Review of Belle Results

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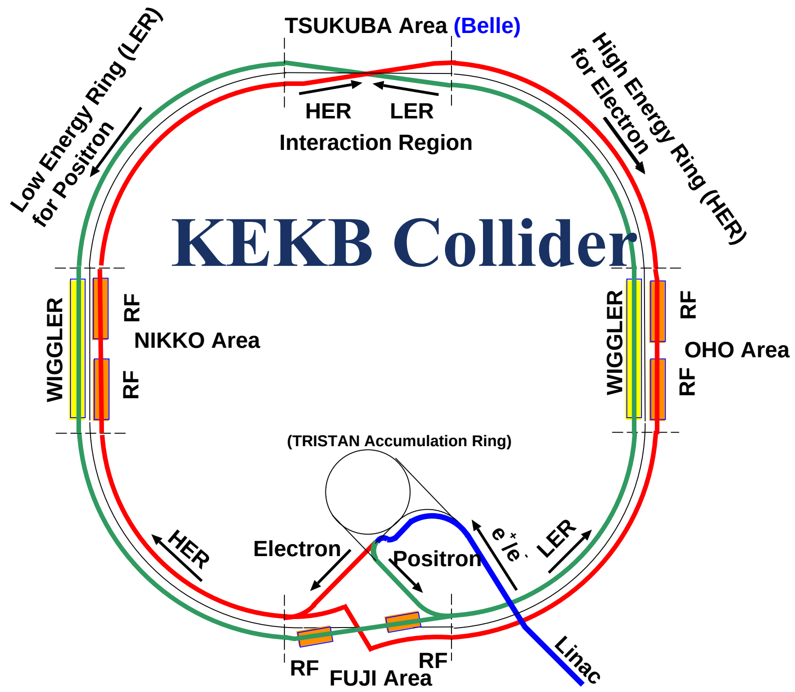
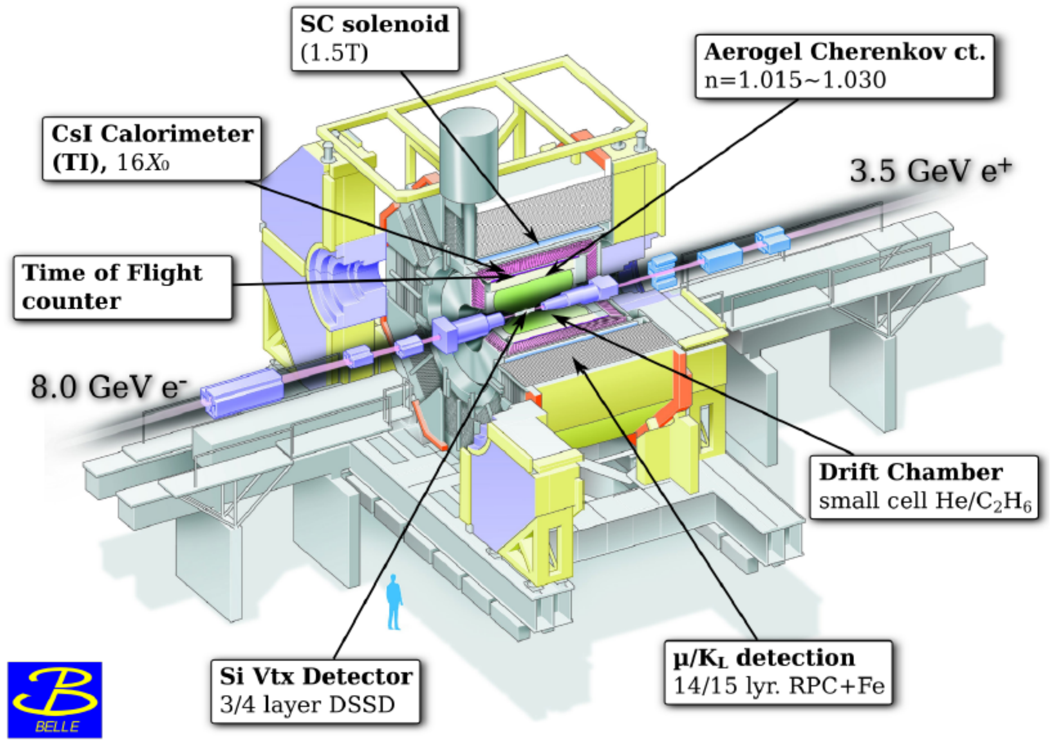
- Introduction
- UT measurements
  - $\phi_1(\beta)$
  - $\phi_2\ (\alpha)$
  - $\phi_3\ (\gamma)$
  - $|V_{xb}|$
- Summary

KEKB and Belle





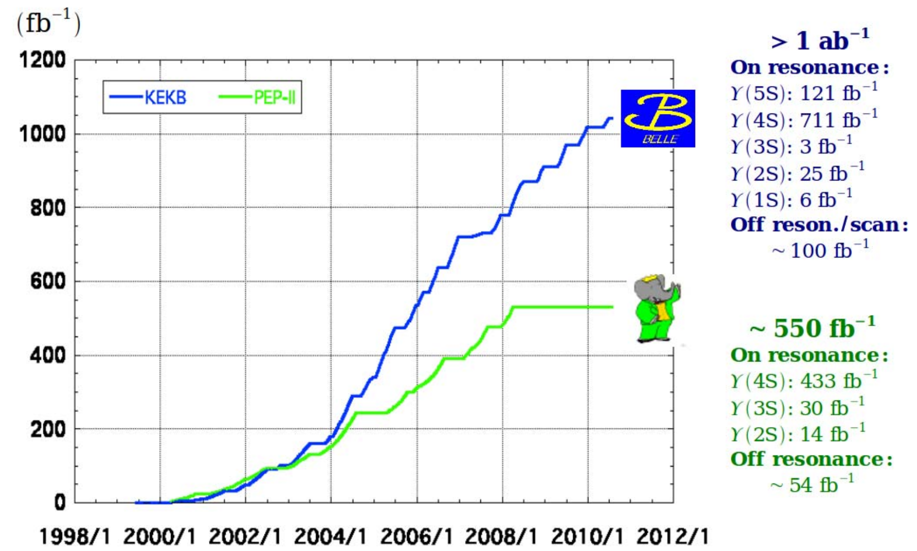
3.5 GeV e^+ & 8 GeV e^- beams

3 km circ, 22 mrad crossing angle

L = 2.1 x 10^{34} $cm^{-2}s^{-1}$

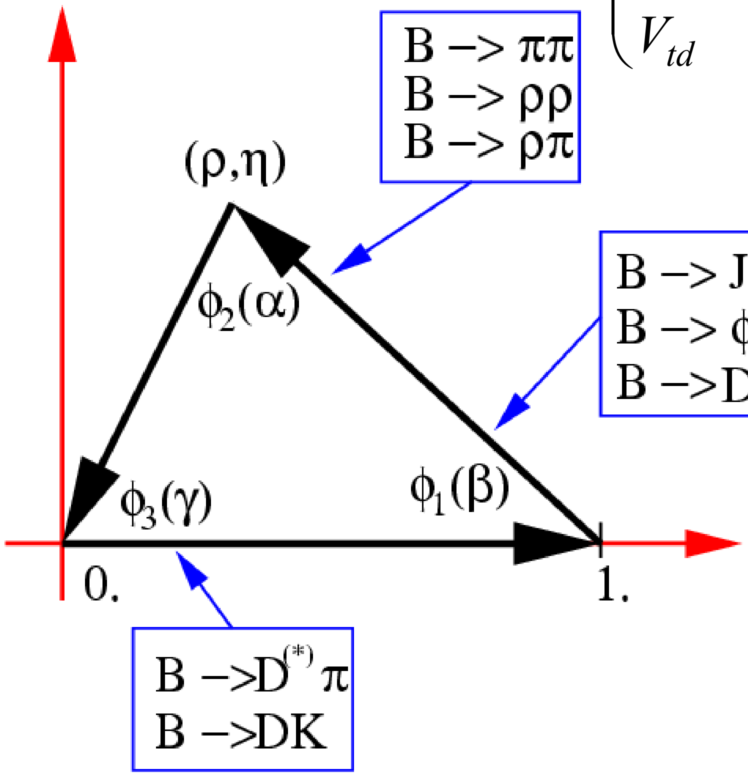
$\int L\,dt$ = 1.04 ab^{-1}

Integrated luminosity of B factories



Unitarity triangle

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \approx \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$



Using unitarity requirement:

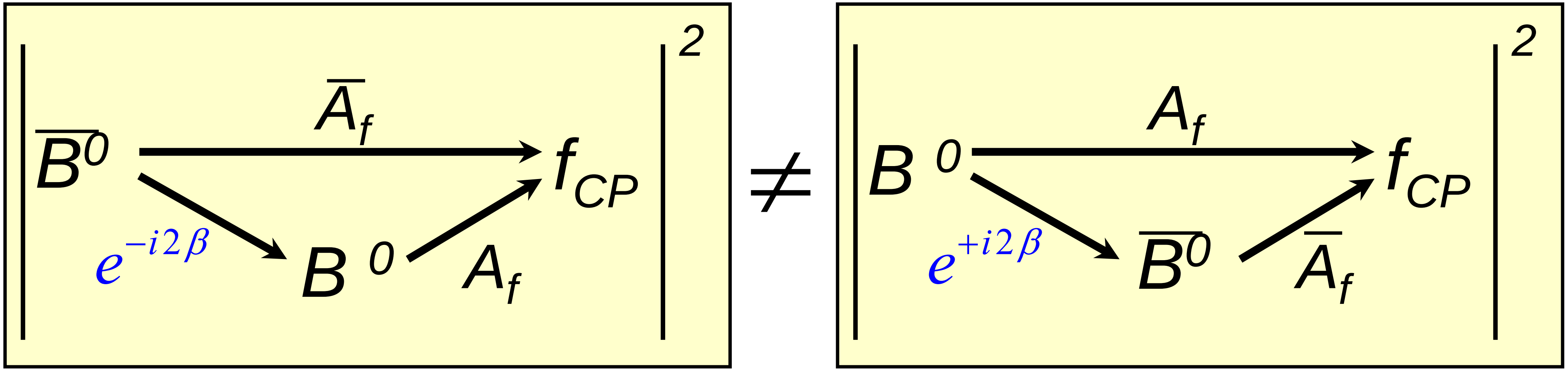
$$\{i = 1, k = 3\} : V_{ub}^{*} V_{ud} + V_{cb}^{*} V_{cd} + V_{tb}^{*} V_{td} = 0$$

$$\Rightarrow \frac{V_{ub}^{*} V_{ud}}{V_{cb}^{*} V_{cd}} + 1 + \frac{V_{tb}^{*} V_{td}}{V_{cb}^{*} V_{cd}} = 0$$

$\sin 2\phi_1$ is measured with a good accuracy at B-factories.

Measurement of all the angles needed to test SM.

CPV in Mixing



$$A_{f_{CP}}(t) = \frac{\Gamma(\overline{B}^0 \to f_{CP}) - \Gamma(B^0 \to f_{CP})}{\Gamma(\overline{B}^0 \to f_{CP}) + \Gamma(B^0 \to f_{CP})}$$

$$= -\eta_{CP}[S_{f_{CP}} \sin(\Delta m\, t) - C_{f_{CP}} \cos(\Delta m\, t)]$$

$$S_{f_{CP}} = \frac{2 \mathrm{Im} \lambda_{f_{CP}}}{1 + |\lambda_{f_{CP}}|^2}$$

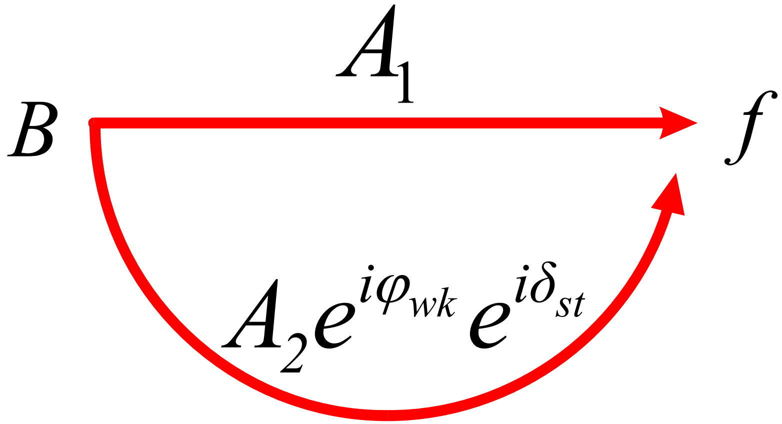
$$C_{f_{CP}} = \frac{1 - |\lambda_{f_{CP}}|^2}{1 + |\lambda_{f_{CP}}|^2}$$

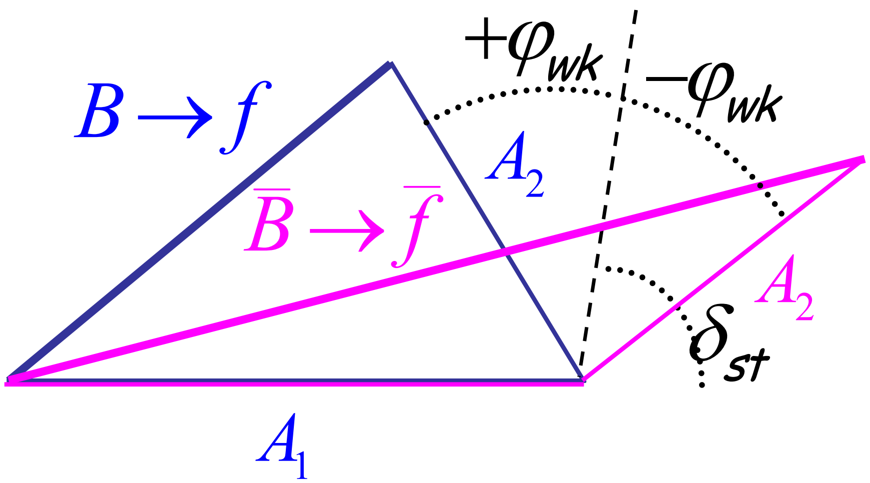
$$\lambda_{f_{CP}} = -e^{-i2\beta} \frac{A(\overline{B}^0 \to f_{CP})}{A(B^0 \to f_{CP})}$$

Difference in decay rate for B^0 and $\overline{B}^0$
→ CP Violation

Direct CPV in charged B Decays

CP violation through interference of decay amplitudes





$$\Gamma(B \to f) = \left| A_1 + A_2 e^{i\varphi_{wk}} e^{i\delta_{st}} \right|^2$$

$$\Gamma(\bar{B} \to \bar{f}) = \left| A_1 + A_2 e^{-i\varphi_{wk}} e^{i\delta_{st}} \right|^2$$

$$\Gamma(\mathrm{B} \to \mathrm{f}) \neq \Gamma(\bar{\mathrm{B}} \to \bar{\mathrm{f}}) \text{ for } \varphi_{\mathrm{wk}} \neq 0 \text{ and } \delta_{\mathrm{st}} \neq 0$$

$$A_{CP} = \frac{G(f) - G(\bar{f})}{G(f) + G(\bar{f})} = \frac{2|A_1||A_2|\sin(j_{wk})\sin(d_{st})}{|A_1|^2 + |A_2|^2 + 2|A_1||A_2|\cos(j_{wk})\cos(d_{st})}$$

One rate asymmetry is not sufficient to extract physical parameters:

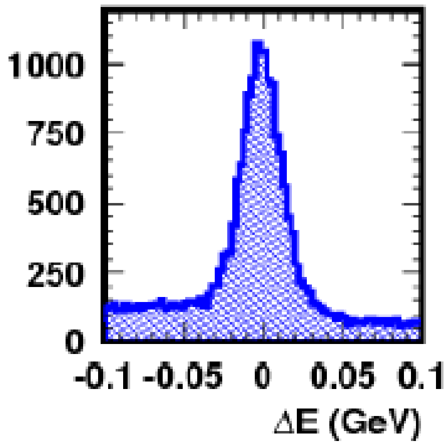
- *measure A and $\bar{A}$, but need A_1, A_2, ϕ_{wk}, δ_{st}*

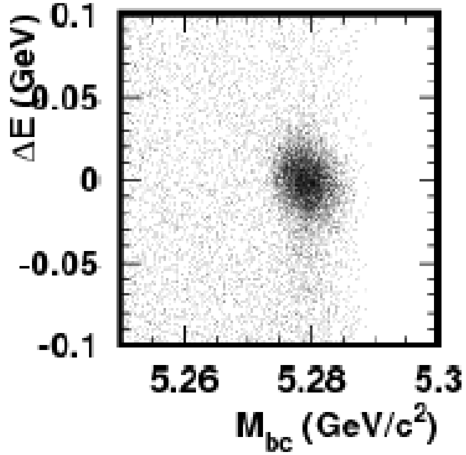
Kinematics and event shape

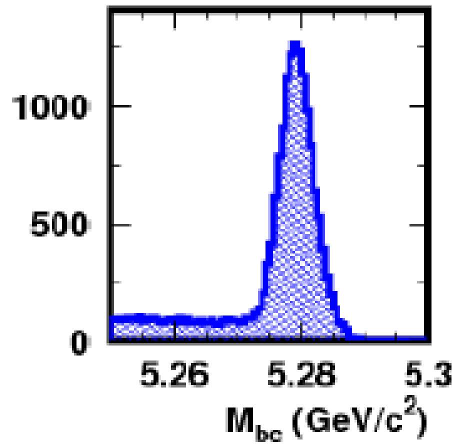
In $\Upsilon(4S)$ decays, pairs of B mesons are produced near threshold.
$E_B = E_{\mathrm{CM}}/2$, small CM momentum (300 MeV$/c$).

Selection variables:

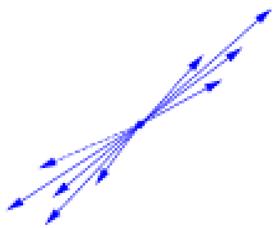
- CM energy difference
  $\Delta E = \sum E_i - E_{\mathrm{CM}}/2$
- B-meson beam-constrained mass
  $M_{\mathrm{bc}} = \sqrt{(E_{\mathrm{CM}}/2)^2 - (\sum p_i)^2}$
- Event shape variables:



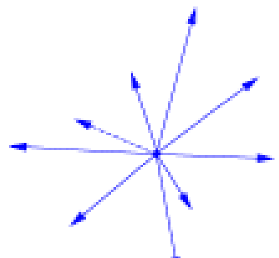




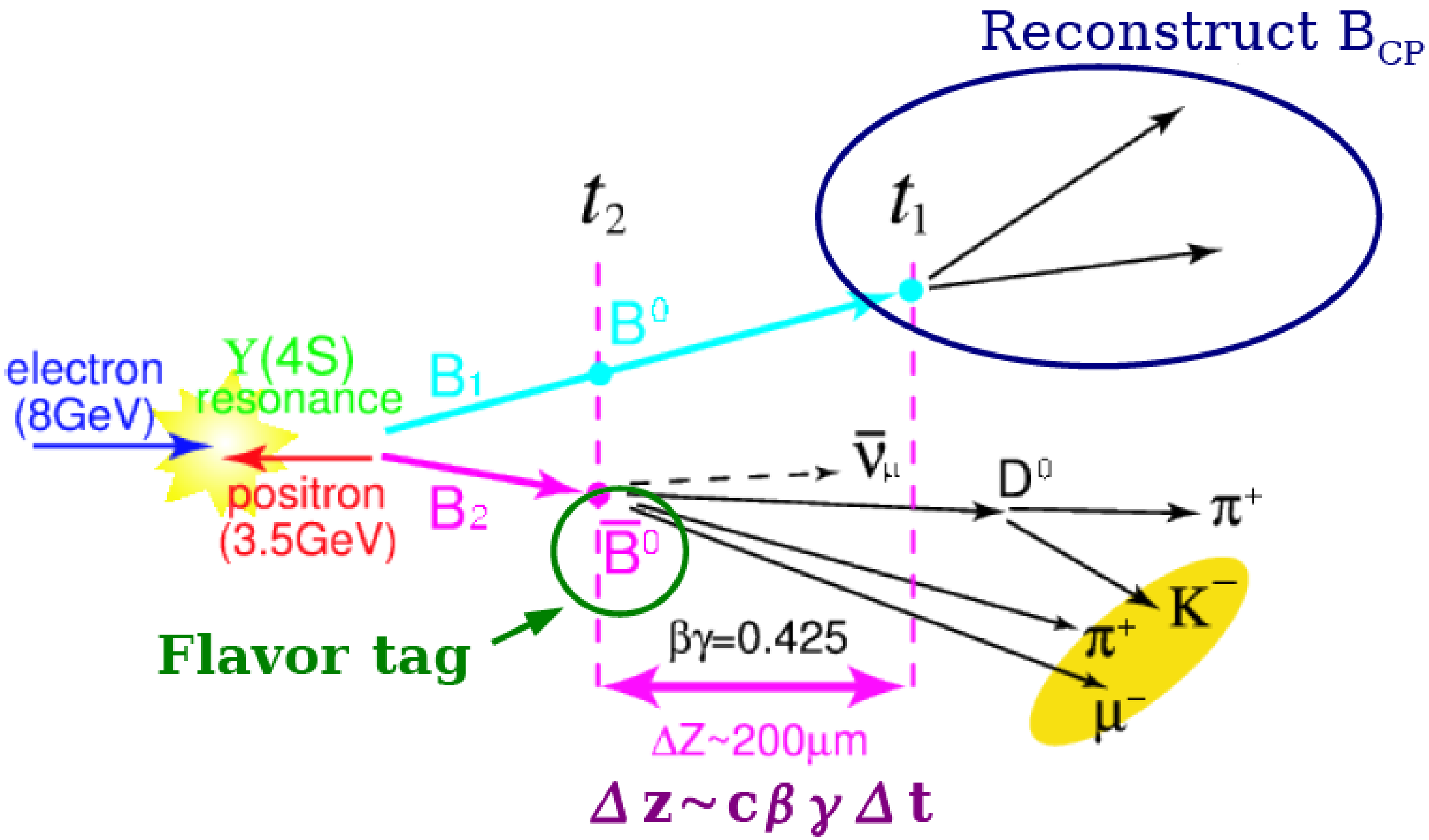
$e^+e^- \to u\bar{u}, d\bar{d}, s\bar{s}, c\bar{c}$:



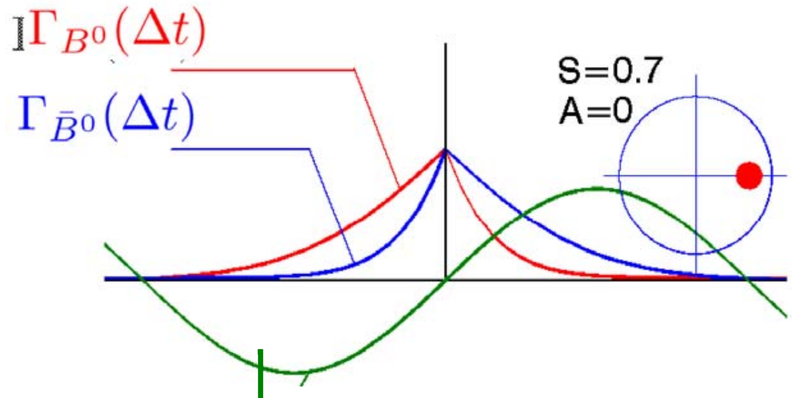
$e^+e^- \to b\bar{b}$:



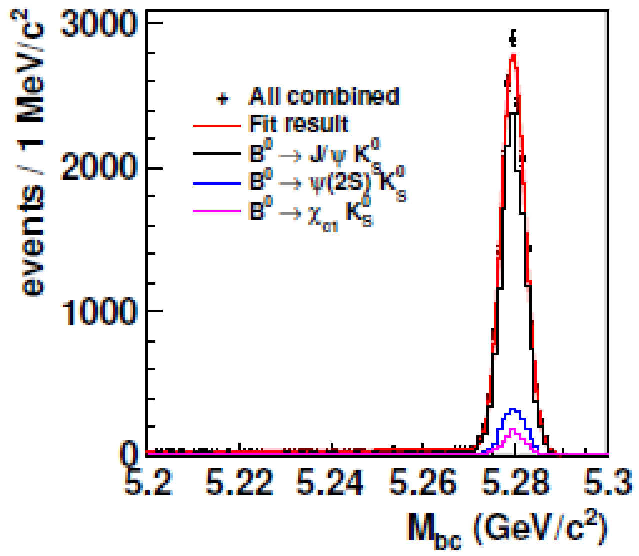
How to measure TCPV



$$
\begin{aligned}
&A_{CP}(\Delta t)\\
&\equiv \frac{\Gamma_{\bar{B}^0}(\Delta t)-\Gamma_{B^0}(\Delta t)}{\Gamma_{\bar{B}^0}(\Delta t)+\Gamma_{B^0}(\Delta t)}\\
&= S\sin\Delta m\Delta t + A\cos\Delta m\Delta t
\end{aligned}
$$

