Time-dependent Wigner inequalities in QFT and elementary particle physics

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Talk is based on the following works:
1) N. Nikitin, K. Toms, "Wigner's inequalities in quantum field theory", Phys. Rev. A 82, 032109 (2010);
2) N. Nikitin, V. Sotnikov, K. Toms, "Time-dependent Bell inequalities in a Wigner form", Phys. Rev. A 90, 042124 (2014);
3) N. Nikitin, V. Sotnikov, K. Toms, "Experimental test of the time-dependent Wigner inequalities for neutral pseudoscalar meson systems", arXiv:1503.05332 (Accepted in Phys. Rev. D).

Bohr's complementarity principle

One of the main questions of quantum physics, existing both in non-relativistic quantum mechanics (NQM) and in quantum field theory (QFT), is may or may not the physical characteristics of a micro-system exist independently of a measurement procedure and/or design of a macro-device, i.e. to be the "elements of reality" by A. Einstein?

Yet another question is whether the probabilistic nature of quantum physics is a fundamental property of our world, or the probabilities arise due to the "roughness" of macro-devices that we use to study the characteristics of micro-objects?

Copenhagen interpretation of quantum mechanics and its base stone Bohr's complementarity principle give a negative answer to the first question and a positive answer to the second. However this is not a single point of view. For example, theories with hidden variables give a positive answer to the first question and a negative answer to the second. That is why test of the complementarity principle, especially in the relativistic area where QFT works, is an interesting task.

Locality and non-locality in quantum physics

Locality in NQM and QFT at the macro-level is provided by Eberhard's theorem (Eberhard, P.H., "Bells theorem and the different concepts of nonlocality", Nuovo Cimento 46B, 392-419 (1978)): let there be a quantum system constituted of two subsystems A and B. Then no measurement of the observables, belonging only to the subsystem A, has any influence on the measurement results of any observables belonging only to the subsystem B. Strictly speaking it means

$$\sum_k w\left(a_i,\, D^{(A)} \middle|\, b_k,\, D^{(B)}\right) = w\left(a_i,\, D^{(A)}\right),$$

where $w(x|y)$ – is a conditional probability of the event x, given that the event y already happened; $D^{(A)}$ and $D^{(B)}$ – are states of the macro-devices that measures the subsystems A and B accordingly.

If the locality is violated at macro-level, then changing for example the state of the macro-device $D^{(B)}$, it would be possible to instantly affect the result of a measurement in the subsystem A.

Locality in NQM at micro-level does not exist by construction of the theory itself. I.e. in NQM any change in the subsystem A leads to an instant change in the subsystem B, given that these subsystems were entangled.

Let us give an example. Let the subsystems A and B – be two spines $s^{(A)} = 1/2$ and $s^{(B)} = 1/2$, in a singlet Bell state

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}\Big(|+\rangle^{(A)}|-\rangle^{(B)} - |-\rangle^{(A)}|+\rangle^{(B)}\Big).$$

The probability density matrix of the whole system $\hat{\rho} = |\Psi^-\rangle\langle\Psi^-|$. Let us now measure in the subsystem B the value of the spin $s_z^{(B)} = +1/2$. Then according to the von Neumann's projection postulate the density matrix of the subsystem A is:

$$\hat{\rho}^{(A)} = \mathbf{Tr}_B\left(\frac{\hat{P}_+^{(B)}\hat{\rho}\,\hat{P}_+^{(B)}}{\mathbf{Tr}\left(\hat{P}_+^{(B)}\hat{\rho}\right)}\right) = \hat{P}_-^{(A)},$$

where $\hat{P}_\pm^{(\alpha)} = |\pm\rangle^{(\alpha)}\langle\pm|^{(\alpha)}$ – the corresponding projection operators and $\alpha = \{A, B\}$ – the subsystem index.

According to von Neumann's formula the conditional probability to measure $s_z^{(A)} = +1/2$ for the subsystem A given that in the subsystem B it was measured $s_z^{(B)} = +1/2$ is:

$$w\left(+^{(A)}\middle|+^{(B)}\right) = \mathbf{Tr}\left(\hat{P}_+^{(A)}\hat{\rho}^{(A)}\right) = \mathbf{Tr}\left(\hat{P}_+^{(A)}\hat{P}_-^{(A)}\right) = 0.$$

In analogy, the conditional probability for $s_z^{(A)} = -\frac{1}{2}$ and $s_z^{(B)} = +\frac{1}{2}$ is

$$w\left(-^{(A)}\middle|+^{(B)}\right) = \mathbf{Tr}\left(\hat{P}_-^{(A)}\hat{\rho}^{(A)}\right) = \mathbf{Tr}\left(\hat{P}_-^{(A)}\hat{P}_-^{(A)}\right) = 1.$$

The changes in the subsystem A happen instantly "immediately after" changes in the subsystem B.

What happens with locality in QFT at the micro-level? Any renormalizable QFT, which describes physical interactions is local by construction. Does that imply that if the entangled systems A and B are separated by a spacelike interval, then any measurement performed on the micro-system A should lead at the micro-level to a change in the subsystem B only after the time interval $\tau > L/c$? In principle the answer is yes.

Static Bell inequalities and CHSH-inequalities

If one suppose that the observable characteristics of a micro-system, which in quantum mechanics correspond to non-commuting operators simultaneously are the "elements of reality" independently of any means of measurement (mathematically they are described as non-negative and normalized joint distributions of probabilities), and that these characteristics are local at the micro-level, then for the correlators of these observables it is possible to write so-called Bell inequalities:
J. S. Bell, Physics 1, 195 (1964); Rev. Mod. Phys. 38, 447 (1966)

$$|\langle AB\rangle - \langle AC\rangle| - \langle BC\rangle \leq 1.$$

and consequently (J. F. Clauser, M. A. Horne, A. Shimony, R. A. Holt, Phys. Rev. Lett. 23, 880 (1969)),

$$|\langle AB\rangle + \langle A' B\rangle + \langle AB'\rangle - \langle A' B'\rangle| \leq 2.$$

Dichotomic variables A, A', B, B' and C may be implemented as spin $1/2$ projections onto non-parralel directions $\vec{a}$, $\vec{a}\,'$, $\vec{b}$, $\vec{b}\,'$ and $\vec{c}$.

Tsirelson's bound

What is the upper bound of correlations, reachable for the Bell inequalities in quantum theory? The answer is given by the Tsirelson theorem (or Cirel'son, in a different transliteration) (B. S. Cirel'son, Lett. Math. Phys. 4, 93 (1980)): let us consider hermitian operators $\hat{A}$, $\hat{B}$, $\hat{A}'$ and $\hat{B}'$, with the spectrum of two values "$+1$" and "-1" (operators of dichotomic observables, i.e. $\hat{A}^2 = \hat{A}'^{\,2} = \hat{B}^2 = \hat{B}'^{\,2} = \hat{I}$). Let also

$$\left[\hat{A}, \hat{B}\right] = \left[\hat{A}, \hat{B}'\right] = \left[\hat{A}', \hat{B}\right] = \left[\hat{A}', \hat{B}'\right] = 0,$$

but $\left[\hat{A}, \hat{A}'\right] \neq 0$ and $\left[\hat{B}, \hat{B}'\right] \neq 0$. Then:

$$|\langle AB\rangle + \langle A'\,B\rangle + \langle AB'\rangle - \langle A'\,B'\rangle| \leq 2\sqrt{2}.$$

Note, that the commutation conditions for the operators $\hat{A}$, $\hat{B}$, $\hat{A}'$ and $\hat{B}'$, actually are another way to express the Eberhard's theorem and lead to the locality of NQM and QFT at the macro-level.

