Phenomenological aspects of Higgs-Radion mixing

Viacheslav Bunichev

In collaboration with: E. Boos, M.Perfilov, M. Smolyakov, I. Volobuev
(SINP Moscow State University)

The work was supported by grant 14-12-00363 of the Russian Science Foundation.
Randall-Sundrum model with two branes stabilized by a bulk scalar field

We discuss a model, where a Higgs-radion coupling naturally arises due to a mechanism of spontaneous symmetry breaking on the brane involving the stabilizing scalar field. Our approach takes into account the influence of the KK tower of higher scalar excitations on the parameters of the Higgs-radion mixing, which turns out to be of importance. It also has the advantage that it modifies only the scalar sector of the model and leaves intact the masses and the coupling constants of the graviton KK excitations.

A stabilized brane world model in five-dimensional space-time $E = M_4 \times S^1 / \mathbb{Z}_2$ with coordinates $\{x^M\} \equiv \{x^\mu, y\}$, $M=0,1,2,3,4$, $\mu=0,1,2,3$, the coordinate $x^4 = y$, $-L \leq y \leq L$ parameterizing the fifth dimension, is defined by the action:

$$S = S_g + S_{\phi+SM}$$

where

$$S_{\phi+SM} = \int d^4x \int_{-L}^{L} dy \left( \frac{1}{2} g^{MN} \partial_M \phi \partial_N \phi - V(\phi) \right) \sqrt{g}$$

$$- \int_{y=0} \sqrt{-g} V_1(\phi) d^4x + \int_{y=L} \sqrt{-g} (-V_2(\phi) + L_{SM-HP} + L_{int}(\phi, H)) d^4x$$

Lagrangian $L_{SM-HP}$ is the SM Lagrangian without the Higgs potential that is replaced by the interaction Lagrangian

$$L_{int}(\phi, H) = -\lambda \left( |H|^2 - \frac{\xi}{M} \phi^2 \right)^2$$

where $\lambda$ and $\xi$ are positive dimensionless parameters.

This leads to relation between the vacuum value of the Higgs field and the value of the field $\phi$ on the brane at $y = L$:

$$\phi^2(L) = \frac{M v^2}{2\xi}$$

This means that in such a scenario the Higgs field vacuum expectation value, being proportional to the value of the stabilizing scalar field on the TeV brane, arises dynamically as a result of the gravitational bulk stabilization.
Integrating out radion excitation fields

Expanding the bulk scalar field in KK modes, substituting this expansion into the second variation Lagrangian and integrating over the extra dimension coordinate $y$ we get a four-dimensional Lagrangian:

\[
L_{\text{part}} = \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} 2\lambda v^2 \sigma^2 + \frac{1}{2} \sum_{n=1}^{\infty} \partial_\mu \phi_n \partial^\mu \phi_n - \frac{1}{2} \sum_{n=1}^{\infty} \mu_n^2 \phi_n^2 \\
+ \sum_{n=1}^{\infty} \mu_n^2 a_n \phi_n \sigma - \sum_f \frac{m_f}{v} \bar{\psi}_f \psi_f \sigma - \sum_{n=1}^{\infty} b_n \phi_n (T^\mu_\mu + \Delta T^\mu_\mu) \\
+ \frac{2M_W^2}{v} W_\mu W^\mu + \sigma + \frac{M_Z^2}{v^2} Z_\mu Z^\mu \sigma + \frac{M_W^2}{v^2} W_\mu W^\mu + \sigma^2 + \frac{M_Z^2}{2v^2} Z_\mu Z^\mu \sigma^2
\]

where $T^\mu_\mu$ is the trace of the SM energy-momentum tensor and $\Delta T^\mu_\mu$ is the conformal anomaly of massless vector fields.

We are going to consider the phenomenology of the Higgs boson and the radion in the energy range much lower that the masses of the radion excitations. In this case we can pass to a low energy approximation for this Lagrangian by dropping the kinetic terms of the radion excitation fields and integrating them out, which gives the following effective Lagrangian for the interactions of the Higgs and radion fields with the SM fields:

\[
L_{\text{part-eff}} = \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} (2\lambda v^2 - d^2) \sigma^2 + \frac{1}{2} \partial_\mu \phi_1 \partial^\mu \phi_1 - \frac{1}{2} \mu_1^2 \phi_1^2 + \mu_1^2 a_1 \phi_1 \sigma \\
- \frac{1}{\Lambda_r} \phi_1 (T^\mu_\mu + \Delta T^\mu_\mu) - \frac{c}{\Lambda_r} \sigma (T^\mu_\mu + \Delta T^\mu_\mu) - \sum_f \frac{m_f}{v} \bar{\psi}_f \psi_f \sigma \\
+ \frac{2M_W^2}{v} W_\mu W^\mu + \sigma + \frac{M_Z^2}{v^2} Z_\mu Z^\mu \sigma + \frac{M_W^2}{v^2} W_\mu W^\mu + \sigma^2 + \frac{M_Z^2}{2v^2} Z_\mu Z^\mu \sigma^2,
\]

dimensionless parameters:
\[
b_1 = \frac{1}{\Lambda_r}, \quad b_n = \frac{\alpha_n}{\Lambda_r}
\]

dimensionless quantities:
\[
a_n = a_1 \alpha_n, \quad \alpha_n = \frac{\chi_n(L)}{\chi_1(L)}
\]

We will determine the grav. scale parameters:
\[
b_1 = \frac{1}{\Lambda_r}, \quad b_n = \frac{\alpha_n}{\Lambda_r}
\]

V. Bunichev, SINP MSU  
QFTHEP 2015
Physical mass eigenstate fields

Then we diagonalize mass matrix $M$ of the fields $\sigma$ and $\varphi_1$ and turn to the physical mass eigenstate fields $h(x), r(x)$:

\[
\begin{align*}
    h(x) &= \cos \theta \sigma(x) + \sin \theta \phi_1(x) \\
    r(x) &= -\sin \theta \sigma(x) + \cos \theta \phi_1(x)
\end{align*}
\]

where rotation angle $\theta$: \[\tan 2\theta = \frac{\mu_1^2 a_1}{\mu_1^2 - 2\lambda v^2 + d^2}\]

The original unobservable parameters we express in terms of the physical masses and the mixing angle:

\[
a_1 = \frac{(m_r^2 - m_h^2) \sin 2\theta}{m_r^2 \cos^2 \theta + m_h^2 \sin^2 \theta} \quad c = \frac{(m_r^2 - m_h^2) \sin 2\theta}{m_r^2 \cos^2 \theta + m_h^2 \sin^2 \theta} \left( \sum_{n=2}^{\infty} \alpha_n^2 \right)
\]

The sum of the wave function ratios $\alpha_n^2$ are model dependent, and they should fall off with $n$ in order for the sum to be convergent. In certain models, several first ratios can be of the order of unity which gives the estimate for the parameter $c$:

The upper level of the parameter $c$ as a function of $m_{\text{Radion}}$ and $\sin \Theta$

\[
0 \leq c < \frac{(m_r^2 - m_h^2) \sin 2\theta}{m_r^2 \cos^2 \theta + m_h^2 \sin^2 \theta}
\]
Effective Lagrangian

We get the following effective Lagrangian for the interactions of the fields \( h(x) \), \( r(x) \) with the SM fields:

\[
L_{h-r} = \frac{1}{2} \partial_\mu h \partial^\mu h - \frac{1}{2} m_h^2 h^2 + \frac{1}{2} \partial_\mu r \partial^\mu r - \frac{1}{2} \mu_r^2 r^2
\]

\[
- \frac{(c \cos \theta + \sin \theta)}{\Lambda_r} h(T_\mu^\mu + \Delta T_\mu^\mu) + \frac{(c \sin \theta - \cos \theta)}{\Lambda_r} r(T_\mu^\mu + \Delta T_\mu^\mu)
\]

\[
- \sum_f \frac{m_f}{v} \bar{\psi}_f \psi_f (\cos \theta h - \sin \theta r) + \frac{2M_W^2}{v} W^- W^{\mu+} (\cos \theta h - \sin \theta r)
\]

\[
+ \frac{M_Z^2}{v} Z_\mu Z^\mu (\cos \theta h - \sin \theta r) + \frac{M_W^2}{v^2} W^- W^{\mu+} (\cos \theta h - \sin \theta r)^2
\]

\[
+ \frac{M_Z^2}{2v^2} Z_\mu Z^\mu (\cos \theta h - \sin \theta r)^2.
\]

The field \( h(x) \) will be called the \textbf{Higgs-dominated field} and the field \( r(x) \) will be called the \textbf{radion-dominated field}, because \( \cos \theta > |\sin \theta| \) in the interval \(-\pi/4 < \theta < \pi/4\) (we recall that \( \theta < 0 \) for \( m_r^2 < m_h^2 \)).