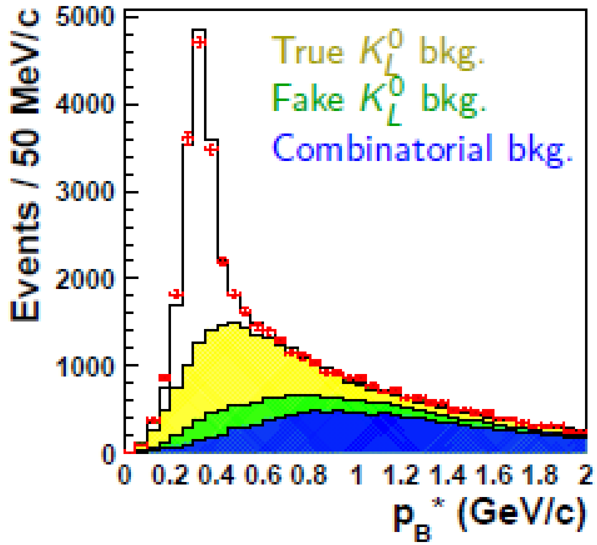


Selection of $B \to (c\bar{c})K^0$



$CP = -1$ modes:

Mode	Signal yield
$B \to J/\psi K^0_S,\ J/\psi \to l^+ l^-$	12681 ± 114
$B \to \psi(2S) K^0_S,\ \psi(2S) \to l^+ l^-$	908 ± 31
$\psi(2S) \to J/\psi \pi^+ \pi^-$	1072 ± 33
$B \to \chi_{c1} K^0_S,\ \chi_{c1} \to J/\psi \gamma$	943 ± 33



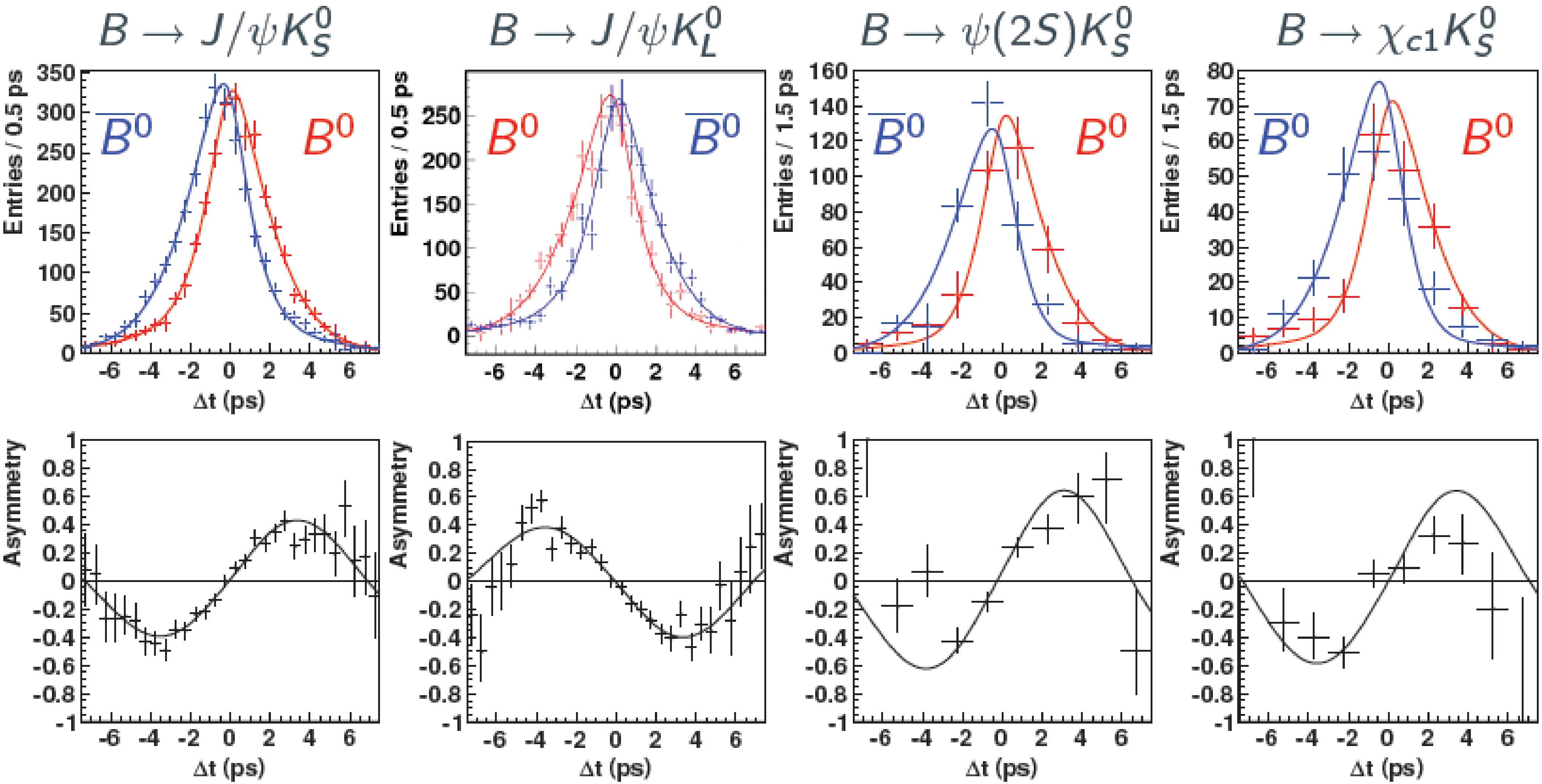
$CP = +1$ mode:

$B \to J/\psi K^0_L$ Signal yield: 10041 ± 154

Missing information about K^0_L momentum: K^0_L cluster reconstructed in ECL or KLM, match it with the K^0_L direction from kinematical constraints.

PRL 108, 171802 (2012)

CP asymmetry in B→(c$\underline{c}$)K^0

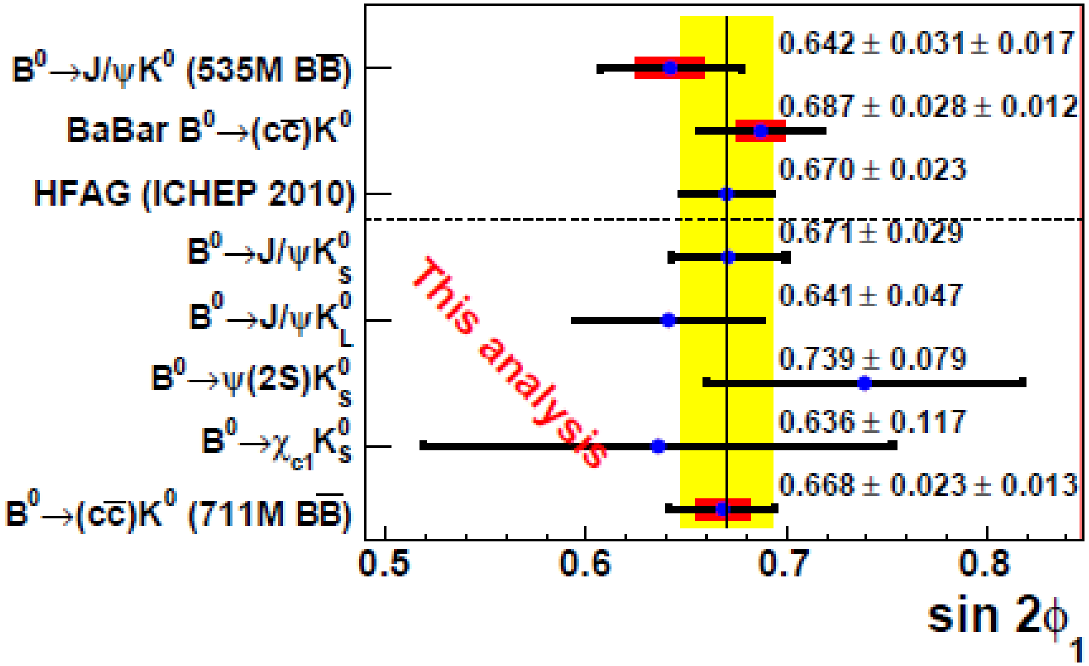


$S = 0.671 \pm 0.029$ $S = 0.641 \pm 0.047$ $S = 0.739 \pm 0.079$ $S = 0.636 \pm 0.117$

$A = -0.014 \pm 0.021$ $A = 0.019 \pm 0.026$ $A = 0.103 \pm 0.055$ $A = -0.023 \pm 0.083$

PRL 108, 171802 (2012)

Measurement of $\sin 2\phi_1$ ($\sin 2\beta$)



Combination of four modes:

$$S = 0.668 \pm 0.023 \pm 0.013\ (\text{syst})$$

$$A = 0.007 \pm 0.016 \pm 0.013\ (\text{syst})$$

PRL 108, 171802 (2012)

Systematic errors:

	ΔS	ΔA
Vertexing	$^{+0.008}_{-0.009}$	± 0.008
Flavor tagging	$^{+0.004}_{-0.003}$	± 0.003
Resolution function	± 0.007	± 0.001
Physics parameters	± 0.001	< 0.001
Fit bias	± 0.004	± 0.005
$J/\psi K^0_S$ signal fraction	± 0.002	± 0.001
$J/\psi K^0_L$ signal fraction	± 0.004	$^{+0.000}_{-0.002}$
$\psi(2S) K^0_S$ signal fraction	< 0.001	< 0.001
$\chi_{c1} K^0_S$ signal fraction	< 0.001	< 0.001
Background Δt	± 0.001	< 0.001
Tag-side interference	± 0.001	± 0.008
Total	± 0.013	± 0.013

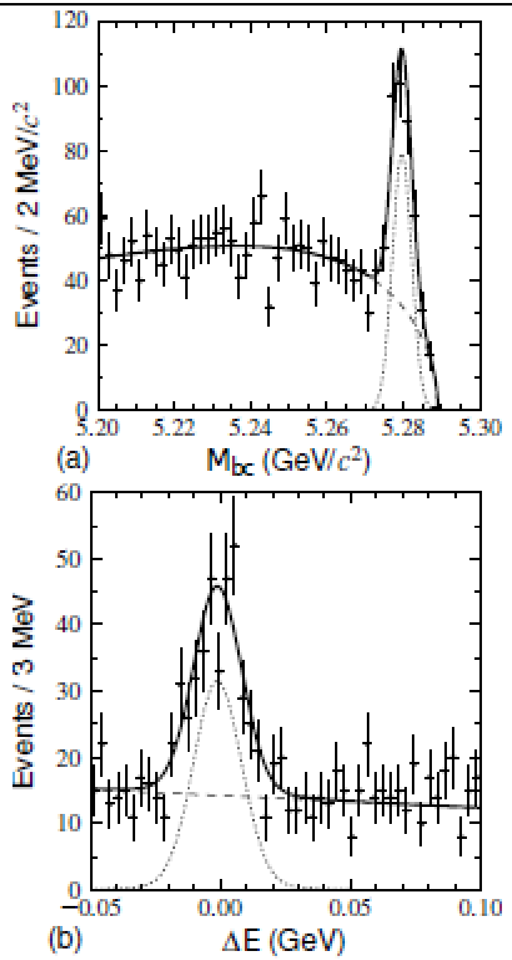
Significant improvement in sys. error

CPV in double charm

Final Belle data sample of 772×10^6 $B\bar{B}$ *pairs*

$B^0 \to D^+ D^-$

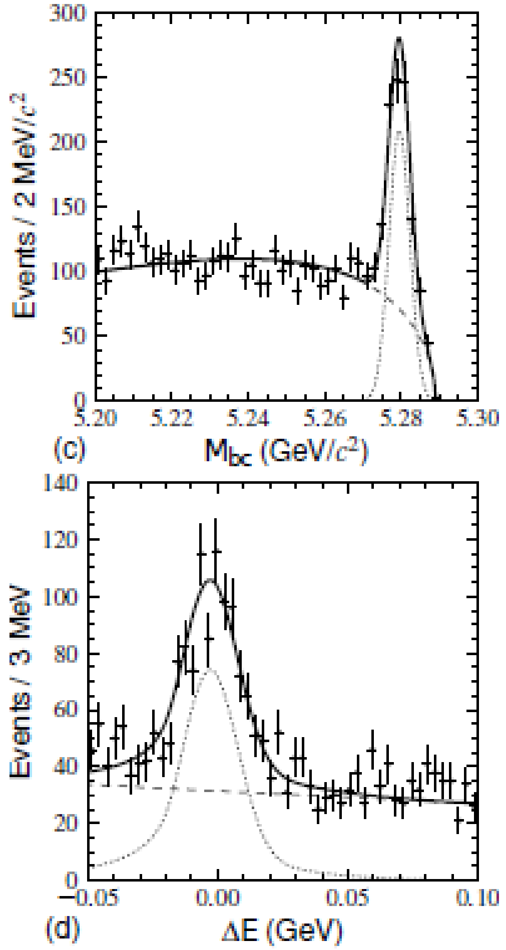
Showed huge direct CP-violation



$N_{\rm sig} = 269 \pm 21$

$B^0 \to D^{*\pm} D^{\mp}$

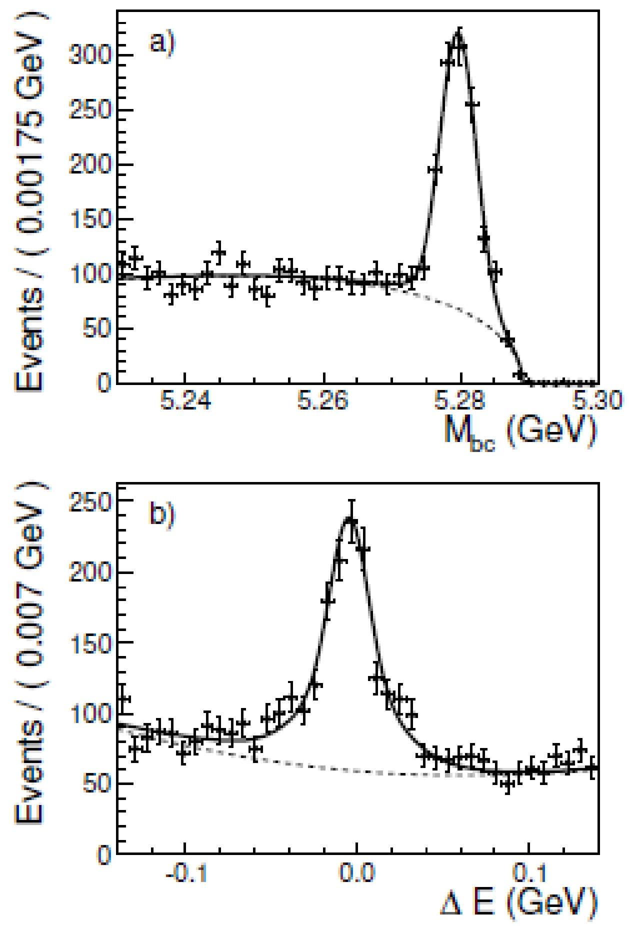
No CP-eigenstate



$N_{\rm sig} = 887 \pm 39$

$B^0 \to D^{*+} D^{*-}$

Admixture of CP-eigenstates

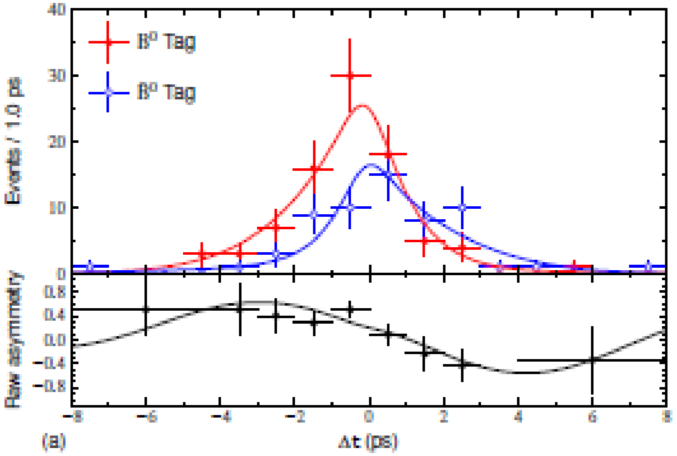


$N_{\rm sig} = 1225 \pm 59$

CPV in double charm

CP parameters

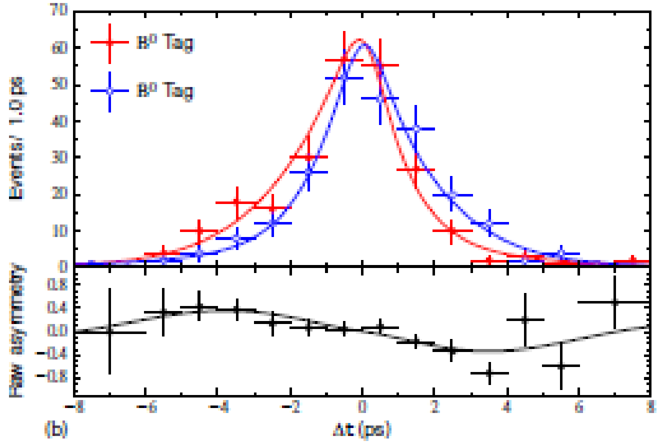
$B^0 \to D^+ D^-$



$$\mathcal{S} = -1.06^{+0.21}_{-0.14} \pm 0.08$$
$$\mathcal{A} = 0.43 \pm 0.16 \pm 0.05$$

Significance: 4.2σ

$B^0 \to D^{*\pm} D^{\mp}$

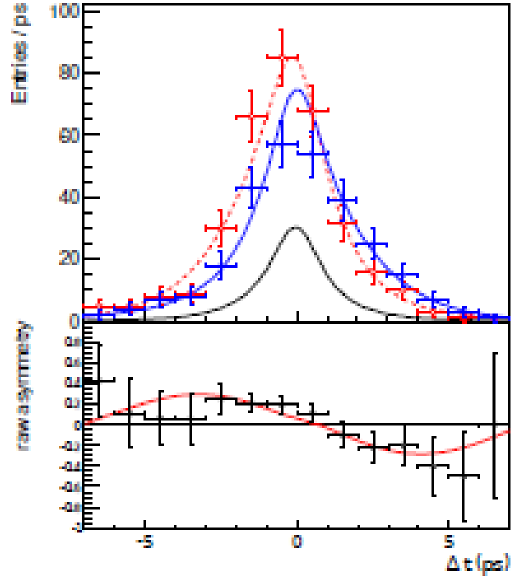


$$\begin{aligned} \mathcal{S} &= -0.78 \pm 0.15 \pm 0.05 \\ \mathcal{A} &= 0.01 \pm 0.11 \pm 0.04 \\ A &= 0.06 \pm 0.05 \pm 0.02 \\ \Delta\mathcal{S} &= -0.13 \pm 0.15 \pm 0.04 \\ \Delta\mathcal{A} &= 0.12 \pm 0.11 \pm 0.03 \end{aligned}$$

Significance: 4.0σ

PRD 85, 091106(R) (2012)

$B^0 \to D^{*+} D^{*-}$



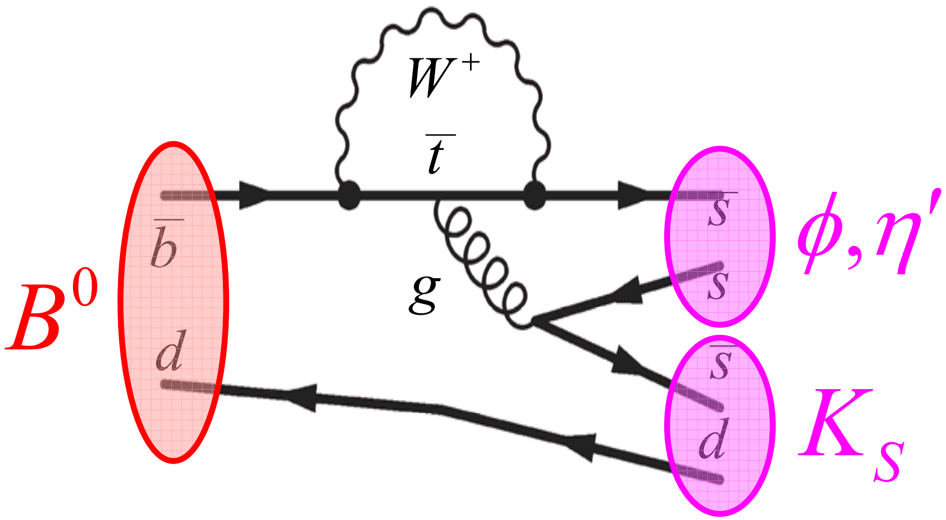
$$\mathcal{S} = -0.79 \pm 0.13 \pm 0.03$$
$$\mathcal{A} = 0.15 \pm 0.08 \pm 0.02$$

Significance: 5.4σ

PRD 86, 071103(R) (2012)

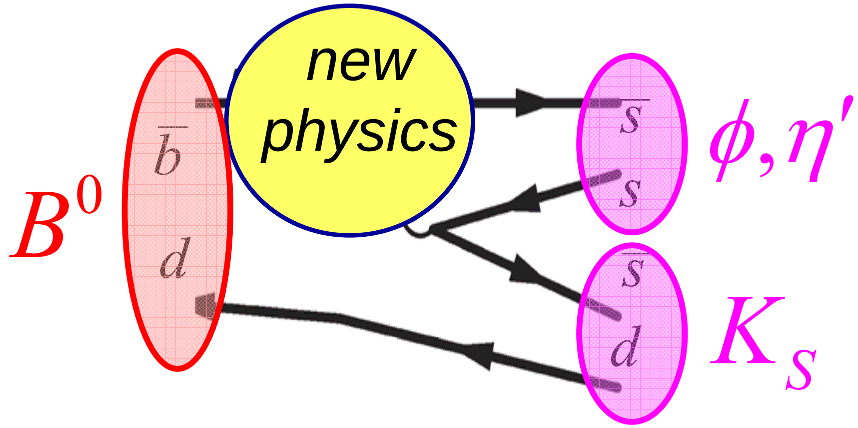
$\sin 2\phi_1^{\mathrm{eff}}$ from Penguin Decays

Standard Model



no weak phase from decay

New Physics



NP weak phase

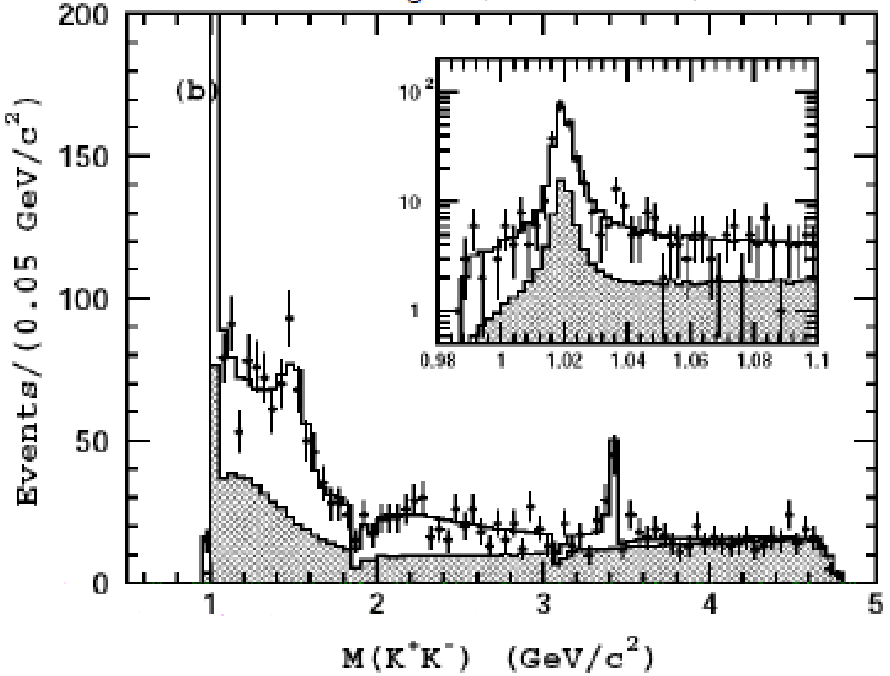
- no weak phase in $b \rightarrow (q\underline{q})s$ penguin decays
  - expect to measure $S = \sin(2\phi_1)$ [just as in $B \rightarrow \psi K_S$]
  - contributions from suppressed diagrams expected to be small ($\Delta\sin(2\phi_1) = \sin(2\phi_1^{\mathrm{eff}}) - \sin(2\phi_1) \sim 0.01\text{-}0.1$)
- if new physics introduces weak phase in decay, we could measure something different than $\sin(2\phi_1)$

