The XXII International Workshop
High Energy Physics and Quantum Field Theory
June 24– July 1, 2015
Samara, Russia

Radiative decays V → P γ* in the instant form of relativistic quantum mechanics

Krutov A.F., **Polezhaev R.G.**, (Samara State University)
Troitsky V.E., (SINP, Moscow State University)

Problems description of bound state

1. Quantum chromodynamics (QCD) gives a reliable description of the so-called "hard" processes (at large momentum transfers). In this regard, it is necessary to consider different options for composite quark models.[1,2,3]

2. The construction of matrix element of the current according to the conditions of covariance and conservation.[1,2,3]

[1] A.V. Anisovich, V.V. Anisovich, L.G. Dakhno, M.A. Matveev, V.A. Nikonov, A.V. Sarantsev. Phys.Atom.Nucl. Vol. 73, 2010, 462-477.

[2] Shan Cheng, Zhen-Jun Xiano. Phys.Rev. D. 90, 2014, 076001.

[3] Jianghao Yu, Bo-Wen Xiano, Bo-Qian Ma. J.Phys. G. 34, 2007, 1845-1860.

Relativistic quantum mechanics [4]

- The construction of the electromagnetic current operator with the conditions of current conservation and Lorentz – covariance.
- Good description of the meson electroweak properties of meson and deuteron in the frame of instant form of dynamics.
- Asymptotic of the electromagnetic form factors at $Q^2 \to \infty$ coinciding with predictions QCD and quark counting rules.
- The impulse approximation does not violate the conditions of covariates and current conservation .

[4] A.F. Krutov, V.E. Troitsky. Physics of Particles and Nuclei. Vol. 40, 2009, 136

The construction basis in the frame of instant form of relativistic quantum mechanics

In one-particle basis:

$$|\vec{p}, M, j, m\rangle \qquad \langle \vec{p_1}, m|\vec{p_1}', m'\rangle = 2p_0\delta(\vec{p_1} - \vec{p_1}')\delta_{mm'} \tag{1}$$

In two-particle basis:

$$|\vec{P}, \sqrt{s}, J, L, S, m\rangle \qquad \vec{P} = \vec{p_1} + \vec{p_2}; \tag{2}$$

$$\langle \vec{P}, \sqrt{s}, J, L, S, m|\vec{P}\,', \sqrt{s'}, J, L', S', m'\rangle = N\delta(\vec{P} - \vec{P}\,')\delta(\sqrt{s} - \sqrt{s'})\delta_{mm'}\delta_{LL'}\delta_{SS'}$$

$$N = \frac{(2P_0)^2}{8\,k\,\sqrt{s}} \qquad k = \frac{\sqrt{\lambda(s,M_1^2,M_2^2)}}{2\sqrt{s}} \qquad \lambda(a,b,c) = a^2 + b^2 + c^2 - 2(ab + ac + bc)$$

The description of the processes with spin-diagonal matrix element

$$\langle \vec{P}, \sqrt{s}, J, l, S, m_J | j_\mu^{(0)} | \vec{P}', \sqrt{s'}, J, l', S', m'_J \rangle =$$

$$= \sum_{m''_J} \langle m_J | D^J(P, P') | m''_J \rangle \langle m''_J | \sum_{i=1}^{3} \left\{ F_i^{ll'SS'} A_\mu^i(s, Q^2, s') \right\}_+ | m'_J \rangle$$

$$F_i^{ll'SS'} = \sum_{n=0}^{2J} f_{in}^{ll'SS'}(s\,,\,Q^2\,,\,s')(iP_\mu \Gamma^\mu(P'))^n$$

$$F_\pi(Q^2) = \int\int d\sqrt{s}\, d\sqrt{s'}\, g_0(s, Q^2, s')\varphi(s)\varphi(s') \qquad (3)$$

modified impulse approximation

Free electromagnetic form factor with (J=J'= L=L'= S=S' =0)

$$g_0(s, Q^2, s') = \frac{(s + s' + Q^2)^2 Q^2 [\vartheta(s' - s_1) - \vartheta(s' - s_2)]}{2\sqrt{s - 4M^2}\sqrt{s' - 4M^2}[\lambda(s, Q^2, s')]^{3/2}}$$

$$\left[\cos(\omega_1 + \omega_2) f_{10}(Q^2) - 2M\xi(s, s', Q^2)\sin(\omega_1 + \omega_2) f_{30}(Q^2))\right] \qquad (4)$$