$2\sqrt{2} > 2$, so in quantum theory the Bell and CHSH-inequalities may be violated. The maximum violation (i.e. $2\sqrt{2}$) is reached for the Bell's state $|\,\Psi^-\,\rangle$, if one choses the angles between the axes as following: $\theta_{a'\,b} = \theta_{b\,a} = \theta_{a\,b'} = \theta$ and $\theta_{a'\,b'} = 3\theta$, where $\theta = \{\pi/8,\, 3\pi/8\}$.

Static Bell inequalities in Wigner form (static Wigner inequalities)

The Static Bell inequalities are not well suited for the relativistic generalization. First, the correlators, which depend on the chosen renormalization scheme already at the lowest order of the perturbative theory. Second, the operators $\hat{A}$, ..., $\hat{B}'$ (usually the spin $s = 1/2$) need to be relativistically generalized themselves. And the correlators depend on the chosen generalization of the spin.

We believe that the most natural for the relativistic generalization are Bell inequalities in Wigner form or Wigner inequalities (WI), E. P. Wigner, Am. J. Phys. 38, 1005 (1970):

$$w\left(s_{\vec{a}}^{(2)} = +\frac{1}{2}, s_{\vec{b}}^{(1)} = +\frac{1}{2}\right) \leq w\left(s_{\vec{c}}^{(2)} = +\frac{1}{2}, s_{\vec{b}}^{(1)} = +\frac{1}{2}\right) + w\left(s_{\vec{a}}^{(2)} = +\frac{1}{2}, s_{\vec{c}}^{(1)} = +\frac{1}{2}\right).$$

These inequalities contain only the probabilities. Calculation of probabilities is a well defined procedure in both NQM and QFT. Also, in the framework of probability theory one can obtain rigor mathematical relations between the probabilities without any links to the quantum paradigm.

Why should we abandon static WI in QFT?

Static Bell inequalities and WI for correlated micro-systems are obtained using the hypothesis of locality (for example, hidden variables) at the microscopic level, while NQM is derived as non-local at the micro-level. Hence, violation of static Bell inequalities in NQM leads to one of two possibilities:

a) either NQM is non-local at the micro-level, but physical values, corresponding to the non-commuting operators, may be simultaneously "elements of reality";

b) or NQM is non-local at the micro-level, but Bohr's complementarity principle is valid.

I.e., in NQM, we believe, it is impossible to consider separately the non-locality at the micro-level and the existence of "elements of reality".

In order to separately test the existence of "elements of reality", one needs to exclude from consideration theories, which contain non-locality at the micro-level. It is possible if we consider experiments, that are described with QFT, which is local at the micro-level by construction. Then violation of Bell inequalities or WI in QFT may be an evidence for impossibility of existence of "elements of reality" in the relativistic area.

Note, that the real experimental tests of Bell inequalities include, as usual, relativistic particles. That is one more argument for using QFT.

However in QFT it is impossible to use the static Bell inequalities or WI, because in QFT it is impossible to ignore the interaction between quantum fields. Consequently the Bell inequalities should be modified for test in QFT, and should include apparent time dependence. This dependence should be combined with the hypothesis of microscopic locality.

Bonus: the same inequalities may be used in NQM for the micro-systems in external classical fields.

Derivation of time-dependent WI from Kolmogorov's axiomatic of probability

Let a pseudoscalar particle decays at time t_0 into a fermion-antifermion pair. Antifermion is denoted by index "1", and fermion – by index "2". Let spin projections of fermion and antifermion on three non-parallel directions $\vec{a}$, $\vec{b}$ and $\vec{c}$ be simultaneously the elements of reality. Let us denote the spin projections $1/2$ on arbitrary axis $\vec{n}$ as

$$s_{\vec{n}} = \pm\frac{1}{2} \equiv n_{\pm}.$$

Let the indices $\{\alpha, \beta, \gamma\} = \{+, -\}$. Then the spin projections at the initial time $t = t_0$ on each direction by construction obey the anticorrelation condition:

$$n_{\pm}^{(1)}(t_0) = -n_{\mp}^{(2)}(t_0). \quad (1)$$

Let us denote a space Ω of elementary outcomes ω_i that the element of physics reality is the whole set of the spin projections $\{a^{(1)}_{\alpha} b^{(1)}_{\beta} c^{(1)}_{\gamma}\ a^{(2)}_{\alpha'} b^{(2)}_{\beta'} c^{(2)}_{\gamma'}\}$. This space does not depend on time.

Let us call an event $\mathcal{K}_{a^{(1)}_{+} b^{(1)}_{+} c^{(1)}_{-}\ a^{(2)}_{-} b^{(2)}_{-} c^{(2)}_{-}}$ a subset of all elementary outcomes ω_k of the set Ω (i.e. $\mathcal{K}_{a^{(1)}_{+} b^{(1)}_{+} c^{(1)}_{-}\ a^{(2)}_{-} b^{(2)}_{-} c^{(2)}_{-}} \subseteq \Omega$ and $\omega_k \in \mathcal{K}_{a^{(1)}_{+} b^{(1)}_{+} c^{(1)}_{-}\ a^{(2)}_{-} b^{(2)}_{-} c^{(2)}_{-}}$) when the element of a physical reality is the whole set of the spin projections $\{a^{(1)}_{+} b^{(1)}_{+} c^{(1)}_{-}\ a^{(2)}_{-} b^{(2)}_{-} c^{(2)}_{-}\}$.

For the initial time $t = t_0$ let us define a special class of correlated events $\mathcal{K}_{a^{(1)}_{\alpha} b^{(1)}_{\beta} c^{(1)}_{\gamma}\ a^{(2)}_{-\alpha} b^{(2)}_{-\beta} c^{(2)}_{-\gamma}}(t_0) \subseteq \Omega$ that the elements of reality are the (anticorrelated as (1)) sets of spin projections of fermion-antifermion pairs on three non-parallel directions $\{a^{(1)}_{\alpha} b^{(1)}_{\beta} c^{(1)}_{\gamma}\ a^{(2)}_{-\alpha} b^{(2)}_{-\beta} c^{(2)}_{-\gamma}\}$. The whole set of such events by definition compose a σ-algebra $\mathcal{F}(t_0)$.

