Renormalization–group inflationary scenarios confronted with recent observation data

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based on
E. Elizalde, S.D. Odintsov, E.O. Pozdeeva, S.Yu. V.,
Phys. Rev. D **90** (2014) 084001, *arXiv:1408.1285*

QFTHEP'2015, Samara, Russia

Models with scalar fields are very useful to describe the observable evolution of the Universe as the dynamics of the spatially flat Friedmann–Lemaître–Robertson–Walker (FLRW) background with

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There are models of inflation, where the role of the inflaton is played by the Higgs field nonminimally coupled to gravity. (F.L. Bezrukov and M. Shaposhnikov, *Phys. Lett.* B **659** (2008) 703–706, arXiv:0710.3755).

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In my talk I consider the possible inflationary scenarios connected with the quantum field theory.

MODEL WITH NON-MINIMAL COUPLING

Models with nonminimally coupled scalar fields are described by the following action:

$$S = \int d^4x\sqrt{-g}\left[U(\phi)R - \frac{1}{2}g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} - V(\phi)\right], \tag{1}$$

where $U(\phi)$ and $V(\phi)$ are differentiable functions of the scalar field ϕ. We assume that $U(\phi) \geqslant 0$.

In the spatially flat FLRW metric with the interval:
$ds^2 = -\,dt^2 + a^2(t)\left(dx_1^2 + dx_2^2 + dx_3^2\right)$,
we get the following system of equations:

$$6UH^2 + 6\dot{U}H = \frac{1}{2}\dot{\phi}^2 + V \,, \tag{2}$$

$$2U\left(2\dot{H} + 3H^2\right) = -\frac{\dot{\phi}^2}{2} - 2\ddot{U} - 4H\dot{U} + V \,, \tag{3}$$

$$\ddot{\phi} + 3H\dot{\phi} - 6U'\left(\dot{H} + 2H^2\right) + V' = 0 \,. \tag{4}$$

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From Eqs. (2)–(4) we get the following system:

$$\begin{aligned}
\dot{\phi} &= \psi, \\
\dot{\psi} &= -3H\psi - \frac{(6U''+1)U'}{2\,(3U'^2+U)}\psi^2 + \frac{UV'-2VU'}{3U'^2+U}, \\
\dot{H} &= -\frac{2U''+1}{4(3U'^2+U)}\psi^2 + \frac{2U'H\psi}{3U'^2+U} - \frac{6U'^2H^2}{3U'^2+U} + \frac{U'V'}{2(3U'^2+U)}.
\end{aligned} \tag{5}$$

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\end{aligned} \tag{5}$$

Note that equation (2) is not a consequence of system (5).
The system (5) is equivalent to the initial system of equations (2)–(4) if and only if we choose such initial data that equation (2) is satisfied.
In other words, if equation (2) is satisfied in the initial moment of time, then from system (5) it follows that equation (2) is satisfied at any moment of time.

DE SITTER SOLUTIONS

De Sitter solutions corresponds to $\psi = 0$, hence,

$$V'(\phi_{dS})U(\phi_{dS}) = 2V(\phi_{dS})U'(\phi_{dS}).$$

The corresponding Hubble parameter is

$$H_{dS}^2 = \frac{V(\phi_{dS})}{6U(\phi_{dS})} = \frac{V'(\phi_{dS})}{12U'(\phi_{dS})}. \quad (6)$$

I only mention that points with $\psi = 0$ correspond to critical points of the effective potential

$$V_{\mathrm{Eff}} = \frac{V}{U^2},$$

because

$$V'_{\mathrm{Eff}} = \frac{V'U - 2VU'}{U^3}, \qquad V'_{\mathrm{Eff}}(\phi_{dS}) = 0.$$

Also, if $U(\phi_{dS}) > 0$, then the stable de Sitter solutions correspond to minima of V_{Eff}.

