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Static electromagnetic moments of the ρ-meson in the instant form of relativistic quantum mechanics

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Experimental information

K.A. Olive (ParticleData Group). Chin.Phys. C. 38, 2014, 090001.

$$f_\rho^{exp} = 221 \pm 1\,MeV$$

The problems to construct matrix element of current, according to the conditions of covariant and conservation

1. E.P. Briernat, W. Schweiger. Phys. Rev. C. 89, 2014, 055205.

2. J.P.B.C.de Melo, A.N. da Silva, C.S. Melo, T. Frederico. ArXiv:1504.07175v1, 2015, 4.

3. G.H.S. Yabusaki, I. Ahmed, M.Ali Paracha, J.P.B.C.de Melo, Bruno EL-Bennich. Phys. Rev. D, 2015, 8.

The description of the processes with spin-diagonal matrix element

$$\langle \vec{p}_c\,, m_{Jc} | j_\mu(0) | \vec{p}_c\,'\,, m'_{Jc} \rangle = \langle\, m_{Jc} |\, D^1(p_c\,, p'_c) \sum_{i=1,3} \tilde{\mathcal{F}}\,^i_c(t)\, \tilde{A}^i_\mu\, | m'_{Jc} \rangle.$$

$$\tilde{\mathcal{F}}\,^1_c(t) = \tilde{f}^c_{10} + \tilde{f}^c_{12} \Big\{ [ip_{c\nu}\,\Gamma^\nu(p'_c)]^2 - \qquad (1)$$

$$-\frac{1}{3}\,\mathrm{Sp}[ip_{c\nu}\,\Gamma^\nu(p'_c)]^2 \Big\} \frac{2}{\mathrm{Sp}[p_{c\nu}\,\Gamma^\nu(p'_c)]^2}, \quad \tilde{\mathcal{F}}\,^3_c(t) = \tilde{f}^c_{30}.$$

$$\tilde{A}^1_\mu = (p_c + p'_c)_\mu, \quad \tilde{A}^3_\mu = \frac{i}{M_c} \varepsilon_{\mu\nu\lambda\sigma}\, p^\nu_c\, p'_c{}^\lambda\, \Gamma^\sigma(p'_c), \quad t = -Q^2$$

Electromagnetic form factors of the rho-meson

$$G_C(Q^2) = \int d\sqrt{s}\, d\sqrt{s'}\, \varphi(s)\, g_{0C}(s\,, Q^2\,, s')\, \varphi(s'),$$

$$G_Q(Q^2) = \frac{2\, M_c^2}{Q^2} \int d\sqrt{s}\, d\sqrt{s'}\, \varphi(s)\, g_{0Q}(s\,, Q^2\,, s')\, \varphi(s'), \qquad (1.1)$$

$$G_M(Q^2) = -M_c \int d\sqrt{s}\, d\sqrt{s'}\, \varphi(s)\, g_{0M}(s\,, Q^2\,, s')\, \varphi(s').$$

$g_{0C}(s\,, Q^2\,, s')$ ***Free electric formfactor***

$g_{0Q}(s\,, Q^2\,, s')$ ***Free quadrupole formfactor***

$g_{0M}(s\,, Q^2\,, s')$ ***Free magnetic formfactor***

$$M_c \;=\; 763.0 \pm 1.3 \;\; \text{MeV}$$

Wave functions

$$\varphi(k(s)) = \sqrt[4]{s}\, u(k)\, k, \quad k = \frac{1}{2}\sqrt{s - 4M^2} \tag{2.1}$$

$$\int u^2(k)\, k^2\, dk = 1, \tag{2.2}$$

Model wave functions

$$u(k) = N_{HO} \exp\left(-k^2/2\,b^2\right) \tag{2.3}$$

$$u(k) = N_{PL}\,(k^2/b^2 + 1)^{-n}, \quad n = 2, 3. \tag{2.4}$$

$$u(r) = N_T\, \exp(-\alpha r^{3/2} - \beta r), \quad \alpha = \frac{2}{3}\sqrt{2\, M_r\, a}, \quad \beta = M_r\, b, \tag{2.5}$$

The parameters of constituent quarks

$$G_E^q(Q^2) = e_q\, f_q(Q^2), \quad G_M^q(Q^2) = (e_q + \kappa_q)\, f_q(Q^2)$$

Sachs form factor

$$f_q(Q^2) = \frac{1}{1 + \ln(1 + \langle r_q^2 \rangle Q^2/6)}, \quad \langle r_q^2 \rangle \simeq 0.3/M^2$$ [1]

$$\bar{M} = 0.25 \ \text{GeV} \qquad \kappa_u + \kappa_{\bar{d}} = 0.09$$ [2]

quark magnetic moments

[1] A.F. Krutov and V.E. Troitsky, Teor. Mat. Fiz. 1998, 116- 215.