On the $(\Omega, \mathcal{F})$ it is possible to introduce a probability measure w, which is always real and non-negative. Also it is additive (σ-additive) for non-intersecting events. Then it is possible to prove the WI in form

$$w\left(a_+^{(2)}, b_+^{(1)}, t_0\right) \le w\left(c_+^{(2)}, b_+^{(1)}, t_0\right) + w\left(a_+^{(2)}, c_+^{(1)}, t_0\right). \qquad (2)$$

If in (2) one drops the time t_0, it will match with the well known static WI

$$w\left(a_+^{(2)}, b_+^{(1)}\right) \le w\left(c_+^{(2)}, b_+^{(1)}\right) + w\left(a_+^{(2)}, c_+^{(1)}\right). \qquad (3)$$

To prove (2) and, hence, (3), it is necessary to consider events

$$\begin{aligned}
\mathcal{A}(t_0) &= \mathcal{K}_{a_-^{(1)} b_+^{(1)} c_+^{(1)}\, a_+^{(2)} b_-^{(2)} c_-^{(2)}}(t_0) \cup \mathcal{K}_{a_-^{(1)} b_+^{(1)} c_-^{(1)}\, a_+^{(2)} b_-^{(2)} c_+^{(2)}}(t_0), \\
\mathcal{B}(t_0) &= \mathcal{K}_{a_-^{(1)} b_+^{(1)} c_-^{(1)}\, a_+^{(2)} b_-^{(2)} c_+^{(2)}}(t_0) \cup \mathcal{K}_{a_+^{(1)} b_+^{(1)} c_-^{(1)}\, a_-^{(2)} b_-^{(2)} c_+^{(2)}}(t_0), \qquad (4) \\
\mathcal{C}(t_0) &= \mathcal{K}_{a_-^{(1)} b_+^{(1)} c_+^{(1)}\, a_+^{(2)} b_-^{(2)} c_-^{(2)}}(t_0) \cup \mathcal{K}_{a_-^{(1)} b_-^{(1)} c_+^{(1)}\, a_+^{(2)} b_+^{(2)} c_-^{(2)}}(t_0),
\end{aligned}$$

from the σ-algebra $\mathcal{F}(t_0)$.

In the whole set of indices $\{a^{(1)}_{\alpha} b^{(1)}_{\beta} c^{(1)}_{\gamma}\; a^{(2)}_{-\alpha} b^{(2)}_{-\beta} c^{(2)}_{-\gamma}\}$ one can hold not six but only three indices, which fully describe the system. Then

$$w\left(a^{(2)}_{+}, b^{(1)}_{+},\ t_0\right) = \sum_{\omega_i \in \mathcal{A}(t_0)} \left(w\left(a^{(2)}_{+}, b^{(1)}_{+}, c^{(2)}_{+}, \omega_i\right) + w\left(a^{(2)}_{+}, b^{(1)}_{+}, c^{(2)}_{-}, \omega_i\right)\right);$$

$$w\left(c^{(2)}_{+}, b^{(1)}_{+},\ t_0\right) = \sum_{\omega_j \in \mathcal{B}(t_0)} \left(w\left(a^{(2)}_{+}, b^{(1)}_{+}, c^{(2)}_{+}, \omega_j\right) + w\left(a^{(2)}_{-}, b^{(1)}_{+}, c^{(2)}_{+}, \omega_j\right)\right);$$

$$w\left(a^{(2)}_{+}, c^{(1)}_{+},\ t_0\right) = \sum_{\omega_k \in \mathcal{C}(t_0)} \left(w\left(a^{(2)}_{+}, b^{(1)}_{+}, c^{(2)}_{-}, \omega_k\right) + w\left(a^{(2)}_{+}, b^{(1)}_{-}, c^{(2)}_{-}, \omega_k\right)\right).$$

Sum $w\left(c^{(2)}_{+}, b^{(1)}_{+},\ t_0\right) + w\left(a^{(2)}_{+}, c^{(1)}_{+},\ t_0\right)$ **is defined on set**

$$\mathcal{B}(t_0) \cup \mathcal{C}(t_0) =$$

$$= \left(\mathcal{K}_{a^{(1)}_{-} b^{(1)}_{+} c^{(1)}_{-} a^{(2)}_{+} b^{(2)}_{-} c^{(2)}_{+}}(t_0) \cup \mathcal{K}_{a^{(1)}_{+} b^{(1)}_{+} c^{(1)}_{-} a^{(2)}_{-} b^{(2)}_{-} c^{(2)}_{+}}(t_0)\right) \cup$$

$$\cup \left(\mathcal{K}_{a^{(1)}_{-} b^{(1)}_{+} c^{(1)}_{+} a^{(2)}_{+} b^{(2)}_{-} c^{(2)}_{-}}(t_0) \cup \mathcal{K}_{a^{(1)}_{-} b^{(1)}_{-} c^{(1)}_{+} a^{(2)}_{+} b^{(2)}_{+} c^{(2)}_{-}}(t_0)\right)$$

the subset of which is the event $\mathcal{A}(t_0)$. Hence, due to non-negativity of probabilities the (2) is proven.

Changing the directions of axes $\vec{a}$ and $\vec{b}$ to opposite, it is possible to obtain three inequalities in analogy to (2):

$$\begin{aligned} w\left(a_+^{(2)}, b_-^{(1)}, t_0\right) &\le w\left(c_+^{(2)}, b_-^{(1)}, t_0\right) + w\left(a_+^{(2)}, c_+^{(1)}, t_0\right); \\ w\left(a_-^{(2)}, b_+^{(1)}, t_0\right) &\le w\left(c_+^{(2)}, b_+^{(1)}, t_0\right) + w\left(a_-^{(2)}, c_+^{(1)}, t_0\right); \\ w\left(a_-^{(2)}, b_-^{(1)}, t_0\right) &\le w\left(c_+^{(2)}, b_-^{(1)}, t_0\right) + w\left(a_-^{(2)}, c_+^{(1)}, t_0\right). \end{aligned} \tag{5}$$

Let the fermion and antifermion after the time $\Delta t = t - t_0$ become separated by a large enough distance. Then it is possible to write the following time evolution of "elements of reality", which is consistent with Kolmogorov's axiomatic and the proposition of locality at the micro-level:

$$\begin{aligned} & w\left(a_+^{(2)}, b_+^{(1)}, t\right) = \\ = \; & w\left(a_+^{(2)}(t_0) \to a_+^{(2)}(t)\right) w\left(b_+^{(1)}(t_0) \to b_+^{(1)}(t)\right) w\left(a_+^{(2)}, b_+^{(1)}, t_0\right) + \\ + \; & w\left(a_-^{(2)}(t_0) \to a_+^{(2)}(t)\right) w\left(b_+^{(1)}(t_0) \to b_+^{(1)}(t)\right) w\left(a_-^{(2)}, b_+^{(1)}, t_0\right) + \\ + \; & w\left(a_+^{(2)}(t_0) \to a_+^{(2)}(t)\right) w\left(b_-^{(1)}(t_0) \to b_+^{(1)}(t)\right) w\left(a_+^{(2)}, b_-^{(1)}, t_0\right) + \\ + \; & w\left(a_-^{(2)}(t_0) \to a_+^{(2)}(t)\right) w\left(b_-^{(1)}(t_0) \to b_+^{(1)}(t)\right) w\left(a_-^{(2)}, b_-^{(1)}, t_0\right). \end{aligned}$$

