Reproducing the Standard Model in 5D brane worlds (arXiv:1503.09074)

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Gauge fields

The background metric:

$$ds^2 = e^{2\sigma(z)}\eta_{\mu\nu}dx^\mu dx^\nu - dz^2$$

$SU(2) \times U(1)$ gauge invariant theory in this background:

$$S = \int d^4x dz \sqrt{g} \left(-\frac{\xi^2}{4} F^{a,MN} F^a_{MN} - \frac{\xi^2}{4} B^{MN} B_{MN} \right.$$
$$\left. + g^{MN} \left(D_M H\right)^\dagger D_N H - V(H^\dagger H) \right)$$

where

$$F^a_{MN} = \partial_M A^a_N - \partial_N A^a_M + g\epsilon^{abc} A^b_M A^c_N,$$
$$B_{MN} = \partial_M B_N - \partial_N B_M,$$
$$D_M H = \left(\partial_M - ig\frac{\tau^a}{2} A^a_M - i\frac{g'}{2} B_M \right) H$$

The orbifold symmetry conditions

$A^a_\mu, B_\mu(x,-z) = A^a_\mu, B_\mu(x,z)$, $\quad A^a_5, B_5(x,-z) = -A^a_5, B_5(x,z)$,
$H(x,-z) = H(x,z)$.

The vacuum solution for these fields:

$$A^a_M \equiv 0, \qquad B_M \equiv 0, \qquad H_0 \equiv \begin{pmatrix} 0 \\ \frac{v(z)}{\sqrt{2}} \end{pmatrix}$$

We retain only the four-vector components of the fields.

The physical degrees of freedom are

$$Z_\mu = \frac{1}{\sqrt{g^2 + g'^2}} \left(g A^3_\mu - g' B_\mu \right), \; A_\mu = \frac{1}{\sqrt{g^2 + g'^2}} \left(g B_\mu + g' A^3_\mu \right)$$

$$W^\pm_\mu = \frac{1}{\sqrt{2}} \left(A^1_\mu \mp i A^2_\mu \right)$$

Let us consider only the quadratic part of effective action in terms of these new fields:

$$S_{eff} = \int d^4xdz \left(-\frac{\xi^2}{2}\eta^{\mu\nu}\eta^{\alpha\beta}W^+_{\mu\alpha}W^-_{\nu\beta} - \frac{\xi^2}{4}\eta^{\mu\nu}\eta^{\alpha\beta}F_{\mu\alpha}F_{\nu\beta} \right.$$
$$-\frac{\xi^2}{4}\eta^{\mu\nu}\eta^{\alpha\beta}Z_{\mu\alpha}Z_{\nu\beta} + e^{2\sigma}\xi^2\eta^{\mu\nu}\partial_5 W^+_\mu \partial_5 W^-_\nu$$
$$+e^{2\sigma}\frac{\xi^2}{2}\eta^{\mu\nu}\partial_5 A_\mu \partial_5 A_\nu + e^{2\sigma}\frac{\xi^2}{2}\eta^{\mu\nu}\partial_5 Z_\mu \partial_5 Z_\nu$$
$$\left. +e^{2\sigma}v^2(z)\frac{g^2}{4}\eta^{\mu\nu}W^+_\mu W^-_\nu + e^{2\sigma}v^2(z)\frac{g^2+g'^2}{8}\eta^{\mu\nu}Z_\mu Z_\nu \right)$$