Stable and unstable de Sitter solutions

We study the stability of solutions that tend to fixed points for the nonminimally coupled gravity models in FLRW metric.
To consider the stability of the fixed point we use the Lyapunov theorem and consider the corresponding linearize system. Supposing that

$$\phi(t)=\phi_f+\varepsilon\phi_1(t),\qquad \psi(t)=\varepsilon\psi_1(t),\qquad H(t)=H_{dS}+\varepsilon H_1(t),$$

$$\begin{aligned} U &= U_f+\varepsilon U'_f\phi_1(t), & U' &= U'_f+\varepsilon U''_f\phi_1,\\ V &= V_f+\varepsilon V'_f\phi_1(t), & V' &= V'_f+\varepsilon V''_f\phi_1, \end{aligned}$$

and substituting it to (5) we obtain the following linear system:

$$\begin{aligned} \dot{\phi}_1 &= \psi_1,\\ \dot{\psi}_1 &= -3H_{dS}\psi_1+\frac{V'_fU'_f+2V_fU''_f-U_fV''_f}{3(U'_f)^2+U_f}\phi_1,\\ \dot{H}_1 &= \frac{(U'_fV''_f-V'_fU''_f)\phi_1+4H_{dS}U'_f\psi_1-24H_{dS}(U'_f)^2H_1}{2(3(U'_f)^2+U_f)}. \end{aligned} \qquad (7)$$

For the case of a generic $U(\phi)$ we the characteristic equation

$$\det(A-\tilde{\lambda}I) = \left[-\frac{12H_{dS}\left(U_f'\right)^2}{3(U_f')^2+U_f} - \tilde{\lambda}\right]\left[\tilde{\lambda}(3H_{dS}+\tilde{\lambda}) - \frac{V_f'U_f'+2V_fU_f''-U_fV_f''}{3(U_f')^2+U_f}\right] = 0,$$

that has the following roots:

$$\tilde{\lambda}_{\pm} = -\frac{3H_{dS}}{2} \pm \sqrt{\frac{9H_{dS}^2}{4} + \frac{V_f'U_f'+2V_fU_f''-U_fV_f''}{3(U_f')^2+U_f}},$$

$$\tilde{\lambda}_3 = -\frac{12H_{dS}\left(U_f'\right)^2}{3(U_f')^2+U_f}.$$

The de Sitter solution is stable if the real parts of $\tilde{\lambda}_{\pm} < 0$. The real part of $\tilde{\lambda}_-$ is always negative, hence, just $\tilde{\lambda}_+$ defines the stability.

$$\tilde{\lambda}_{+} = -\frac{3H_{dS}}{2} + \sqrt{\frac{9H_{dS}^2}{4} + \frac{V_f'U_f'+2V_fU_f''-U_fV_f''}{3(U_f')^2+U_f}}, \qquad (8)$$

$$K_f \equiv \frac{V_f'U_f'+2V_fU_f''-U_fV_f''}{3(U_f')^2+U_f} = \frac{2\left(\frac{U_f'}{U_f}\right)' - \left(\frac{V_f'}{V_f}\right)'}{\frac{3}{4}\left(\frac{V_f'}{V_f}\right)^2\frac{U_f}{V_f} + \frac{1}{V_f}}. \qquad (9)$$

The de Sitter solution ($H_{dS} > 0$) is stable at $K_f < 0$ and unstable at $K_f > 0$.

OBSERVATION DATA

There are a few main parameters that can be obtained by the observation data:

- The tensor-to-scalar ratio r.
  There was a disagreement in the Planck2013 (+ WMAP) data analysis $r < 0.13$ and the BISEP2 data analysis $r \simeq 0.2$.
  The resulting joint BICEP2+Planck2013 analysis yields that the upper limit of the tensor-to-scalar ratio is $r < 0.11$, a slight improvement relative to the Planck analysis alone, which gives $r < 0.13$ (95% c.l.). Models with $r > 0.14$ are excluded with 99.5% confidence [*M.J. Mortonson and U. Seljak*, JCAP **1410** (2014) 035, **arXiv:1405.5857**]

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$$n_{\rm s} = 0.9603 \pm 0.0073.$$

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  Planck2015:

$$n_{\mathrm{s}} = 0.9655 \pm 0.0063.$$

- The associated running of the spectral index α_{s} should be small.