Using the (2) and (5) one can obtain the following (quite bulky) inequality:

$$
\begin{aligned}
& w\left(a_+^{(2)}, b_+^{(1)}, t\right) \le \\
\le \quad & w\left(a_+^{(2)}(t_0) \to a_+^{(2)}(t)\right) w\left(b_+^{(1)}(t_0) \to b_+^{(1)}(t)\right) \left(w\left(c_+^{(2)}, b_+^{(1)}, t_0\right) + w\left(a_+^{(2)}, c_+^{(1)}, t_0\right)\right) + \\
+ \quad & w\left(a_-^{(2)}(t_0) \to a_+^{(2)}(t)\right) w\left(b_+^{(1)}(t_0) \to b_+^{(1)}(t)\right) \left(w\left(c_+^{(2)}, b_+^{(1)}, t_0\right) + w\left(a_-^{(2)}, c_+^{(1)}, t_0\right)\right) + \\
+ \quad & w\left(a_+^{(2)}(t_0) \to a_+^{(2)}(t)\right) w\left(b_-^{(1)}(t_0) \to b_+^{(1)}(t)\right) \left(w\left(c_+^{(2)}, b_-^{(1)}, t_0\right) + w\left(a_+^{(2)}, c_+^{(1)}, t_0\right)\right) + \\
+ \quad & w\left(a_-^{(2)}(t_0) \to a_+^{(2)}(t)\right) w\left(b_-^{(1)}(t_0) \to b_+^{(1)}(t)\right) \left(w\left(c_+^{(2)}, b_-^{(1)}, t_0\right) + w\left(a_-^{(2)}, c_+^{(1)}, t_0\right)\right) = \\
= \quad & w\left(a_+^{(2)}(t_0) \to a_+^{(2)}(t)\right) \left(w\left(b_+^{(1)}(t_0) \to b_+^{(1)}(t)\right) + w\left(b_-^{(1)}(t_0) \to b_+^{(1)}(t)\right)\right) w\left(a_+^{(2)}, c_+^{(1)}, t_0\right) + \\
+ \quad & w\left(a_-^{(2)}(t_0) \to a_+^{(2)}(t)\right) \left(w\left(b_+^{(1)}(t_0) \to b_+^{(1)}(t)\right) + w\left(b_-^{(1)}(t_0) \to b_+^{(1)}(t)\right)\right) w\left(a_-^{(2)}, c_+^{(1)}, t_0\right) + \\
+ \quad & w\left(b_+^{(1)}(t_0) \to b_+^{(1)}(t)\right) \left(w\left(a_+^{(2)}(t_0) \to a_+^{(2)}(t)\right) + w\left(a_-^{(2)}(t_0) \to a_+^{(2)}(t)\right)\right) w\left(c_+^{(2)}, b_+^{(1)}, t_0\right) + \\
+ \quad & w\left(b_-^{(1)}(t_0) \to b_+^{(1)}(t)\right) \left(w\left(a_+^{(2)}(t_0) \to a_+^{(2)}(t)\right) + w\left(a_-^{(2)}(t_0) \to a_+^{(2)}(t)\right)\right) w\left(c_+^{(2)}, b_-^{(1)}, t_0\right).
\end{aligned}
$$

Note, that in this inequality the condition of the full anticorrelation is used only at the initial time t_0. At time t the anticorrelation is (generally speaking) not supposed to exist.

Simple regrouping of the summands gives:

$$
\begin{aligned}
& w\left(a_+^{(2)}, b_+^{(1)}, t\right) \leq \qquad (6)\\
\leq\ & w\left(a_+^{(2)}(t_0) \to a_+^{(2)}(t)\right)\\
& \left(w\left(b_+^{(1)}(t_0) \to b_+^{(1)}(t)\right) + w\left(b_-^{(1)}(t_0) \to b_+^{(1)}(t)\right)\right) w\left(a_+^{(2)}, c_+^{(1)}, t_0\right) +\\
+\ & w\left(a_-^{(2)}(t_0) \to a_+^{(2)}(t)\right)\\
& \left(w\left(b_+^{(1)}(t_0) \to b_+^{(1)}(t)\right) + w\left(b_-^{(1)}(t_0) \to b_+^{(1)}(t)\right)\right) w\left(a_-^{(2)}, c_+^{(1)}, t_0\right) +\\
+\ & w\left(b_+^{(1)}(t_0) \to b_+^{(1)}(t)\right)\\
& \left(w\left(a_+^{(2)}(t_0) \to a_+^{(2)}(t)\right) + w\left(a_-^{(2)}(t_0) \to a_+^{(2)}(t)\right)\right) w\left(c_+^{(2)}, b_+^{(1)}, t_0\right) +\\
+\ & w\left(b_-^{(1)}(t_0) \to b_+^{(1)}(t)\right)\\
& \left(w\left(a_+^{(2)}(t_0) \to a_+^{(2)}(t)\right) + w\left(a_-^{(2)}(t_0) \to a_+^{(2)}(t)\right)\right) w\left(c_+^{(2)}, b_-^{(1)}, t_0\right).
\end{aligned}
$$

The time-dependent (non-static) WI (6) is the main result of the present talk.

The inequality (6) is proven on the set of elementary outcomes Ω, which does not depend on time. In the absence of interactions

$$w\left(a_{-}^{(2)}(t_0) \to a_{+}^{(2)}(t)\right) = w\left(b_{-}^{(1)}(t_0) \to b_{+}^{(1)}(t)\right) = 0,$$

while

$$w\left(a_{+}^{(2)}(t_0) \to a_{+}^{(2)}(t)\right) = w\left(b_{+}^{(1)}(t_0) \to b_{+}^{(1)}(t)\right) = 1.$$

Hence (6) is reduced to (2), as it should be from the physical point of view. The inequality (2), is equivalent to the static inequality (3).

Let us note once more that the static WI may be applied not only to relativistic (QFT), but to non-relativistic open quantum systems (NQM), where the correlations between particles are apparently time-dependent. That is why the inequality (6) widens the possibility of experimental tests of quantum correlations.

Below we will demonstrate a few examples of application of the time-dependent inequality (6) and its advantages over the static inequality (3).

Relativistic generalization of spin $s = 1/2$ operator

For calculation of probabilities from (6) in the framework of QFT it is suitable to use the relativistic spin $s = 1/2$ generalization:

$$\vec{O} = -\gamma^5 \vec{\gamma} + \gamma^5 \frac{\vec{p}}{\varepsilon_p} + \frac{\vec{p}\,\gamma_5\,(\vec{\gamma}, \vec{p}\,)}{\varepsilon_p\,(\varepsilon_p + m)},$$

where $\vec{p}$ – momentum of particle, ε_p – its energy, m – mass in the rest frame. We use the following definition:

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3.$$

Components of this operator satisfy the standard commutation relations for doubled components of non-relativistic spin $s = 1/2$ operator

$$\left[O^i, O^j\right] = 2\,i\,\epsilon^{ijk}\,O^k.$$

S. Stech, Zs. f. Phys. 144, 214 (1956).

QFT: decay of a pseudoscalar particle into fermion-antifermion pair

Let us consider decay of a pseudoscalar particle at rest of mass M into a fermion-antifermion pair. Let us denote the momentum of the antifermion as $\vec{k}_1$, and of the fermion – as $\vec{k}_2 = -\vec{k}_1$, and their masses as m_1 and m_2 accordingly (actually, $m_1 = m_2 = m$).