The effective four-dimensional interaction Lagrangian involves only five parameters in addition to those of the SM:

- the masses of the Higgs-dominated and radion-dominated fields \( m_h \) and \( m_r \),
- the mixing angle \( \theta \),
- the (inverse) coupling constant of the radion to the trace of the energy-momentum tensor of the SM fields \( \Lambda_r \),
- and the parameter \( c \) that accommodates the contributions of the integrated out heavy scalar modes.

V. Bunichev, SINP MSU

QFTHEP 2015
**Feynman rules**

<table>
<thead>
<tr>
<th>Triple vertices</th>
<th>Feynman rules</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{t} \ t \ h$</td>
<td>$-M_t \cdot \left( \frac{C_h}{\Lambda_r} + \frac{\cos \theta}{v} \right)$</td>
</tr>
<tr>
<td>$\bar{t} \ t \ r$</td>
<td>$-M_t \cdot \left( \frac{C_r}{\Lambda_r} - \frac{\sin \theta}{v} \right)$</td>
</tr>
<tr>
<td>$Z_\mu \ Z_\nu \ h$</td>
<td>$2 \cdot M_Z^2 \cdot \left( \frac{C_h}{\Lambda_r} + \frac{\cos \theta}{v} \right) \cdot g^{\mu \nu}$</td>
</tr>
<tr>
<td>$Z_\mu \ Z_\nu \ r$</td>
<td>$2 \cdot M_Z^2 \cdot \left( \frac{C_r}{\Lambda_r} - \frac{\sin \theta}{v} \right) \cdot g^{\mu \nu}$</td>
</tr>
<tr>
<td>$W_\mu^+ \ W_\nu^- \ h$</td>
<td>$2 \cdot M_W^2 \cdot \left( \frac{C_h}{\Lambda_r} + \frac{\cos \theta}{v} \right) \cdot g^{\mu \nu}$</td>
</tr>
<tr>
<td>$W_\mu^+ \ W_\nu^- \ r$</td>
<td>$2 \cdot M_W^2 \cdot \left( \frac{C_r}{\Lambda_r} - \frac{\sin \theta}{v} \right) \cdot g^{\mu \nu}$</td>
</tr>
<tr>
<td>$G_\mu \ G_\nu \ h$</td>
<td>$\frac{g_2^2}{8\pi^2} \cdot \left[ b_{QCD} \cdot \frac{C_h}{\Lambda_r} + F_t \cdot \left( \frac{C_h}{\Lambda_r} + \frac{\cos \theta}{v} \right) \right] \cdot \left( g^{\mu \nu} p_1 p_2 - p_1^\nu p_2^\mu \right)$</td>
</tr>
<tr>
<td>$G_\mu \ G_\nu \ r$</td>
<td>$\frac{g_2^2}{8\pi^2} \cdot \left[ b_{QCD} \cdot \frac{C_r}{\Lambda_r} + F_t \cdot \left( \frac{C_r}{\Lambda_r} - \frac{\sin \theta}{v} \right) \right] \cdot \left( g^{\mu \nu} p_1 p_2 - p_1^\nu p_2^\mu \right)$</td>
</tr>
<tr>
<td>$A_\mu \ A_\nu \ h$</td>
<td>$\frac{e^2}{8\pi^2} \cdot \left[ (b_2 + b_Y) \cdot \frac{C_h}{\Lambda_r} + (F_W + \frac{8}{3} F_t) \cdot \left( \frac{C_h}{\Lambda_r} + \frac{\cos \theta}{v} \right) \right] \cdot \left( g^{\mu \nu} p_1 p_2 - p_1^\nu p_2^\mu \right)$</td>
</tr>
<tr>
<td>$A_\mu \ A_\nu \ r$</td>
<td>$\frac{e^2}{8\pi^2} \cdot \left[ (b_2 + b_Y) \cdot \frac{C_r}{\Lambda_r} + (F_W + \frac{8}{3} F_t) \cdot \left( \frac{C_r}{\Lambda_r} - \frac{\sin \theta}{v} \right) \right] \cdot \left( g^{\mu \nu} p_1 p_2 - p_1^\nu p_2^\mu \right)$</td>
</tr>
</tbody>
</table>