Predictions of simplest inflationary models will minimally coupled scalar field are in disagreement with the Planck2013 results and the resulting joint BICEP2+Planck2013 analysis.

At the same time many of these inflationary scenarios can be improved by adding a tiny nonminimal coupling of the inflaton field to gravity.

F. Bezrukov, D. Gorbunov, J. High Energy Phys. **1307** (2013) 140, arXiv:1303.4395,

R. Kallosh, A. Linde, J. Cosmol. Astropart. Phys. **1306** (2013) 027, arXiv:1306.3211.

The Planck2013 data analysis as well as the joint BICEP2+Planck2013 analysis confirm the prediction of the Starobinsky R^2 inflationary model and the Bezrukov-Shaposhnikov Higgs-driven inflation.

INFLATIONARY PARAMETERS

Our goal is to construct an inflationary model using the RG-improved potentials and to examine if the inflationary model with this potential is compatible with the Planck13 and BICEP2 data.

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Our goal is to construct an inflationary model using the RG-improved potentials and to examine if the inflationary model with this potential is compatible with the Planck13 and BICEP2 data.
Much of the formalism developed for calculating the parameters of inflation, for example, the primordial spectral index n_s, assume General Relativity models with minimally coupled scalar fields.
It has been shown by D.I. Kaiser in 1994, that in the case of quasi de Sitter expansion there is no difference between spectral indexes calculated either in the Jordan frame directly, or in the Einstein frame after conformal transformation.
The standard way to use this formalism for models with nonminimal coupling is to perform a conformal transformation and to consider the model in the Einstein frame.
See, for example, *F.L. Bezrukov and M. Shaposhnikov*, *Phys. Lett.* B **659** (2008) 703–706, arXiv:0710.3755;
A. De Simone, M.P. Hertzberg and F. Wilczek, Phys. Lett. B **678** (2009) 1 (arXiv:0812.4946)

These two frames are related by conformal transformation

$$g_{\mu\nu} = \Omega^2 g^{(E)}_{\mu\nu}.$$

$$\Rightarrow \quad R = \Omega^{-2}\left[R^{(E)} - 6\left(\Box^{(E)}\ln\Omega + g^{\mu\nu(E)}\nabla^{(E)}_{\mu}\ln\Omega\nabla^{(E)}_{\nu}\ln\Omega\right)\right]$$

$$\text{If} \qquad \Omega^{-2} = 2\kappa^2 U \qquad \rightarrow \qquad \Omega = \frac{1}{\kappa\sqrt{2U}},$$

where $\kappa^2 \equiv 8\pi/M^2_{\rm Pl}$,
$M_{\rm Pl}$ is the Planck mass.

We also introduce a new scalar field φ, such that

$$\frac{d\varphi}{d\phi} = \frac{\sqrt{U + 3U'^2}}{\sqrt{2}\kappa U} \quad \Rightarrow \quad \varphi = \frac{1}{\sqrt{2}\kappa} \int \frac{\sqrt{U + 3U'^2}}{U} d\phi. \qquad (10)$$

We get a model with for a minimally coupled scalar field:

$$S = \int d^4x \sqrt{-g^{(E)}} \left[\frac{1}{2\kappa^2} R^{(E)} - \frac{1}{2} g^{\mu\nu(E)} \varphi_{,\mu} \varphi_{,\nu} + V_{\mathrm{E}}(\varphi) \right], \qquad (11)$$

where

$$V_{\mathrm{E}}(\varphi) = \frac{V(\phi(\varphi))}{4\kappa^4 U^2(\phi(\varphi))}. \qquad (12)$$

Inflationary universe models are based upon the possibility of a slow evolution of some scalar field φ in the potential $V_{\mathrm{E}}(\varphi)$.