If the P-parity and the full momentum of the system are conserved, then: $-1 = (-1)^{L_{f\bar{f}}+1}$. I.e. for $J_{f\bar{f}} = 0$ we have $L_{f\bar{f}} = S_{f\bar{f}} = 0$. That leads to the full anticorrelation of the fermion "2" and antifermion "1" spin projections on any direction.

Effective hamiltonial of the decay has the form:

$$\mathcal{H}^{(PS)}(x) = g\,\varphi(x)\,\left(\bar{f}(x)\,\gamma^5\,f(x)\right)_N, \tag{7}$$

where g – effective coupling constant, $\varphi(x)$ – field of the pseudoscalar particle, $f(x)$ and $\bar{f}(x)$ – fermionic fields.

In the simplest case the pseudoscalar particle rests at the origin of coordinates, while the spin measurement devices are set to infinity and measure the spin projections in the planes parallel to the plane (xz). The fermion and antifermion propagate along the axis "y". If spin projection of the fermion on the direction $\vec{a}$ and spin projection of the antifermion on the direction $\vec{b}$ are equal to $+1/2$, then the decay amplitude is

$$A\left(a_+^{(2)},\, b_+^{(1)}\right) \,=\, g\,\sqrt{\frac{\varepsilon_2 + m_2}{\varepsilon_1 + m_1}}\,(M + m_1 - m_2)\,\chi_+^\dagger(\vec{a}\,)\,\chi_-(\vec{b}\,).$$

Taking the explicit form of two-component spinors ($\phi_a = \phi_b = 0$)

$$\chi_+^\dagger(\vec{a}\,) \,=\, \begin{pmatrix} \cos{}^{\theta_a}/_2 \\ \sin{}^{\theta_a}/_2 \end{pmatrix}, \quad \chi_-(\vec{b}\,) \,=\, \begin{pmatrix} -\sin{}^{\theta_b}/_2 \\ \cos{}^{\theta_b}/_2 \end{pmatrix},$$

it is possible to write the following:

$$w\left(a_+^{(2)},\, b_+^{(1)}\right) \,=\, g^2\, f(M,\, m_1,\, m_2)\, \sin^2\frac{\theta_{ab}}{2},$$

where $\theta_{ab} = \theta_a - \theta_b$. The function $f(M,\, m_1,\, m_2)$ can be easily calculated. It does not became null and does not depend on the directions on which the spins of the fermion and antifermion are projected.

QFT: adjustment for non-parallel momentums of fermion and antifermion

Finite distance to analysers, emission of soft photon by one of the fermions, thermal motion of the pseudoscalar particle, interaction of the fermions between each other or interaction with an external field – all may lead to violation of parallelity of the vectors $\vec{k}_2$ and $\vec{k}_1$.

The following is correct. Let $\vec{k}_1 = |\vec{k}_1|\vec{n}_1$ and $\vec{k}_2 = |\vec{k}_2|\vec{n}$ to be linked by a conservation law

$$|\vec{k}_1|\,\vec{n}_1 \,+\, |\vec{k}_2|\,\vec{n} \,=\, |\vec{p}\,|\,\vec{\ell},$$

where vector $\vec{\ell}$ is not parallel to the vectors $\vec{n}_1$ and $\vec{n}$. Also, $E = \varepsilon_1 + \varepsilon_2$. And let $|\vec{p}\,|/M \ll 1$ – be a small parameter.

Then

$$w\left(a_+^{(2)},\, b_+^{(1)}\right) \;=\; g^2\, f(M, m_1, m_2)\, \sin^2\frac{\theta_{ab}}{2} \;+\; O\left(\frac{|\vec{p}\,|^2}{M^2}\right).$$

This result follows from the expansion by the small parameter $|\vec{p}\,|/M$ of the exact amplitude.

Numerical estimate of the non-parallelity effect for the soft photon emission with the decay $\pi^0 \to e^+e^-$.

Let the photon energy be $E_\gamma \sim |\vec{p}_\gamma| \sim 10$ **KeV**. Note that the decay $\pi^0 \to e^+e^-\gamma$ adds an additional α_{em}. Then the suppression factor is:

$$\alpha_{em}\left(\frac{E_\gamma}{M_\pi}\right)^2 \sim \frac{1}{137}\left(\frac{10}{135000}\right)^2 \sim 10^{-10}.$$

What about the thermal motion? Let $T = 300$ **K**. Then

$$\left(\frac{k\,T}{M_\pi}\right)^2 \sim \left(\frac{3\,\times\,10^{-2}}{135\,\times\,10^6}\right)^2 \sim 10^{-19}.$$

What about the finite distance to the analysers? Let us consider two spin analysers, which measure the spin projections in the planes parallel to the plane (xz). Let the distance between the analysers be L. Uncertainty of the momenta of the fermions is $\Delta k \sim 1/L$. As L should be macro-scopic by definition the $1/L \ll M_\pi$. Hence

$$\left(\frac{\Delta k}{M}\right)^2 \sim \frac{1}{(M_\pi\, L)^2} \ll 1.$$

For example for $L \sim 2$ cm and $M_\pi \sim 135$ MeV it is $1/(M_\pi\, L)^2 \sim 10^{-26}$.

As one can see any effects of antiparallelity are negligible in the considered case.

QFT: calculations in finite time

Standard methods of QFT allow to calculate the probability of decay $W(t) = \partial w(t)/\partial t$ only with perturbative theory. Let us continue with the decay of a neurtal pseudoscalar particle $P \to f^+ f^-$. Hamiltonian of the decay is (7). Let us neglect the masses of the fermions. This approximation does not affect the final result, but simplifies the calculations. Width of the decay of a pseudoscalar meson Γ_0 in limit $t_0 \to -\infty$ and $t \to +\infty$ is equal $\Gamma_0 = \dfrac{g^2 M}{8\pi}$. However in the inequality (6) there are the probabilities for finite values of t and t_0.

Using the technique for calculations in QFT in the finite time for the probability of the decay $P \to f^+ f^-$ in the first order of perturbative theory we have:

$$W^{(1)}(a_+^{(2)}, b_+^{(1)}, \tau) = \frac{\Gamma_0}{2}\left(1 + \frac{\mathbf{si}(M\tau)}{\pi} + \frac{\sin(M\tau)}{\pi(M\tau)^2} + \frac{\cos(M\tau)}{\pi(M\tau)}\right) \times \sin^2\frac{\theta_{ab}}{2}, \quad (8)$$

where τ – current measurement time.

Integral sine is defined as follows:

$$\mathbf{si}(x) = -\int\limits_{x}^{+\infty} \frac{\sin\zeta}{\zeta}\, d\zeta.$$

For $\tau \to +\infty$, the expression (8) comes to $W^{(1)}(a_+^{(2)}, b_+^{(1)}) = \Gamma_0 \times \sin^2 \frac{\theta_{ab}}{2}$, as it should be. For $M \to 0$ the $W^{(1)}(a_+^{(2)}, b_+^{(1)}, \tau) \to 0$ because of the decreased phase space. For $\tau \to 0$ (8) has a pole for τ:

$$W^{(1)}(a_+^{(2)}, b_+^{(1)}, \tau \to 0) \approx \frac{\Gamma_0}{2}\left(\frac{1}{2} + \frac{2}{\pi(M\tau)}\right) \times \sin^2 \frac{\theta_{ab}}{2}.$$

In the work N.N.Bogoliubov and D.V.Shirkov "Introduction to theory of Quantized Fields" it was shown that such poles can not be removed by a renormalization procedure. However in our case times τ are cut by the resolution Δt of a macro-device. It is obvious that $\Delta t \gg 1/M$. Hence the pole for τ in (8) is not significant for the considered case.

