Testing extra dimensions hypothesis in high energy physics

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Prologue

- Bernhard Riemann, "Über die Hypothesen, welche der Geometrie zu Grunde liegen" (1854 г.)

  (*On the Hypotheses which lie at the Bases of Geometry*, *Translated by William Kingdon Clifford*)

  The questions about the infinitely great are for the inter-pretation of nature useless questions. But this is not the case with the questions about the infinitely small. It is upon the exactness with which we follow phenomena into the infinitely small that our knowledge of their causal relations essentially depends…

- Now it seems that the empirical notions on which the metrical determinations of space are founded, the notion of a solid body and of a ray of light, cease to be valid for the infinitely small. We are therefore quite at liberty to suppose that the metric relations of space in the infinitely small do not conform to the hypotheses of geometry; and we ought in fact to suppose it, if we can thereby obtain a simpler explanation of phenomena…

- This leads us into the domain of another science, of physic, into which the object of this work does not allow us to go today.

- Gunnar Nordström, "Über die Möglichkeit, das elektromagnetische Feld und das Gravitationsfeld zu vereinigen", (*On the possibility of unifying the electromagnetic and the gravitational fields*) (1914 г.)

The first physical theory, in which an attempt was made to unify electromagnetism and relativistic scalar gravity in five-dimensional space-time.

1 Kaluza-Klein theory

Kaluza T. "Zum Unitätsproblem in der Physik", (*On the problem of unity in physics*) Sitzungsber. Preuss. Akad. Wiss. Berlin. (Math. Phys.). 1921. P. 966-972;

Klein O. "Quantentheorie und fünfdimensionale Relativitätstheorie", (*Quantum theory and five-dimensional theory of relativity*) Zeit. Physik. 1926. V. 37. P. 895-906.

The space-time has an extra space dimension, which is macroscopically unobservable.

The unobservability of the extra dimension was explained by its compactness and extremely small size, - of the order of the Planck length $\mathbf{l_{Pl} = 1/M_{Pl}}$

The original Kaluza-Klein model: gravity in five-dimensional space-time $E=M^4 \times S^1$ with action

$$S = \frac{1}{16\pi \hat{G}} \int_E \hat{R}\sqrt{-g}\, d^5X, \quad X^N = \{x^\nu, y\}, \quad 0 \leq y < L,$$

$\hat{G}$ being the five dimensional gravitational constant, $\hat{R}$ being the five-dimensional scalar curvature and the signature of the metric being

sign g_{MN} = (-,+,+,+, +), M,N = 0,1,2,3,4.

The five-dimensional metric g_{MN} can be decomposed as

$$g_{MN} = \begin{pmatrix} \gamma_{\mu\nu} + \phi A_\mu A_\nu & \phi A_\mu \\ \phi A_\nu & \phi \end{pmatrix}, \quad \phi = g_{44}, \, \phi A_\mu = g_{\mu 4}.$$

If g_{MN} does not depend on the extra dimension coordinate y, then

$$\begin{aligned} \hat{R} &= R_{(4)} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \phi^{-1} - \frac{1}{2} \phi^{-2} \partial_\mu \phi \partial^\mu \phi \\ F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu, \end{aligned}$$

$R_{(4)}$ being the scalar curvature of the four-dimensional space-time M^4 with metric $\gamma_{\mu\nu}$.

In the papers by Kaluza and Klein the field φ was assumed to be constant, and the field A_μ was identified with the electromagnetic field.

The theory gives a relation between the five-dimensional (M) and four-dimensional (M_{Pl}) Planck masses:

$$M_{Pl}^2 = M^3 L.$$

Any field in space-time $E=M^4 \times S^1$ can be expanded in a Fourier series in the coordinate y.

Thus, for a scalar field

$$\phi(x,y) = L^{-\frac{1}{2}} \sum_n \phi^{(n)}(x) \exp(i\frac{2\pi n y}{L}).$$

The Lagrangian in five-dimensional space-time is

$$\mathcal{L} = -\frac{1}{2}\partial_M \phi \partial^M \phi - \frac{m^2}{2}\phi^2$$

and the equations of motion look like

$$(\partial_M \partial^M - m^2)\phi = 0.$$

The modes $\varphi^{(n)}$ satisfy the equations

$$(\partial_\mu \partial^\mu - m_n^2)\phi^{(n)} = 0, \quad m_n^2 = m^2 + \frac{4\pi^2 n^2}{L^2}.$$

Since *L* is of the order of the Planck length, the observable fields may be only the “zero modes”, i.e. they do not depend on the coordinate of the extra dimension.

- For every four-dimensional field there exists a tower of fields with the same quantum numbers and the masses of the order of M_{Pl}, which cannot be observed at the energies available nowadays.

2 Large extra dimensions

Localization of fields:

V.A.Rubakov and M.E. Shaposhnikov,
"Do We Live Inside A Domain Wall?"
Phys. Lett. 125 (1983) 136.

"Extra Space-Time Dimensions: Towards A Solution Of The Cosmological Constant Problem",
Phys. Lett. 125 (1983) 139.

The fields of the SM can be localized on a domain wall in a multidimensional space. If the thickness of the domain wall goes to zero, then it turns into a membrane, or just a “brane”.

- ADD scenario

N. Arkani-Hamed, S.Dimopoulos and G.R. Dvali, “The hierarchy problem and new dimensions at a millimeter”, Phys. Lett. B 429 (1998) 263

A single brane without tension (i.e. energy density) in a space-time with an arbitrary number of compact extra dimensions.

The scenario provides a solution to the hierarchy problem: it gives a strong gravity in the multidimensional space-time and a weak gravity on the brane

$$M_{Pl}^2 = M^{(2+n)} V_n$$

- The approximation of the zero brane tension turns out to be rather too rough, and the proper gravitational field of the brane cannot be taken into account perturbatively.

- For the equations of Einstein gravity in space–time with compact extra dimensions to be consistent, there should exist at least two branes with tension, and the number of extra dimensions can be either one or two.

L. Randall and R. Sundrum,
“A large mass hierarchy from
a small extra dimension”,
Phys. Rev. Lett. 83 (1999) 3370

3 The Randall-Sundrum model

Two branes with tension at the fixed points of the orbifold S^1/Z_2:

$$\begin{aligned} S &= \int d^4x \int_{-L}^{L} dy \left(2M^3 R - \Lambda\right) \sqrt{-g} - \\ &- \lambda_1 \int_{y=0} \sqrt{-\tilde{g}} d^4x - \lambda_2 \int_{y=L} \sqrt{-\tilde{g}} d^4x . \end{aligned}$$

The solution for the background metric:

$$ds^2 = \gamma_{MN} dx^M dx^N = e^{-2\sigma(y)} \eta_{\mu\nu} dx^\mu dx^\nu + (dy)^2, \; \sigma(y) = k|y| + c.$$

The parameters k, Λ и $\lambda_{1,2}$ satisfy the fine tuning conditions:

$$\Lambda = -24M^3k^2, \quad \lambda_1 = -\lambda_2 = 24M^3k.$$

The linearized gravity is obtained by the substitution

$$g_{MN} = \gamma_{MN} + \frac{1}{\sqrt{2M^3}} h_{MN}$$

The field h_{MN} can be transformed to the gauge

$$h_{\mu 4} = 0,\ h_{44} = \phi(x).$$

The distance between the branes along the geodesic

x = const

$$l = \int_0^L \sqrt{ds^2} \simeq \int_0^L \left(1 + \frac{1}{2\sqrt{2M^3}} h_{44}\right) dy = L\left(1 + \frac{1}{2\sqrt{2M^3}} \phi(x)\right).$$

