 0,$$

$$\left(\frac{1}{4M}\frac{d^2\lambda_2}{d\phi_2^2} + \frac{\phi_2''}{\phi_2'}\right) g_n' - \mu_n^2 e^{2A} g_n \Big|_{y=L-0} = 0.$$

Using the results of the paper:

Edward E. Boos, Yuri S. Mikhailova, Mikhail N. Smolyakov, Igor P. Volobuev, "Physical degrees of freedom in stabilized brane world models",

in the case $uL << 1$ we get the following mass of the lowest excitation of the scalar field identified as the Higgs boson:

$$m_H^2 = \frac{\upsilon^2 u^2}{3M^2} \frac{\beta_2 \upsilon^2 - uM}{\beta_2 \upsilon^2 + uk}.$$

If we choose $M = 2TeV$ and $\beta_2 \to \infty$ we get the model parameters as follows:

$$u \approx 1.76TeV, \qquad \phi_1 = 345TeV,$$

$$k \approx 186TeV, \qquad L = 0.2TeV^{-1} \approx 2 \cdot 10^{-18}\, cm.$$

The Higgs boson can now interact with the energy-momentum tensor: $\varepsilon_H h T_\mu^{\ \mu}$, where $h \equiv g_1$.

The coupling: $\varepsilon_H = -\sqrt{\dfrac{k}{24M^3}} \sim 1TeV^{-1}.$

Conclusion

- The stabilization of the size of the extra dimension in the Randall-Sundrum model and the spontaneous symmetry breaking on “our” brane are explained simultaneously with help of the five-dimensional Higgs field.
- The equation of motion for this field is found and a solution is obtained.

- In this case the Higgs boson is the radion at the same time, and it now has an interaction with the energy-momentum tensor that can affect its properties significantly.
- The possible values of the model parameters are estimated, which give the correct value of the Higgs boson mass.