First order of the perturbative theory is applicable when the full width of the decay is significantly less than the mass of the decayed particle.

Let us suppose that the decay $P \to f^+ f^-$ is absolutely dominant among all the other decays of P. Then the applicability condition of the perturbative theory will be:

$$\frac{\Gamma_0}{2}\left(1 + \frac{\mathbf{si}(M\,\tau)}{\pi} + \frac{\sin(M\,\tau)}{\pi(M\,\tau)^2} + \frac{\cos(M\,\tau)}{\pi(M\,\tau)}\right) = \\ = \quad W^{(1)}(\tau) \approx \Gamma(\tau) \ll M. \tag{9}$$

For small enough coupling constant g one can easily obtain

$$\frac{\Gamma_0}{M} = \frac{g^2}{8\,\pi} \ll 1$$

with any required degree of smallness. Hence, the condition (9) is satisfied in a wide range of values of the variable $M\,\tau \gg 1$.

Times t_0 and t in (6) should be interpreted as time intervals for measurement of the probabilities. The probabilities of the fermion $w\left(a_-^{(2)}(t_0) \to a_+^{(2)}(t)\right)$ and antifermion $w\left(b_-^{(1)}(t_0) \to b_+^{(1)}(t)\right)$ spin flip in the right part of (6) have a higher degree of smallness by the coupling constant g. Hence in the first order of the perturbative theory

$$w\left(a_-^{(2)}(t_0) \to a_+^{(2)}(t)\right) \approx w\left(b_-^{(1)}(t_0) \to b_+^{(1)}(t)\right) \approx 0.$$

In analogy we can write for the probabilities without the fermion spin flip

$$w\left(a_+^{(2)}(t_0) \to a_+^{(2)}(t)\right) \approx w\left(b_+^{(1)}(t_0) \to b_+^{(1)}(t)\right) \approx 1.$$

Integrating (8) by $d\tau$ from t_i to t_f given that $\frac{M}{\Gamma_0} \gg M\,\tau \gg 1$, gives

$$w\left(a_+^{(2)}, b_+^{(1)}, t_f - t_i\right) \approx \int_{t_i}^{t_f} d\tau\, W^{(1)}(a_+^{(2)}, b_+^{(1)}, \tau) \approx \frac{\Gamma_0}{2}\sin^2\frac{\theta_{ab}}{2}\int_{t_i}^{t_f} d\tau = \frac{(t_f - t_i)\,\Gamma_0}{2}\sin^2\frac{\theta_{ab}}{2}.$$

Setting in the right part of (6) measurement time $t_f - t_i = t_0$, and in the left $t_f - t_i = t$, we obtain the following time-dependent WI:

$$\frac{t}{t_0}\sin^2\left(\frac{\theta_{ba}}{2}\right) \leq \sin^2\left(\frac{\theta_{ca}}{2}\right) + \sin^2\left(\frac{\theta_{bc}}{2}\right). \tag{10}$$

Note that in the left part of (10) it is always possible to divide by t_0 because $t_0 \geq \Delta t \gg 0$. Let us compare the obtained result with the static inequality (3).

It is easy to show that in the relativistic case and the approximation of antiparallelity of momenta of fermion an and antifermion the static inequality (3) comes to well known non-relativistic inequality

$$\sin^2\left(\frac{\theta_{ba}}{2}\right) \leq \sin^2\left(\frac{\theta_{ca}}{2}\right) + \sin^2\left(\frac{\theta_{bc}}{2}\right), \tag{11}$$

which is maximally violated for $\theta_{bc} = \theta_{ca} = \pi/3$ and $\theta_{ba} = 2\pi/3$.

The time-dependent inequality (10) differ from the static inequality (11) by the ratio $\frac{t}{t_0} \geq 1$ in the left part. The ratio t/t_0 may exceed unity by a few times, widening the angular range where the WI is violated. Experimentally, instead of the ratio of times t/t_0 it is more suitable to consider the ratio of distances between the analysers L/L_0, i.e. in experiments for measurement the spin projections on directions $\vec{a}$ and $\vec{c}$ and directions $\vec{b}$ and $\vec{c}$ it should be equal L_0, and for measurement of the spin projections on directions $\vec{a}$ and $\vec{b}$ it should be equal L. The following condition should be satisfied: $\frac{M}{\Gamma_0} \gg M\{L,\, L_0\} \gg 1$. It is always true for macro-scopic distances.

Hamiltonian (7) does not affect the inequality (10) in any way, meaning that this inequality is valid for any QFT hamiltonian, which provides the condition of anticorellation (1) and is restricted only by the validity of the first order of perturbative theory. In this sense (10) may be considered as universal time-dependent WI in QFT without external fields.

Time-dependent WI for oscillations of neutral pseudoscalar mesons

Let us apply the WI to the problem of oscillation neutral pseudoscalar mesons $M = \{K,\, D,\, B_q\}$, where $q = \{d,\, s\}$. In this case the static inequality (3) either not violated at all, or its violation can not be experimentally measured. At the same time it is always possible to find the conditions when the time-dependent inequality (6) is violated, and can be tested in experiment.

Key idea here is that in the systems of neutral pseudoscalar mesons there are the natural "directions", with non-commuting projection operators of meson states. First, this is the flavor "direction" of pseudoscalar meson ($|\, M\,\rangle$ and $\left|\, \bar{M}\,\right\rangle$). Second, "direction" is the states with certain values of the CP-parity ($|\, M_1\,\rangle$ and $|\, M_2\,\rangle$). Third "direction" is the states with certain values of mass and lifetime ($|\, M_L\,\rangle$ and $|\, M_H\,\rangle$). In the decays of the vector mesons (for example in the decay $\Upsilon(4S) \to B^0 \bar{B}^0$) at the $t = t_0$ we have the flavor entangled state

$$\left|\, \Psi^-(t_0)\,\right\rangle = \frac{1}{\sqrt{2}} \left(|\, M\,\rangle^{(1)} \left|\, \bar{M}\,\right\rangle^{(2)} - \left|\, \bar{M}\,\right\rangle^{(1)} |\, M\,\rangle^{(2)} \right).$$

