An effective theory for QCD with an axial chemical potential

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Based on

- A. A. Andrianov, V. A. Andrianov, D. Espriu & X. Planells, Phys. Lett. B 710 (2012) 230.
- A. A. Andrianov, D. Espriu & X. Planells, Eur. Phys. J. C (2013) 73:2294.

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Outline

- Motivation of local parity breaking (LPB)
- Axial baryon charge and axial chemical potential
- Effective scalar/pseudosalar meson theory with μ_5
- Vector Meson Dominance (VMD) approach to LPB
- Conclusions

Motivation of local parity breaking

Parity is one of the well established global symmetries of strong interactions. Yet there are reasons to believe that it may be broken in a finite volume since no fundamental principle forbids spontaneous parity breaking for $\mu \neq 0$.

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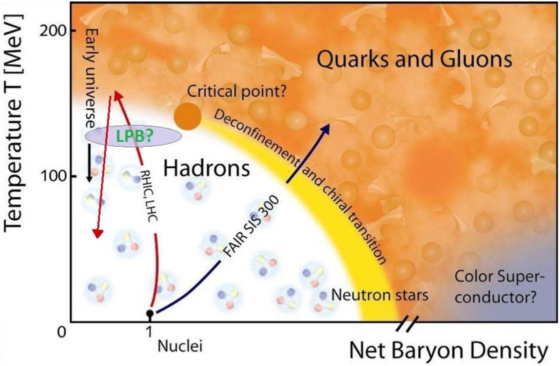
Motivation of local parity breaking

Local large fluctuations in the topological charge probably exist in a hot environment.

For *peripheral* Heavy Ion Collisions (HIC) they lead to the Chiral Magnetic Effect (CME): Large $\vec{B} \Rightarrow$ large $\vec{E} \Rightarrow$ charge separation.

For *central* collisions (and light quarks) they correspond to an ephemeral phase with axial chemical potential $\mu_5 \neq 0$.



Axial baryon charge and axial chemical potential

Topological charge T_5 may arise in a finite volume due to quantum fluctuations in a hot medium due to sphaleron transitions [Manton, McLerran, Rubakov, Shaposhnikov]

$$T_5 = \frac{1}{8\pi^2} \int_{\text{vol.}} d^3x \varepsilon_{jkl} \mathrm{Tr}\left(G^j \partial^k G^l - i\frac{2}{3} G^j G^k G^l \right)$$

and survive for a sizeable lifetime in a heavy-ion fireball. One can control the value of $\langle \Delta T_5 \rangle$ introducing into the QCD Lagrangian a topological chemical potential μ_θ via $\Delta\mathcal{L}_{top} = \mu_\theta \Delta T_5$, where

$$\Delta T_5 = T_5(t_f) - T_5(0) = \frac{1}{8\pi^2} \int_0^{t_f} dt \int_{\text{vol.}} d^3x \mathrm{Tr}\left(G^{\mu\nu} \widetilde{G}_{\mu\nu} \right).$$

The PCAC (broken by gluon anomaly) predicts the induced axial charge to be conserved during $\tau_{fireball}$ (in the chiral limit):

$$\frac{d}{dt}\left(Q_5^q - 2N_f T_5 \right) \simeq 0, \quad Q_5^q = \int_{\text{vol.}} d^3x \bar{q}\gamma_0\gamma_5 q = \langle N_L - N_R \rangle$$

Axial baryon charge and axial chemical potential

The characteristic left-right oscillation time is governed by inverse quark masses.

- For u, d quarks $1/m_q \sim 1/5\ \text{MeV}^{-1} \sim 40$ fm $\gg \tau_{\text{fireball}}$ and the left-right quark mixing can be neglected.
- For s quark $1/m_s \sim 1/150\ \text{MeV}^{-1} \sim 1$ fm $\ll \tau_{\text{fireball}}$ and $\langle Q_5^s \rangle \simeq 0$ due to left-right oscillations.