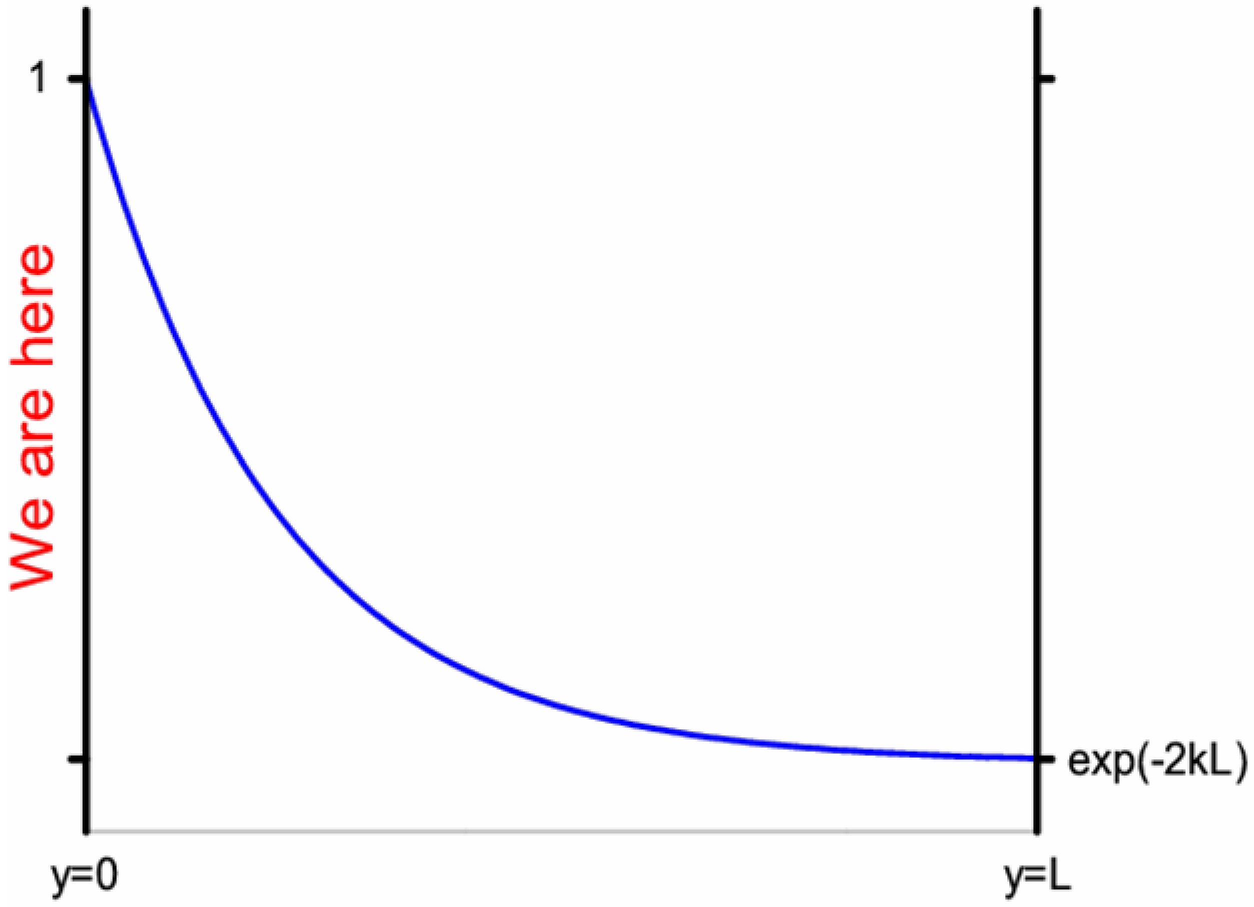
The metric in the zero mode approximation looks like

$$ds^2 = e^{-2\sigma(y)}(\eta_{\mu\nu}+\bar{h}_{\mu\nu}(x))dx^\mu dx^\nu + dy^2 = e^{-2\sigma(y)}\bar{g}_{\mu\nu}(x)dx^\mu dx^\nu + dy^2.$$

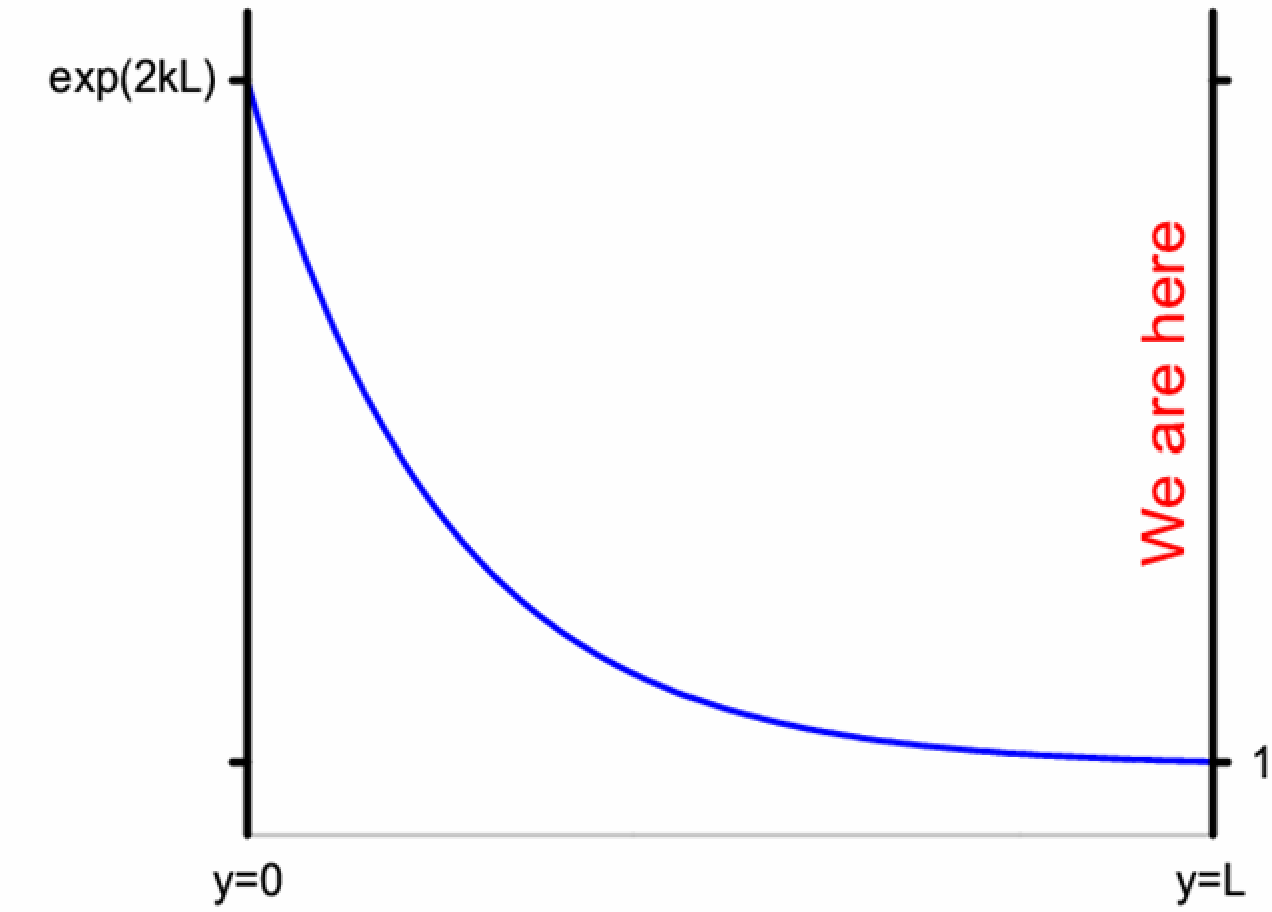
Substituting this metric into the action and integrating over the coordinate of the extra dimension one gets an effective action