Let us use the following definitions:

$$\hat{C}\hat{P}\,|\,M\,\rangle = e^{i\alpha}\,|\,\bar{M}\,\rangle \quad \textbf{and} \quad \hat{C}\hat{P}\,|\,\bar{M}\,\rangle = e^{-i\alpha}\,|\,M\,\rangle,$$

where α – nonphysical arbitrary real phase of CP-conjugation. Then let us define the states:

$$|\,M_1\,\rangle = \frac{1}{\sqrt{2}}\,\left(|\,M\,\rangle + e^{i\alpha}\,|\,\bar{M}\,\rangle\right), \quad |\,M_2\,\rangle = \frac{1}{\sqrt{2}}\,\left(|\,M\,\rangle - e^{i\alpha}\,|\,\bar{M}\,\rangle\right),$$

with a positive and negative CP-parity accordingly. Finally, let us write for the states with certain values of masses and lifetimes:

$$|\,M_L\,\rangle = p\left(|\,M\,\rangle + e^{i\alpha}\,\frac{q}{p}\,|\,\bar{M}\,\rangle\right) \quad \textbf{and} \quad |\,M_H\,\rangle = p\left(|\,M\,\rangle - e^{i\alpha}\,\frac{q}{p}\,|\,\bar{M}\,\rangle\right).$$

Let us choose the definition for the mass difference and the decay width difference of "heavy" (H) and "light" (L) states:

$$\Delta M = M_H - M_L,$$
$$\Delta\Gamma = \Gamma_H - \Gamma_L.$$

Note that our definition of $\Delta\Gamma$ is of the different sign comparing to the definition of $\Delta\Gamma$ from PDG.

The coefficients p and q obey the standard normalization condition:

$$\langle M_L \mid M_L \rangle = \langle M_H \mid M_H \rangle = |p|^2 + |q|^2 = 1.$$

In order to automatically satisfy this condition let us use new variable β, which is useful for analysis of the violation of the time-dependent WI.

$$|p| = \cos\beta; \qquad |q| = \sin\beta \qquad \textbf{and} \qquad \frac{q}{p} = r\, e^{i\zeta} \equiv \tan\beta\, e^{i\zeta}.$$

From above it follows that $\beta \in [0, \pi/2]$.

If the CP-violation due to oscillation of pseudoscalar mesons does not exist, then $\beta = \beta_0 \equiv \pi/4$.

For subsequent analysis of the time-dependent WI (6) it is suitable to introduce two new dimensionless parameters: $x = \Delta\Gamma t$ (dimensionless time) and ratio $\lambda = \Delta M/\Delta\Gamma$.

Table: Experimental values of the oscillation and CP-violation parameters for neurtal pseudoscalar mesons (PDG data). The minus sign in numerical values is related to the difference in the definition of $\Delta\Gamma$.

Meson	$\Delta\Gamma$ (MeV)	ΔM (MeV)	$\tan\beta \equiv \lvert q/p \rvert_M^{exp}$
B_s^0	$-6.0 \cdot 10^{-11}$	$1.2 \cdot 10^{-8}$	1.0039 ± 0.0021
K^0	$-7.3 \cdot 10^{-12}$	$3.5 \cdot 10^{-12}$	0.99668 ± 0.00004
D^0	$-2.1 \cdot 10^{-11}$	$-6.3 \cdot 10^{-12}$	$0.92^{+0.12}_{-0.09}$

From the table one can see that for all the neutral mesons $\beta \approx \beta_0 = \pi/4$. More detailed analysis shows that: $\cos\zeta_K \approx +1$, $\cos\zeta_D \approx +1$ and $\cos\zeta_{B_s} = -1$.

During the derivation of (6) the condition of normalization of probabilities to unity was not used. So the inequality (6) is still correct for non-stable particles, when the normalization of the state vectors explicitly depends on time.

In the time-dependent inequalities there are eight possible choices of the "directions". We are interested only in the ones that lead to the violation of the WI for $\Delta\Gamma \le 0$.

Analysis of (6) for the approximation when $\beta = \beta_0 = \pi/4$ and $\cos\zeta = \pm 1$ (i.e. when there are no effects of CP-violation) shows that for the violation of (6) in systems of K- and D-mesons one needs to choose the cases N5 and N6. For studies of the violation of (6) in B_s-mesons one can choose the cases N7 and N8.

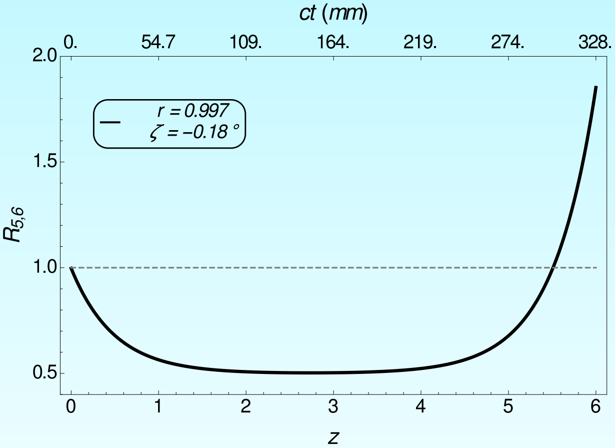
If one calculates the time-dependent WI (6) in the framework of quantum theory then all the inequalities may be written as follows:

$$1 \le \mathbf{R}_N(x,\, r,\, \zeta,\, \lambda), \tag{12}$$

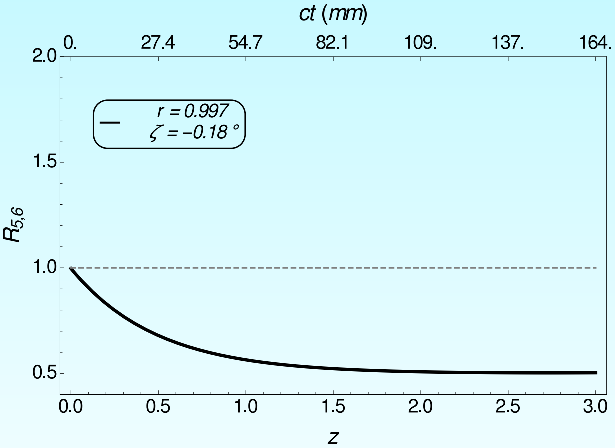
where $\mathbf{R}_N$ – functions of the arguments $x = \Delta\Gamma t$, r, ζ (remind that $q/p = r\, e^{i\zeta}$) and $\lambda = \Delta M/\Delta\Gamma$.

Table: Time-dependent WI (6) for pseudoscalar mesons at $\beta = \beta_0 = {}^{\pi}/_{4}$ and $\cos\zeta = \pm 1$.