where: $b_{QCD} = 7$, $b_2 = \frac{19}{6}$, $b_Y = -\frac{41}{6}$, $C_h = \sin \theta + c \cdot \cos \theta$, $C_r = \cos \theta - c \cdot \sin \theta$, $F_W = -(2 + 3y_W + 3y_W(2 - y_W)f(y_W))$, $F_t = y_t(1 + (1 - y_t)f(y_t))$, $y_t = 4m_t^2/(2p_1 \cdot p_2)$,

$$f(z) = \begin{cases} 
\sin^{-1} \left( \frac{1}{\sqrt{z}} \right)^2, & z \geq 1 \\
-\frac{1}{4} \left[ \log \frac{1 + \sqrt{1 - z}}{1 - \sqrt{1 - z}} - i \pi \right]^2, & z < 1.
\end{cases}$$
Table calculations in CompHEP

The Feynman rules have been implemented as a «new model» in a special version of the CompHEP package wich includes functions for table calculations and also a routine for $\chi^2$-analysis of the Higgs signal strength.

Table calculations

$\chi^2$ analysis and contour plots
Similarity of the Higgs boson and the radion production and decay amplitudes

The NNLO corrections are taken into account in the CompHEP computations and in the corresponding analysis by multiplying the involved vertices by correction factors for each model parameter point such that the partial and the total SM Higgs decay widths and the Higgs production cross sections in gluon–gluon fusion are exactly equal to those presented by the LHC Higgs Cross Section Working Group. Due to the similarity of the Higgs boson and the radion production and decay amplitudes including loops the same correction factors have been used for the Higgs- and the radion-dominated states.
We examine two possible scenarios, where the observed 125 GeV boson is either the Higgs-dominated state or the radion-dominated state.

At first we consider the case, where the higgs-dominated state has mass 125 GeV
Radion-dominated state branch fractions as functions of $m_{\text{Radion}}$

(Higgs-dominated state mass fixed at 125 GeV, $\Lambda_r=3$ TeV, $c=c_{\text{max}}$)

$$\sin \theta = 0$$

$$\sin \theta = 0.7$$

As well known the main decay mode of the light radion is the decay to two gluons due to the anomaly enhancement. However due to a compensation in the gluon-gluon-radion vertex between the trace anomaly part proportional to $1/\Lambda_r$ and the part proportional to $\sin \theta/\nu$ the vertex can be very small for some particular regions of the parameter space.

$$\text{gluon-gluon-radion vertex:} \quad \frac{g_s^2}{8\pi^2} \cdot \left[ b_{QCD} \cdot \frac{C_r}{\Lambda_r} + F_t \cdot \left( \frac{C_r}{\Lambda_r} - \frac{\sin \theta}{\nu} \right) \right] \cdot \left( g^{\mu\nu} p_1 p_2 - p_1^\nu p_2^\mu \right)$$

where $b_{QCD} = 7$, $C_r = \cos \theta - c \cdot \sin \theta$
Higgs-dominated and Radion-dominated states width as a function of $m_{\text{Radion}}$ and $\sin \Theta$

*(Higgs-dominated state mass fixed at 125 GeV, $\Lambda_r = 3$ TeV, $c = c_{\text{max}}$)*

Because of the compensation between various parts of the interaction vertex of the radion-dominated state with gluons the behavior of the total width modes in gluon-gluon fusion have some minima.
Partial cross sections as a functions of $m_{\text{Radion}}$ and $\sin \Theta$

(\textit{Higgs-dominated state mass fixed at 125 GeV, } $\Lambda_r = 3$ TeV, $c = c_{\text{max}}$, \textit{LHC } \sqrt{s} = 8$ TeV)

\[ gg \rightarrow \gamma \gamma \quad \text{and} \quad gg \rightarrow ZZ^* \]
Signal strength:

\[ \mu_i = \frac{\sigma_{i, \text{signal teor}}}{\sigma_{i, \text{signal SM}}} \quad \text{where } i \text{ is a number of signal channel} \]

Available experimental data provides the signal strength \( \hat{\mu}_i \) and corresponding error \( \Delta \hat{\mu}_i \):

\[ \hat{\mu}_i = \frac{N_{i, \text{obs}} - N_{i, \text{bkgr}}}{N_{i, \text{signal SM}}} \]

Global \( \chi^2 \):

\[ \chi^2(\mu_i) = \sum_{i}^{N_{\text{ch}}} \left( \frac{\mu_i - \hat{\mu}_i}{\Delta \hat{\mu}_i} \right)^2 \]
Exclusion contours for the partial $\chi^2$ as a functions of $m_{\text{Radion}}$ and $\sin\theta$

(Higgs-dominated state mass fixed at 125 GeV, $\Lambda_r=3$ TeV, $c=c_{\text{max}}$, LHC: $\sqrt{s}=8$ TeV. The dark medium and light shaded areas correspond to CL of the fit 65%, 90% and 99%)

$g \ g \rightarrow \gamma \gamma$

$\mu = 1.12 \pm 0.24$

$g \ g \rightarrow ZZ^*$

$\mu = 1 \pm 0.29$

The ZZ* mode gives restrictions on the radion mass region above the Z-boson pair threshold, where the cross section is increased.