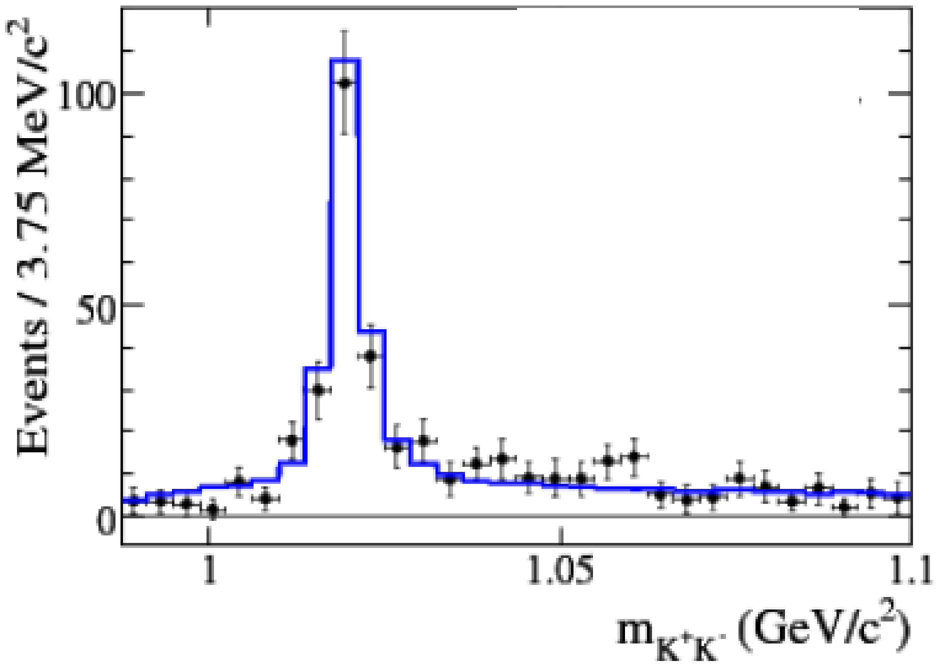
B->$K_S K^+ K^-$ time-dependent Dalitz analysis

PRD 82.073011 (2010)

465×10^6 $B\bar{B}$ pairs
[ArXiv:0808.0700]

$N_S = (1269\pm51)$ evts





ϕK_S

$$\beta_{eff} = (21.2\ ^{+9.8}_{-10.4} \pm 2.0 \pm 2.0)^\circ$$
$$A_{CP} = +0.31\ ^{+0.21}_{-0.23} \pm 0.04 \pm 0.09$$

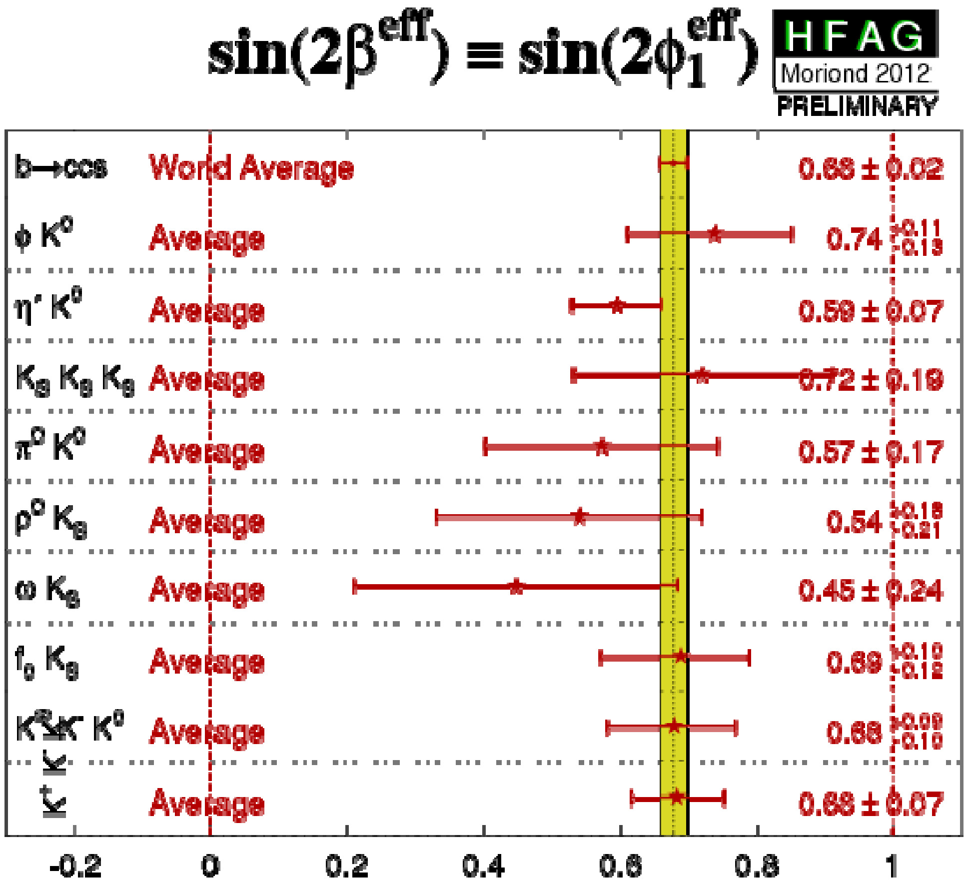
$$\beta_{eff} = (7.7 \pm 7.7 \pm 0.9)^\circ$$
$$A_{CP} = +0.14 \pm 0.19 \pm 0.02$$

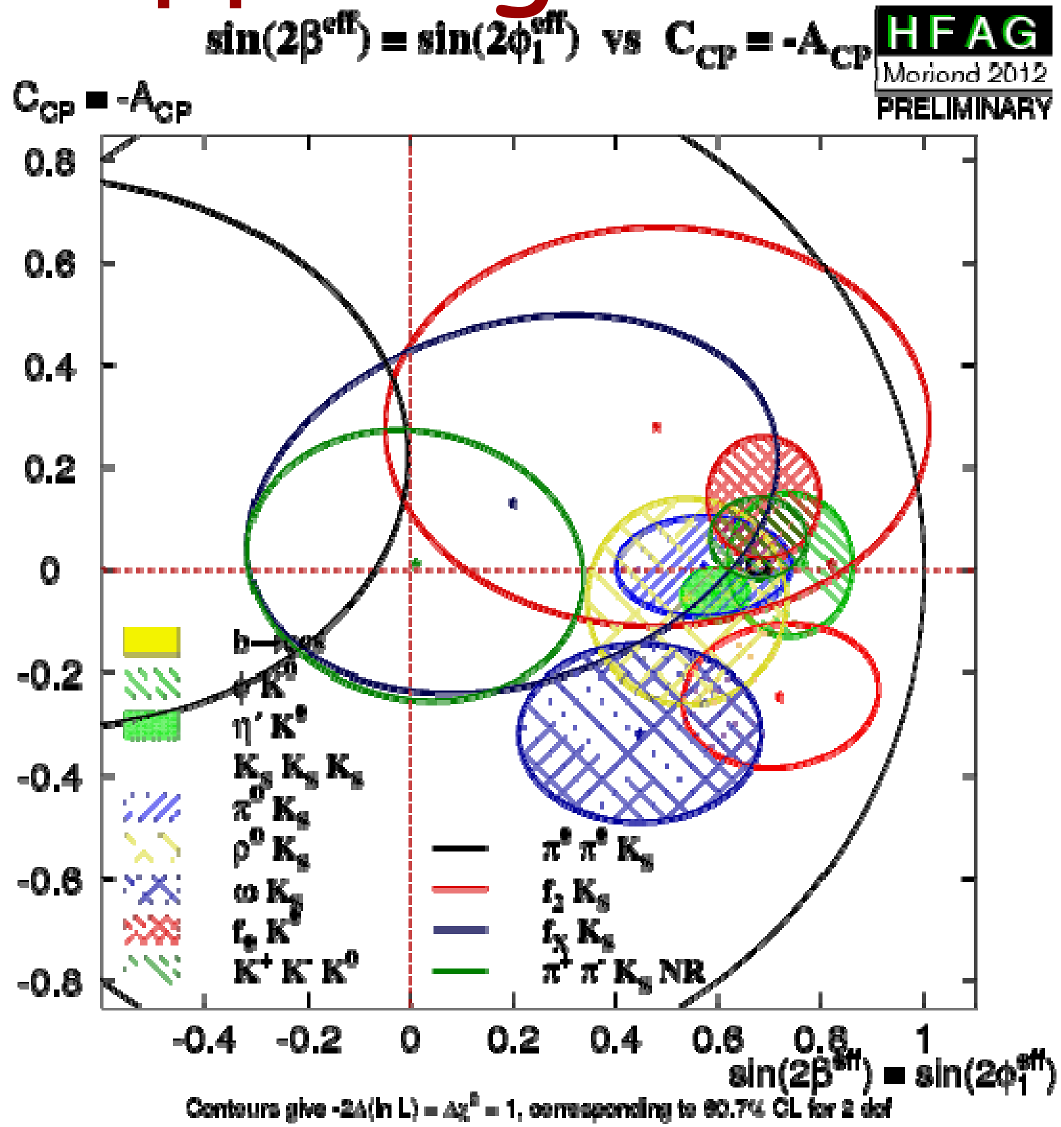
$f_0(980)K_S$

$$\beta_{eff} = (28.2\ ^{+9.9}_{-9.8} \pm 2.0 \pm 2.0)^\circ$$
$$A_{CP} = -0.02 \pm 0.34 \pm 0.08 \pm 0.09$$

$$\beta_{eff} = (8.5 \pm 7.5 \pm 1.8)^\circ$$
$$A_{CP} = +0.01 \pm 0.26 \pm 0.07$$

$\sin 2\phi_1^{eff}$ in $b \to s q\bar{q}$ Penguins



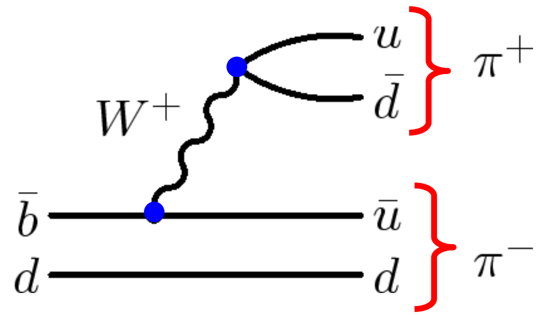


No significant deviations from the value in the b→ccs modes: $\sin 2\phi_1 = 0.679 \pm 0.020$

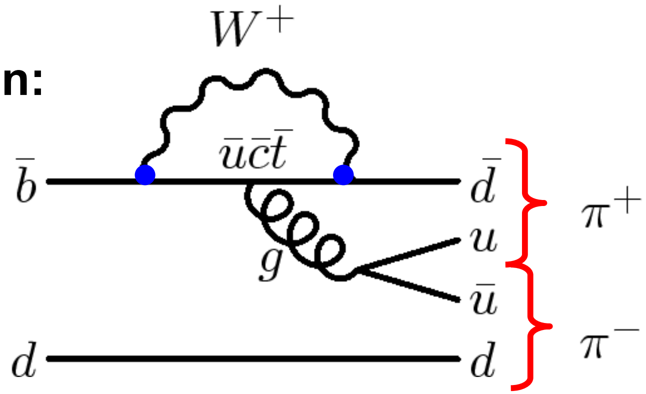
Determination of $\phi_2(\alpha)$

Time-dependent CP asymmetry: $A(\Delta t) = S\sin(\Delta m_d \Delta t) - C\cos(\Delta m_d \Delta t)$

Tree:



Penguin:



Without penguin:

$C = 0 \qquad S = \sin(2\varphi_2)$

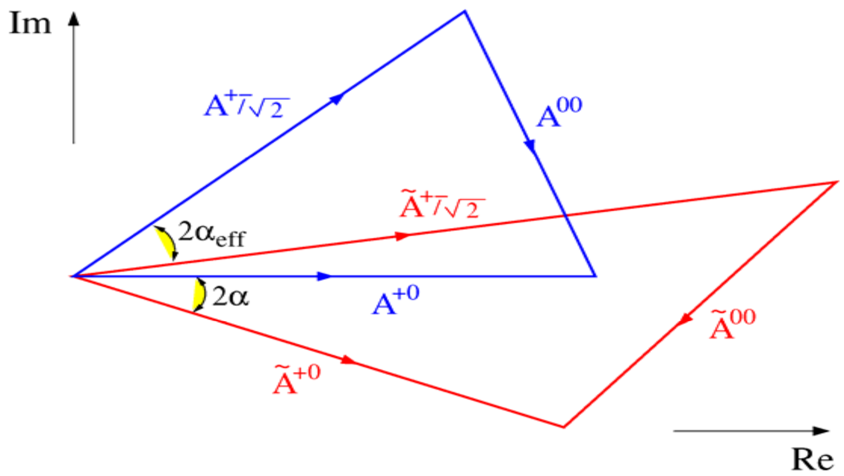
Including penguin:

$C \neq 0 \qquad S = \sqrt{1 - C^2}\sin(2\varphi_2^{\rm eff}) \neq \sin(2\varphi)$

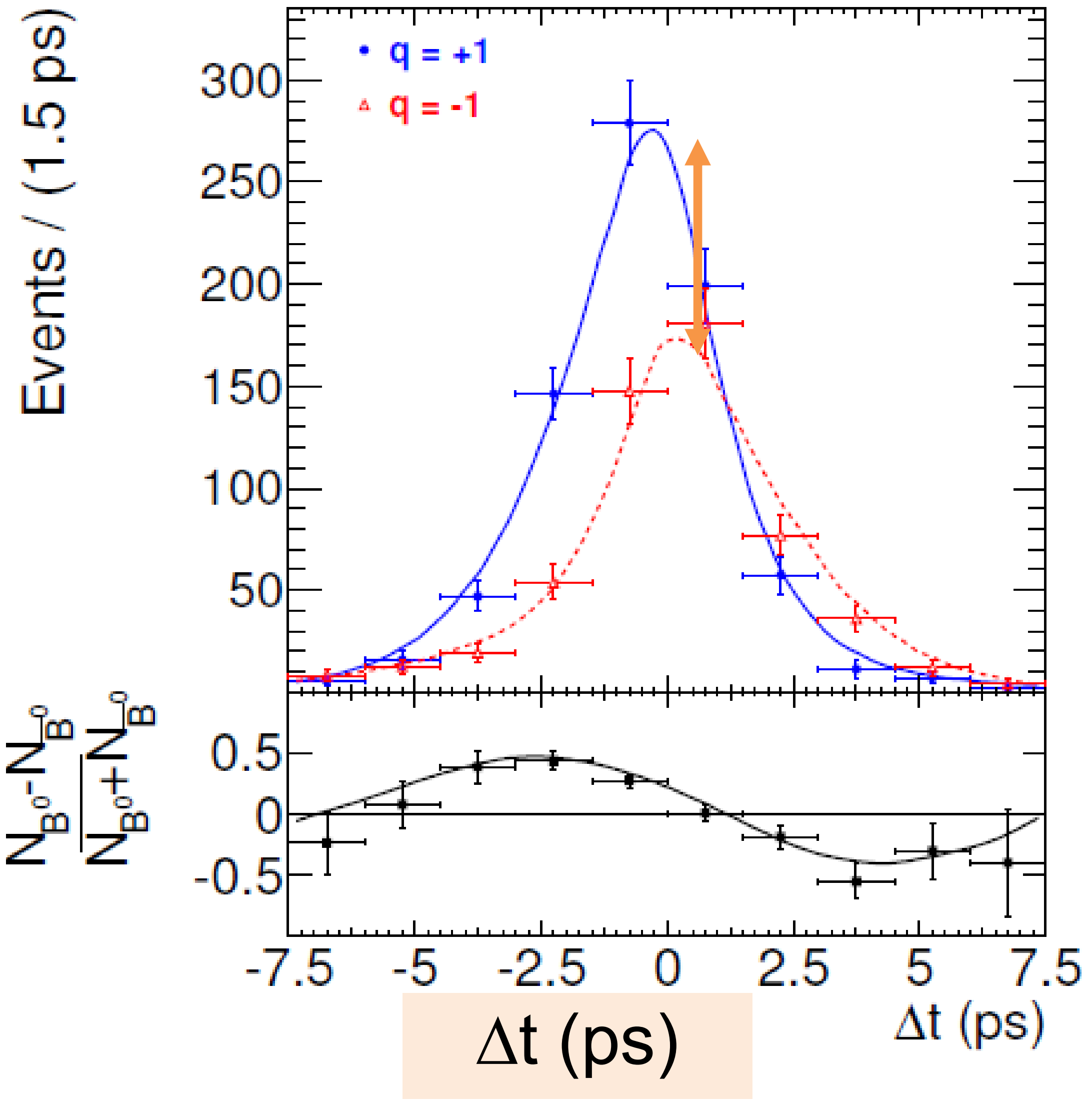
Use isospin relations to estimate the penguin contribution: Gronau-London, PRL, 65, 3381 (1990) Lipkin *et al.*, PRD 44, 1454 (1991)

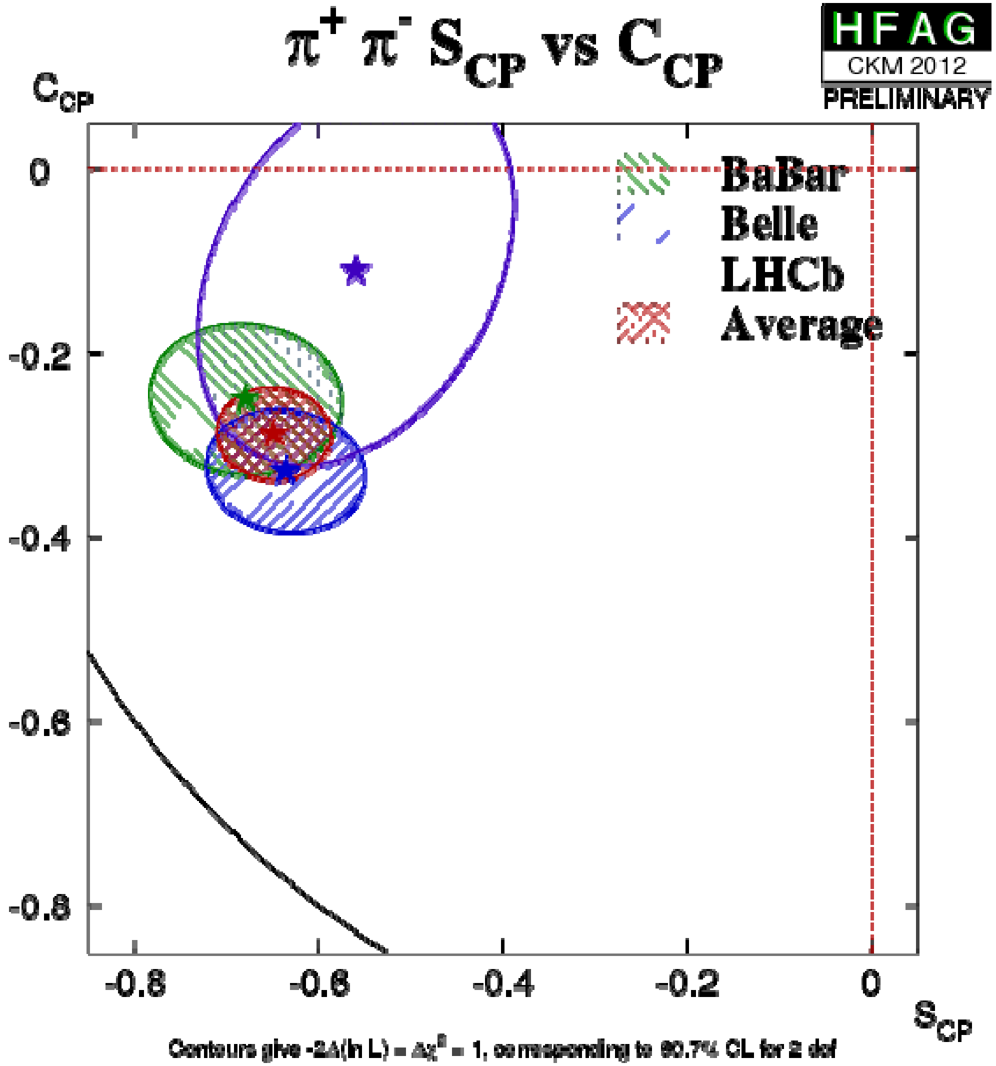
$$A^{+0} = 1/\sqrt{2}\cdot A^{-+} + A^{00}$$

Neglecting EWP, $h^+ h^0$ (I=2)=pure tree $\quad A^{+0} = A^{-0}$



TCPV in B → ππ





$S_{\pi\pi} = -0.64 \pm 0.08 \pm 0.03$

$C_{\pi\pi} = -0.33 \pm 0.06 \pm 0.03$

PRD 87, 031103(R) (2013).

Isospin analysis

Exploit isospin correlated decays e.g.
$B^0 \to \pi^+\pi^-$, $B^+ \to \pi^+\pi^0$ and $B^0 \to \pi^0\pi^0$

for all final states

- $3\ \pi$ final state $I = 0, 1$ or 2
- $I \neq 1$ because of Bose-statistics $\to I = 0, 2$

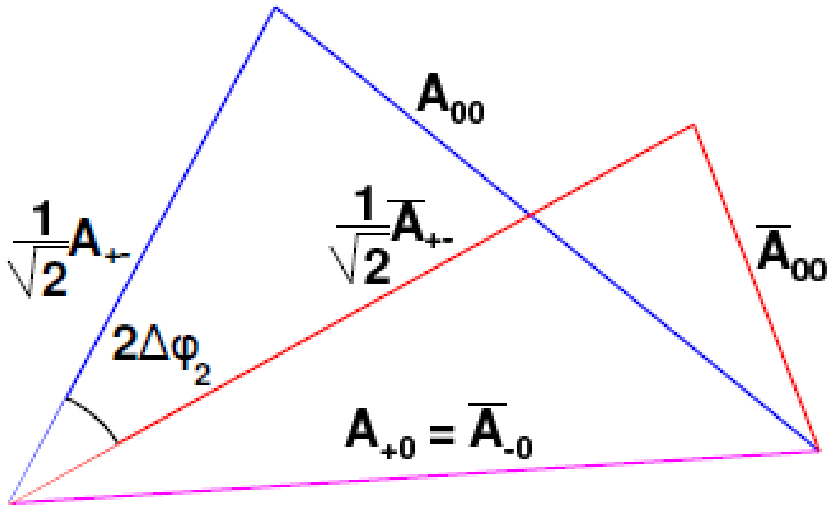
for $\pi^+\pi^0$ final states

- $I_3 = +1 \to I = 1, 2$
- In the penguin the gluon carries $I = 0$ therefore $I = 0, 1$ (excluded by I_3 and Bose stat.)
- $\to$ no penguin in A_{+0}

To eliminate the penguin contributions we use the isospin relations:
(M. Gronau and D. London, PRL **65**, 3381 (1990))

$$A^{+0} = \frac{1}{\sqrt{2}} A^{+-} + A^{00},\ A^{-0} = \frac{1}{\sqrt{2}} \bar{A}^{+-} + \bar{A}^{00}.$$

$$\phi_2^{eff} = \phi_2 + \Delta\phi_2$$



Quantities needed:

- $\mathcal{B}(B^0 \to \pi^+\pi^-)$
- $\mathcal{B}(B^+ \to \pi^+\pi^0)$
- $\mathcal{B}(B^0 \to \pi^0\pi^0)$
- $\mathcal{A}_{CP}(B^0 \to \pi^+\pi^-)$
- $\mathcal{A}_{CP}(B^0 \to \pi^0\pi^0)$
- $\mathcal{S}_{CP}(B^0 \to \pi^+\pi^-)$

four-fold ambiguity

$\phi_2(\alpha)$ result from *isospin analysis*

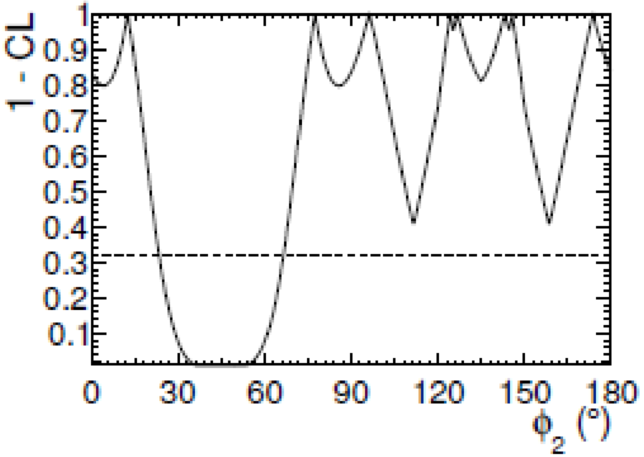
Isospin analysis Belle only data

$B^0 \to \pi^+\pi^-(772 \cdot 10^6 B\overline{B})$

$B^0 \to \pi^+\pi^0(772 \cdot 10^6 B\overline{B})$

$B^0 \to \pi^0\pi^0(253 \cdot 10^6 B\overline{B})$

8 fold solution
large penguin contribution



$23.8^\circ < \phi_2 < 66.8^\circ$

Isospin analysis

$B^0 \to \rho^+\rho^-(W.A.)$

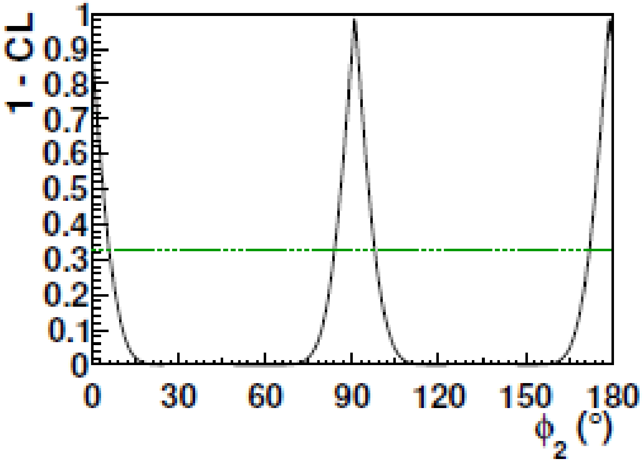
$B^0 \to \rho^+\rho^0(W.A.)$

$B^0 \to \rho^0\rho^0(772 \cdot 10^6 B\overline{B})$

A_{CP}, S_{CP} from BaBar

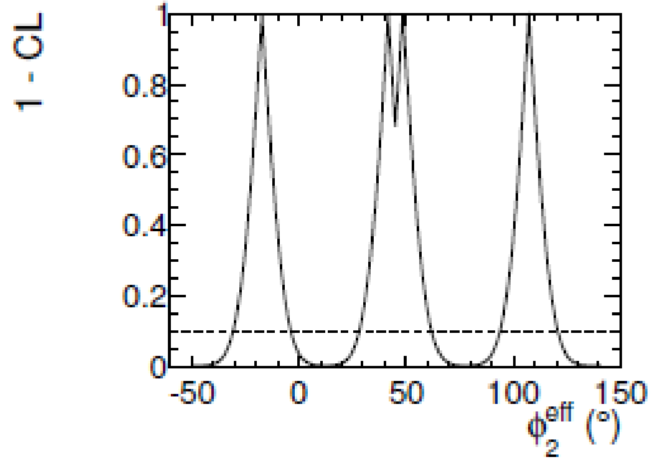
B. Aubert et al. (BaBar Collaboration), Phys. Rev. D 76, 052007 (2007)

small penguin



$\phi_2 = (91.0 \pm 7.2)^\circ$

Deacy $B^0 \to a_1(1260)\pi$
Determination of the effective angle ϕ_2^{eff}
4 fold solution



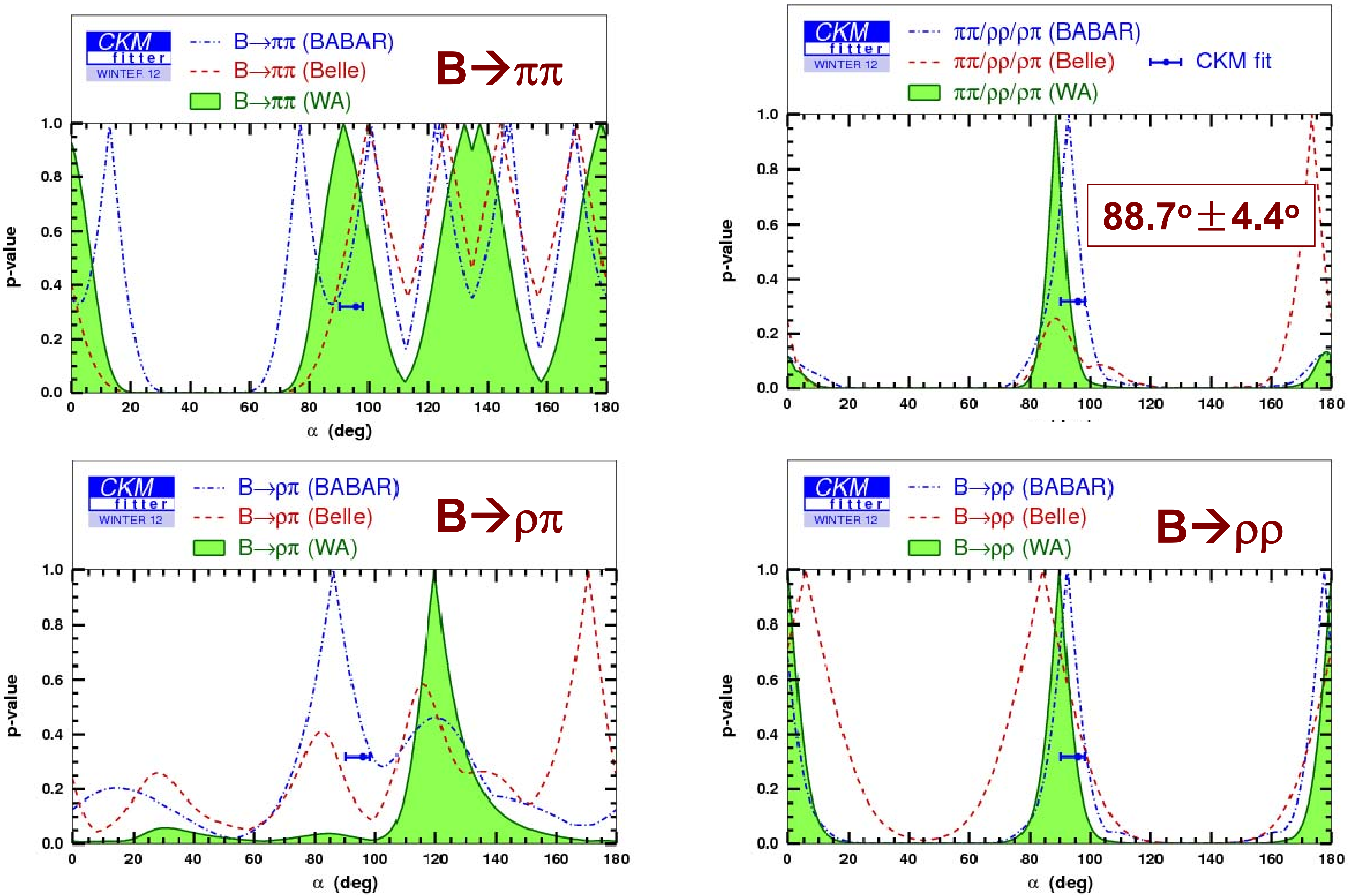
$\phi_2^{eff} = (-17.3 \pm 6.6(\text{stat}) \pm 4.8(\text{syst}))^\circ$

$\phi_2^{eff} = (41.6 \pm 6.2(\text{stat}) \pm 3.4(\text{syst}))^\circ$

$\phi_2^{eff} = (48.4 \pm 6.2(\text{stat}) \pm 3.4(\text{syst}))^\circ$

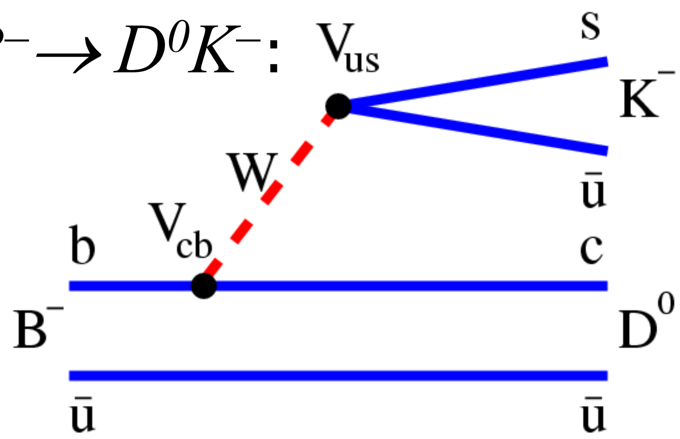
$\phi_2^{eff} = (107.3 \pm 6.6(\text{stat}) \pm 4.8(\text{syst}))^\circ$

$\phi_2(\alpha)$ results: $\pi\pi,\rho\pi,\rho\rho$



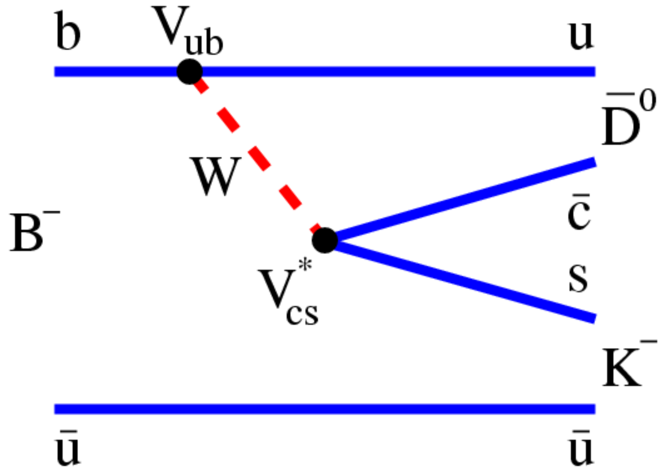
Determination of $\phi_3(\gamma)$

$B^- \to D^0 K^-$:



$\mathbf{A}_1 \sim V_{cb} V_{us}^* \sim A\lambda^3$

$B^- \to \bar{D}^0 K^-$:



$\mathbf{A}_2 \sim V_{ub} V_{cs}^* \sim A\lambda^3(\rho - i\eta) \sim e^{-i\varphi_3}$

If D^0 and $\bar{D}^0$ decay into the same final state, $\left|\tilde{D}^0\right\rangle = \left|D^0\right\rangle + re^{i\theta}\left|\bar{D}^0\right\rangle$

Relative phase: $\theta = -\varphi_3 + \delta$ $(B^- \to DK^-)$, $\theta = +\varphi_3 + \delta$ $(B^+ \to DK^+)$

includes weak (ϕ_3) and strong (δ) phase.

Amplitude ratio: $r = |A(B^- \to \bar{D}^0 K^-)|/|A(B^- \to D^0 K^-)| \sim 0.1$

Possible D^0 / $\bar{D}^0$ final states:

CP eigenstates ($\pi\pi$, KK)

Flavor eigenstates (Kπ)

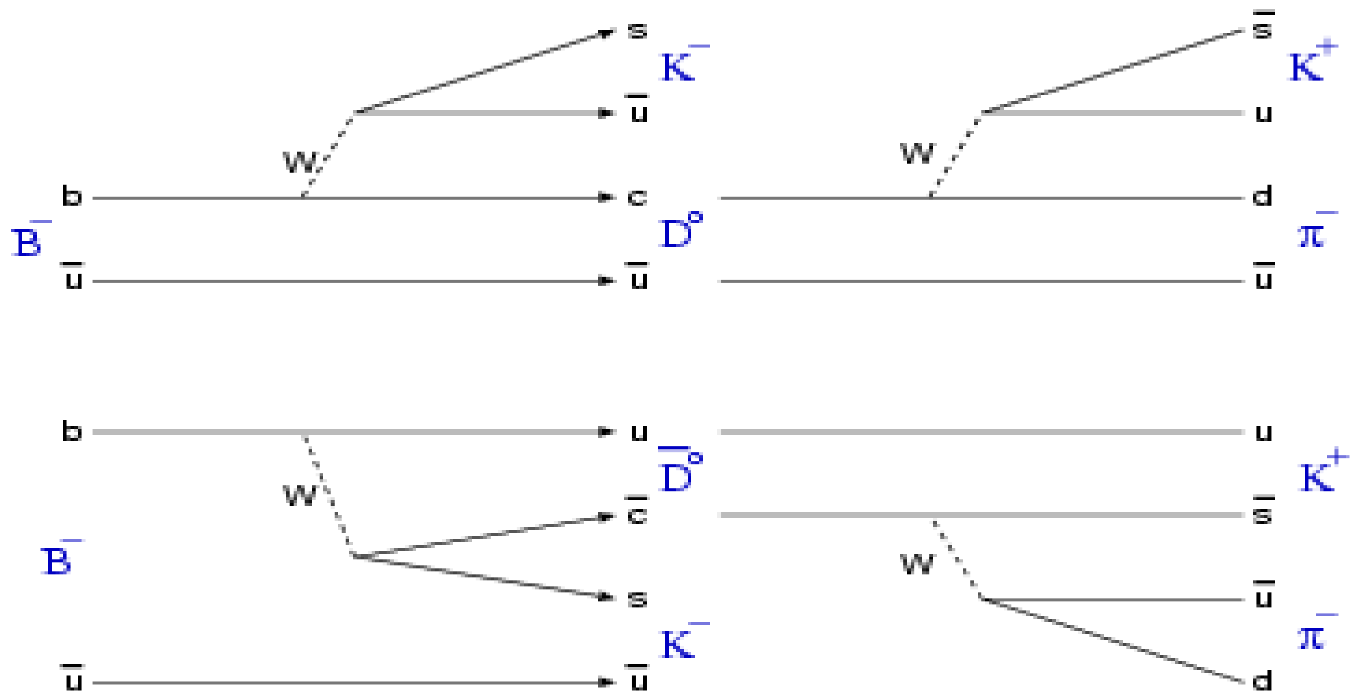
Three-body decays ($K_S\pi\pi$)

Gronau & London, PLB 253, 483 (1991)
Gronau & Wyler, PLB 265, 172 (1991)
Atwood, Dunietz, & Soni, PRL 78, 3257 (1997),
Atwood, Dunietz, & Soni, PRD 63, 036005 (2001)
Giri, Grossman, Soffer, & Zupan, PRD 68, 054018 (2003)
Bondar, PRD 70, 072003 (2004)

Atwood-Dunietz-Soni method

D. Atwood, I. Dunietz and A. Soni, PRL **78**, 3357 (1997);
PRD **63**, 036005 (2001)

Enhancement of CP-violation due to use of Cabibbo-suppressed D decays



$B^-\to D^0K^-$ - color allowed

$D^0\to K^+\pi^-$ - doubly Cabibbo-suppressed

$B^-\to \bar{D}^0K^-$ - color suppressed

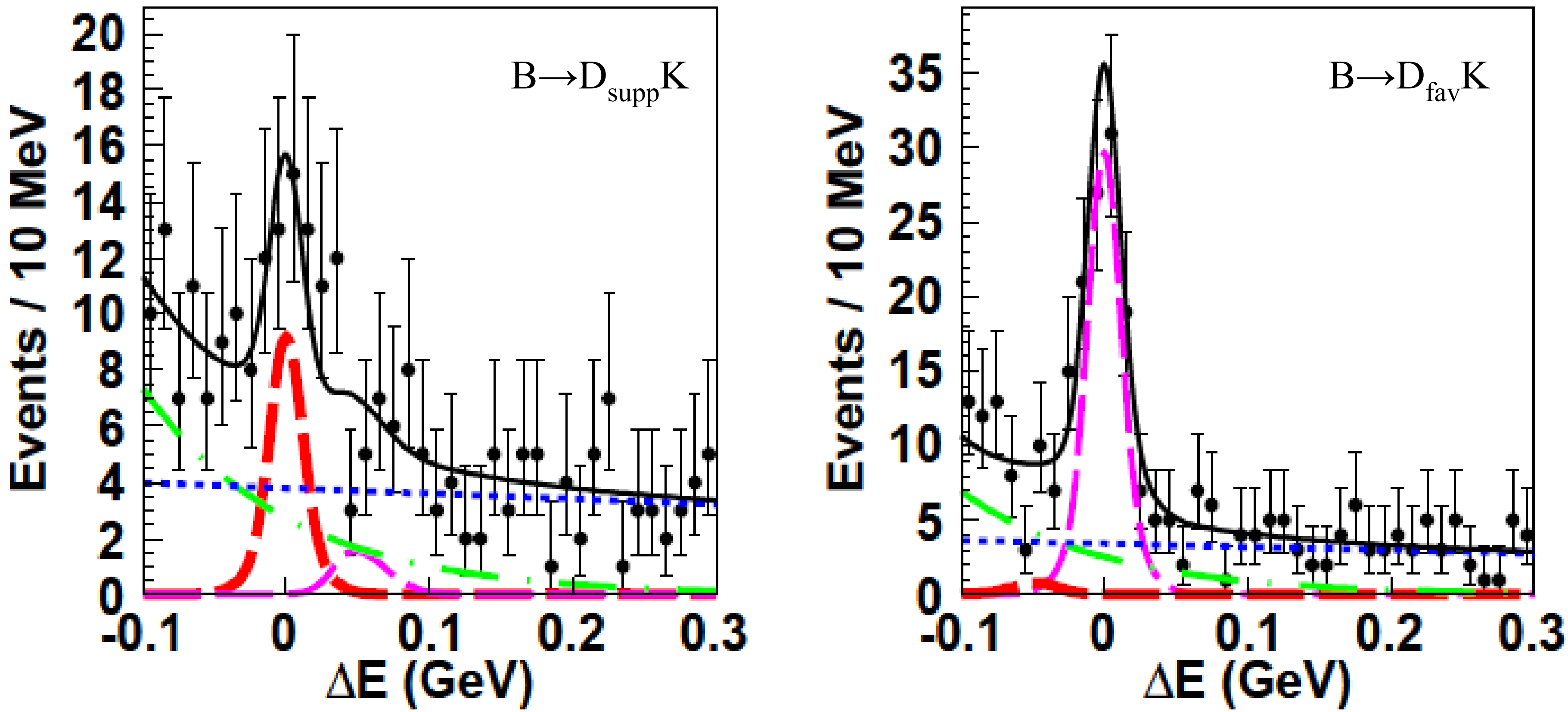
$\bar{D}^0\to K^+\pi^-$ - Cabibbo-allowed

$\Rightarrow$ Interfering amplitudes are comparable

ADS method (Belle)

Belle collaboration, 772M BB pairs PRL 106, 231803 (2011)

$B^{-}\to[K^{+}\pi^{-}]_D K^{-}$ (suppressed) and $B^{-}\to[K^{-}\pi^{+}]_D K^{-}$ (favored) modes are selected.



CP asymmetry:

$$\mathcal{R}_{DK} = [1.63^{+0.44}_{-0.41}(\text{stat})^{+0.07}_{-0.13}(\text{syst})] \times 10^{-2}$$

$$\mathcal{R}_{D\pi} = [3.28^{+0.38}_{-0.36}(\text{stat})^{+0.12}_{-0.18}(\text{syst})] \times 10^{-3}$$

$$\mathcal{A}_{DK} = -0.39^{+0.26}_{-0.28}(\text{stat})^{+0.04}_{-0.03}(\text{syst}),$$

$$\mathcal{A}_{D\pi} = -0.04 \pm 0.11(\text{stat})^{+0.02}_{-0.01}(\text{syst})$$

ADS using B→D*K

study both modes: $D^* \to D\pi^0$, $D\gamma$:

[see "On φ_3 Measurements Using $B \to D^* K^-$ Decays", arXiv:hep-ph/0409281]

Signal seen with a significance of 3.5σ for $D^* \to D\gamma$ mode

Ratio to favored mode:

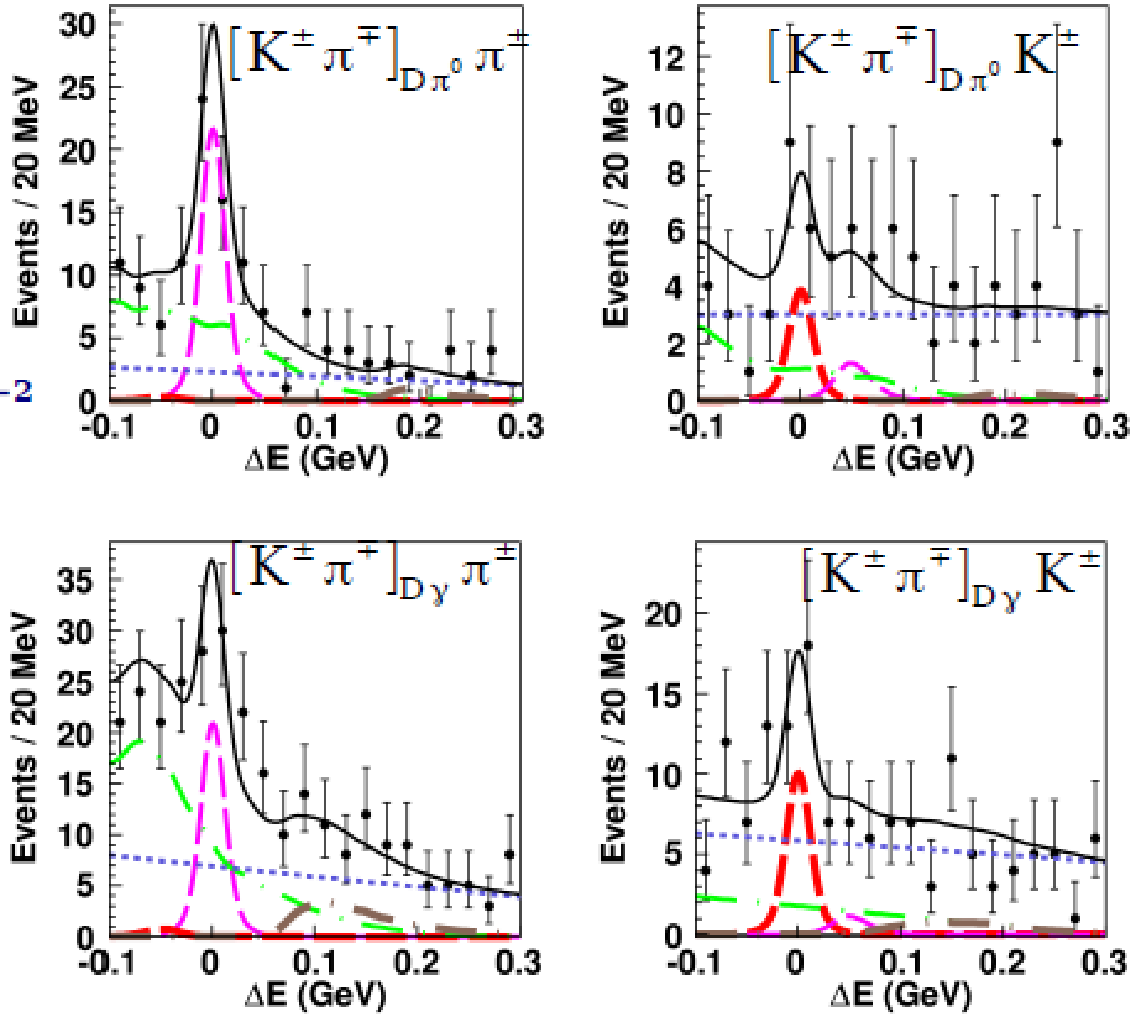
$$R_{D\pi^\bullet} = (1.0\,^{+0.8}_{-0.7}(\text{stat})\,^{+0.1}_{-0.2}(\text{syst})) \times 10^{-2}$$

$$R_{D\gamma} = (3.6\,^{+1.4}_{-1.2}(\text{stat}) \pm 0.2(\text{syst})) \times 10^{-2}$$