The invariant parameterization of the e.m. current for the two particles system (non-diagonal case)

Breit system

$$\tilde{P} = (\tilde{P}_0, \vec{q}) \ , \quad \tilde{P}' = (\tilde{P}'_0, -\vec{q}) \qquad K'_\mu = (\sqrt{K'^2}, 0, 0, 0)$$

$$q = \sqrt{\lambda(M_1^2, M_2^2, Q^2)/[8(M_1^2 + M_2^2) + 4Q^2]} \qquad (5)$$

$$\rho \longrightarrow \pi\gamma^*$$

$$\langle \vec{P}, \sqrt{s} | j_0(0) | \vec{P}\,', \sqrt{s'}, 1, 0, 1, m' \rangle =$$

$$= \sum_{\tilde{m}', l', k'} D^1_{m', \tilde{m}'}(P', w) \langle 1 \tilde{m}' l' k' | 00 \rangle \, Y_{l'k'}(\vec{q}) \, G_{01}^{0, l'}(s, Q^2, s') \qquad (6)$$

The invariant parameterization of the e.m. current for the two particles system (non-diagonal case)

the transition to the canonical basis[5]

$$j^0{}_r(0) = a_{rt} j^{0}{}^{1}_{t}(0) \quad \Longrightarrow \quad a_{rt} = \sqrt{\frac{2\pi}{3}} \begin{pmatrix} -1 & 0 & 1 \\ i & 0 & i \\ 0 & \sqrt{2} & 0 \end{pmatrix} \qquad (7)$$

$$t = -1, 0, 1$$

$$\langle \vec{P}, \sqrt{s} | j^1_t(0) | \vec{P}\,', \sqrt{s'}, 1, 0, 1, m' \rangle =$$

$$= \sum_{\tilde{m}', l, k, j, n} D^1_{m', \tilde{m}'}(P', w) \langle 1 \tilde{m}' j n | 00 \rangle \langle 1 t l k | j n \rangle \, Y_{lk}(\vec{q}) \, G^{1,l,j}_{01}(s, Q^2, s') \qquad (8)$$

[5] A.R. Edmonds. Geneva, 1955

Free non-diagonal form factors

In one-particle basis:

$$\langle \vec{P}, \sqrt{s}|j^0_\mu(0)|\vec{P}\,', \sqrt{s'}, 1, 0, 1, m'\rangle = \int \frac{d^3\vec{p_1}}{2p_{10}}\frac{d^3\vec{p_2}}{2p_{20}}\frac{d^3\vec{p_1}\,'}{2p'_{10}}\frac{d^3\vec{p_2}\,'}{2p'_{20}} \cdot$$

$$\cdot\langle \vec{P}, \sqrt{s}|\vec{p_1}, m_1; \vec{p_2}, m_2\rangle \, \langle \vec{p_1}, m_1; \vec{p_2}, m_2|j^0_\mu(0)|\vec{p_1}\,', m'_1; \vec{p_2}\,', m'_2\rangle \cdot \qquad (9)$$

$$\cdot\langle \vec{p_1}\,', m'_1; \vec{p_2}\,', m'_2|\vec{P}\,', \sqrt{s'}, 1, 0, 1, m'\rangle \,,$$

$$\langle \vec{p_1}, m_1; \vec{p_2}, m_2|j^0_\mu(0)|\vec{p_1}\,', m'_1; \vec{p_2}\,', m'_2\rangle = \langle \vec{p_1}, m_1|j^0_{\mu 1}(0)|\vec{p_1}\,', m'_1\rangle \cdot \qquad (10)$$