The slow-roll approximation, which neglects the most slowly changing terms in the equations of motion, is used.
As known, the slow-roll parameters ϵ, η and ζ are connected with the potential in the Einstein frame as follows:

$$\epsilon \equiv \frac{1}{2\kappa^2}\left(\frac{V'_{\mathrm{E},\varphi}(\varphi)}{V_{\mathrm{E}}(\varphi)}\right)^2, \qquad \eta \equiv \frac{1}{\kappa^2}\frac{V''_{\mathrm{E},\varphi}(\varphi)}{V_{\mathrm{E}}(\varphi)},$$

$$\zeta^2 \equiv \frac{1}{\kappa^4}\frac{V'_{\mathrm{E}}(\varphi)V'''_{\mathrm{E},\varphi}(\varphi)}{V_{\mathrm{E}}(\varphi)^2}.$$

We add the additional subscript $_{,\varphi}$ to denote derivatives with respect to φ.
During inflation, each of these parameters should remain to be less than one.

It is suitable to calculate the slow-roll parameters as functions of the initial scalar field ϕ:
where the prime denotes now derivative with respect to ϕ. We get

$$\epsilon(\phi) = \frac{1}{2\kappa^2}\frac{(V_{\rm E}')^2}{V_{\rm E}^2 Q}\,, \qquad \eta(\phi) = \frac{1}{\kappa^2 V_{\rm E} Q}\left[V_{\rm E}'' - \frac{V_{\rm E}' Q'}{2Q}\right]\,,$$

$$\text{where} \qquad Q = \frac{U + 3U'^2}{2\kappa^2 U^2}.$$

Similar calculations yield

$$\zeta^2 = \frac{V_E'}{\kappa^4 V_{\rm E}^2 Q^2}\left[V_{\rm E}''' - \frac{3V_{\rm E}'' Q'}{2Q} - \frac{V_{\rm E}' Q''}{2Q} + \frac{V_{\rm E}'(Q')^2}{Q^2}\right].$$

The number of e-foldings of a slow-roll inflation is

$$N_e(\phi) = \kappa^2 \int_{\varphi_{\rm end}}^{\varphi} \left| \frac{V_{\rm E}(\tilde{\varphi})}{V'_{{\rm E},\varphi}(\tilde{\varphi})} \right| d\tilde{\varphi} = \kappa^2 \int_{\phi_{\rm end}}^{\phi} \left| \frac{V_{\rm E}}{V'_{\rm E}} \right| Q\, d\tilde{\phi} = \frac{\kappa}{\sqrt{2}} \int_{\phi_{\rm end}}^{\phi} \left(\frac{d\varphi}{d\tilde{\phi}} \right) \frac{d\tilde{\phi}}{\sqrt{\epsilon(\tilde{\phi})}} \,,$$

where $\phi_{\rm end}$ is the value of the field at the end of inflation, defined by $\epsilon = 1$. The number of e-foldings must be matched with the appropriate normalization of the data set and the cosmic history, a typical value being $50 \leqslant N_e \leqslant 65$.

The tensor-to-scalar ratio r, the scalar spectral index of the primordial curvature fluctuations $n_{\rm s}$, and the associated running of the spectral index $\alpha_{\rm s}$, are given, to very good approximation, by

$$r = 16\epsilon \,, \qquad n_{\rm s} - 1 \simeq -6\epsilon + 2\eta \,, \qquad \alpha_{\rm s} \equiv \frac{dn_{\rm s}}{d\ln k} \simeq 16\epsilon\eta - 24\epsilon^2 - 2\zeta^2 \,.$$

We describe the inflationary dynamics for two models that have unstable de Sitter solutions with $U_f > 0$. Note that the existence of an unstable de Sitter solution **is not** *a necessary condition* for inflation.

INFLATIONARY POTENTIALS

The standard potential for nonminimally coupled cosmological models is

$$W^{(0)}(\phi) = a\lambda\phi^4 - b\xi\phi^2 R = V_0 - U_0 R, \tag{13}$$

where a and b are positive constants and ξ is the conformal coupling.
The potential $W^{(0)}$ includes both the potential V_0 and the function U_0 multiplied by the scalar curvature.
The term proportional to $\phi^2 R$ is called in the induced gravity term.

$$V_{\mathrm{E}}^{(0)} = \frac{V_0}{4\kappa^4 U_0^2} = \frac{a\lambda}{4\kappa^4 b^2 \xi^2} = const.$$

This model is not suitable for inflation.