The equations for the wave functions and the masses of the KK modes are:

$$-m_{W,n}^2 f_{W,n} - \partial_5(e^{2\sigma}\partial_5 f_{W,n}) + \frac{g^2}{4\xi^2}e^{2\sigma}v^2(z)f_{W,n} = 0,$$

$$-m_{Z,n}^2 f_{Z,n} - \partial_5(e^{2\sigma}\partial_5 f_{Z,n}) + \frac{g^2+g'^2}{4\xi^2}e^{2\sigma}v^2(z)f_{Z,n} = 0,$$

$$-m_{A,n}^2 f_{A,n} - \partial_5(e^{2\sigma}\partial_5 f_{A,n}) = 0.$$

The independence of the wave functions $f_{W,0}(z)$ and $f_{Z,0}(z)$ on the coordinate of the extra dimension can be achieved only when

$$v(z) = \xi\tilde{v}e^{-\sigma},$$

$\tilde{v}$ is a constant. The masses of the zero mode gauge bosons:

$$m_{W,0} = \frac{g\tilde{v}}{2}, \qquad m_{Z,0} = \frac{\sqrt{g^2+g'^2}\,\tilde{v}}{2}.$$

Modification of the shapes of gauge boson wave functions has an influence on the electroweak observables:

C. Csaki, J. Erlich and J. Terning, Phys. Rev. D **66** (2002) 064021

G. Burdman, Phys. Rev. D **66** (2002) 076003

For example, in the case of the Randall-Sundrum model such a modification leads to restrictions on the value of the five-dimensional energy scale, which put the theory out of the reach of the present day experiments.

Fermions

V.A. Rubakov, M.E. Shaposhnikov, Phys. Lett. B **125** (1983) 136.

$$S_{scalar} = \int d^4xdz \left(\frac{1}{2}\partial_M\Phi\partial^M\Phi - \frac{\lambda}{4}\left(\Phi^2 - v^2\right)^2 \right),$$

$$v = \frac{m}{\sqrt{\lambda}}, \quad M = 0, 1, 2, 3, 5$$

Kink solution:

$$\Phi_0 = \frac{m}{\sqrt{\lambda}} \tanh\left(\frac{m}{\sqrt{2}} z \right)$$

$$S = \int \left[i\bar{\Psi}\Gamma^M\partial_M\Psi - h\Phi\bar{\Psi}\Psi \right] d^4xdz, \qquad \Gamma^\mu = \gamma^\mu,\ \Gamma^5 = i\gamma^5$$

$$\Psi_L(x,z) \to C_L\psi_L(x)e^{-\frac{hm}{\sqrt{\lambda}}|z|}, \qquad i\gamma^\mu\partial_\mu\psi_L = 0$$

$$\Psi_R(x,z) \to C_R\psi_R(x)e^{\frac{hm}{\sqrt{\lambda}}|z|}, \qquad i\gamma^\mu\partial_\mu\psi_R = 0$$

For a nonzero mass it is necessary to take two five-dimensional spinor fields, satisfying the orbifold symmetry conditions

$$\Psi_1(x,-z) = \gamma^5 \Psi_1(x,z),$$
$$\Psi_2(x,-z) = -\gamma^5 \Psi_2(x,z).$$

S.L. Dubovsky, V.A. Rubakov and P.G. Tinyakov, Phys. Rev. D **62** (2000) 105011,
C. Macesanu, Int. J. Mod. Phys. A **21** (2006) 2259,
R. Casadio and A. Gruppuso, Phys. Rev. D **64** (2001) 025020

$$S = \int d^4 x dz \sqrt{g} \left(E^M_N i \bar{\Psi}_1 \Gamma^N \nabla_M \Psi_1 + E^M_N i \bar{\Psi}_2 \Gamma^N \nabla_M \Psi_2 \right.$$
$$\left. - F_1(z) \bar{\Psi}_1 \Psi_1 - F_2(z) \bar{\Psi}_2 \Psi_2 - G(z) \left(\bar{\Psi}_2 \Psi_1 + \bar{\Psi}_1 \Psi_2 \right) \right),$$

where $M, N = 0, 1, 2, 3, 5$, $\Gamma^\mu = \gamma^\mu$, $\Gamma^5 = i\gamma^5$, ∇_M is the covariant derivative containing the spin connection, E^M_N is the vielbein, $F_{1,2}(z)$ and $G(z)$ are some functions satisfying the symmetry conditions $F_{1,2}(-z) = -F_{1,2}(z)$ and $G(-z) = G(z)$

$$G(z) = hv(z),$$

4D Dirac equation

$$i\gamma^{\mu}\partial_{\mu}\psi - m\psi = 0 \quad \rightarrow \quad \Box\psi + m^2\psi = 0$$