The narrow allowed region on the right plot appears due to the above-mentioned numerical compensation in the interaction vertices leading to a smaller production cross section.
Exclusion contours for the combined $\chi^2$ fit ($gg \rightarrow \gamma\gamma$ and $gg \rightarrow ZZ^*$) as a functions of $m_{\text{Radion}}$ and $\sin \Theta$

*Higgs-dominated state mass fixed at 125 GeV, $\Lambda_r=3$ TeV, $c=c_{\text{max}}$, LHC: $\sqrt{s}=8$ TeV.*

The dark medium and light shaded areas correspond to CL of the fit 65%, 90% and 99%.

Here an allowed region of heavier masses of the radion-dominated state is present, where its production rate is decreased so that the corresponding state may be again allowed.
Exclusion contours for the combined $\chi^2$ fit (gg → γγ and gg → ZZ*) as a functions of $m_{\text{Radion}}$ and $\sin\Theta$

(Higgs-dominated state mass fixed at 125 GeV, $\Lambda_r=5$ TeV, $c=c_{\text{max}}$, LHC: $\sqrt{s}=8$ TeV. The dark medium and light shaded areas correspond to CL of the fit 65%, 90% and 99%)

If one considers larger values of the parameter $\Lambda_r$, the cross section of the radion-dominated state gets smaller and the allowed region for such a state is increased.
Now we consider the case, where the radion-dominated state has mass 125 GeV
Exclusion contours for the combined $\chi^2$ fit ($gg \rightarrow \gamma\gamma$ and $gg \rightarrow ZZ^*$) as a functions of $m_{Higgs}$ and $\sin\Theta$

(Radion-dominated state mass fixed at 125 GeV, $c=c_{\text{max}}$, LHC: $\sqrt{s}=8$ TeV. The dark medium and light shaded areas correspond to CL of the fit 65%, 90% and 99%)

One may see that due to specific cancellations between the anomaly and the SM-like pieces in the couplings to gluons and photons an additional small allowed region appears for some value sinus of the mixing angle.
Conclusion

- In the present work we have considered the Higgs-radion mixing arising in stabilized brane world models due to merging the mechanism of stabilization of the extra dimension size and the Higgs mechanism of spontaneous symmetry breaking on the TeV brane and discussed its phenomenological consequences. This mixing is, similar in many aspects to the one arising due to the Higgs-curvature term on the brane. However, an important difference is the presence of an extra coupling at low energies of the Higgs-dominated field to the trace of the SM energy-momentum tensor originating from the coupling of this field to the heavy scalar states of the radion KK tower.

- In order to present the physics of the Higgs-radion mixing in stabilized brane world models, we derived the effective Lagrangian and gave a qualitative description of the phenomena taking for the masses, the coupling constants and the mixing angle consistent values. It turned out that, though the interaction of an individual higher excited scalar state with the Higgs field may be weak, their cumulative effect on the Higgs-radion mixing may be observable.

- Our results show that the interpretation of the 125 GeV scalar state as a Higgs-dominated state is the preferred one, although the radion component in this state can be rather large. Depending on the value of the radion coupling constant $\Lambda_r$, the allowed regions for the mass of the radion-dominated state have been found. It turned out that the radion-dominated state can either have a mass close to 125 GeV, or a mass above 300 GeV, the allowed regions growing with the growth of the radion coupling constant $\Lambda_r$.

- We have also shown that the interpretation of the 125 GeV scalar state as a radion dominated state is not completely excluded by two leading signal strength measurements, though in this case the restrictions on the allowed masses of the Higgs-dominated state are very stringent. The mass of the Higgs-dominated state can either be close to 125 GeV, which is in accord with our analysis of this state at 125 GeV, or have a value somewhat above 160 GeV for a non-zero value of the mixing angle.