For u, d quarks QCD with a background topological charge leads to the generation of an axial chemical potential μ_5, conjugate to Q_5^q

$$\langle \Delta T_5 \rangle \simeq \frac{1}{2N_f} \langle Q_5^q \rangle \iff \mu_5 \simeq \frac{1}{2N_f} \mu_\theta,$$

$$\Delta \mathcal{L}_{top} = \mu_\theta \Delta T_5 \iff \Delta \mathcal{L}_q = \mu_5 Q_5^q$$

In the scalar sector the spurion technique can be used by taking μ_5 as the time component of some external axial-vector field

$$D_\nu \Longrightarrow D_\nu - i\{\mathbf{I}_q \mu_5 \delta_{0\nu}, \cdot\} = D_\nu - 2i\mathbf{I}_q \mu_5 \delta_{0\nu}.$$

Note the breaking of Lorentz symmetry.

Two new processes are likely to appear inside the fireball: the decays $\eta, \eta' \to \pi\pi$ that are strictly forbidden in QCD on parity grounds.

In a medium where parity is broken: are these processes relevant within the fireball? Can these particles reach thermal equilibrium?!

Effective Lagrangian:

$$\mathcal{L} = \frac{1}{4}\mathrm{Tr}\left(D_\mu H D^\mu H^\dagger\right) + \frac{b}{2}\mathrm{Tr}\left[M(H+H^\dagger)\right] + \frac{M^2}{2}\mathrm{Tr}\left(HH^\dagger\right)$$
$$-\frac{\lambda_1}{2}\mathrm{Tr}\left[(HH^\dagger)^2\right] - \frac{\lambda_2}{4}\left[\mathrm{Tr}\left(HH^\dagger\right)\right]^2 + \frac{c}{2}(\det H + \det H^\dagger)$$
$$+\frac{d_1}{2}\mathrm{Tr}\left[M(HH^\dagger H + H^\dagger H H^\dagger)\right] + \frac{d_2}{2}\mathrm{Tr}\left[M(H+H^\dagger)\right]\mathrm{Tr}\left(HH^\dagger\right)$$

where

$$H = \xi\Sigma\xi, \quad \xi = \exp\left(i\frac{\Phi}{2f}\right), \quad \Phi = \lambda^a\phi^a, \quad \Sigma = \lambda^b\sigma^b.$$

The v.e.v. of the neutral scalars are defined as $v_i = \langle\Sigma_{ii}\rangle$ where $i = u, d, s$, and satisfy the following gap equations:

$$M^2 v_i - 2\lambda_1 v_i^3 - \lambda_2 \vec{v}^2 v_i + c\frac{v_u v_d v_s}{v_i} = 0.$$

For further purposes we need the non-strange meson sector and η_s

$$\Phi = \begin{pmatrix} \eta_q + \pi^0 & \sqrt{2}\pi^+ & 0 \\ \sqrt{2}\pi^- & \eta_q - \pi^0 & 0 \\ 0 & 0 & \sqrt{2}\eta_s \end{pmatrix}, \Sigma = \begin{pmatrix} v_u + \sigma + a_0^0 & \sqrt{2}a_0^+ & 0 \\ \sqrt{2}a_0^- & v_d + \sigma - a_0^0 & 0 \\ 0 & 0 & v_s \end{pmatrix}$$

$$\begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix} = \begin{pmatrix} \cos\psi & \sin\psi \\ -\sin\psi & \cos\psi \end{pmatrix} \begin{pmatrix} \eta \\ \eta' \end{pmatrix}$$

For $\mu_5 = 0$, we assume $v_u = v_d = v_s = v_0 \equiv f_\pi \approx 92$ MeV. The coupling constants (in units of MeV) are fitted to phenomenology assuming isospin symmetry via χ^2 minimization (MINUIT):

$$b = -3510100/m,\ M^2 = 1255600,\ c = 1252.2,\ \lambda_1 = 67.007,$$

$$\lambda_2 = 9.3126,\ d_1 = -1051.7/m,\ d_2 = 523.21/m,$$

where $m \equiv m_q = (m_u + m_d)/2$ and $m/m_s \simeq 1/25$.