$$\cdot\delta(\vec{p_2} - \vec{p_2}\,')\,\delta_{m_2 m_{2'}} + \langle \vec{p_2}, m_2|j^0_{\mu 2}(0)|\vec{p_2}\,', m'_2\rangle\delta(\vec{p_1} - \vec{p_1}\,')\,\delta_{m_1 m_{1'}} \,.$$

$$\langle \vec{p}, m|j^0_\mu(0)|\vec{p}\,', m'\rangle = \sum_{m''} D^{1/2}_{m,m''}(p,p')\langle m''|[f_{10}(Q^2)K_\mu + i f_{30}(Q^2)R_\mu]|m'\rangle \qquad (11)$$

$$K'_\mu = (p+p')_\mu \,, \quad K_\mu = (p-p')_\mu \,, \quad R_\mu = \varepsilon_{\mu\nu\lambda\rho}p^\nu p'^\lambda \Gamma^\rho(p') \qquad (12)$$

Form factor of the free system

$$G_{01}^{111}(s,Q^2,s') = \frac{\Theta(s,Q^2,s')(s+s'+Q^2)^2}{\sqrt{2}\sqrt{s-4M^2}\sqrt{s'-4M^2}\sqrt{4M^2+Q^2}[\lambda(s,-Q^2,s')]^{1/2}} \cdot$$
$$\cdot \cos(\omega_1+\omega_2)/2 \left(\frac{s'(s'-s+3Q^2)}{[\lambda(s,-Q^2,s')]^{1/2}} (G_M^u(Q^2) + G_M^{\bar{d}})(Q^2) \right) +$$
$$+\sin(\omega_1+\omega_2)/2 \left(\frac{(s'-s-Q^2)}{(s+s'+Q^2)} \frac{\xi(s,s',Q^2)}{\sqrt{s'}} (G_M^u(Q^2) + G_M^{\bar{d}}(Q^2)) - \right.$$
$$\left. -\xi(s,s',Q^2) \frac{4M}{(s+s'+Q^2)} (G_E^u(Q^2) + G_E^{\bar{d}}(Q^2)) \right), \quad (13)$$

Form factor of the composite system

$$G_{0,1}^{1,1,1}(Q^2) = \int\int d\sqrt{s}\, d\sqrt{s'}\, G_{0,1}^{1,1,}(s,Q^2,s')\varphi_\pi(s)\varphi_\rho(s') \quad (14)$$

The variational method

$$\hat{M}_I \psi = M_c \psi \qquad \hat{M}_I = \hat{M}_0 + \hat{V} \tag{15}$$

$$\varphi(s) = \sqrt[4]{s} k \psi(k)$$

$$V(r) = -\frac{4}{3}\frac{\alpha_c}{r} + \beta r + \Lambda + \frac{a_c}{m_q m_Q}\left(\vec{S}_q \vec{S}_Q\right) \delta^3(\vec{r}) \tag{16}$$

$$\psi(k) = \frac{2}{b^{3/2}\pi^{1/4}} \exp\left(\frac{-k^2}{2b^2}\right) \tag{17}$$

Harmonic oscillation wave function

$$\frac{dM_c}{db} = 0, \frac{d^2 M_c}{db^2} > 0 \tag{18}$$

Transition form factors measured in experiment

$$\langle \vec{P}_\pi | j^c_\mu(0) | \vec{P}_\rho, 1, m_\rho \rangle = F_{\pi\rho}(Q^2)\varepsilon_{\mu\nu\sigma\delta}\xi^\nu(m_\rho)P^\sigma_\pi P^\delta_\rho \quad \xi^\nu(m_\rho) = -\frac{1}{\sqrt{2}}(0,0,1,i) \qquad (19)$$

$$\langle \vec{\tilde{P}}_\pi | \tilde{j}^c_1(0) | \vec{\tilde{P}}_\rho, 1, \tilde{m}_\rho \rangle = -\frac{q(\tilde{P}^0_\pi + \tilde{P}^0_\rho)}{\sqrt{2}} F_{\pi\rho}(Q^2) \qquad (20)$$