Our case

$$\begin{aligned} -\Box\Psi_1 + e^{\sigma}(\partial_5 + 2\sigma')e^{\sigma}(\partial_5 + 2\sigma')\Psi_1 + e^{\sigma}\partial_5(e^{\sigma}F_1)\gamma^5\Psi_1 \\ -e^{2\sigma}(F_1^2(z) + h^2v^2(z))\Psi_1 \\ +he^{\sigma}\partial_5(e^{\sigma}v(z))\gamma^5\Psi_2 - he^{2\sigma}v(z)\left(F_1(z) + F_2(z)\right)\Psi_2 = 0, \end{aligned}$$

$$\begin{aligned} -\Box\Psi_2 + e^{\sigma}(\partial_5 + 2\sigma')e^{\sigma}(\partial_5 + 2\sigma')\Psi_2 + e^{\sigma}\partial_5(e^{\sigma}F_2)\gamma^5\Psi_2 \\ -e^{2\sigma}(F_2^2(z) + h^2v^2(z))\Psi_2 \\ +he^{\sigma}\partial_5(e^{\sigma}v(z))\gamma^5\Psi_1 - he^{2\sigma}v(z)\left(F_1(z) + F_2(z)\right)\Psi_1 = 0. \end{aligned}$$

Possible problems (for simplicity, $\sigma \equiv 0$):

$$\left[\Box - \partial_5^2 - F_2' + F_2^2 + h^2 v^2\right] \frac{\left[\Box - \partial_5^2 - F_1' + F_1^2 + h^2 v^2\right] \Psi_1^{(1)}}{hv(F_1 + F_2) - hv'} - \left(hv(F_1 + F_2) - hv'\right) \Psi_1^{(1)} = 0.$$

Exception 1

$$F_1(z) \equiv -F_2(z),$$
$$\partial_5(e^{\sigma} v(z)) \equiv 0.$$

$$\Psi_1 = C_f \exp\left[-\int_0^z F_1(y)dy - 2\sigma(z)\right]\psi_L(x), \quad i\gamma^{\mu}\partial_{\mu}\psi_L - \tilde{h}\tilde{v}\psi_R = 0,$$
$$\gamma^5\psi_L = \psi_L,$$
$$\Psi_2 = C_f \exp\left[-\int_0^z F_1(y)dy - 2\sigma(z)\right]\psi_R(x), \quad i\gamma^{\mu}\partial_{\mu}\psi_R - \tilde{h}\tilde{v}\psi_L = 0,$$
$$\gamma^5\psi_R = -\psi_R,$$

Exception 2

$$F_1(z) \equiv F_2(z)$$

For simplicity, $\sigma \equiv 0$.

$$\begin{aligned} &-\Box(\Psi_1+\Psi_2)+\partial_5^2(\Psi_1+\Psi_2)-h^2v^2(z)(\Psi_1+\Psi_2)\\ &\qquad\qquad +hv'(z)\gamma^5(\Psi_1+\Psi_2)=0,\\ &-\Box(\Psi_1-\Psi_2)+\partial_5^2(\Psi_1-\Psi_2)-h^2v^2(z)(\Psi_1-\Psi_2)\\ &\qquad\qquad -hv'(z)\gamma^5(\Psi_1-\Psi_2)=0. \end{aligned}$$