$$S_{eff} = 2M^3 e^{-2c}\,\frac{1-e^{-2kL}}{k}\int d^4x R_4(\bar{g})\sqrt{-\bar{g}}.$$

Galilean coordinates: $g_{\mu\nu}$ = diag(-1,1,1,1).



Coordinates {x} are Galilean for c=0, $M_{Pl}^2 = \frac{M^3}{k}(1 - e^{-2kL})$.



Coordinates {x} are Galilean for c = - kL, $M_{Pl}^2 = \frac{M^3}{k}(e^{2kL} - 1)$.

The hiearrchy problem is solved, if M ~ k ~ 1 TeV и kL~ 35.

There appears a tower of tensor fields on the brane with the lowest mass of the order of M and the coupling to the SM fields of the order of 1/M.

The coupling of the radion to matter on the brane is too strong and contradicts the experimental restrictions even at the level of classical gravity.

The Randall-Sundrum model must be stabilized!

4 Stabilized Randall-Sundrum model

- Stabilization mechanisms:

W. D. Goldberger and M.B. Wise,
“Modulus stabilization with bulk fields”,
Phys. Rev. Lett. **83** (1999) 4922

O. DeWolfe, D.Z. Freedman, S.S. Gubser and A. Karch,
“Modeling the fifth dimension with scalars and gravity”,
Phys. Rev. D **62** (2000) 046008

The second model is more consistent. We consider such values of the model parameters that the background metric of the stabilized model is close to that of the unstabilized model.

The physical degrees of freedom of the model in the linear approximation were isolated in the paper

E.E. Boos, Y.S. Mikhailov,
M.N. Smolyakov and I.P. Volobuev,
"Physical degrees of freedom
in stabilized brane world models",
Mod. Phys. Lett. A **21** (2006) 1431

They are:

- tensor fields $b_{\mu\nu}{}^{n}(x)$, n=0,1, … with masses m_n (m_0 = 0) and wave functions in the space of extra dimension $\psi_n(y)$,
- scalar fields $\varphi_n(x)$, n=1,2, … with masses μ_n and wave functions in the space of extra dimension $g_n(y)$.

The interaction with the SM fields is described by the Lagrangian

$$L_{int} = -\frac{1}{\sqrt{8M^3}}\left(\psi_0(L)b^0_{\mu\nu}(x)T^{\mu\nu} + \sum_{n=1}^{\infty}\psi_n(L)b^n_{\mu\nu}(x)T^{\mu\nu} + \frac{1}{2}\sum_{n=1}^{\infty}g_n(L)\varphi_n(x)T^{\mu}_{\mu}\right),$$

$T_{\mu\nu}$ being the energy-momentum tensor of the SM.

At low energies this leads to contact interactions of SM fields

$$
\begin{aligned}
L_{eff} &= \frac{1.82}{\Lambda_\pi^2 m_1^2} T^{\mu\nu} \tilde{\Delta}_{\mu\nu,\rho\sigma} T^{\rho\sigma}, \\
\tilde{\Delta}_{\mu\nu,\rho\sigma} &= \frac{1}{2}\eta_{\mu\rho}\eta_{\nu\sigma} + \frac{1}{2}\eta_{\mu\sigma}\eta_{\nu\rho} - \left(\frac{1}{3} - \frac{\delta}{2}\right)\eta_{\mu\nu}\eta_{\rho\sigma},
\end{aligned}
$$

m_1 and Λ_π being the mass and the coupling constant of the first tensor mode and the constant δ describing the contribution of the scalar modes.

For M ≈ 2 TeV, k ≈ 1 TeV, k L = 35 and the mass of the first scalar mode of the order of 2 TeV these parameters turn out to be

$$
\Lambda_\pi \simeq 8\, TeV, \quad m_1 \simeq 3.83\, TeV.
$$

5 Models with more than one extra dimension

Sean M. Carroll, Monica M. Guica,
“Sidestepping the cosmological constant
with football shaped extra dimensions .
e-Print: hep-th/0302067

In Einstein gravity it is impossible to find braneworld solutions with more than two extra dimensions.

A possible solution – to pass to Lovelock gravity, based on the Lovelock Lagrangian:

$$16\pi\, G_d\, \mathcal{L}_L = \sum_{0 \le p \le (d+1)/2} \alpha_p\, \lambda^{2(p-1)}\, \mathcal{L}_{(p)}$$

$$\mathcal{L}_{(p)} = \frac{1}{2^p}\, \delta^{I_1 I_2 \cdots I_{2p}}_{J_1 J_2 \cdots J_{2p}}\, R^{J_1 J_2}{}_{I_1 I_2} \cdots R^{J_{2p-1} J_{2p}}{}_{I_{2p-1} I_{2p}}$$

$$\mathcal{L}_{(0)} = 1, \quad \mathcal{L}_{(1)} = R, \quad \mathcal{L}_{(2)} = R^{PR}{}_{MN} R^{MN}{}_{PR} - 4 R_{MN} R^{MN} + R^2$$