There are models of inflation, where the role of the inflaton is played by the Higgs field nonminimally coupled to gravity. (F.L. Bezrukov and M. Shaposhnikov, *Phys. Lett.* B **659** (2008) 703–706, arXiv:0710.3755). They add $W^{(0)}(\phi)$ to the standard GR term and get the following action:

$$S = \int d^4x \sqrt{-g} \left[\left(\frac{M_{PL}^2}{2} + \xi\phi^2 \right) R - \frac{1}{2}(\partial\phi)^2 - \lambda \left(\phi^2 - \phi_0^2\right)^2 \right].$$

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This model have been actively studied

A.O. Barvinsky, A.Y. Kamenshchik, and A.A. Starobinsky, J. Cosmol. Astropart. Phys. **0811** (2008) 021 (arXiv:0809.2104);

F. Bezrukov, D. Gorbunov and M. Shaposhnikov, J. Cosmol. Astropart. Phys. **0906** (2009) 029, arXiv:0812.3622;

A. De Simone, M.P. Hertzberg and F. Wilczek, Phys. Lett. B **678** (2009) 1 (arXiv:0812.4946);

A.O. Barvinsky, A.Y. Kamenshchik, C. Kiefer, A.A. Starobinsky, and C.F. Steinwachs, J. Cosmol. Astropart. Phys. **0912** (2009) 003 (arXiv:0904.1698);

J. Garcia-Bellido, D.G. Figueroa, and J. Rubio, Phys. Rev. D **79** (2009) 063531 (arXiv:0812.4624);

F. Bezrukov, Class. Quant. Grav. **30** (2013) 214001 (arXiv:1307.0708)

RG-improved potentials

In particular, in
A.O. Barvinsky, A.Yu. Kamenshchik, C. Kiefer, A.A. Starobinsky, and C.F. Steinwachs, Eur. Phys. J. C **72** (2012) 2219 (arXiv:0910.1041)
the renormalization-group improved potentials for this models has been studied.
The renormalization-group improved effective potential for an arbitrary renormalizable massless gauge theory in curved space-time was discussed in detail in
I.L. Buchbinder, S.D. Odintsov, Class. Quant. Grav. **2** (1985) 721–731;
I.L. Buchbinder, S.D. Odintsov and I.M. Lichtzier, Class. Quant. Grav. **6** (1989) 605;
E. Elizalde and S.D. Odintsov, Phys. Lett. B **303** (1993) 240 (arXiv:hep-th/9302074);
E. Elizalde and S.D. Odintsov, Phys. Lett. B **321** (1994) 199–204 (arXiv:hep-th/9311087);
E. Elizalde and S.D. Odintsov, Phys. Lett. B **333** (1994) 331, (arXiv:hep-th/9403132)

The standard flat-space renormalization-group equation is modified in curved space-time, for instance, it has an additional term related with the contribution from the nonminimal coupling constant ξ and the corresponding β_ξ function.
It is natural to split W into two parts, namely

$$W \equiv V - UR \equiv af_1(p, \phi, \mu)\phi^4 - bf_2(p, \phi, \mu)\phi^2 R, \tag{14}$$

where f_1 and f_2 are some unknown functions, and $p = \{\widetilde{g}, \alpha, \xi\}$.
The renormalization-group equation for the effective potential in curved space-time has the form

$$\left(\mu \frac{\partial}{\partial \mu} + \beta_{\widetilde{g}} \frac{\partial}{\partial \widetilde{g}} + \delta \frac{\partial}{\partial \alpha} + \beta_\xi \frac{\partial}{\partial \xi} - \gamma \phi \frac{\partial}{\partial \phi}\right) W = 0, \tag{15}$$

Actually, the authors imposed the additional restriction that, not only the function W satisfies (15), but also that the functions V and U satisfy it, separately.