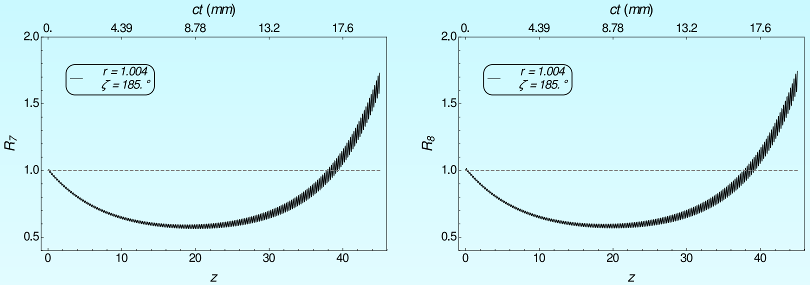
N	Correspondence of variables	Time-dependent WI	Violation condition
...	...	...	...
4	$a_+ \to M_2$, $b_+ \to M$, $c_+ \to M_H$, $a_- \to M_1$, $b_- \to \bar{M}$, $c_- \to M_L$	$1 \le e^{-\Delta\Gamma\, t}$ **for** $\cos\zeta = +1$	**if** $\Delta\Gamma \ge 0$
5	$a_+ \to M_1$, $b_+ \to \bar{M}$, $c_+ \to M_L$, $a_- \to M_2$, $b_- \to M$, $c_- \to M_H$	$1 \le e^{\Delta\Gamma\, t}$ **for** $\cos\zeta = +1$	**if** $\Delta\Gamma \le 0$
6	$a_+ \to M_1$, $b_+ \to M$, $c_+ \to M_L$, $a_- \to M_2$, $b_- \to \bar{M}$, $c_- \to M_H$	$1 \le e^{\Delta\Gamma\, t}$ **for** $\cos\zeta = +1$	**if** $\Delta\Gamma \le 0$
7	$a_+ \to M_2$, $b_+ \to \bar{M}$, $c_+ \to M_L$, $a_- \to M_1$, $b_- \to M$, $c_- \to M_H$	$1 \le e^{\Delta\Gamma\, t}$ **for** $\cos\zeta = -1$	**if** $\Delta\Gamma \le 0$
8	$a_+ \to M_2$, $b_+ \to M$, $c_+ \to M_L$, $a_- \to M_1$, $b_- \to \bar{M}$, $c_- \to M_H$	$1 \le e^{\Delta\Gamma\, t}$ **for** $\cos\zeta = -1$	**if** $\Delta\Gamma \le 0$



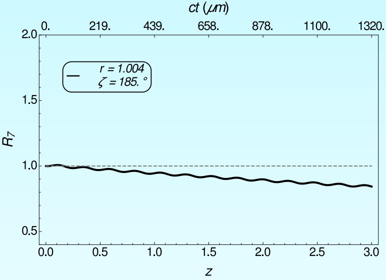
Functions $\mathbf{R}_{5,\,6}(x,\ r,\ \zeta,\ \lambda)$ for neutral K-mesons (both functions are practically consistent because of the high precision of the CP-violation parameter ε). Top scale corresponds to $c\,t$ (in mm), bottom scale – to time in units of the lifetime $z = (\Gamma_H + \Gamma_L)\,t/2 = \Gamma\,t$, where time t is calculated in the kaons rest frame. Time-dependent WI (12) are relaxed for large values of z.

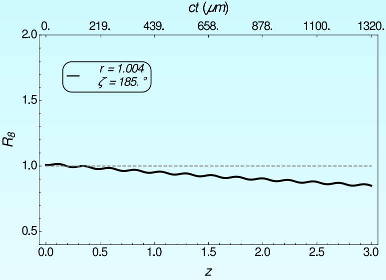


Functions $\mathbf{R}_{5,\,6}(x,\, r,\, \zeta,\, \lambda)$ for neutral K-mesons in the area $z \leq 3$, which is most experimentally interesting. In the considered area the WI (12) are violated in the whole range of z.

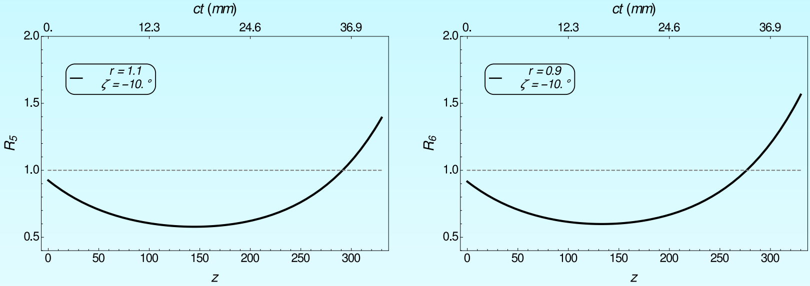


Functions $\mathbf{R}_{7,\,8}(x,\ r,\ \zeta,\ \lambda)$ for B_s-mesons. Top scale corresponds to ct (in mm), bottom scale – to the lifetime in units of $z = (\Gamma_H + \Gamma_L)\, t/2 = \Gamma\, t$, where the time t is calculated in the B_s-meson rest frame. With a proper choice of functions $\mathbf{R}_N$ for $r > 1$ and $r < 1$ the time-dependent WI (12) are violated in almost the whole range of z, which is experimentally accessible. For large values of z one can see the effect of relaxation.

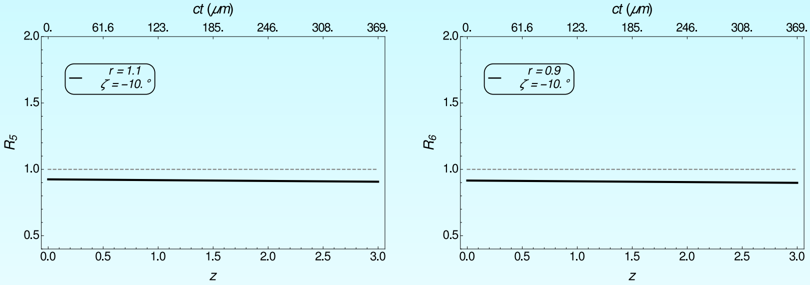




Functions $\mathbf{R}_{7,\,8}(x,\, r,\, \zeta,\, \lambda)$ for B_s-mesons for the range $z \leq 3$, which is the most experimentally interesting. In the considered area both functions are almost consistent with unity. Function $\mathbf{R}_8$ even exceeds the unity for $z \to 0$. The ct is in μm.



Functions $\mathbf{R}_{5,\,6}(x,\, r,\, \zeta,\, \lambda)$ for neutral D-mesons. Top scale corresponds to the value $c\,t$ (in mm), bottom – to the time in the units of the lifetime $z = (\Gamma_H + \Gamma_L)\,t/2 = \Gamma\,t$, where the time t is calculated in the D-meson rest frame. With a proper choice of fuctions $\mathbf{R}_N$ for $r > 1$ and $r < 1$ the time-dependent WI (12) are violated in almost the whole range of z, which is experimentally accessible. For the large values of z one can observe the effect of relaxation.



Functions $\mathbf{R}_{5,\,6}(x,\, r,\, \zeta,\, \lambda)$ for neutral *D*-mesons in the range $z \leq 3$, which is the most experimentally interesting. In this area both functions are almost consistent with unity. The *ct* is in μm.

Conclusion

1) Using Kolmogorov's axiomatic of probability theory and hypothesis of locality of "elements of reality ", a new class of time-dependent Bell inequalities in Wigner form is obtained (6). New time-dependent inequalities may be used both in quantum field theory and in non-relativistic quantum mechanics of open quantum systems.

2) In the framework of QFT the violation of time-dependent WI is studied for the case of the decay of a pseudoscalar particle into fermion-antifermion pair. These inequalities are violated more strongly than static WI.

3) For oscillations of neutral pseudoscalar mesons in non-relativistic quantum mechanics eight new time-dependent WI are obtained, which are violated depending on choice of the values of $\Delta\Gamma$ and q/p. The effect of relaxation time-dependent inequalities for large values of z, is found. It is governed by the CP-violation effects.