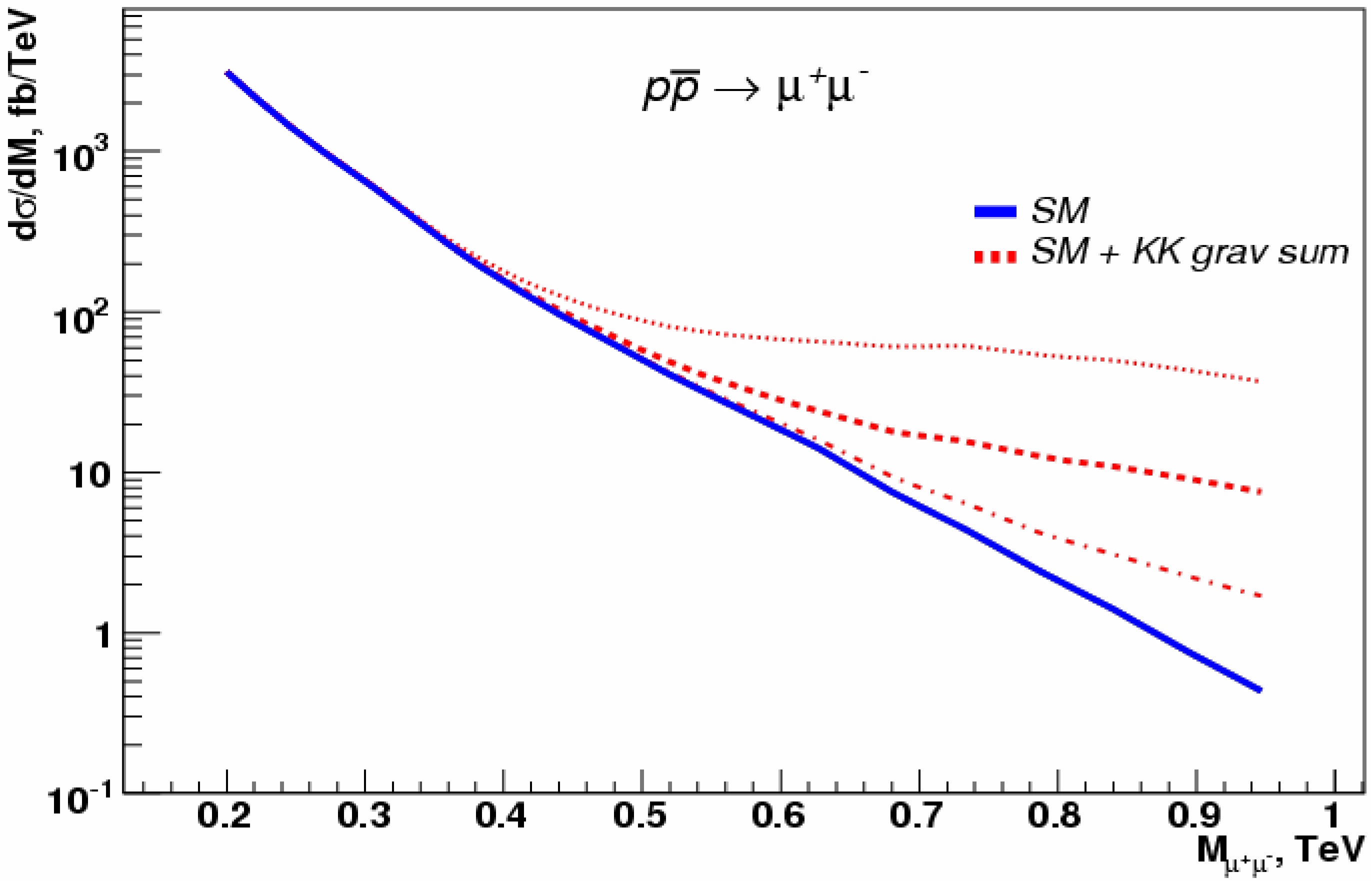
D. Lovelock,
"The Einstein tensor and its generalizations",
J. Math. Phys. **12** (1971) 498 - 501.

6 Processes with Kaluza-Klein gravitons

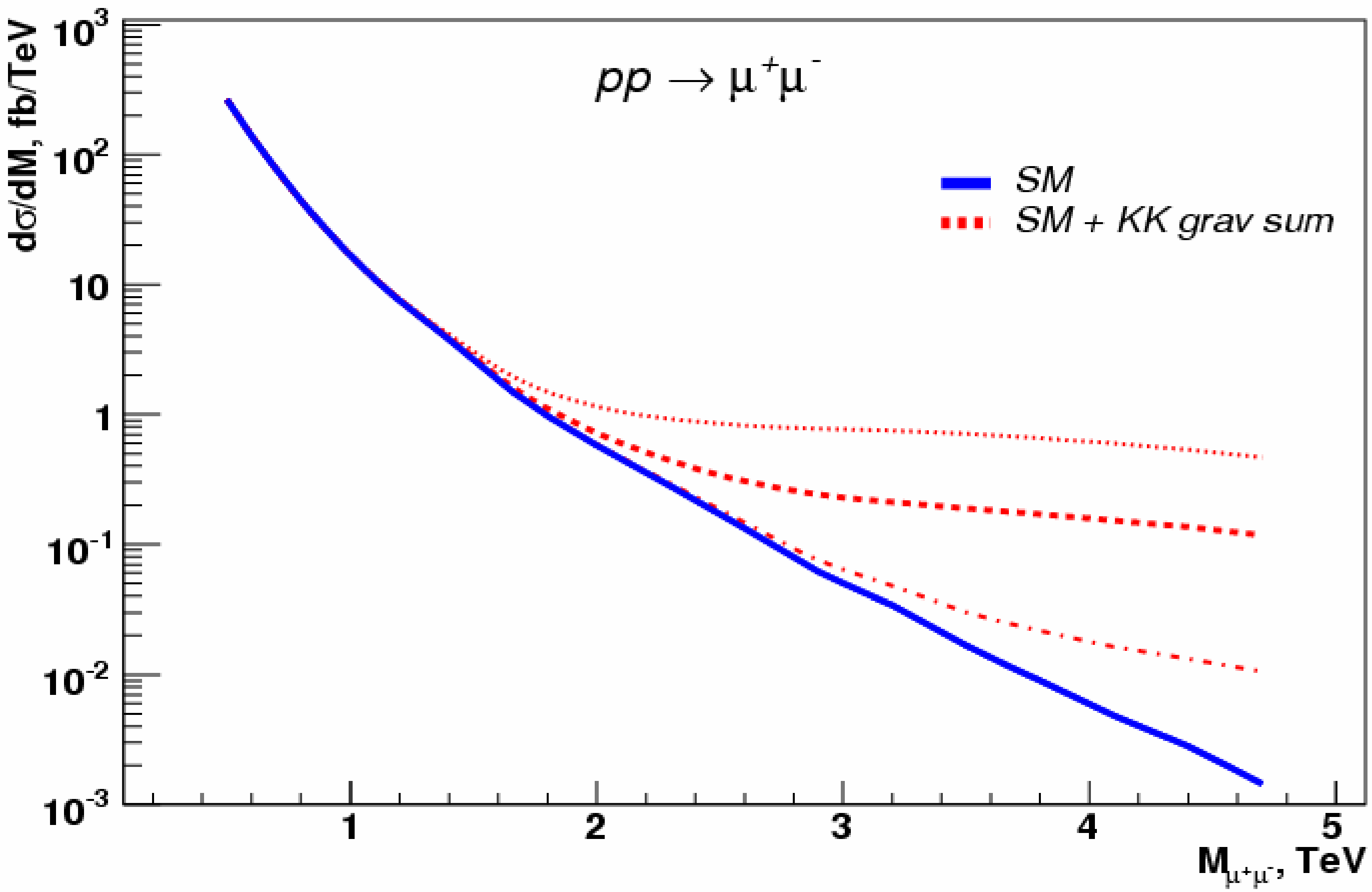
In the first approximation the effective interaction Lagrangian includes a sum of various 4-particle effective operators (not only 4-fermion, but also 2-fermion-2boson and 4-boson), which are invariant with respect to the SM gauge group and lead to a well defined phenomenology.