The RG-improved potential for the scalar electrodynamic, the $SU(2)$ and $SU(5)$ models have been found in *E. Elizalde and S.D. Odintsov*, Phys. Lett. B **303** (1993) 240 (arXiv:hep-th/9302074).
We use these potentials to check the possibility to construct the inflationary models without the Hilbert–Einstein curvature term $\frac{M_{PL}^2}{2}R$.

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In arXiv:1408.1285, we show that inflation is realized both for scalar electrodynamics and for $SU(5)$ RG-improved potentials.

I plan to show in detail that for the $SU(5)$ RG-improved potential the corresponding inflationary models are in good agreement with the most recent observational data provided some reasonable values are taken for the parameters.

The $SU(5)$ model in non-flat metric

E. Elizalde and S.D. Odintsov, Phys. Lett. B **303** (1993) 240 (arXiv:hep-th/9302074)

We study the RG-improved potential for the $SU(5)$ GUT.

The tree-level potential has the form

$$V_{\text{tree}} = \frac{1}{4}\lambda_1(\text{Tr}\,\bar{\phi}^2)^2 + \frac{1}{2}\lambda_2\text{Tr}\,\bar{\phi}^4 - \frac{1}{2}\xi R\text{Tr}\,\bar{\phi}^2,$$

where λ_1 and λ_2 are scalar couplings.

For simplicity we suppose that there are no fermions in the theory.

Even in this case, the system of RG equations for the coupling constants can be solved only numerically.

This is why *E. Elizalde and S.D. Odintsov* considered the vector loop contributions to the β-functions.

The breaking $SU(5) \to SU(3) \times SU(2) \times U(1)$ has taken place.

Then $\bar{\phi} = \phi$ diag $(1, 1, 1, -\frac{3}{2}, -\frac{3}{2})$ and

$$V_{\text{tree}} = \frac{15}{16}(15\lambda_1 + 7\lambda_2)\,\phi^4 - \frac{15}{4}\xi R\phi^2.$$

The $SU(5)$ RG-IMPROVED POTENTIAL

Within this approach

$$\frac{dg(\vartheta)}{d\vartheta} = -\frac{5g^3(\vartheta)}{6\pi^2}, \quad \frac{d}{d\vartheta}\left[\frac{15}{4}(15\lambda_1 + 7\lambda_2)\right] = \frac{5625}{128\pi^2} g^4(\vartheta),$$

$$\frac{d\xi(\vartheta)}{d\vartheta} = -\frac{30}{16\pi^2}\left(\xi(\vartheta) - \frac{1}{6}\right) g^2(\vartheta), \quad \gamma = -\frac{15g^2}{16\pi^2},$$

$\vartheta = \frac{1}{2}\ln(\phi^2/\mu^2)$.

The $SU(5)$ RG-improved potential has been calculated in *E. Elizalde, S.D. Odintsov, 1993*:

$$V = \frac{3375}{512}\left(g^2 - \frac{g^2}{f_5^{16/9}}\right)\phi^4 f_5^4, \qquad U = \frac{15}{4}\left[\frac{1}{6} + \left(\xi - \frac{1}{6}\right)\breve{\Theta}^{-9/8}\right]\phi^2 f_5^2,$$

where $\breve{\Theta} = 1 + \frac{5g^2\vartheta}{3\pi^2}$, $f_5 = \breve{\Theta}^{9/16}$,
g and μ^2 are nonzero constants.

We have found that there is no de Sitter solution for $\xi = 1/6$.
For other values of ξ, the de Sitter solutions are defined by

$$H_{dS}^2 = \frac{225\phi_f^2 \breve{\Theta}_f^{5/4} g^2 (\breve{\Theta}_f - 1)}{128(\breve{\Theta}_f^{9/8} + 6\xi - 1)}.$$

The number $\breve{\Theta}_f$ is a root of

$$\xi = \frac{1}{6} - \frac{2\breve{\Theta}_f^{9/8}}{3(9\breve{\Theta}_f - 5)}.$$

We can eliminate ξ and express H_{dS}^2 as

$$H_{dS}^2 = \frac{25}{128} \breve{\Theta}_f^{1/8} (9\breve{\Theta}_f - 5)\phi_f^2 g^2.$$

The Hubble parameter H is real if and only if $\breve{\Theta}_f \geqslant 5/9$.
It is possible for $\xi < 1/6$ only.