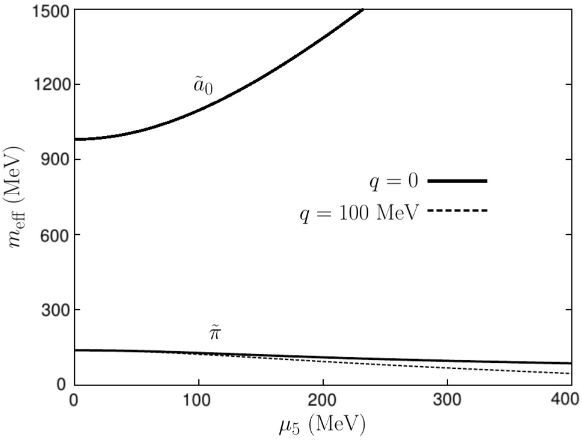
We present a simple case of mixing due to LPB in the isotriplet sector with π and a_0. The kinetic and mixing terms in the Lagrangian are given by

$$\mathcal{L} = \frac{1}{2}(\partial a_0)^2 + \frac{1}{2}(\partial \pi)^2 - \frac{1}{2}m_1^2 a_0^2 - \frac{1}{2}m_2^2\pi^2 - 4\mu_5 a_0 \dot{\pi},$$

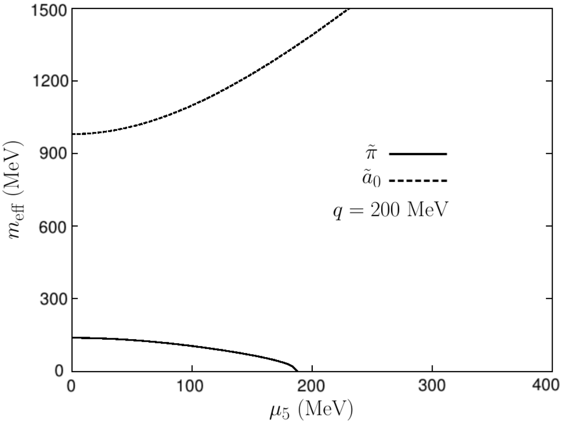
where

$$\begin{aligned} m_1^2 =& -2[M^2 - 2(3\lambda_1 + \lambda_2)v_q^2 - \lambda_2 v_s^2 \\ & - cv_s + 2(3d_1 + 2d_2)mv_q + 2d_2 m_s v_s + 2\mu_5^2] \\ m_2^2 =& \frac{2m}{v_q}\left[b + (d_1 + 2d_2)v_q^2 + d_2 v_s^2\right] \end{aligned}$$

After diagonalization in the momentum representation, the new (momentum-dependent) eigenstates are defined $\tilde{\pi}$ and $\tilde{a}_0$.



For high energies $k_0, |\vec{k}| > m_1 m_2/(4\mu_5) \equiv k^c_{\tilde{\pi}}$, in-medium $\tilde{\pi}$ goes tachyonic. Nevertheless, energies are always positive (no vacuum instabilities). $\tilde{a}_0$ mass shows an enhancement, but μ_5 has to be understood as a perturbatively small parameter.



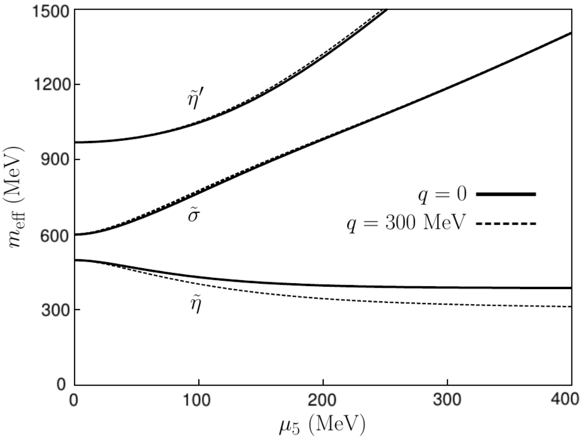
In the isosinglet sector, we show the mixing of η, σ and η'. The kinetic and mixing terms in the Lagrangian are given by