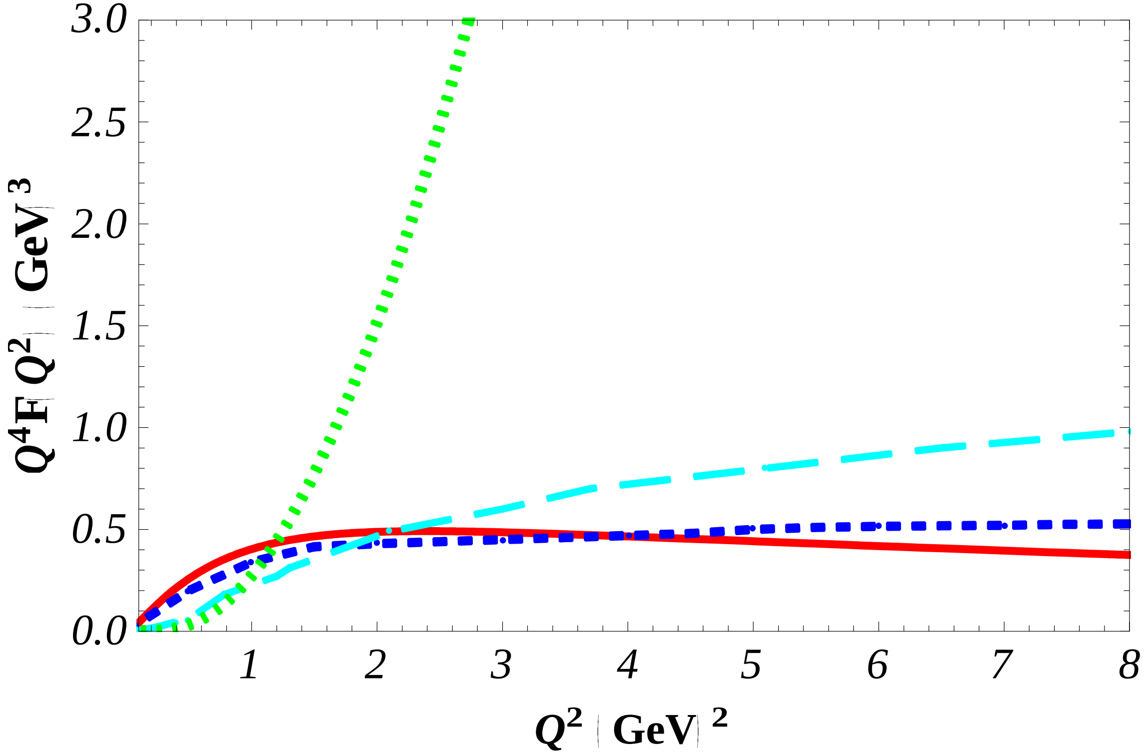
$$\langle \vec{\tilde{P}}_\pi | j^c_1(0) | \vec{\tilde{P}}_\rho, 1, \tilde{m}_\rho \rangle = -\frac{1}{\sqrt{3}} G^{111}_{01}(Q^2) \qquad (20.1)$$

$$F_{\pi\rho}(Q^2) = \sqrt{\frac{2}{3}} \cdot \frac{G^{111}_{01}(Q^2)}{q\left(\sqrt{M^2_\pi + q^2} + \sqrt{M^2_\rho + q^2}\right)} \qquad (21)$$