At the de Sitter point we get

$$U_f = \frac{15}{4}\left(\frac{1}{6} - \frac{2}{3(9\breve{\Theta}_f - 5)}\right)\phi_f^2 \breve{\Theta}_f^{9/8} > 0. \tag{16}$$

for $1 < \breve{\Theta}_f$.

$$V_f = \frac{3375}{512} g^2 \left(1 - \breve{\Theta}_f^{-1}\right) \phi_f^4 \breve{\Theta}_f^{9/4}.$$

We see that $U_f < 0$ at $5/9 < \breve{\Theta}_f < 1$ and $U_f > 0$ for $1 < \breve{\Theta}_f$.

Let us consider the stability of the de Sitter solutions obtained. Note that we used the conditions $\breve{\Theta}_f \neq 1$:

$$2\left(\frac{U'}{U}\right)'|_{\phi=\phi_f} - \left(\frac{V'}{V}\right)'|_{\phi=\phi_f} = \frac{(5-\breve{\Theta}_f)(\breve{\Theta}'_f)^2}{8(\breve{\Theta}_f-1)^2\breve{\Theta}_f^2}\,. \qquad (17)$$

For $1 < \breve{\Theta}_f$, $U_f > 0$ and $V_f > 0$, so the denominator of K_f calculated by (9) is positive, and thus the sign of K_f can be determined by the numerator that was calculated in (17).

We come to the conclusion that

$$K_f > 0 \text{ for } 1 < \breve{\Theta}_f < 5$$

and

$$K_f < 0 \text{ for } 5 < \breve{\Theta}_f.$$

- So, the de Sitter solution is unstable for $U_f > 0$ at $1 < \breve{\Theta}_f < 5$.

In the case of a $SU(5)$ RG-improved potential,

$$V_{\mathrm{E}} = \frac{135 g^2 (\breve{\Theta} - 1)\breve{\Theta}^{5/4}}{32\kappa^4 \left(\breve{\Theta}^{9/8} + 6\xi - 1\right)^2},$$

$$Q = \frac{4(\breve{\Theta}^{9/8} + 6\xi - 1) + \frac{15}{128\pi^2}\left(15 g^2 \breve{\Theta}^{1/8} + 16\pi^2\left(\breve{\Theta}^{9/8} + 6\,\xi - 1\right)\right)^2}{5\left(\breve{\Theta}^{9/8} + 6\,\xi - 1\right)^2 \kappa^2 \phi^2}.$$

In the case of a $SU(5)$ RG-improved potential,

$$V_{\rm E} = \frac{135 g^2 (\breve{\Theta} - 1)\breve{\Theta}^{5/4}}{32\kappa^4 \left(\breve{\Theta}^{9/8} + 6\xi - 1\right)^2},$$

$$Q = \frac{4(\breve{\Theta}^{9/8} + 6\xi - 1) + \frac{15}{128\pi^2}\left(15 g^2 \breve{\Theta}^{1/8} + 16\pi^2\left(\breve{\Theta}^{9/8} + 6\,\xi - 1\right)\right)^2}{5\left(\breve{\Theta}^{9/8} + 6\,\xi - 1\right)^2 \kappa^2\phi^2}.$$

The slow-roll parameter

$$\epsilon = \frac{\frac{125}{288\pi^4} g^4 \left(4\,\breve{\Theta}^{9/8} - 5(6\xi - 1) + 9\breve{\Theta}(6\xi - 1)\right)^2}{\left(\breve{\Theta} - 1\right)^2 \breve{\Theta}^2 \left[4\left(\breve{\Theta}^{9/8} + 6\,\xi - 1\right) + \frac{15}{128\pi^2}\left(15\,g^2\breve{\Theta}^{9/8} + 16\pi^2\left(\breve{\Theta}^{9/8} + 6\xi - 1\right)^2\right)\right]},$$