Various processes due to this Lagrangian were studies with the help of the CompHEP package in the paper

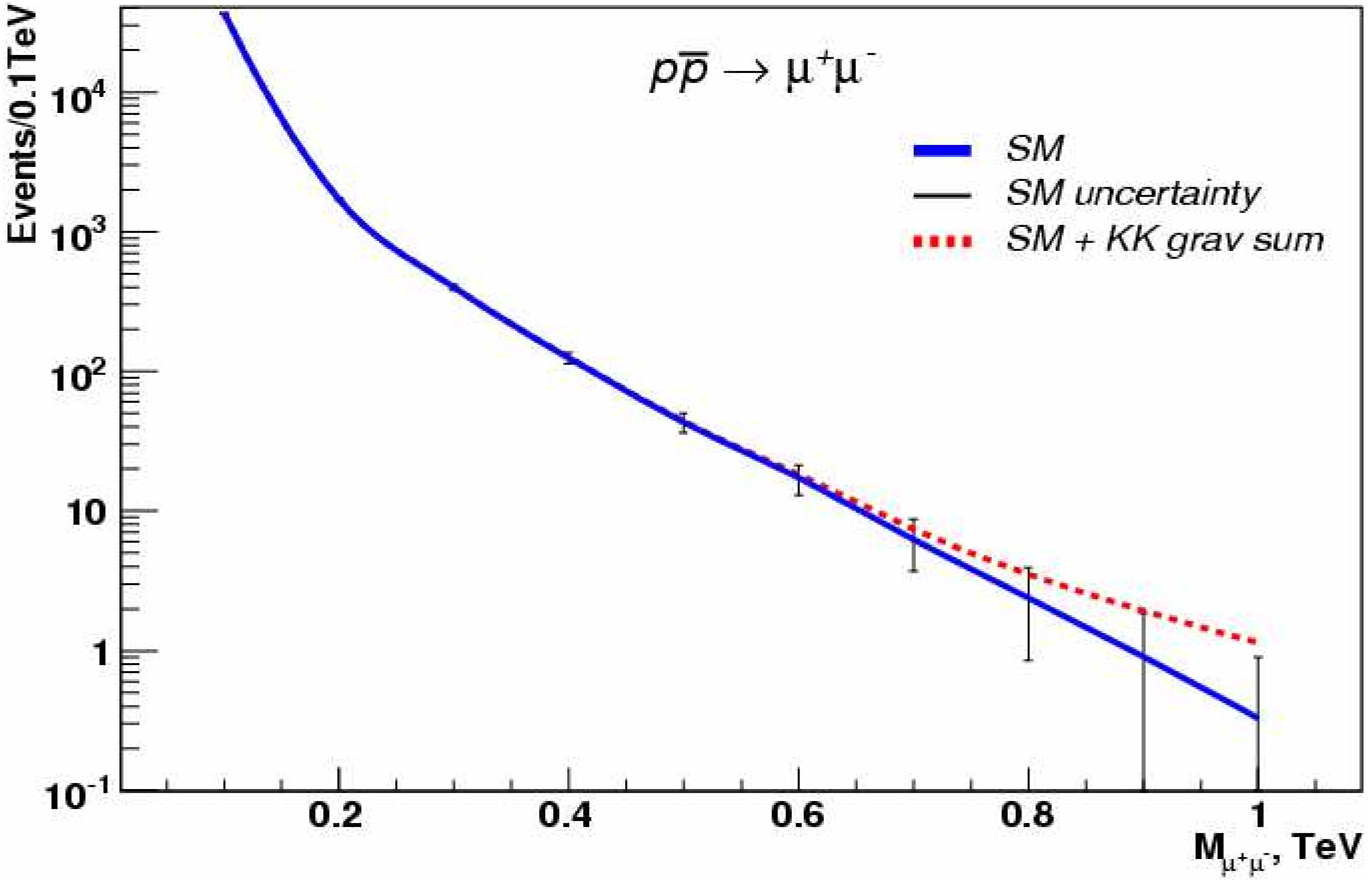
E.E. Boos, V.E. Bunichev, M.N. Smolyakov and I.P. Volobuev, "Testing extra dimensions below the production threshold of Kaluza-Klein excitations" Phys.Rev.D79:104013,2009 arXiv:0710.3100v4 [hep-ph].



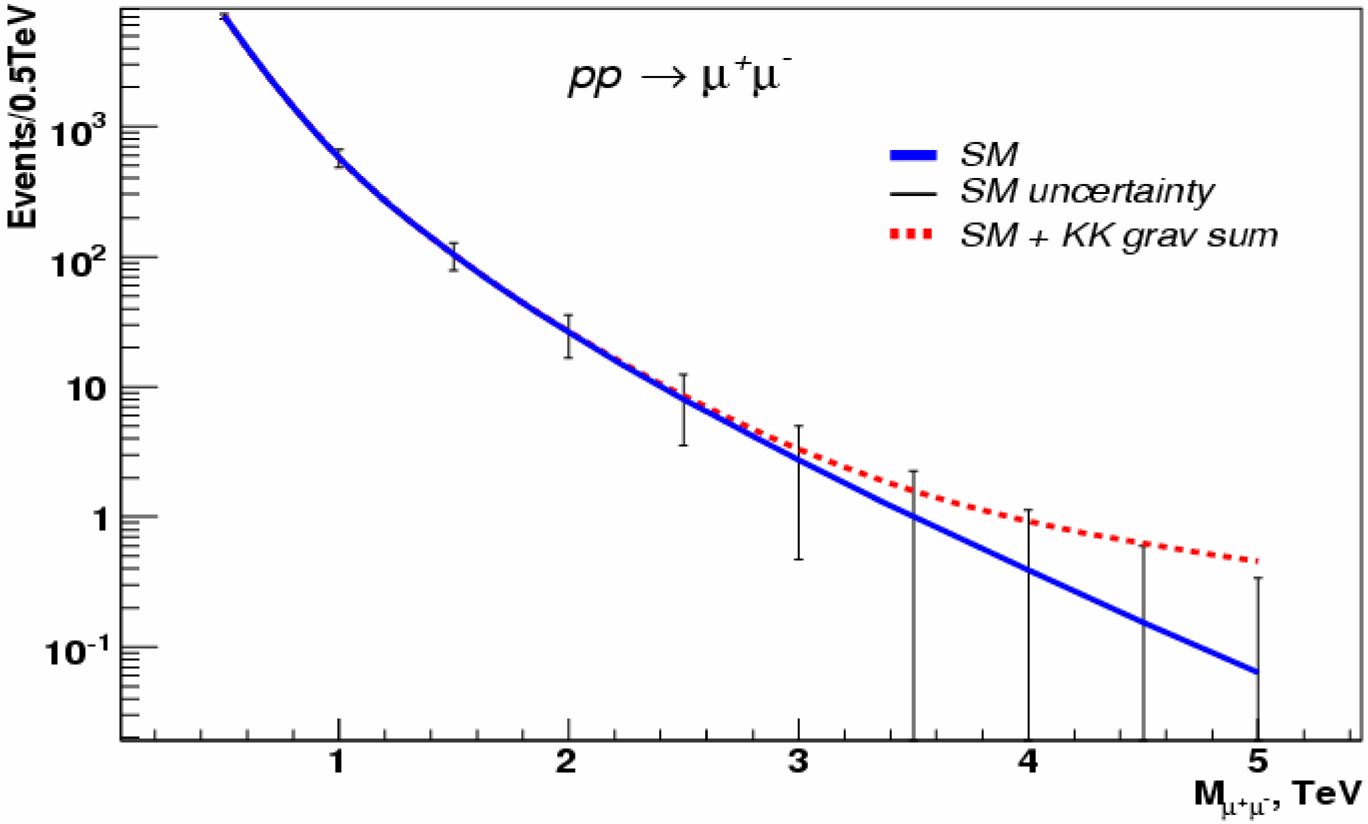
Dilepton invariant mass distribution for parameter $\frac{0.91}{\Lambda_\pi^2\, m_1^2} \times TeV^4$=0.66 (dash-dotted line), 1.82 (dashed line), 4 (dotted line) for the Tevatron