The fields Ψ_1 and Ψ_2 can be decomposed into the Kaluza-Klein modes as

$$\Psi_1 = \sum_n \left(f_+^n(z)\psi_L^n(x) + f_-^n(z)\psi_R^n(x)\right),$$

$$\Psi_2 = \sum_n \left(f_-^n(z)\psi_L^n(x) - f_+^n(z)\psi_R^n(x)\right).$$

$$f_+^n(z) = f^n(z) + f^n(-z), \qquad f_-^n(z) = f^n(z) - f^n(-z),$$

the function $f^n(z)$ is a periodic continuously differentiable solution to the equation

$$m_n^2 f^n + \partial_5^2 f^n - h^2 v^2 f^n + h v' f^n = 0.$$

$SU(2) \times U(1)$ gauge fields in the flat ($\sigma(z) \equiv 0$) space-time with $F_1(z) \equiv F_2(z) \equiv 0$:

$$\int d^4x dz \left(i\bar{\hat{\Psi}}_1 \Gamma^M D_M \hat{\Psi}_1 + i\bar{\Psi}_2 \Gamma^M D_M \Psi_2 - \sqrt{2} h \left[\left(\bar{\hat{\Psi}}_1 H \right) \Psi_2 + \mathrm{h.c.} \right] \right)$$

$SU(2)$ doublet:

$$\hat{\Psi}_1 = \begin{pmatrix} \Psi_1^\nu \\ \Psi_1^\psi \end{pmatrix}$$

$SU(2)$ singlet: Ψ_2.

$$\begin{aligned} D_M \hat{\Psi}_1 &= \left(\partial_M - ig \frac{\tau^a}{2} A_M^a + i \frac{g'}{2} B_M \right) \hat{\Psi}_1, \\ D_M \Psi_2 &= \left(\partial_M + i g' B_M \right) \Psi_2. \end{aligned}$$

The vacuum solution for the Higgs field:

$$H_0 \equiv \begin{pmatrix} 0 \\ \frac{v(z)}{\sqrt{2}} \end{pmatrix}$$

with $v(z) \neq \mathrm{const}$.

Free theory.

$$\hat{\Psi}_1(x,z) = \left(\begin{array}{l} \xi \nu_L(x) \\ f_+(z)\psi_L(x) + f_-(z)\psi_R(x) \end{array} \right),$$
$$\Psi_2(x,z) = f_-(z)\psi_L(x) - f_+(z)\psi_R(x).$$

$$S = \int d^4x \left(i\bar{\nu}_L \gamma^\mu \partial_\mu \nu_L + i\bar{\psi}\gamma^\mu \partial_\mu \psi - m\bar{\psi}\psi \right),$$

where $\psi = \psi_L + \psi_R$. In order to have the canonically normalized kinetic term of the field ψ, the condition

$$a^2 + b^2 = 1, \qquad a^2 = \int dz f_+^2(z), \qquad b^2 = \int dz f_-^2(z).$$

must be fulfilled.

$$A^a_\mu(x,z) \equiv A^a_\mu(x), \quad B_\mu(x,z) \equiv B_\mu(x);$$

Differences with the SM:

1. The coupling constant of the field ψ to the charged gauge bosons is
$$g\frac{1}{\sqrt{2L}}\int dz f_+(z)$$
instead of g in the SM.
2. The axial coupling constant of the field ψ to the neutral gauge boson Z is
$$g_A = g_A^{SM}(a^2 - b^2)$$
instead of g_A^{SM} in the SM.

$a = 1$, $b = 0$, $\frac{1}{\sqrt{2L}}\int dz f_+(z) = 1$ if $f(z) = \mathrm{const} \to v(z) = \mathrm{const}$ (non-flat background: $v(z) \to \sim e^{-\sigma}$)

Conclusions

- $e^{\sigma} v(z) \neq \mathrm{const}$
  1. $F_1(z) \neq -F_2(z)$, $F_1(z) \neq F_2(z)$ — pathologies (higher derivatives)
  2. $F_1(z) \equiv F_2(z)$ — deviations from the SM, no localization

- $e^{\sigma} v(z) \equiv \mathrm{const}$, $F_1(z) \equiv -F_2(z)$ — no pathologies, no deviations from the SM, localization,
  but unnatural fine-tuning in the non-flat case!

Thank you for your attention!