$$\begin{aligned}\mathcal{L} =&\frac{1}{2}[(\partial\sigma)^2 + (\partial\eta_q)^2 + (\partial\eta_s)^2] - \frac{1}{2}m_3^2\sigma^2 - \frac{1}{2}m_4^2\eta_q^2 - \frac{1}{2}m_5^2\eta_s^2 \\ &- 4\mu_5\sigma\dot{\eta}_q - 2\sqrt{2}cv_q\eta_q\eta_s,\end{aligned}$$

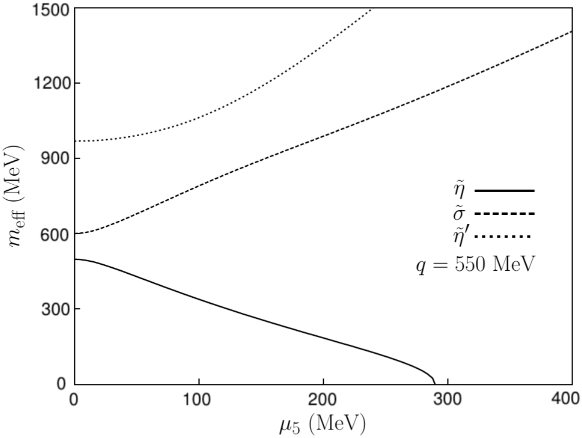
where

$$\begin{aligned}m_3^2 =& - 2(M^2 - 6(\lambda_1 + \lambda_2)v_q^2 - \lambda_2 v_s^2 + cv_s \\ &+ 6(d_1 + 2d_2)mv_q + 2d_2 m_s v_s + 2\mu_5^2), \\ m_4^2 =&\frac{2m}{v_q}\left[b + (d_1 + 2d_2)v_q^2 + d_2 v_s^2\right] + 2cv_s, \\ m_5^2 =&\frac{2m_s}{v_s}[b + 2d_2 v_q^2 + (d_1 + d_2)v_s^2] + \frac{cv_q^2}{v_s}.\end{aligned}$$

After diagonalization, the new eigenstates are $\tilde{\sigma}$, $\tilde{\eta}$ and $\tilde{\eta}'$.



Again, for high energies $k_0, |\vec{k}| > k^c_{\tilde{\eta}}$ with $k^c_{\tilde{\eta}} \equiv \frac{m_3}{4\mu_5 m_5}\sqrt{m_4^2 m_5^2 - 8c^2 v_q^2}$, in-medium $\tilde{\eta}$ goes tachyonic.



The cubic couplings used to calculate the widths $\tilde{\eta}, \tilde{\sigma}, \tilde{\eta}' \to \tilde{\pi}\tilde{\pi}$ from the Lagrangian are given by

$$\mathcal{L}_{\sigma aa} = 2[(3d_1 + 2d_2)m - 2(3\lambda_1 + \lambda_2)v_q]\sigma \vec{a}_0^2,$$

$$\mathcal{L}_{\sigma\pi\pi} = \frac{1}{v_q^2}\left[(\partial\vec{\pi})^2 v_q - (b + 3(d_1 + 2d_2)v_q^2 + d_2 v_s^2)m\vec{\pi}^2\right]\sigma,$$

$$\mathcal{L}_{\eta a\pi} = \frac{2}{v_q^2}\vec{a}_0[\partial\eta_q\partial\vec{\pi}v_q - (b + (3d_1 + 2d_2)v_q^2 + d_2 v_s^2)m\eta_q\vec{\pi}],$$

$$\mathcal{L}_{\sigma a\pi} = -\frac{4\mu_5}{v_q}\sigma\vec{a}_0\dot{\vec{\pi}}, \quad \mathcal{L}_{\eta aa} = -\frac{2\mu_5}{v_q}\dot{\eta}_q\vec{a}_0^2, \quad \mathcal{L}_{\eta\pi\pi} = 0.$$

After diagonalization, one replaces the initial $\{\eta_q, \eta_s, \sigma\}$ to $\{\tilde{\eta}, \tilde{\sigma}, \tilde{\eta}'\}$ and $\{\pi, a_0\}$ to $\{\tilde{\pi}, \tilde{a}_0\}$.