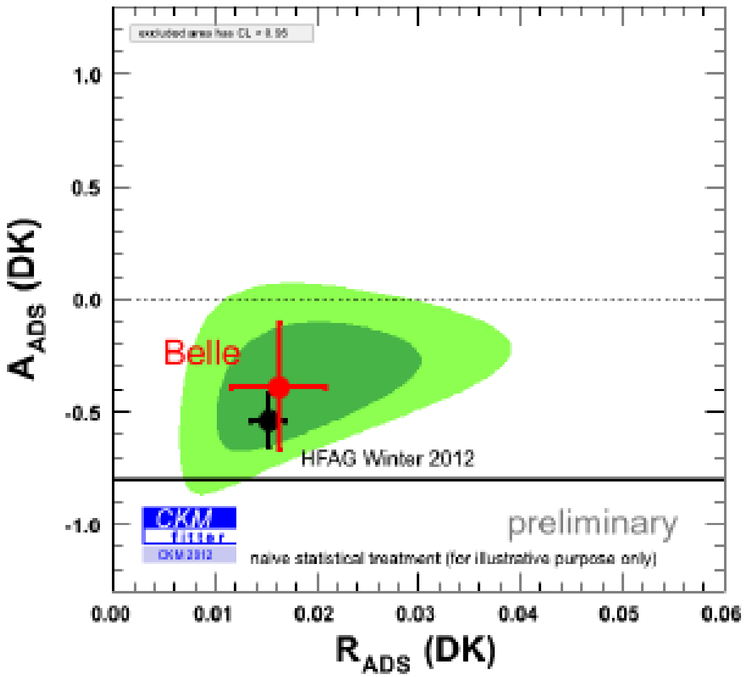
asymmetry:

$$A_{D\pi^\bullet} = 0.4\,^{+1.1}_{-0.7}(\text{stat})\,^{+0.2}_{-0.1}(\text{syst})$$

$$A_{D\gamma} = -0.51\,^{+0.33}_{-0.29}(\text{stat}) \pm 0.08(\text{syst})$$



Comparison of results

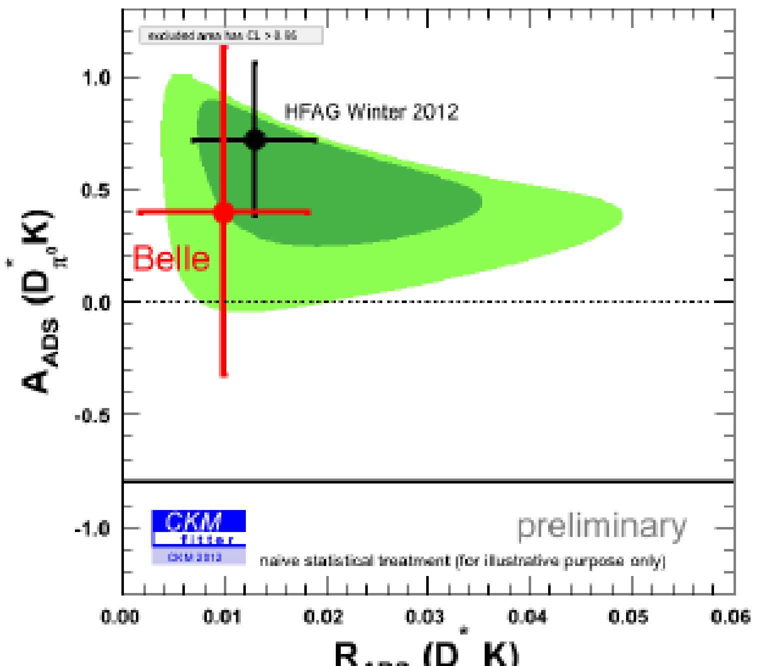


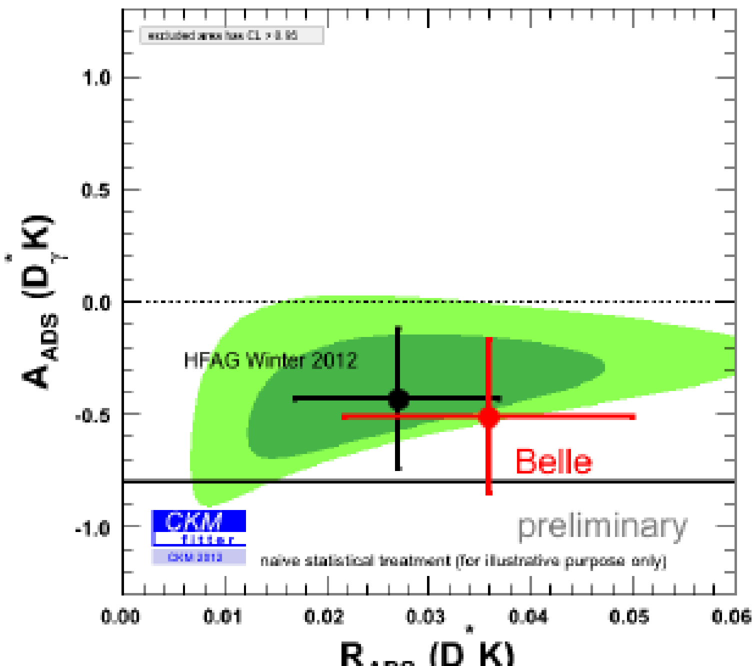
$$R_{ADS}(DK) = r_B^2 + r_D^2 + 2r_B r_D \cos(\delta_B + \delta_D)\cos\gamma$$
$$A_{ADS}(DK) = 2r_B r_D \sin(\delta_B + \delta_D)\sin\gamma / R_{ADS}(DK)$$

$$R_{ADS}(D^*_{\pi^0}K) = r_B^{*2} + r_D^2 + 2r_B^* r_D \cos(\delta_B^* + \delta_D)\cos\gamma$$
$$A_{ADS}(D^*_{\pi^0}K) = 2r_B^* r_D \sin(\delta_B^* + \delta_D)\sin\gamma / R_{ADS}(D^*_{\pi^0}K)$$

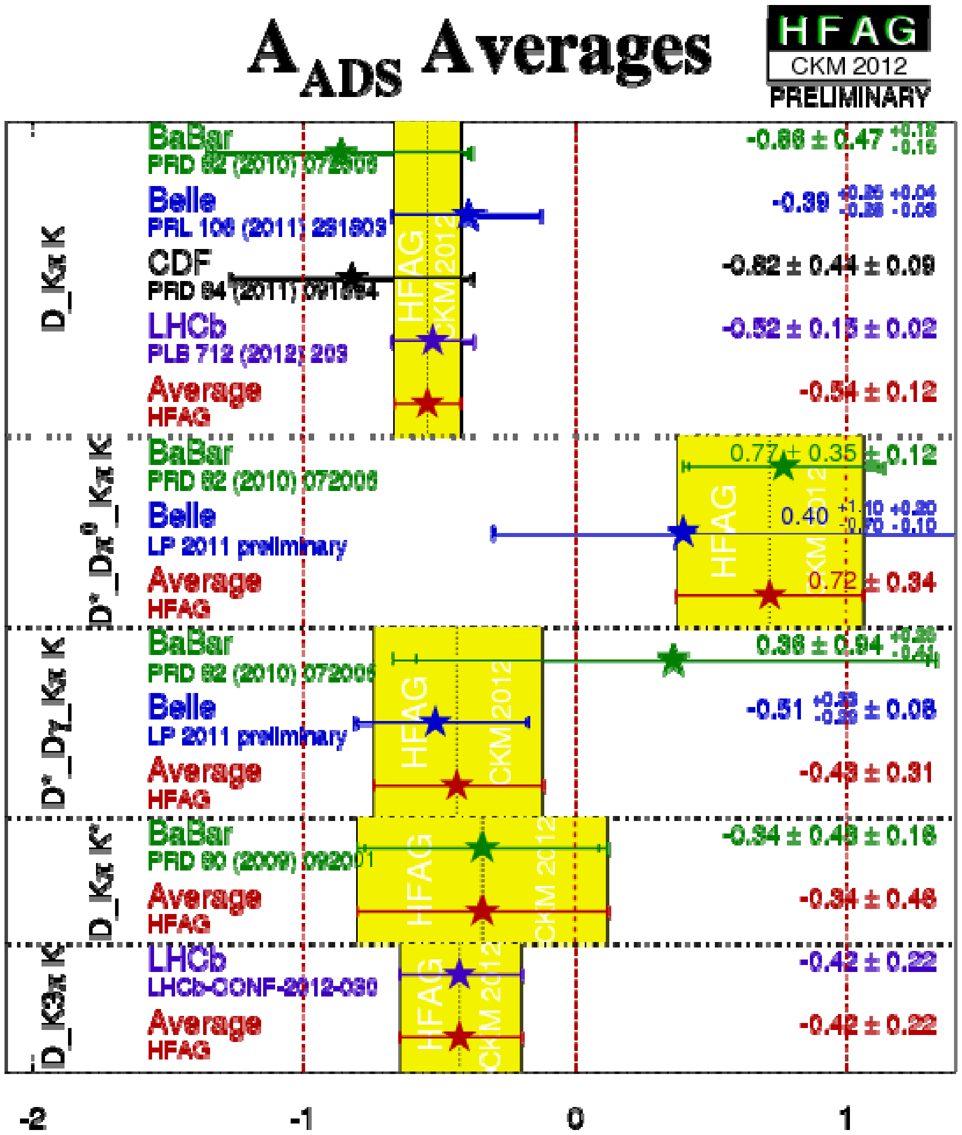
$$R_{ADS}(D^*_{\gamma}K) = r_B^{*2} + r_D^2 - 2r_B^* r_D \cos(\delta_B^* + \delta_D)\cos\gamma$$
$$A_{ADS}(D^*_{\gamma}K) = -2r_B^* r_D \sin(\delta_B^* + \delta_D)\sin\gamma / R_{ADS}(D^*_{\gamma}K)$$

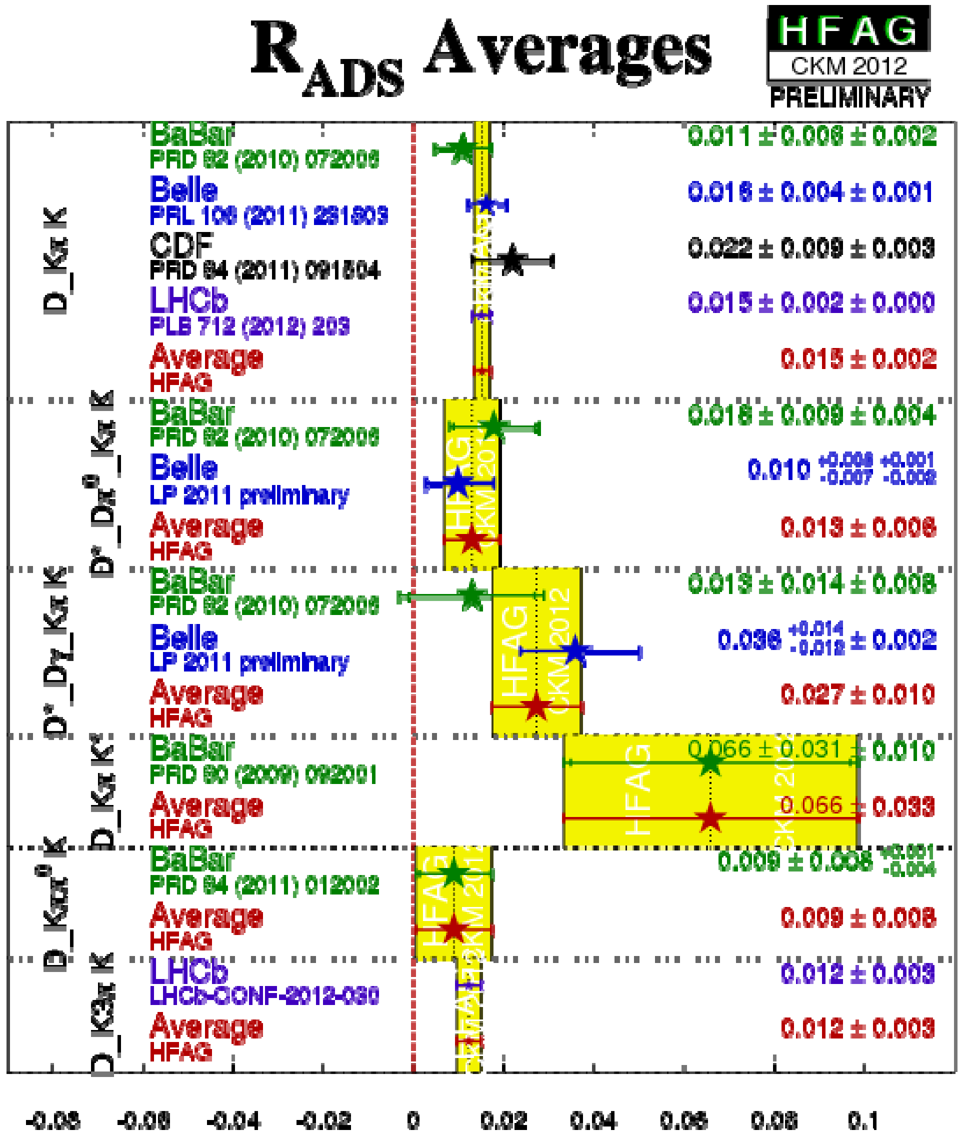
(HFAG winter 2012 includes Belle results)





ADS method

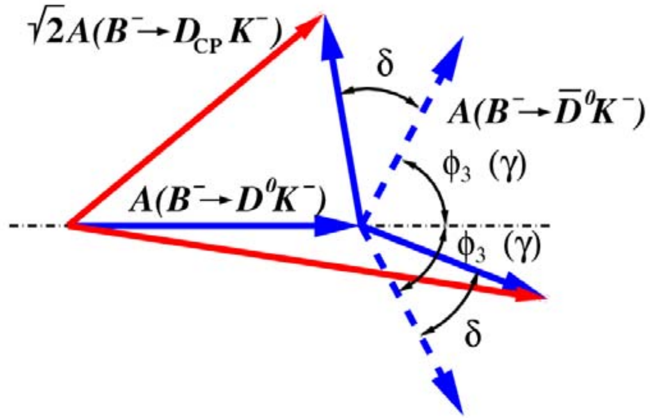




Multiple solutions in $\phi_3(\gamma)$

Gronau-London-Wyler method

[Phys. Lett. B 253 (1991) 483]
[Phys. Lett. B 265 (1991) 172]



CP eigenstate of D-meson is used ($D_{\rm CP}$).

CP-even : $D_1 \to K^+K^-,\ \pi^+\pi^-$

CP-odd : $D_2 \to K_S\pi^0,\ K_S\omega,\ K_S\varphi,\ K_S\eta\ldots$

CP-asymmetry:

$$\mathcal{A}_{1,2} = \frac{Br(B^- \to D_{1,2}K^-) - Br(B^+ \to D_{1,2}K^+)}{Br(B^- \to D_{1,2}K^-) + Br(B^+ \to D_{1,2}K^+)} = \frac{2r_B \sin\delta' \sin\varphi_3}{1 + r_B^2 + 2r_B\cos\delta'\cos\varphi_3}$$

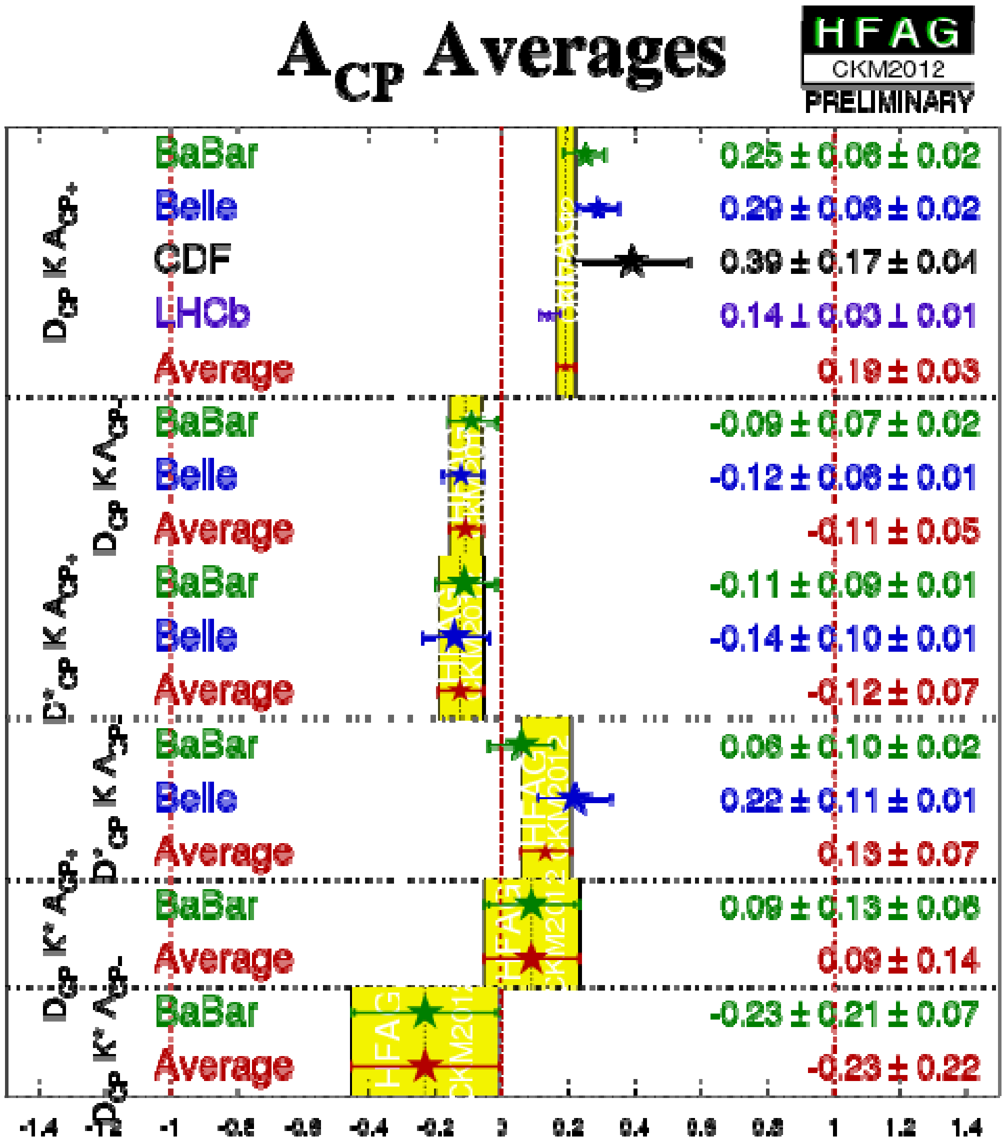
$$\delta' = \begin{cases} \delta & \text{for } D_1 \\ \delta + \pi & \text{for } D_2 \end{cases} \quad \Rightarrow \quad \mathcal{A}_{1,2} \text{ have opposite signs}$$

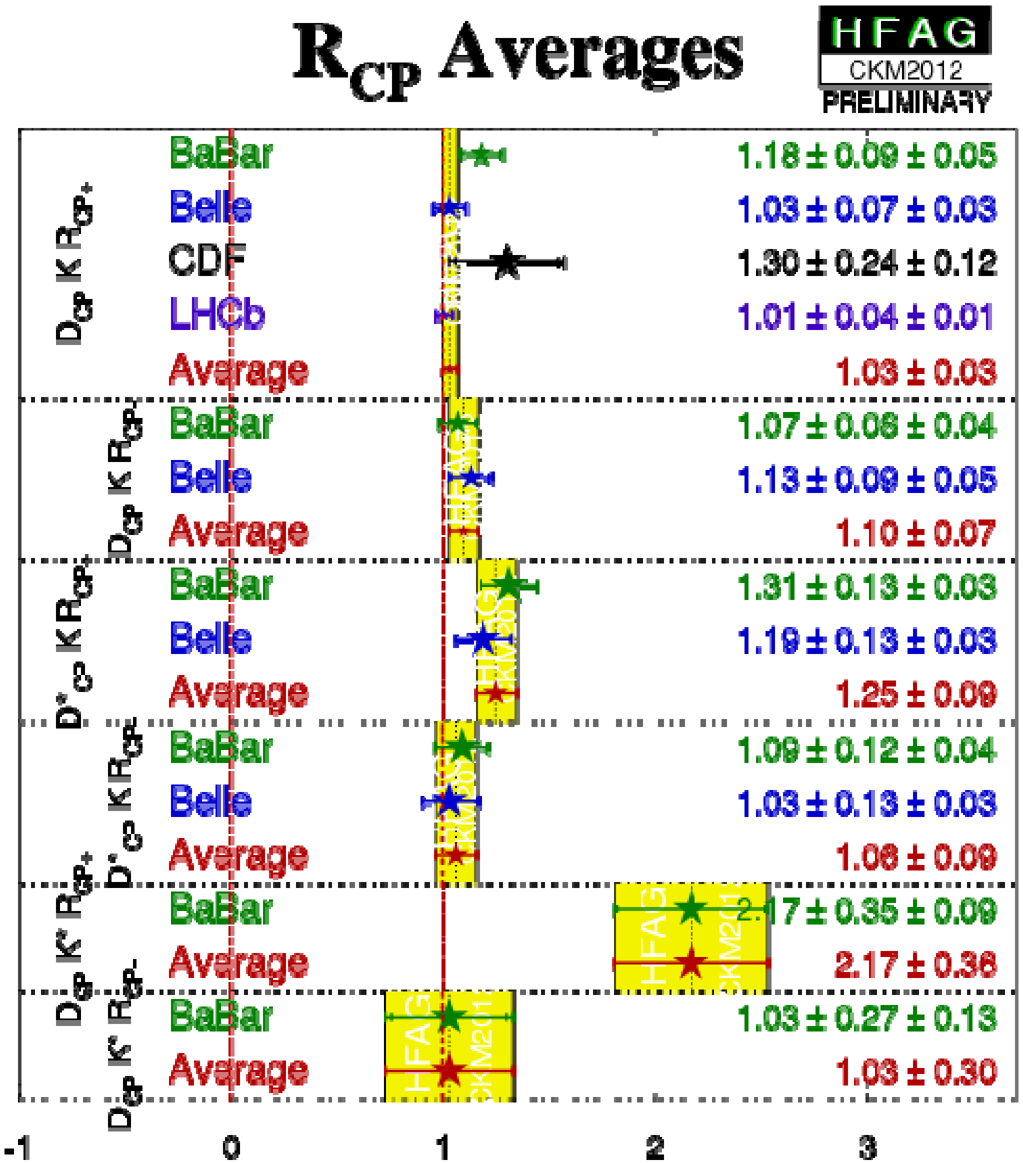
Additional constraint:

$$\mathcal{R}_{1,2} = \frac{Br(B \to D_{1,2}K)/Br(B \to D_{1,2}\pi)}{Br(B \to D^0K)/Br(B \to D^0\pi)} = 1 + r_B^2 + 2r_B\cos\delta'\cos\varphi_3$$

4 equations (3 independent: $\mathcal{A}_1\mathcal{R}_1 = -\mathcal{A}_2\mathcal{R}_2$), 3 unknowns (r_B, δ, φ_3)

GLW method





Multiple solutions in ϕ_3

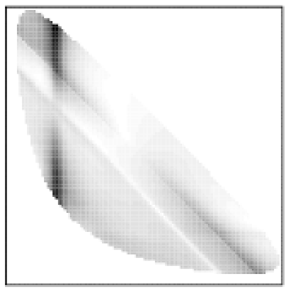
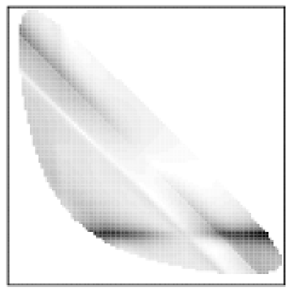
Dalitz analysis method

A. Giri, Yu. Grossman, A. Soffer, J. Zupan, PRD **68**, 054018 (2003)
A. Bondar, Proc. of Belle Dalitz analysis meeting, 24-26 Sep 2002.

$$\left|\tilde{D}^0\right\rangle = \left|D^0\right\rangle + re^{i\theta}\left|\overline{D}^0\right\rangle$$

Using 3-body final state, identical for D^0 and $\overline{D}^0$: $K_s\pi^+\pi^-$.