$$N_e = \int_{\breve{\Theta}_{end}}^{\breve{\Theta}_N} \frac{\left(\breve{\Theta} - 1\right)\breve{\Theta}\left(4\left(\breve{\Theta}^{\frac{9}{8}} + 6\xi - 1\right) + \frac{15}{128}\left(\frac{15\,\breve{\Theta}^{\frac{1}{8}} g^2}{\pi^2} + 16(\breve{\Theta}^{\frac{9}{8}} + 6\xi - 1)\right)^2\right)}{\frac{125}{36\pi^4} g^4 \left(\breve{\Theta}^{\frac{9}{8}} + 6\,\xi - 1\right)\left((9\,\breve{\Theta} - 5)(6\,\xi - 1) + 4\,\breve{\Theta}^{\frac{9}{8}}\right)} d\breve{\Theta}\,.$$

The slow-roll parameters and N_e depend on the dimensionless function $\breve{\Theta}$.

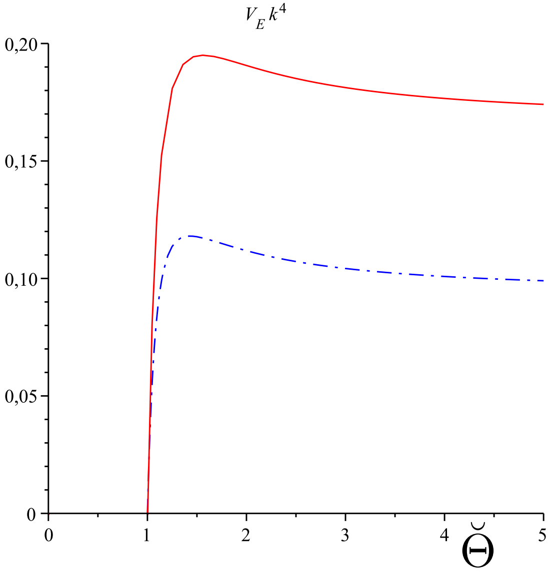

Figure: The potential $V_{\rm E}(\breve{\Theta})$ multiplied by κ^4 in the $SU(5)$ model at $\xi = 0.04$, $g = 0.15$ (blue dashed line) and at $\xi = 0.045$, $g = 0.2$ (red solid line).

Table: Parameter values for the $SU(5)$ inflationary scenario.

ξ	g	N	$\breve{\Theta}_{end}(\epsilon = 1)$	$\breve{\Theta}_N$	n_s	r	α
0.04	0.15	50	1.000868906	1.0121	0.963	0.070	0.00731
0.04	0.15	55	1.000868906	1.0126	0.965	0.063	0.00643
0.04	0.15	60	1.000868906	1.0132	0.968	0.058	0.00660
0.04	0.15	65	1.000868906	1.0137	0.969	0.0535	0.00540
0.045	0.2	50	1.001564816	1.02152	0.958	0.066	0.00699
0.045	0.2	55	1.001564816	1.02252	0.960	0.0595	0.00638
0.045	0.2	60	1.001564816	1.023475	0.963	0.054	0.00579
0.045	0.2	65	1.001564816	1.024388	0.965	0.0495	0.00548

The resulting joint BICEP2+Planck2013 analysis yields that the upper limit of the tensor-to-scalar ratio is $r < 0.11$, a slight improvement relative to the Planck analysis alone, which gives $r < 0.13$ (95% c.l.). We do see that the inflationary parameters of the model considered are in very good agreement with the observational data.

CONCLUSIONS

- Cosmological models with nonminimally coupling scalar fields has been considered.
- We study dynamics of nonminimally coupled scalar field cosmological models with the RG-improved potentials.
- In all cases, the tree-level potential is $\lambda\phi^4 - \xi\phi^2 R$, what corresponds to the cosmological constant in the Einstein frame, and is in no case suitable for inflation.
- In the inflationary models, both for scalar electrodynamics and the $SU(5)$ RG-improved potentials, we have got that these models are in good agreement with the most recent observational data provided some reasonable values are taken for the parameters.
- Our study indicates that inflation could well be caused by quantum effects of the scalar sector of some convenient GUT theory.