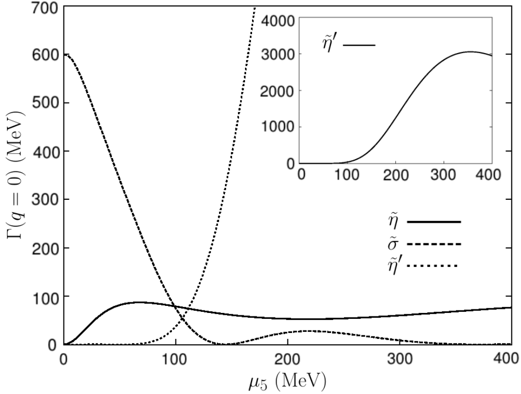
The widths are firstly computed at the rest frame of the decaying particle and then with a boost.

$\tilde{\eta}$ exhibits a smooth behaviour with $\langle \Gamma_{\tilde{\eta}} \rangle \sim 60$ MeV $\leftrightarrow$ mean free path ~ 3 fm $\lesssim L_{\text{fireball}} \sim 5 \div 10$ fm. Possible thermalization!

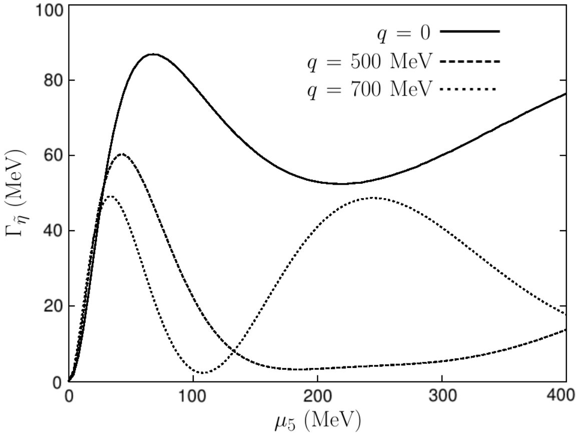
Down to $\mu_5 \sim 100$ MeV, $\tilde{\sigma}$ width decreases and becomes stable.

The bumps seem to reflect the tachyonic nature of the decaying $\tilde{\pi}$.

$\tilde{\eta}'$ width grows up to the GeV scale (violation of unitarity).



Decay widths in the isosinglet case show strong dependences on the 3-momentum (nothing to do with Lorentz time dilatation).



Vector Meson Dominance approach to LPB

If we assume that the vector mesons appear as part of a covariant derivative, no mixing term can be generated. However, such a mixing is not forbidden. This coupling is very model dependent.

Vector mesons can be introduced in the Vector Meson Dominance framework with no mixing of states with different parities. The only LPB effect will be the Chern-Simons term

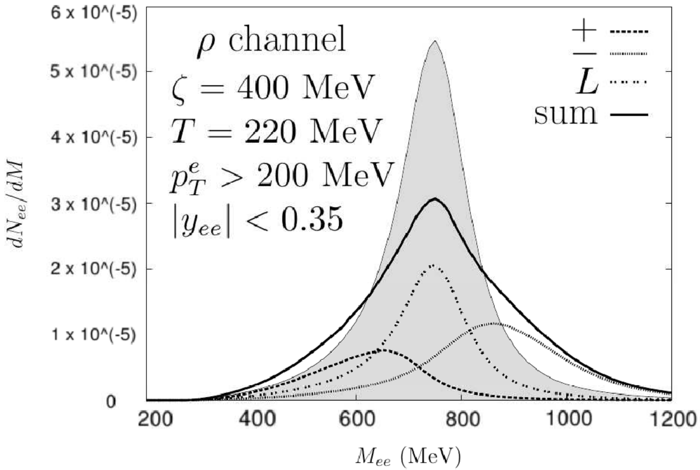
$$\Delta\mathcal{L} \simeq \varepsilon^{\mu\nu\rho\sigma}\mathrm{Tr}\left[\zeta_\mu V_\nu V_{\rho\sigma}\right],$$