Dalitz distribution density: $d\sigma(m^2_{K_s\pi^+}, m^2_{K_s\pi^-}) \propto |\mathbf{A}|^2\, dm^2_{K_s\pi^+}\, dm^2_{K_s\pi^-}$

$|\mathbf{A}(m^2_{K_s\pi^+}, m^2_{K_s\pi^-})|^2 = \Bigg|$  $+\, re^{i\delta \pm i\phi_3}$  $\Bigg|^2$

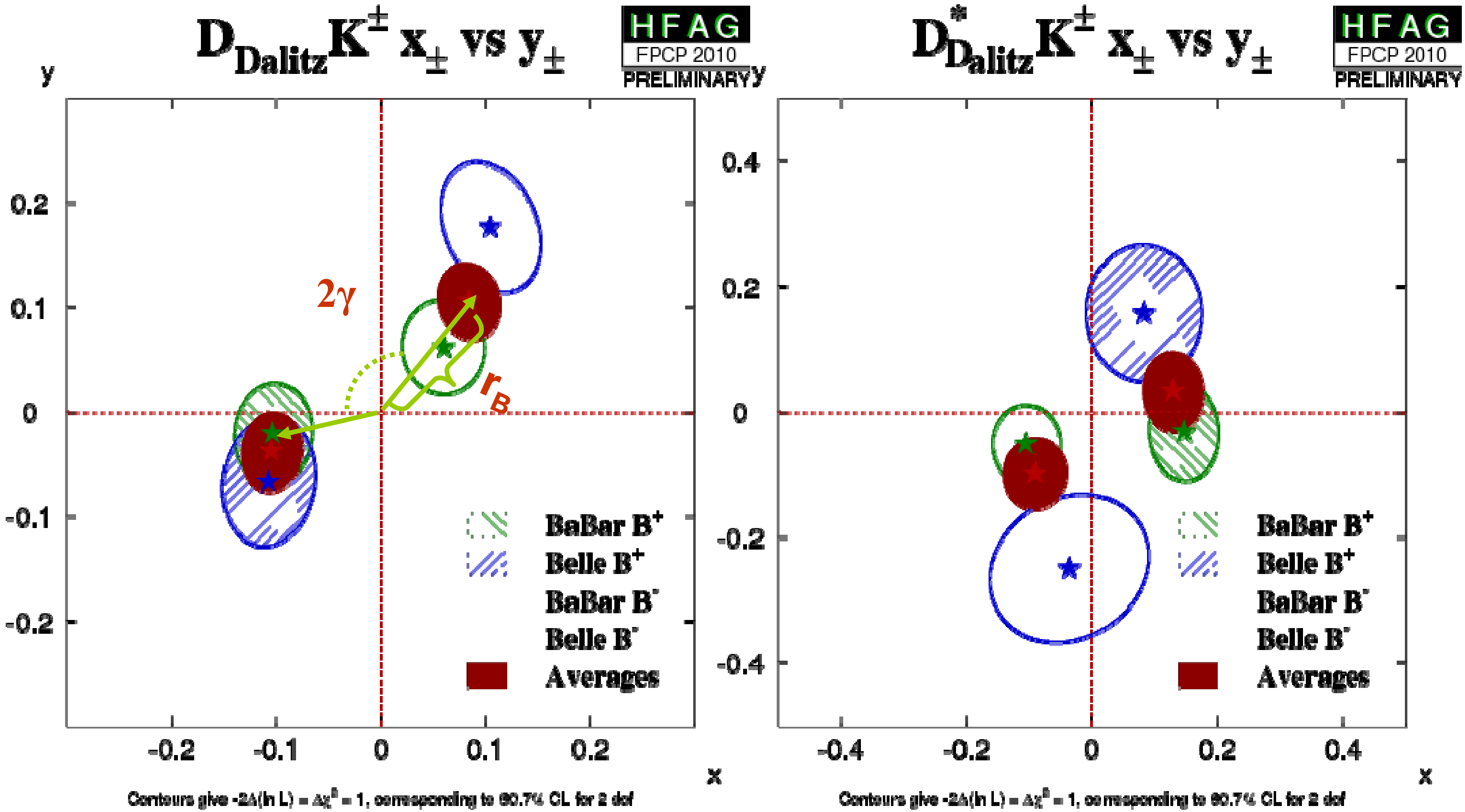
(assuming CP-conservation in D^0 decays)

If $f(m^2_{K_s\pi^+}, m^2_{K_s\pi^-})$ is known, parameters (r_B, δ, φ_3) are obtained from the fit to Dalitz distributions of $D \to K_s\pi^+\pi^-$ from $B^\pm \to DK^\pm$ decays

Only |f|^2 is observable → Model uncertainty

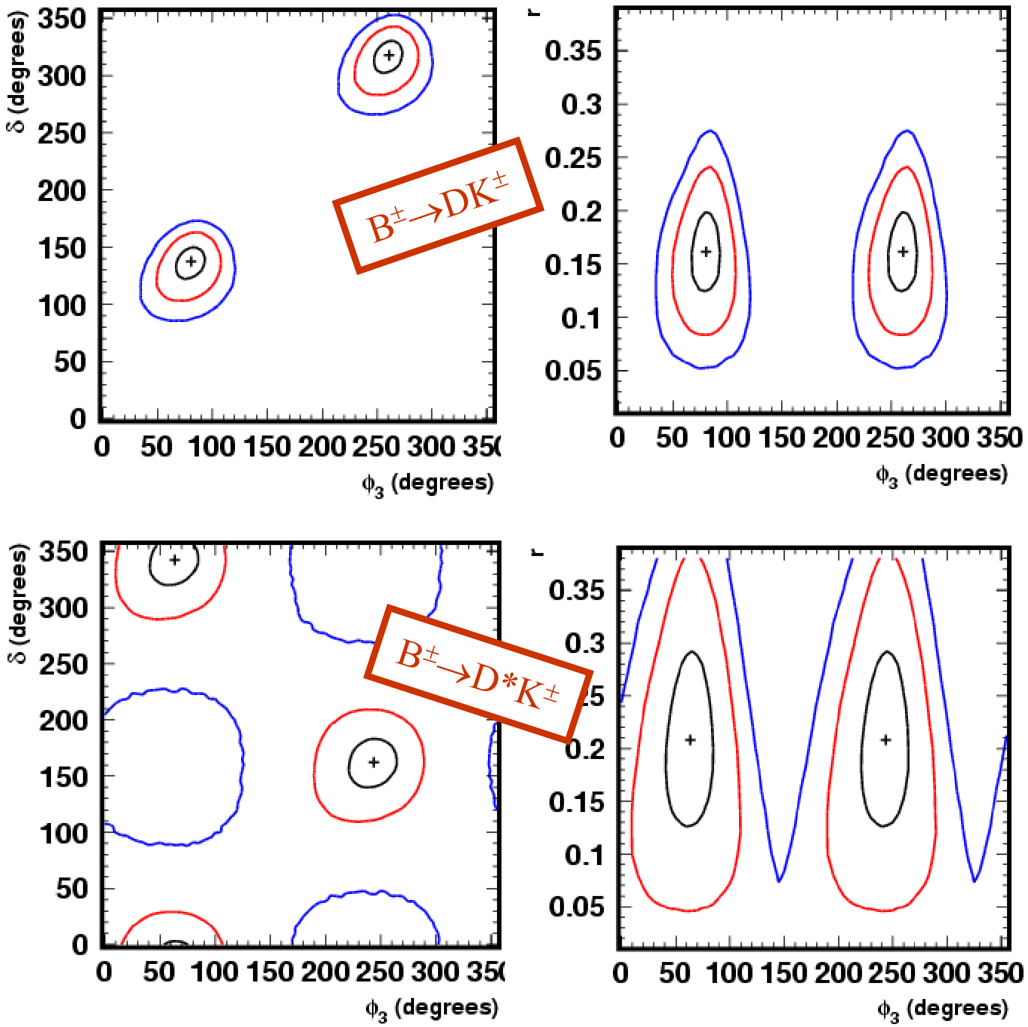
Dalitz: fit results

Fit parameters are $x_{\pm} = r_B \cos(\pm\phi_3+\delta)$ and $y_{\pm} = r_B \sin(\pm\phi_3+\delta)$



Physical parameters r_B, δ and ϕ_3 are extracted from x and y.

Dalitz: ϕ_3 results



$B^{\pm}\to DK^{\pm}$ only:

$\varphi_3=81^{+13}_{-15}{}^{\circ} \pm 5°(\text{syst}) \pm 9°(\text{model})$

$B^{\pm}\to D^{*}K^{\pm}$ only:

$\varphi_3=64^{+21}_{-23}{}^{\circ} \pm 4°(\text{syst}) \pm 9°(\text{model})$

$B^{\pm}\to DK^{\pm}$, $B^{\pm}\to D^{*}K^{\pm}$ combined:

$\varphi_3=78^{+11}_{-12}{}^{\circ} \pm 4°(\text{syst}) \pm 9°(\text{model})$

$r_{DK}=0.16 \pm 0.04 \pm 0.01(\text{syst}) \pm 0.05(\text{model})$

$r_{D^*K}=0.21 \pm 0.08 \pm 0.01(\text{syst}) \pm 0.05(\text{model})$

$\delta_{DK}=136^{+14}_{-16}{}^{\circ} \pm 4°(\text{syst}) \pm 23°(\text{model})$

$\delta_{D^*K}=343^{+20}_{-22}{}^{\circ} \pm 4°(\text{syst}) \pm 23°(\text{model})$

Future improving is limited by the model uncertainty

Stat. confidence level of CPV is $(1-5.5\cdot10^{-4})$ or 3.5σ

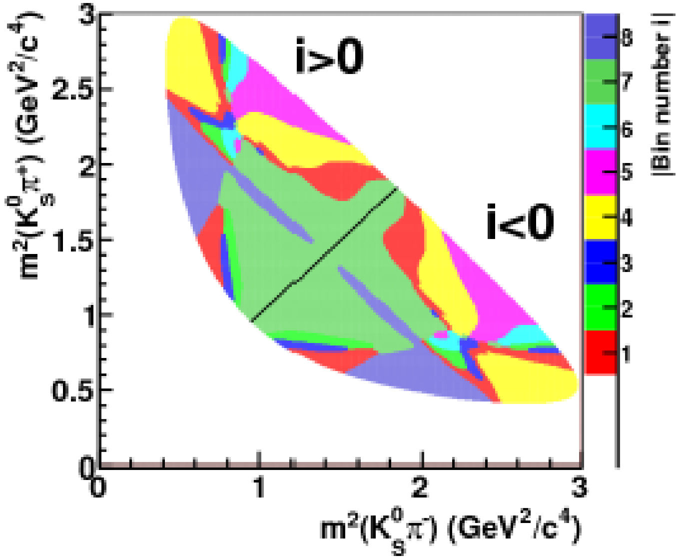
BaBar result:
$\gamma = (68 \pm 15 \pm 4 \pm 3)°$
3.0σ CPV significance

Binned Dalitz analysis method

Solution: use binned Dalitz plot and deal with numbers of events in bins.

[A. Giri, Yu. Grossman, A. Soffer, J. Zupan, PRD **68**, 054018 (2003)]

[A. Bondar, A. P. EPJ C **47**, 347 (2006); EPJ C **55**, 51 (2008)]



$$M_i^{\pm} = h\{K_i + r_B^2 K_{-i} + 2\sqrt{K_i K_{-i}}(x_{\pm} c_i + y_{\pm} s_i)\}$$

$$x_{\pm} = r_B \cos(\delta_B \pm \phi_3) \quad y_{\pm} = r_B \sin(\delta_B \pm \phi_3)$$

$M_i^{\pm}$: numbers of events in $D \to K_S^0 \pi^+ \pi^-$ bins from $B^{\pm} \to DK^{\pm}$

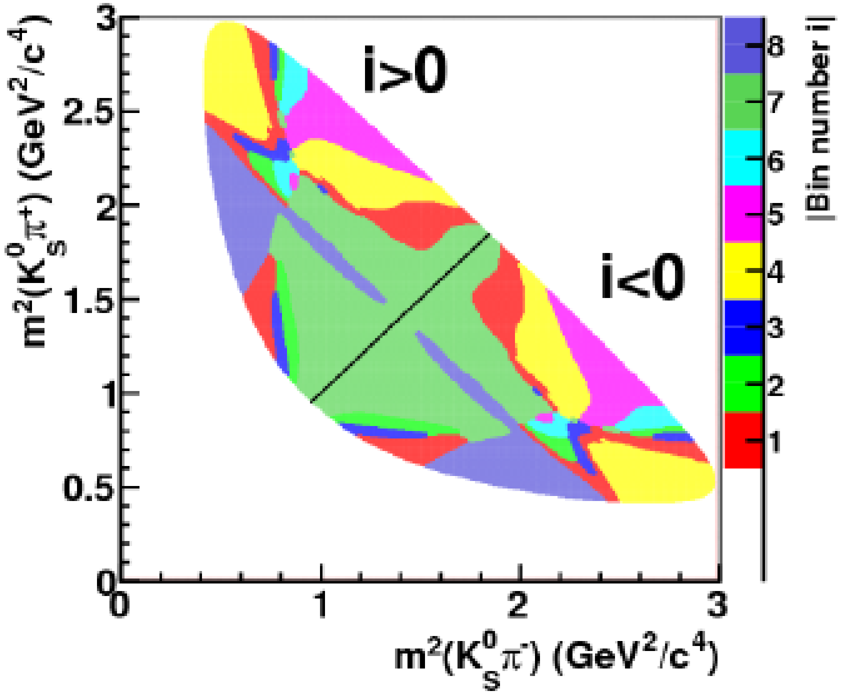
K_i: numbers of events in bins of flavor $\overline{D}^0 \to K_S^0 \pi^+ \pi^-$ from $D^* \to D\pi$.

c_i, s_i contain information about strong phase difference between symmetric Dalitz plot points $(m^2_{K_S^0\pi^+}, m^2_{K_S^0\pi^-})$ and $(m^2_{K_S^0\pi^-}, m^2_{K_S^0\pi^+})$:

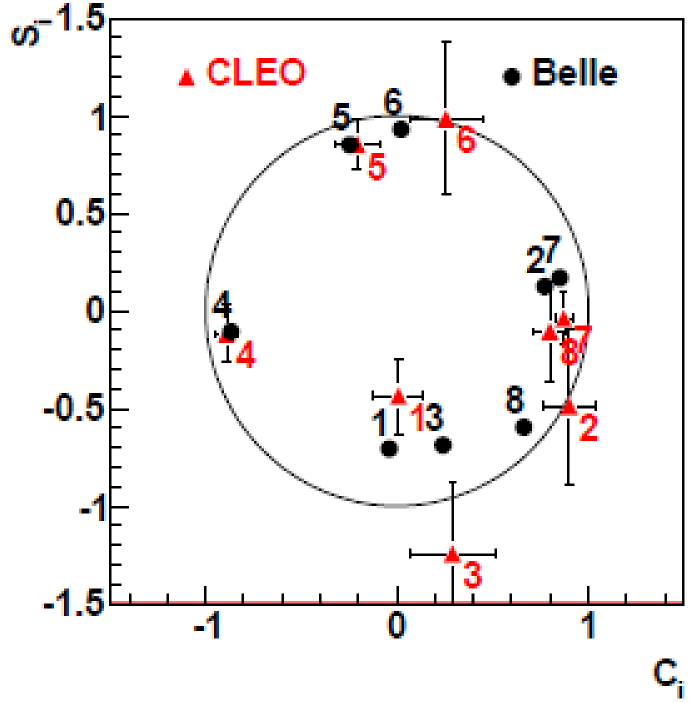
$$c_i = \langle \cos \Delta\delta_D \rangle, \quad s_i = \langle \sin \Delta\delta_D \rangle$$

Binning & measurement of c_i, s_i

Binned analysis reduces stat. precision.
Can improve this by choosing a binning inspired by $\overline{D}^0 \to K_S^0\pi^+\pi^-$ model
[CLEO collaboration, PRD **82**, 112006 (2010)]



Optimized $\overline{D}^0 \to K_S^0\pi^+\pi^-$ binning
using BaBar 2008 measurement.

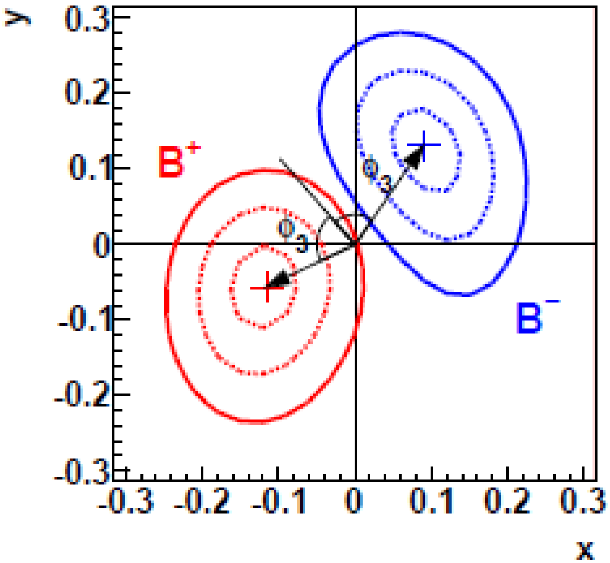


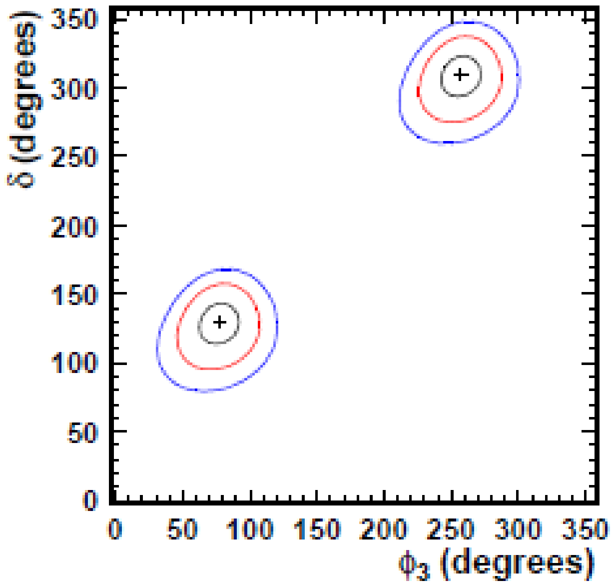
Measured c_i, s_i values and
predictions by Belle model

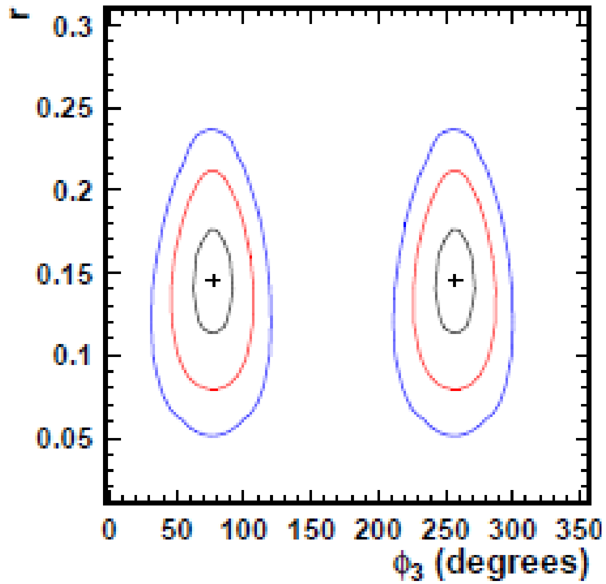
Optimal binning depends on model, but ϕ_3 does not.
Bad model $\Rightarrow$ worse precision, but no bias!

Fit results

Free parameters: (x, y), normalization, background fractions in bins.







$x_- = +0.095 \pm 0.045 \pm 0.014 \pm 0.017$

$y_- = +0.137^{+0.053}_{-0.057} \pm 0.019 \pm 0.029$

$\mathrm{corr}(x_-, y_-) = -0.315$

$x_+ = -0.110 \pm 0.043 \pm 0.014 \pm 0.016$

$y_+ = -0.050^{+0.052}_{-0.055} \pm 0.011 \pm 0.021$

$\mathrm{corr}(x_+, y_+) = +0.059$

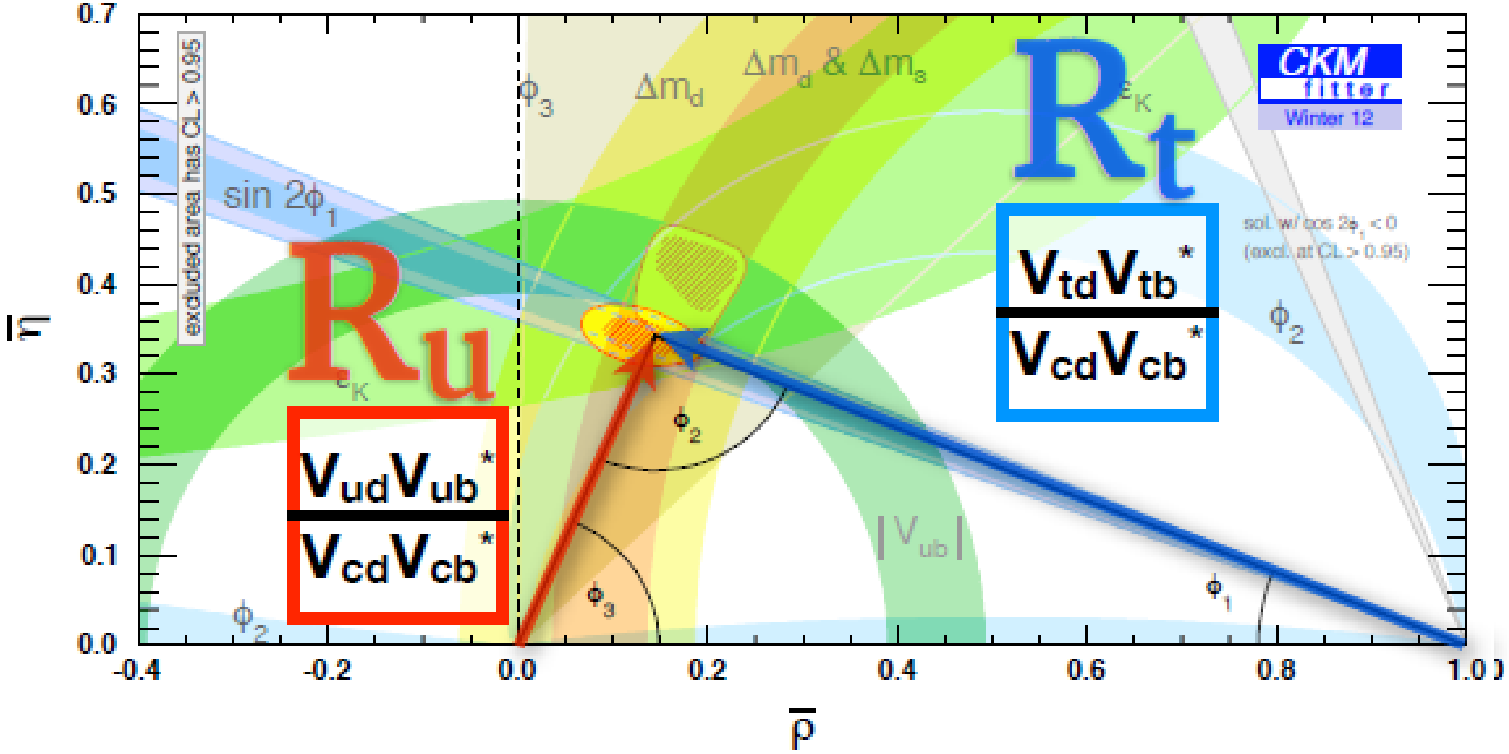
$\phi_3 = (77.3^{+15.1}_{-14.9} \pm 4.2 \pm 4.3)^\circ$

$r_B = 0.145 \pm 0.030 \pm 0.011 \pm 0.011$

$\delta_B = (129.9 \pm 15.0 \pm 3.9 \pm 4.7)^\circ$

1st error is statistical, 2nd — systematic, 3rd — c_i, s_i precision.