Numerical calculation transition form factor

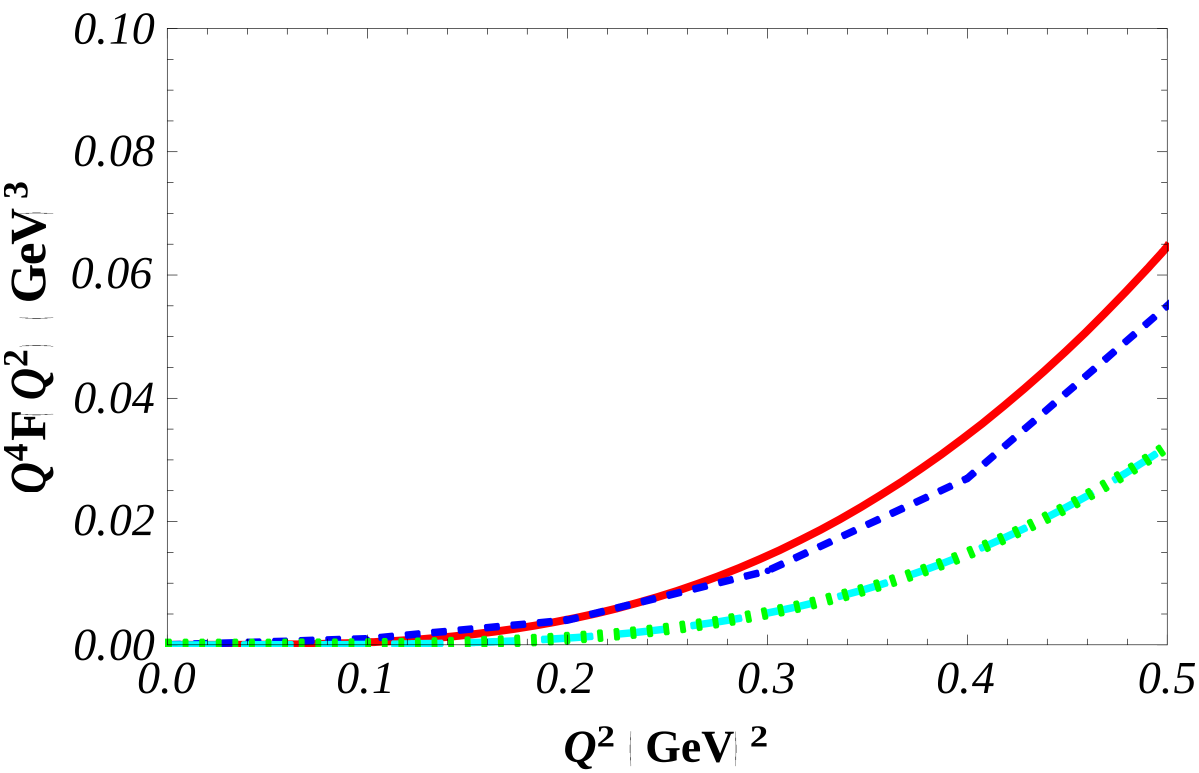
$\rho \rightarrow \pi\gamma^*$

Solid line – our model
Green and blue dashed line- LFQM[6,7]
Dotted line – VMD model[7]



[6] F. Cardarelli, I.L. Grach, I.M. Narodetskii, G.Salme, S. Simula. Phys.Lett. B 359, 1995, 1;
[7] J. Yu, Bo-Wen Xiao, Bo-Qiang Ma, J. Phys. G 34, 2007, 1845.

Numerical calculation transition form factor



$$F_{\pi\,\rho}(0) = 0.736\,\mathrm{GeV}^{-1}$$

experiment [8]

$$\mu_{\pi\rho} = F_{\gamma^*\pi\to\rho}(0) = (0.733 \pm 0.038)\,\mathrm{GeV}^{-1}$$

[8] W.N. Yao (Particle Data Group). J.Phys.G: Nucl.Part. Phys, Vol. 35, 2006, 1

Conclusions

- ✓ ***Matrix element of the electromagnetic current was constructed for the non-diagonal case in the instant form of RQM in the impulse approximation.***

- ✓ ***The electromagnetic current satisfies the conservation law and Lorentz-covariance conditions.***

- ✓ ***Analytic expression was obtained for the transition form factors in the radiative decay rho meson.***

- ✓ ***Numerical calculation of the transition form factor shows good agreement with other approaches at small momentum transfers (is the matching principle).***