where $\zeta_\mu \propto \mu_5 \delta_{\mu 0}$. Vector mesons exhibit the following dispersion relation:

$$m_{V,\epsilon}^2 - m_V^2 \propto \epsilon\mu_5 |\vec{k}|,$$

where $\epsilon = 0, \pm 1$ is the meson polarization. Note the breaking of Lorentz symmetry again.

Massive vector mesons split into three polarizations with masses $m_{V,-}^2 < m_{V,L}^2 < m_{V,+}^2$.



Polarization splitting in $\rho \to e^+e^-$ decay for LPB $\mu_5 = 290$ MeV compared with $\mu_5 = 0$ (shaded region).

Note the polarization asymmetry aside the peak.

Conclusions

- LPB not forbidden by any physical principle in QCD at finite temperature/density.
- Topological fluctuations transmit their influence to hadronic physics via an axial chemical potential.
- LPB leads to unexpected modifications of the in-medium properties of scalar and vector mesons.
- The new eigenstate $\tilde{\eta}$ seems to be in thermal equilibrium with the pion gas in the HIC fireball.

Thank you for your attention!

Backup I

MINUIT input in MeV

$$v_0^{\rm exp} = 92 \pm 5, \quad m_\pi^{\rm exp} = 137 \pm 5, \quad m_a^{\rm exp} = 980 \pm 50,$$
$$m_\sigma^{\rm exp} = 600 \pm 120, \quad m_\eta^{\rm exp} = 548 \pm 50, \quad m_{\eta'}^{\rm exp} = 958 \pm 100,$$
$$\Gamma_a^{\rm exp} = 60 \pm 30, \quad \Gamma_\sigma^{\rm exp} = 600 \pm 120.$$

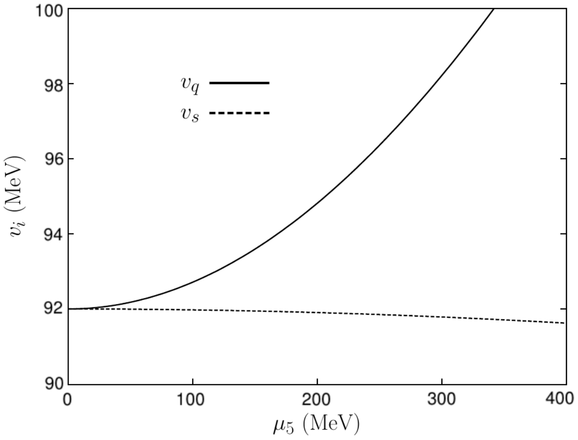
MINUIT output versus experimental values in MeV

Magnitude	MINUIT	Exp. value	Error
v_0	92.00	92	-3.52×10^{-7}
m_π	137.84	137	6.10×10^{-3}
m_a	980.00	980	-1.26×10^{-6}
m_σ	599.99	600	-1.66×10^{-5}
m_η	497.78	548	-9.16×10^{-2}
$m_{\eta'}$	968.20	958	1.06×10^{-2}
Γ_a	60.00	60	2.04×10^{-5}
Γ_σ	600.00	600	6.81×10^{-6}

$\psi^{\rm exp} \simeq -18^\circ + \arctan\sqrt{2} \simeq 36.7^\circ$, while $\psi_{\tt MINUIT} \approx 35.46^\circ$.

Backup II

Vacuum: for non-vanishing isosinglet μ_5 we impose our solutions to be $v_u = v_d = v_q \neq v_s$.



Backup III

μ_5-dependence of the tachyon critical energy for isotriplet $k^c_{\tilde{\pi}}$ and isosinglet case $k^c_{\tilde{\eta}}$.