Sides of UT



UT CKM Parameter	Measurement	δV/V	Ref.
V_{ub}**	$(4.4\pm0.5)10^{-3}$	10%	PDG
V_{cb}	$(4.1\pm0.1)10^{-2}$	3%	
V_{td} (Δmd)**	$(8.4\pm0.6)10^{-3}$	7%	
V_{td}/V_{ts}(mix)		3%	
V_{cd}	0.228±0.006	3%	1209.0085
V_{tb}:single-t	~1.03±0.04	4%	1302.1773

V_{cb} from inclusive $B \to X_c l\nu$

- ***HQE params & $|V_{cb}|$*** from spectral "moments"

$$\langle M_x^n \rangle|_{E\ell>E_0} = \tau_B \int_{E_0} M_x^n d\Gamma = f(E_0, m_b, m_c, \mu_\pi^2, \mu_G^2, \rho_D^3, \rho_{LS}^3)$$

Cut-off

quark masses

Non-perturbative parameters

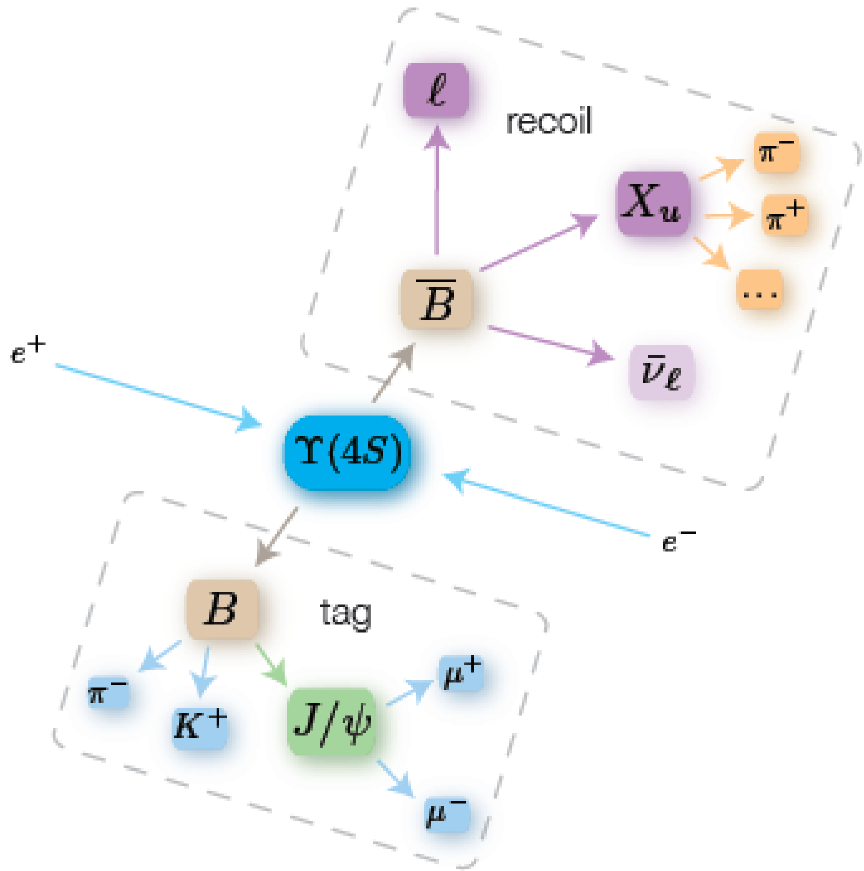
Moments can be calculated for cut-off in E_l

- Need high resolution access to *B* rest frame, unfolded:

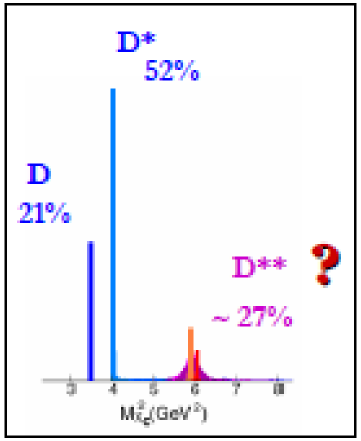
  Hadronic invariant mass

  Lepton momentum

- Use hadronic tag $B_{tag} \to D^{(*)}Y$ ($Y = n\pi, m\pi^0, pK_s, qK\ldots$), to infer signal *B*: flavour, charge, p_4

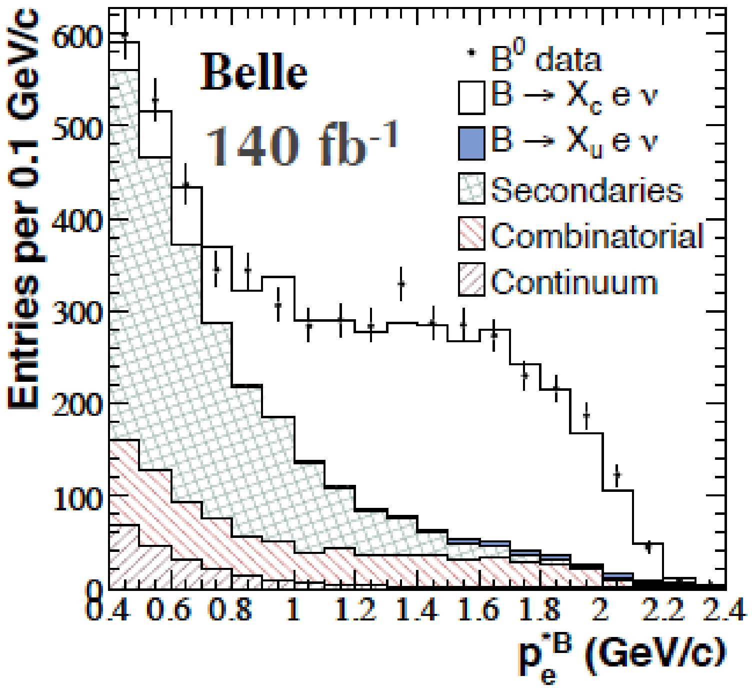


V_{cb} determination



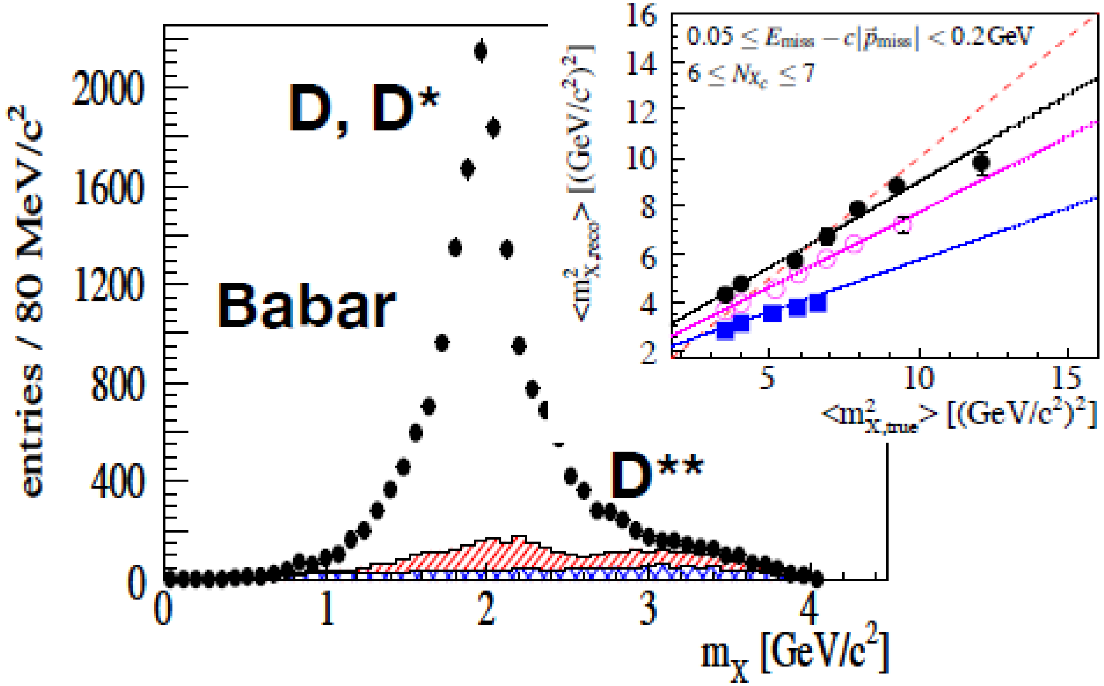
- Inclusive CL decay recoiling against B_{tag}
- Unfold to true
- Extra constraints on m_b from $b \to s\,\gamma$ photon spectrum

Lepton momentum



Belle., PRD.75.032005 (2007)

Hadron mass



BABAR, PRD 81, 032003 (2010)

Belle, PRD.75.032001 (2007)

Exclusive V_{cb}

$B \to D^{(*)} l \nu$ differential decay rates proportional to $|V_{cb}|^2$ & form factors.

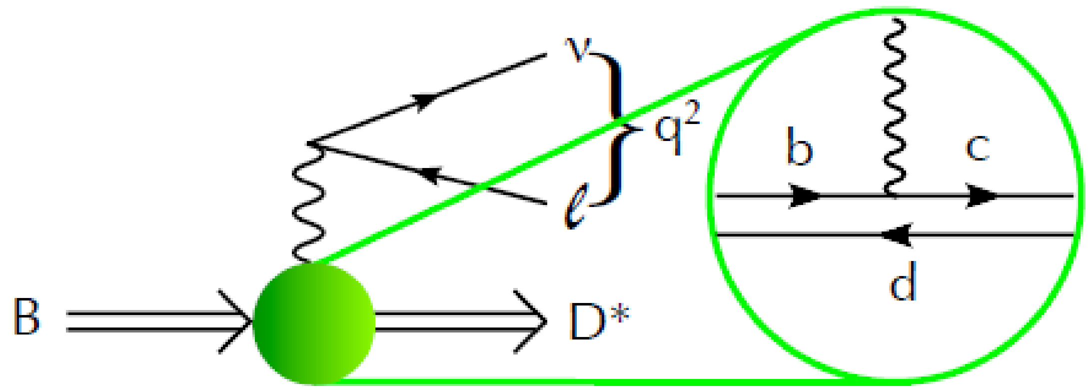
$$w \equiv v_B \cdot v_{D^{(*)}} = \frac{p_B \cdot p_{D^{(*)}}}{m_B \cdot m_{D^{(*)}}} : D^{(*)} \text{ boost}$$

$$\frac{d\Gamma(\bar{B} \to D^* \ell \bar{\nu})}{dw\, d\cos\theta_\ell\, d\cos\theta_V\, d\chi} = \frac{G_F^2 |V_{cb}|^2}{48\pi^3} m_{D^*}^3 \sqrt{w^2 - 1}\, P(w)\, |\mathcal{F}(w, \cos\theta_\ell, \cos\theta_V, \chi)|^2$$

$$\frac{d\Gamma(\bar{B} \to D \ell \bar{\nu})}{dw} = \frac{G_F^2 |V_{cb}|^2}{48\pi^3} (m_B + m_D)^2\, m_D^3 (w^2 - 1)^{3/2} |\mathcal{G}(w)|^2$$

1 normalisation point from

lattice-QCD at 0-recoil (w=1)



From experiment

$|V_{cb}|$ x F.F. @w=1

ρ_D, ρ_{D^*} (F.F. slopes)

$G_{B\to D}(1) = 1.074(18)_{stat}(16)_{sys}$

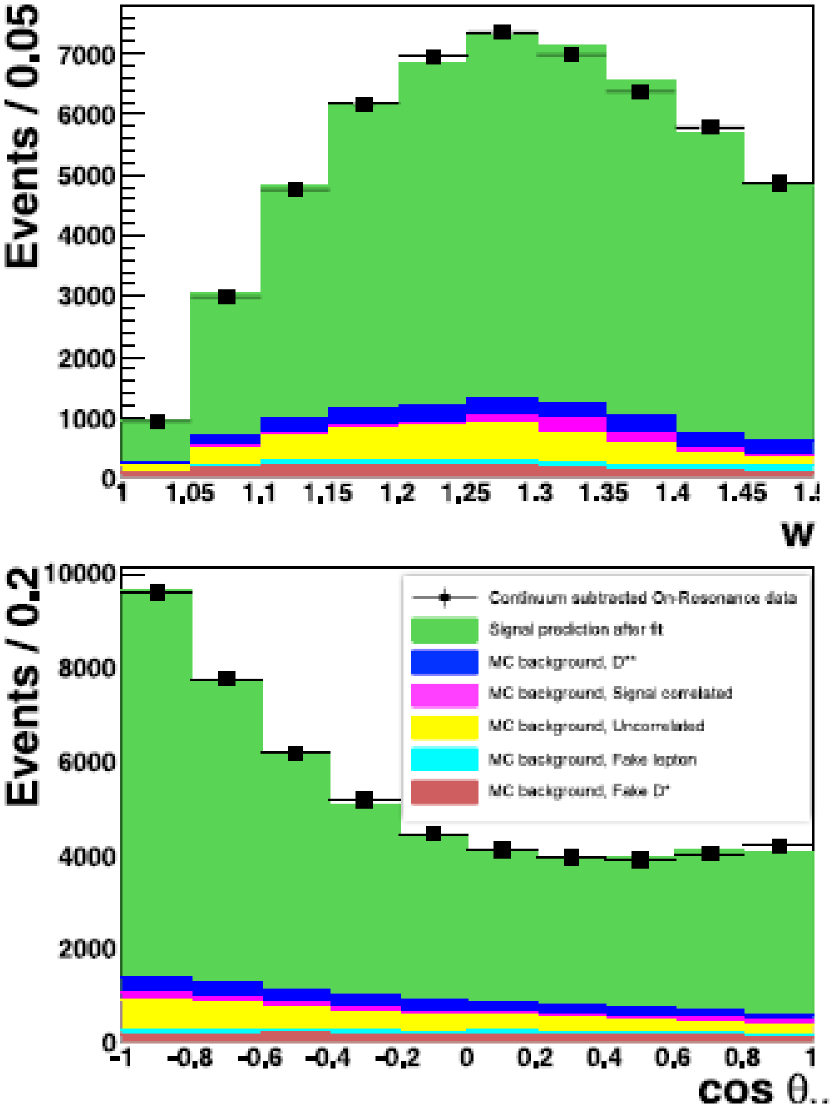
[Fermilab/MILC NPPS 140, 461(2005)]

$F_{B\to D^*}(1) = 0.9077(51)_{stat}(158)_{sys}$

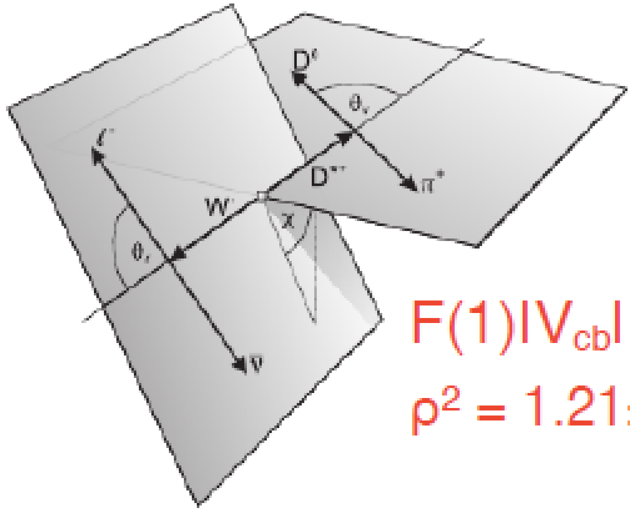
[Fermilab/MILC, arXiv:1011.2166]

B->D*lν

Belle PRD82, 112007(2010)



- **772M BBbar** events, **123K D^{*-} l^+** candidates
- FF pars from fit to 1D hists (10 bins) of ***w*, $\cos\theta_{lep}$, $\cos\theta_V$, χ**
- 40x40 covariance in $F(1)|V_{cb}|$ fit



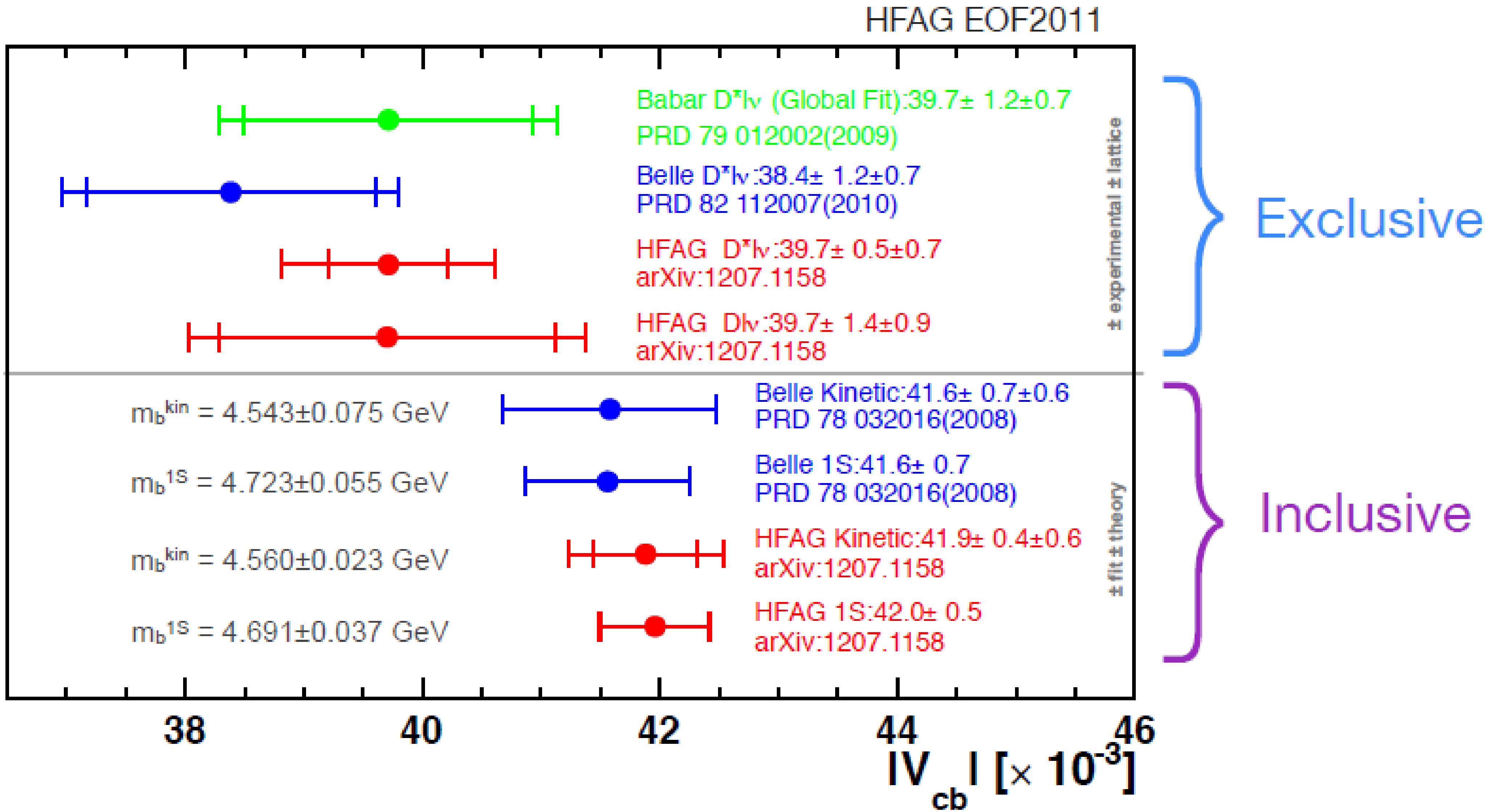
$F(1)|V_{cb}| = (34.7 \pm 0.2 \pm 1.0) \times 10^{-3}$

$\rho^2 = 1.21 \pm 0.02 \pm 0.02$

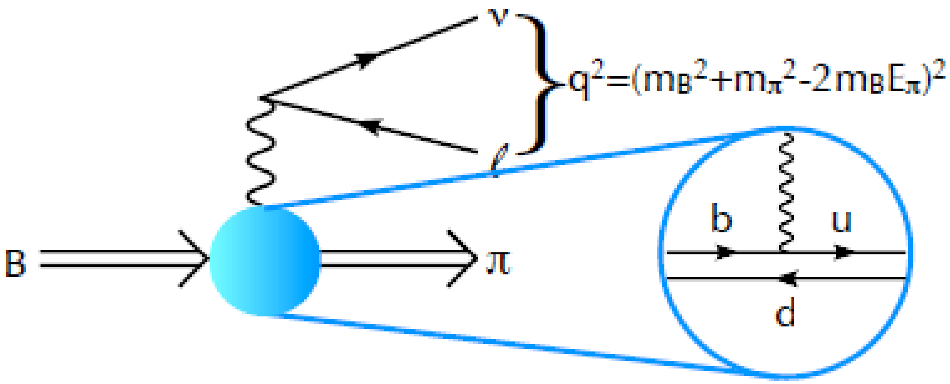
- **Sys:** Limited by track (fast&slow) efficiencies ~0.8%, and B→D**lν
- **B → D*lν FF's**, **1.6%** accuracy (**2.6%** in 2008)

V_{cb} summary

- Small persistent *discrepancy*, up to ~2.4σ; exclusive - inclusive.
- ΔExclusive~2%, ΔInclusive~1-2%



Exclusive V_{ub}

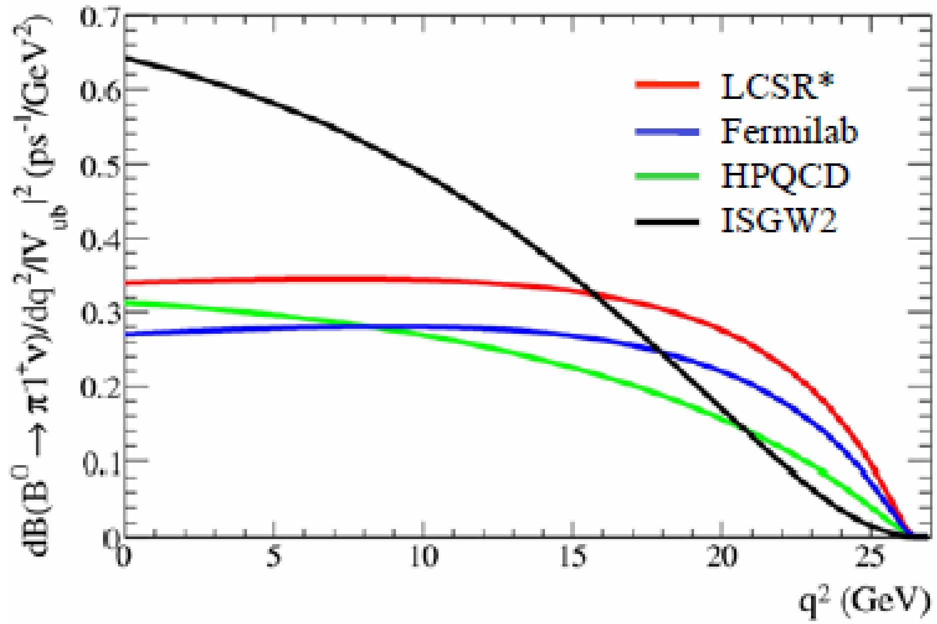


$$\Delta\zeta(0, q^2_{max}) = \frac{G_F^2}{24\pi^3}\int_0^{q^2_{max}} dq^2\, p_\pi^3 |f_+(q^2)|^2$$

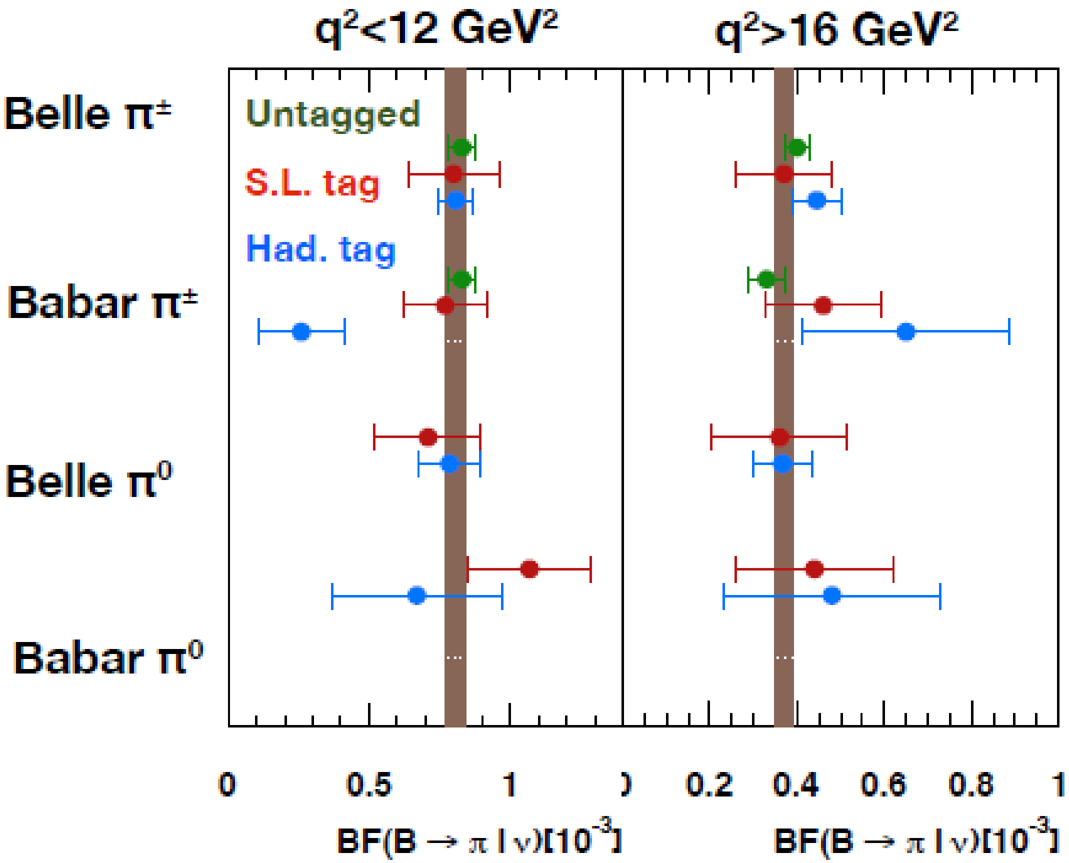
One FF for $B \to \pi l\nu$ with massless lepton

$$= \frac{1}{|V_{ub}|^2 \tau_{B_0}} \int_0^{q^2_{max}} dq^2 \frac{d\mathcal{B}(B \to \pi\ell\nu)}{dq^2}$$

- Rates determined by $|V_{ub}|$ & Form Factors
- Calculable at kinematical limits with **L**ight **C**one **S**um **R**ules or **L**attice **QCD**
- Empirical extrapolation necessary to extract $|V_{xb}|$ from measurements



B->πlν results



	Efficiency	Purity
Untagged	High ↑ Low	Low ↓ High
Tag $B \to D^{(*)}l\nu$		
Tag $B \to$ hadrons		

- *Experimentally robust.*
- Biggest (recent) advance: improved hadron tag for m^2_{miss} analyses.