Dilepton invariant mass distribution for parameter $\frac{0.91}{\Lambda_\pi^2 m_1^2} \times TeV^4$=0.0014 (dash-dotted line), 0.0046 (dashed line), 0.01 (dotted line) for the LHC



Dilepton invariant mass distribution for 95% CL parameter $\frac{0.91}{\Lambda_\pi^2 m_1^2} \times TeV^4$=0.66 for the Tevatron ($L = 10fb^{-1}$)



Dilepton invariant mass distribution for 95% CL parameter $\frac{0.91}{\Lambda_\pi^2\, m_1^2} \times TeV^4$=0.0014 for the LHC ($L = 100\, fb^{-1}$)

- The restrictions on the coupling constant for which the extra dimension cannot be observed at the Tevatron and the LHC:

$$Tevatron: \ \frac{0.91}{\Lambda_\pi^2 m_1^2} \times TeV^4 < 0.66,$$

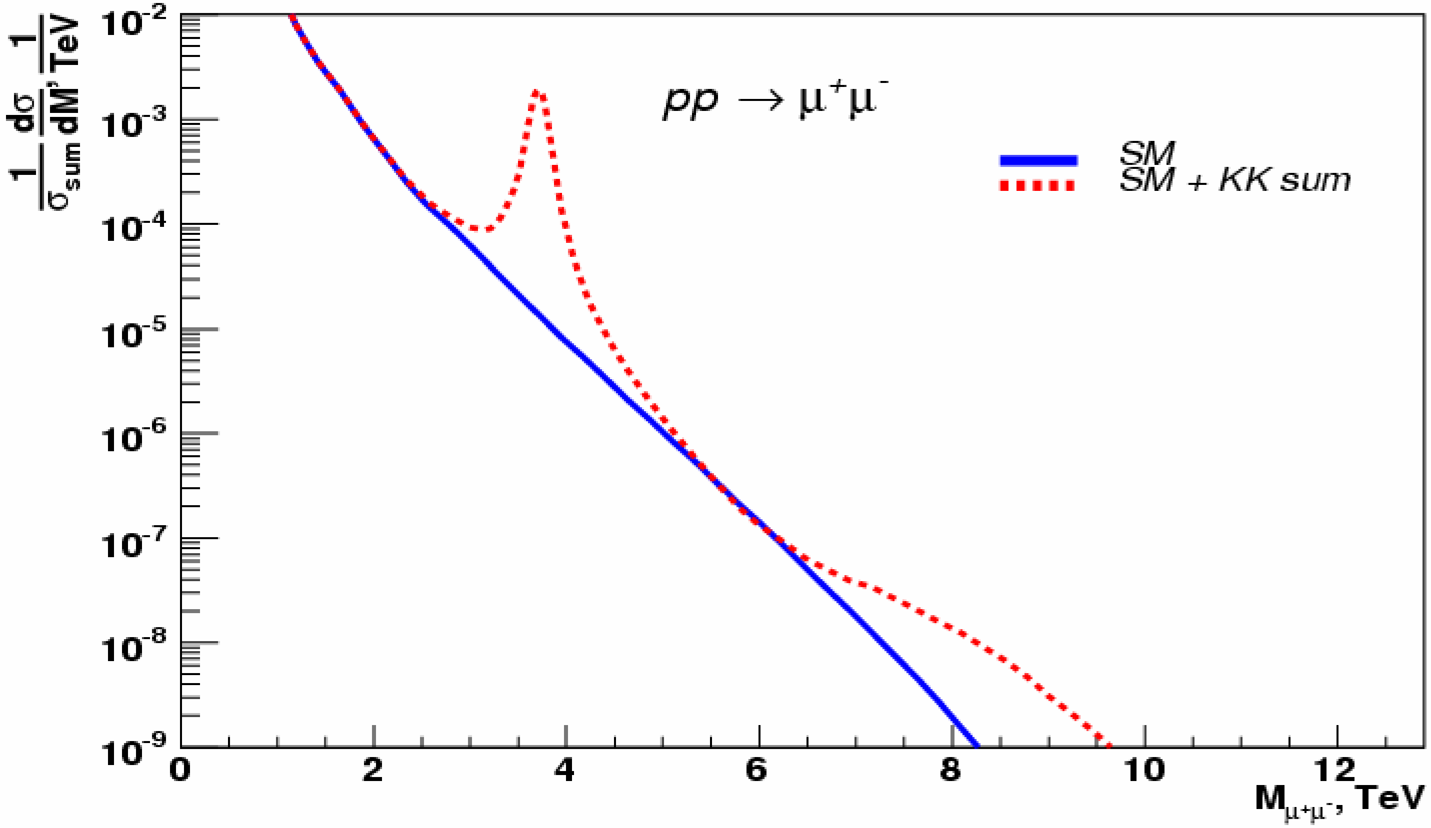
$$LHC: \ \frac{0.91}{\Lambda_\pi^2 m_1^2} \times TeV^4 < 0.0014.$$

- The lowest value of the parameter Λ_π , for which the extra dimension does not manifest itself, can be found form the demand that the resonance width is lesser than its mass, i.e. $\Gamma_1 = m_1/\xi$, where $\xi > 1$:

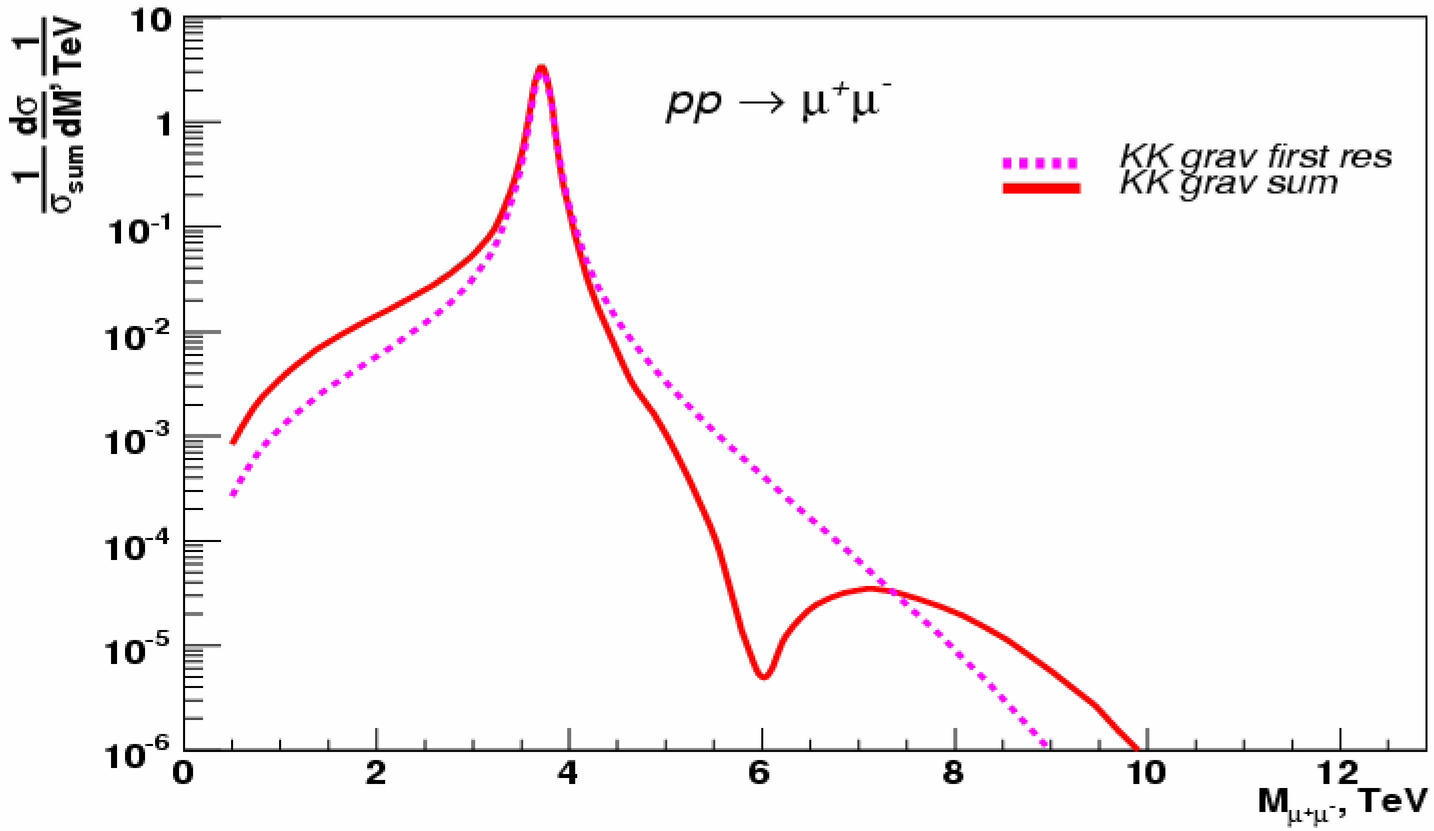
$$Tevatron: \ \Lambda_\pi > 0.61 \cdot \xi^{1/4}\, TeV,$$

$$LHC: \ \Lambda_\pi > 2.82 \cdot \xi^{1/4}\, TeV.$$

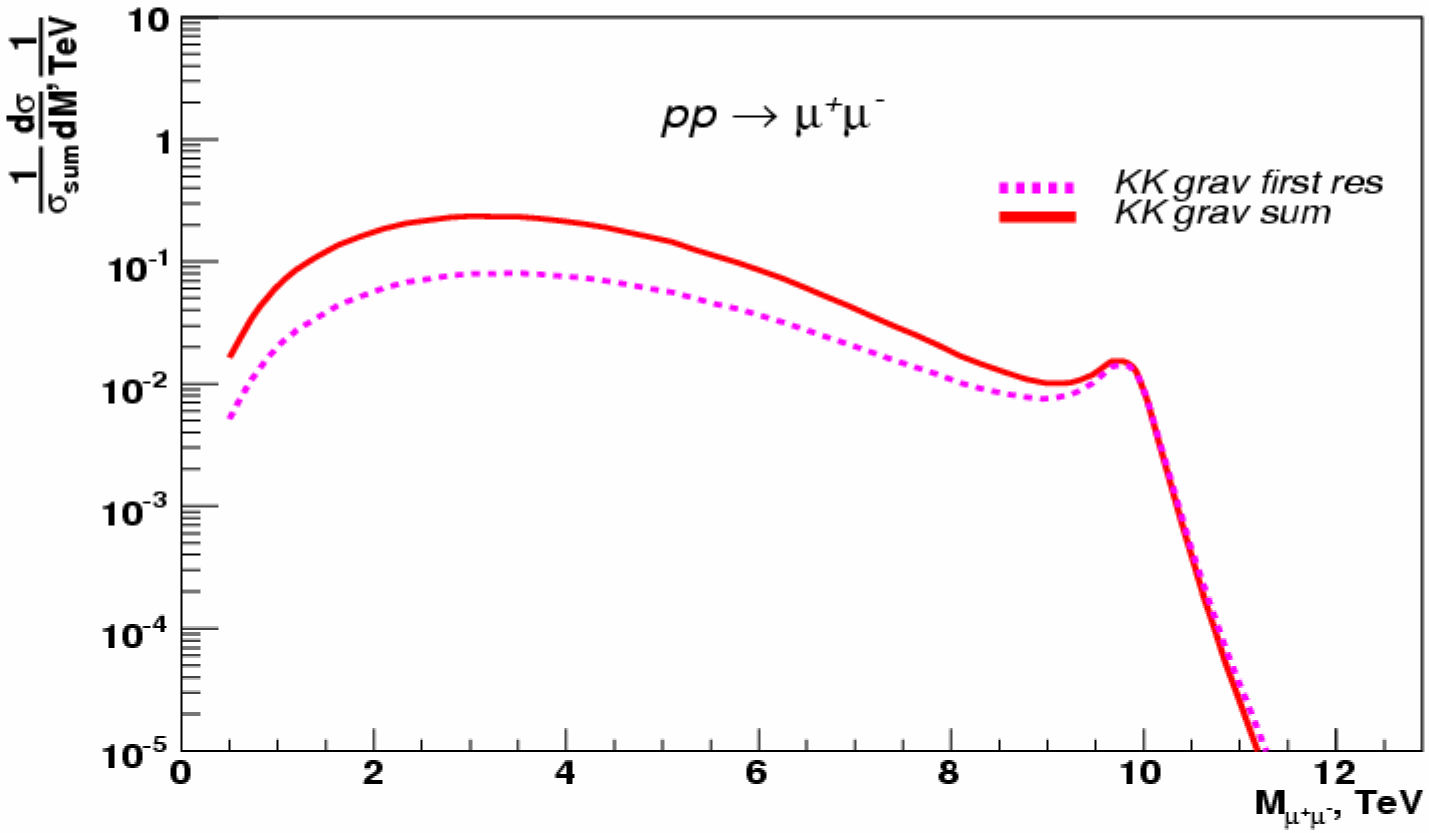
- In a similar way one can consider the situation, where the mass of the first mode lies in the accessible energy range. In this case the contribution of the first mode can be taken into account explicitly and the contribution of all the other modes, starting from the second one, can be again described by the effective contact interaction.