[2] S.B Gerasimov. Phys. Lett. B 357 1995, 666–670.

The parameters of constituent quarks

$$\langle r_\rho^2 \rangle = -6\, G'_C(0) \quad G_Q(0) = M_c^2\, Q_\rho \quad G_M(0) = \frac{M_c}{M}\, \mu_\rho \quad (3)$$ [3]

$$\langle r_\pi^2 \rangle^{1/2} = 0.657 \pm 0.012 \ \text{fm}$$

$$\langle r_\rho^2 \rangle - \langle r_\pi^2 \rangle = 0.11 \pm 0.06 \ \text{fm}^2 \quad (4)$$ [4]

3 R.G. Arnold, C.E. Carlson, and F. Gross, Phys. Rev. C 21, 1980, 1426.

4 F. Cardarelli, I.L. Grach, I.M. Narodetskii, E. Pace, G. Salm´e, and S. Simula, Phys. Rev. D 53, 1996, 6682.

Relativistic static moment of rho-meson

$$\mu_\rho = \frac{1}{2}\int_{2M}^{\infty} d\sqrt{s}\,\frac{\varphi^2(s)}{\sqrt{s-4M^2}}\left\{1 - L(s) + (\kappa_u + \kappa_{\bar{d}})\left[1 - \frac{1}{2}L(s)\right]\right\},$$

$$Q_\rho = -\frac{M}{2}\int_{2M}^{\infty} d\sqrt{s}\,\frac{\varphi^2(s)}{\sqrt{s}}\left[\frac{M}{\sqrt{s}+2M} + \kappa_u + \kappa_{\bar{d}}\right]\frac{L(s)}{2M^2\sqrt{s-4M^2}},\qquad (5)$$

$$L(s) = \frac{2M^2}{\sqrt{s-4M^2}\,(\sqrt{s}+2M)}\left[\frac{1}{2M^2}\sqrt{s(s-4M^2)} + \ln\frac{\sqrt{s}-\sqrt{s-4M^2}}{\sqrt{s}+\sqrt{s-4M^2}}\right]$$

Without spin rotation

$$\tilde{\mu}_\rho = \frac{1}{2}(1+\kappa_u+\kappa_{\bar{d}}) \int_{2M}^{\infty} d\sqrt{s}\, \frac{\varphi^2(s)}{\sqrt{s-4M^2}} \left\{1-\frac{1}{2}L(s)\right\} \quad (6)$$

$$\mu_{\rho\,NR} = 1+\kappa_u+\kappa_{\bar{d}}.$$

MODEL	b, a	$\langle r^2_{NR}\rangle$	$\langle \tilde{r}^2\rangle$	μ_ρ	$\tilde{\mu}_\rho$	Q_ρ
(2.3)	0.231	0.275	0.731	0.852	0.966	-0.0065
(2.4) n=2	0.302	0.319	0.711	0.864	0.972	-0.0059
(2.4) n=3	0.430	0.305	0.710	0.866	0.973	-0.0061
(2.5)	0.028	0.301	0.711	0.865	0.973	-0.0061

The decay constant of rho-meson

$$\langle 0|j_\mu(0)|\vec{P}, M_\rho, 1, m\rangle = \frac{i}{(2\pi)^{3/2}} \frac{f_\rho M_\rho \varepsilon_\mu}{\sqrt{2(\vec{P}^2 + M_\rho^2)}} \quad (7)$$ [5]

$$\varepsilon_\mu(\pm 1) = \frac{1}{\sqrt{2}}(0, \mp 1, -i, 0) \qquad \varepsilon_\mu(0) = (0, 0, 0, 1)$$

Using non-digional parametrization for matrix element (7) , get (8)

$$f_\rho = \frac{1}{\pi\sqrt{2}} \int_0^\infty k^2 \psi(k) dk \frac{(\sqrt{k^2 + M^2} + M)}{\sqrt{(2\sqrt{k^2 + M^2})(k^2 + M^2)}} \times \left(1 + \frac{k^2}{3(\sqrt{k^2 + M^2} + M)^2}\right)$$

$$M = 0.25 \ \text{GeV}$$

$$f_\rho = 219\, MeV \qquad f_\rho^{exp} = 221 \pm 1\, MeV$$

for wave function (2.3)

[5] V.V. Andreev. Physics of Particles and Nuclei. V. 8, 2011, 596

Conclusions

- ✓ ***Matrix element of the electromagnetic current was constructed for the diagonal case in the instant form of RQM in the impulse approximation.***

- ✓ ***The electromagnetic current satisfies the conservation law and Lorentz-covariance conditions.***

- ✓ ***Analytic expression was obtained for the relativistic static moment and decay constant of rho meson.***

- ✓ ***Numerical calculation of the decay constant shows good agreement with experiment.***