  Reduced model dependence - cross check of untagged.

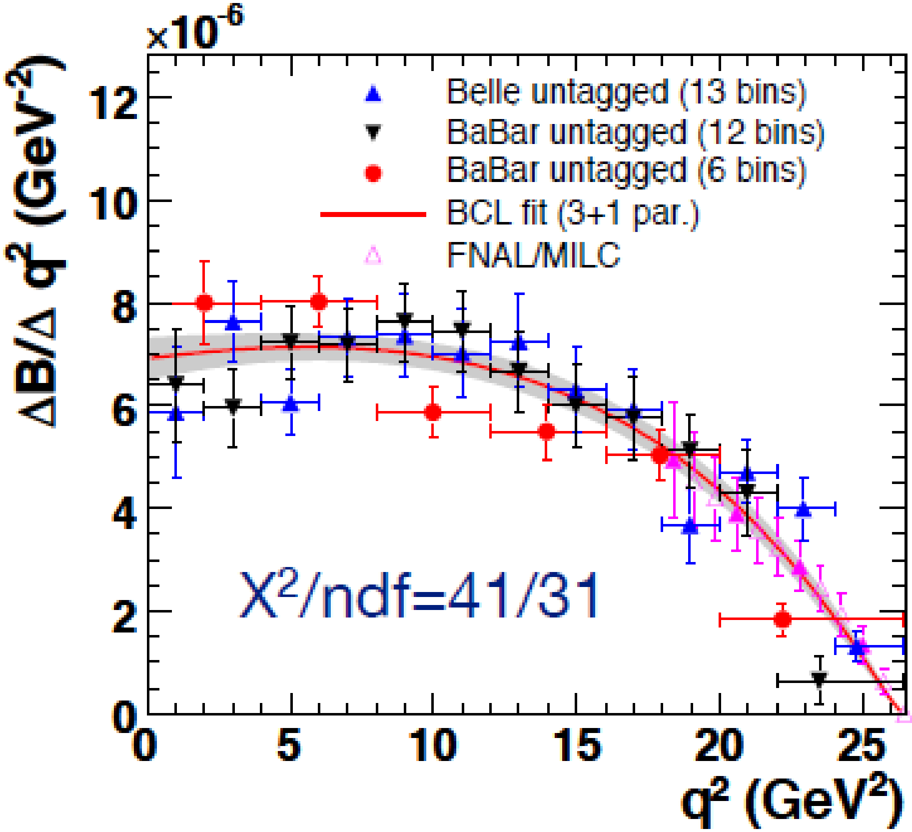
ε_{tag} %	BABAR Cut (2008>)	Belle Cut	Belle NN (2011>)
Modes	1768	-	~1000
B^+_{tag}	0.4	0.14	0.28
B^0_{tag}	0.21	0.1	0.18

Exclusive V_{ub}

1. $|V_{ub}|$ from partial q^2 integral with **FF** (theory/lattice).
2. **Fit** data & theory: LCSR or LQCD in q^2(2-3 shape pars + $|V_{ub}|$, data & LQCD correlations)
   1. Not precise enough to rule out any theory (other than ISGW2)

Error budget:
2% total rate
4% q^2 shape
8% FF normalisation

HFAG PDG 2013	q^2 $(GeV/c)^2$	$\|V_{ub}\| 10^3$
LCSR Siegen	**<12**	**$3.42 \pm 0.06^{+0.37}_{-0.32}$**
LCSR Ball/Zwicky	<16	$3.58 \pm 0.06^{+0.59}_{-0.40}$
LQCD HPQCD	>16	$3.49 \pm 0.09^{+0.60}_{-0.40}$
LQCD FNAL/MILC	>16	$3.33 \pm 0.08^{+0.37}_{-0.31}$
Global Fit (FNAL)	**All**	**3.26 ± 0.29**
Global Fit (LCSR)	All	3.26 ± 0.19



Inclusive V_{ub}

- Total rate can't be measured! Too much **$B \rightarrow X_c l\nu$** background.
- Remove ***$b \rightarrow cl\nu$***: **BUT** lose part of **$b \rightarrow ul\nu$**.

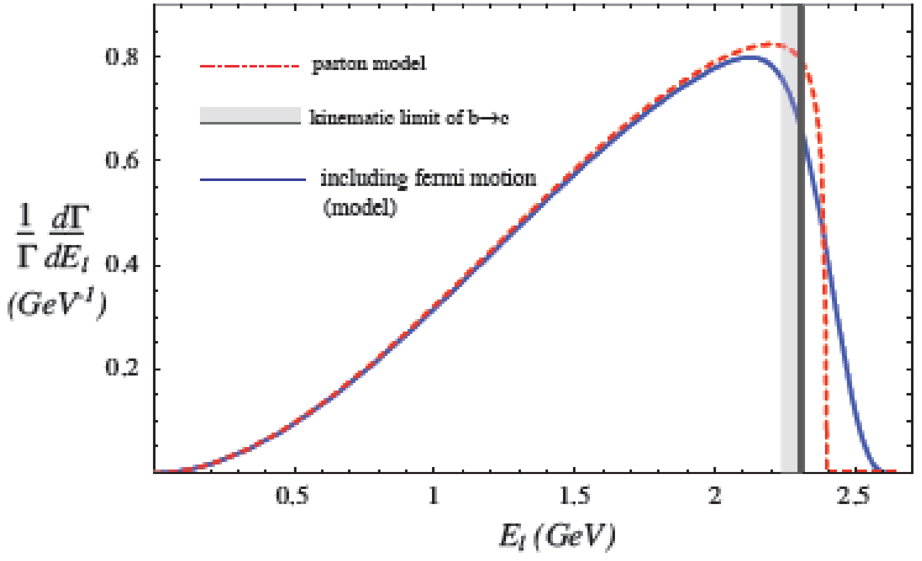
Measure $$\Gamma(B \rightarrow X_u \ell \nu) \times f_C = |V_{ub}|^2 \zeta_C$$

Fraction of the signal that passes the cut
→ corrected for **QCD**, **motion of *b***-quark

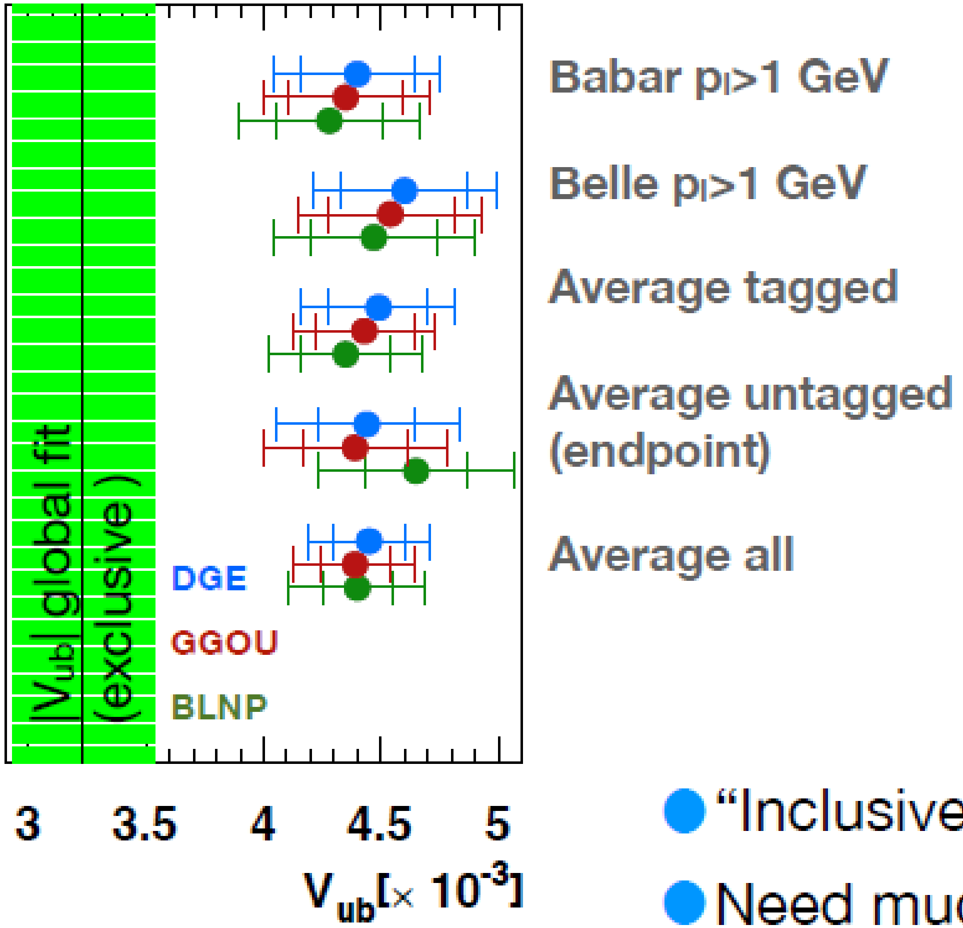
Cut-dependent constant (theory)

Problems: Restriction of phase space **creates complication, need models**

$\Gamma \sim |V_{ub}|^2 m_b^5$, but partial rates $\Delta\Gamma \sim |V_{ub}|^2 m_b^{10}$



Inclusive V_{ub}



Systematics %	Babar ($p_l^*>1$ GeV)	Belle
$B \to X_u \ell \bar{\nu}_\ell$ (SF)	5.6	3.6
$B \to X_u \ell \bar{\nu}_\ell$ $(g \to s\bar{s})$	2.7	1.5
$B \to X_u \ell \bar{\nu}_\ell$ exclusive	1.9	4.0
$B \to X_u \ell \bar{\nu}_\ell$ unmeasured	-	2.9
All $B \to X_u \ell \bar{\nu}_\ell$	6.5	5.8
$B \to X_c \ell \bar{\nu}_\ell$	2.7	1.7
PID and reconstruction	3.4	3.1
BDT	-	3.1
Other	2.1	2
Total	8.4	8.1

- "Inclusive" analyses not inclusive enough...
- Need much better understanding of $X_u l^+ \nu$

Resonant & non-res., light quark **hadronisation.**

Common techniques used in Belle & Babar

$m_{Xu}>1$ GeV (exclusives) next frontier.

B→τν

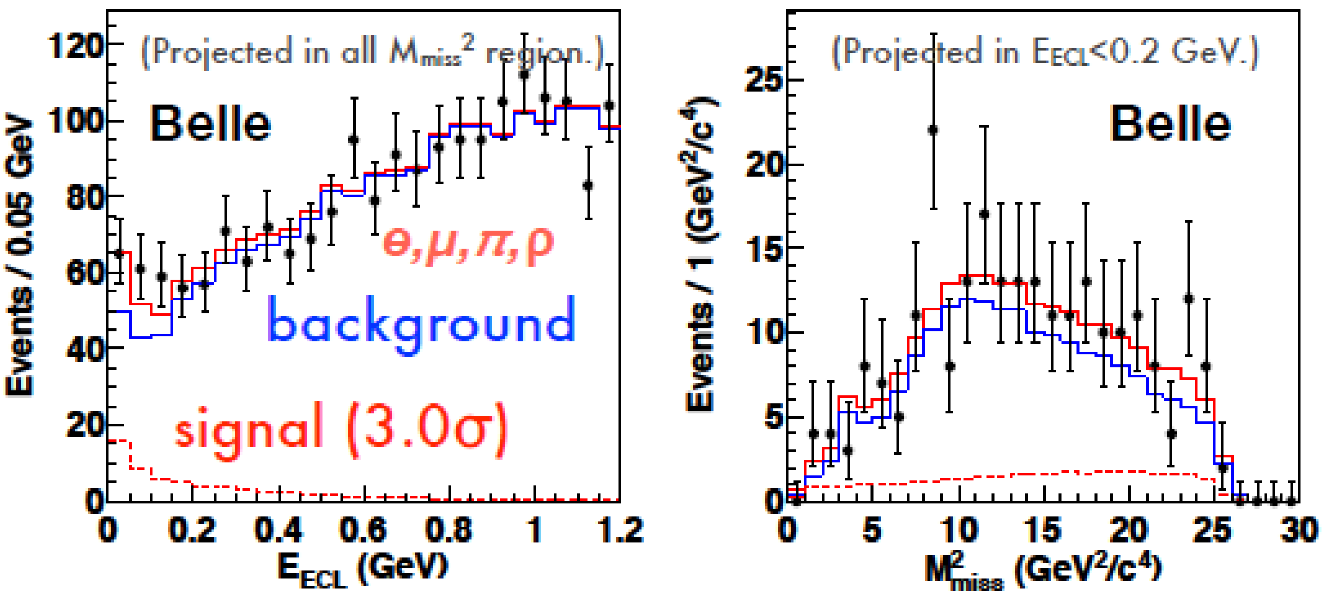
- Enters UT in 2 main ways:
  - B(B→τν) ∝ $f_B^2\,|V_{ub}|^2$ (f_B very precise)

$$\mathcal{B}(B^- \to \tau^- \bar{\nu}_\tau) = \frac{G_F^2 m_B m_\tau^2}{8\pi}\left(1 - \frac{m_\tau^2}{m_B^2}\right)^2 f_B^2 |V_{ub}|^2 \tau_B$$

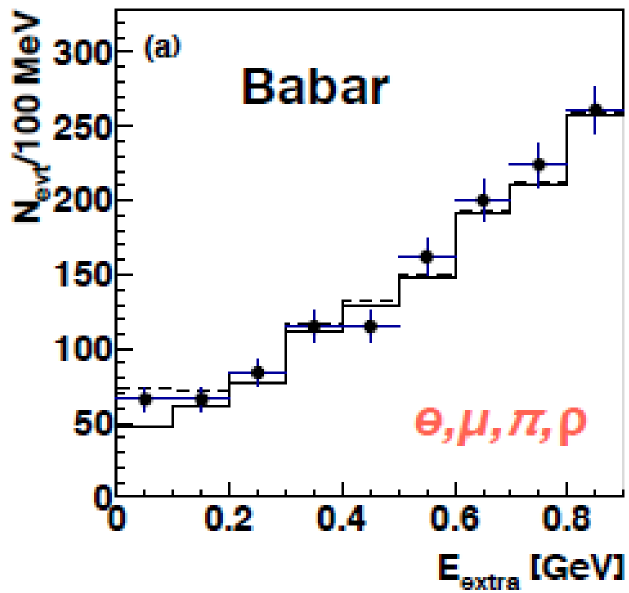
  - B(B→τν) / $\Delta m_d \propto |V_{ub}|^2 / |V_{td}|^2$, Cancels f_B uncertainties.

- *New Had tagged results*

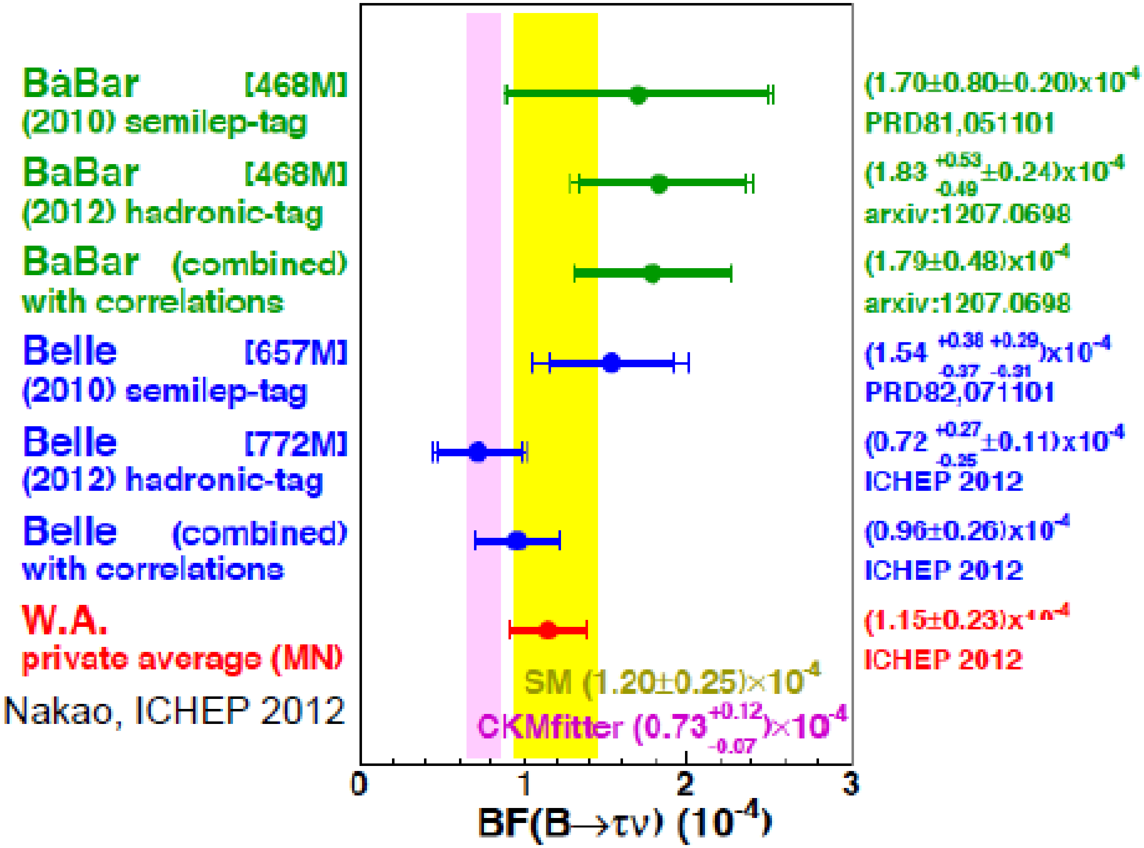
PRL 110, 131801 (2013).



arXiv:1207.0698



f_B and $|V_{ub}|$ from leptonic decays



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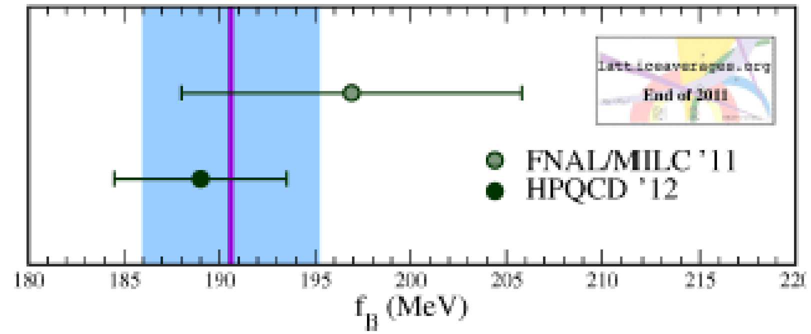
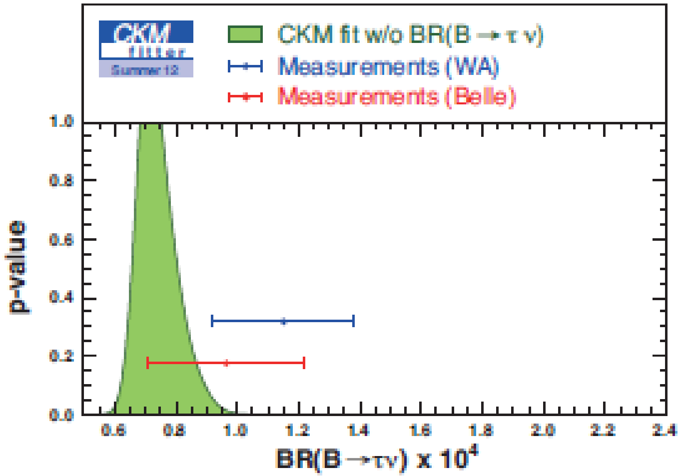
LQCD: ~2.5% error on f_B



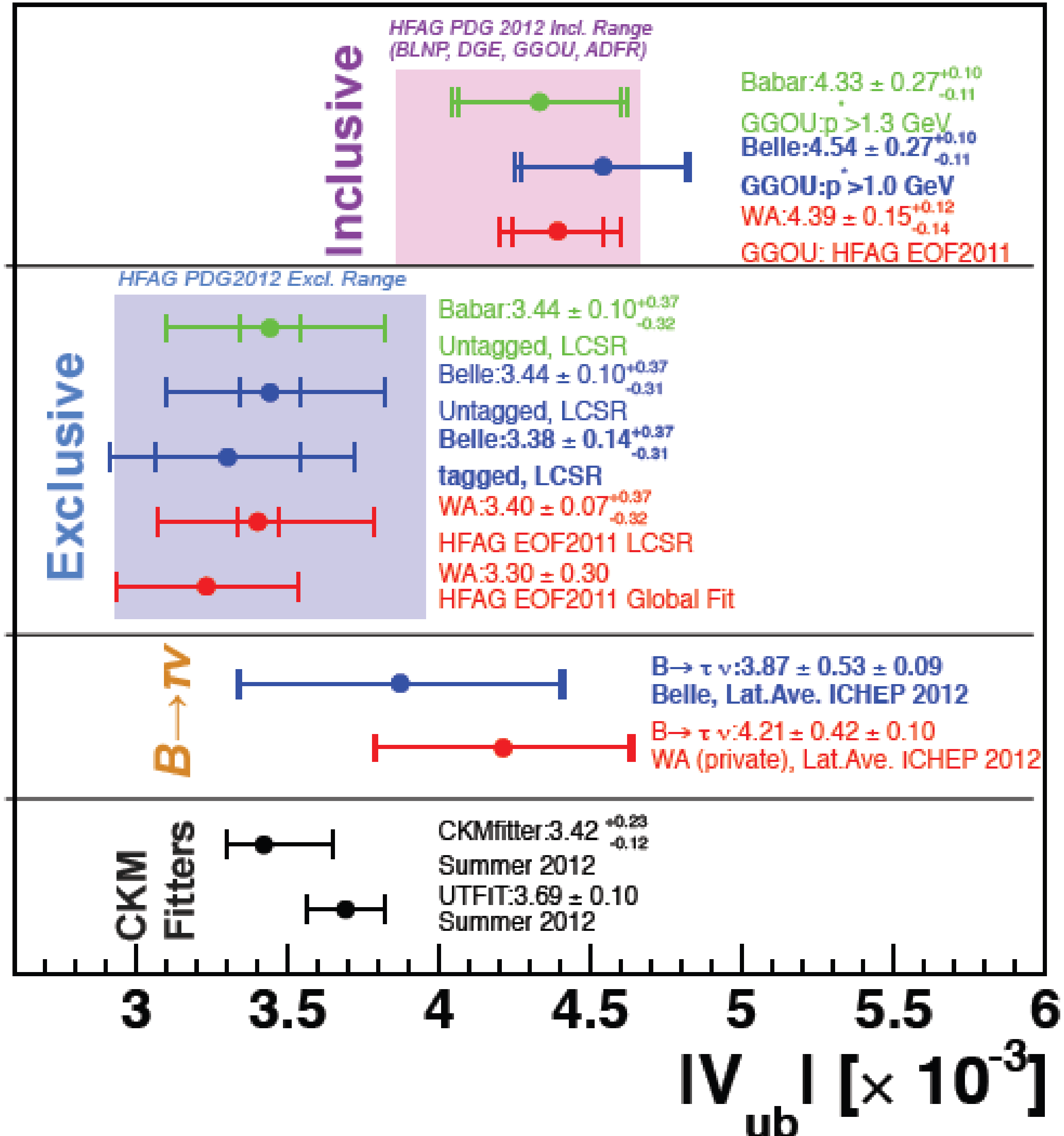
$\otimes$



Differ by 1.5 σ if error uncorrelated

$|V_{ub}|$ (±Exp±LQCD) ($\times 10^{-3}$)	
Babar	5.3±0.7±0.1
Belle	3.9±0.5±0.1
WA	4.2±0.4±0.1

V_{ub} summary



Exclusive / inclusive $|V_{ub}|$ differ by ~3σ

LCSR: Khodjamirian et al. $q^2 < 12$
PRD 83:094031 (2011)

GGOU: Gambino et al.
JHEP 0710:058 (2007)

Summary

- Angle $\phi_1(\beta)$ is measured with 1º accuracy in $b \rightarrow \bar{c}cs$. No deviation from SM is found in $b \rightarrow sq\bar{q}$ penguin and $b \rightarrow c\bar{c}d$ decays.

- Accuracy of O(5º) is achieved in $\phi_2(\alpha)$ measurement using $\rho\rho$ and $\rho\pi$. Still is limited by statistic.

- $\phi_3(\gamma)$ remains the most difficult angle of the UT to measure. Good perspectives with higher statistics since the theoretical uncertainties are very low.

- 3% and 10% accuracy are obtained for $|V_{ub}|$ and $|V_{ub}|$. 2-3σ discrepancy in inclusive / exclusive methods.

- Excellent agreement with Standard Model so far. Next order of statistics is necessary to give an answer for the New Physics existence.