Dilepton invariant mass distribution from the SM (solid line) and from the SM plus sum of KK modes including the first KK resonance with $M_{res} = 3.83\ TeV$, $\Gamma_{res} = 0.08\ TeV$, $\Lambda_\pi = 8\ TeV$ (dashed line) for the LHC



The normalized dilepton invariant mass distribution from the first KK resonance plus the sum of KK tower states starting from the second mode (solid line) and from the first KK resonance only (dashed line) for $M_{res} = 3.83\ TeV$, $\Gamma_{res} = 0.08\ TeV$, $\Lambda_\pi = 8\ TeV$ for the LHC



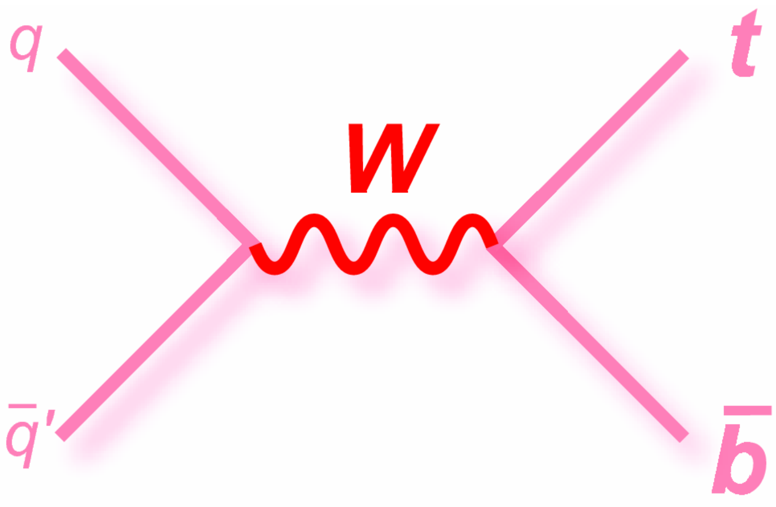
The normalized dilepton invariant mass distribution from the sum of KK tower states starting from the first KK mode (solid line) and from the first KK mode only (dashed line) for

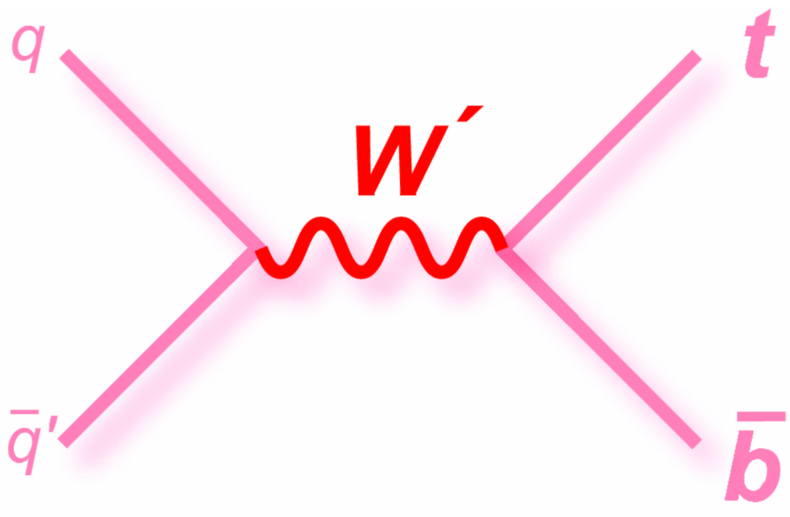
$M_{res} = 10\ TeV$, $\Gamma_{res} = 0.5\ TeV$, $\Lambda_\pi = 14\ TeV$ for the LHC

- Universal Extra Dimensions and processes with the excitations of the SM gauge bosons

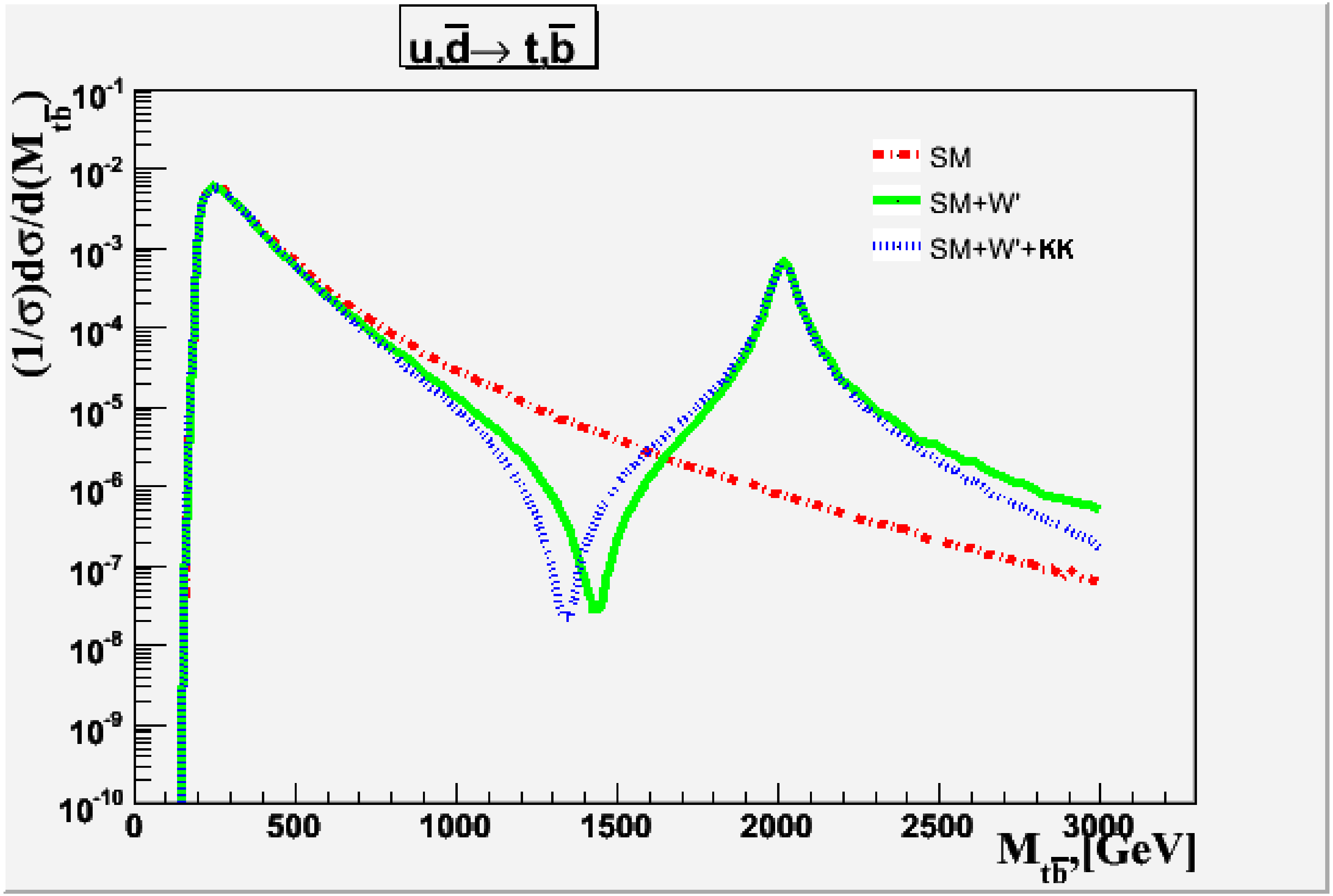
If the SM gauge bosons can propagate in the bulk, there also arise KK towers of their excitations, which may produce similar effects.

Single top production





$$M_{W'} = 2\text{ TeV}, \qquad M_{W'\_sum} = 2.8\text{ TeV}, \qquad \Gamma_{W'} = 65.7\text{ GeV}$$



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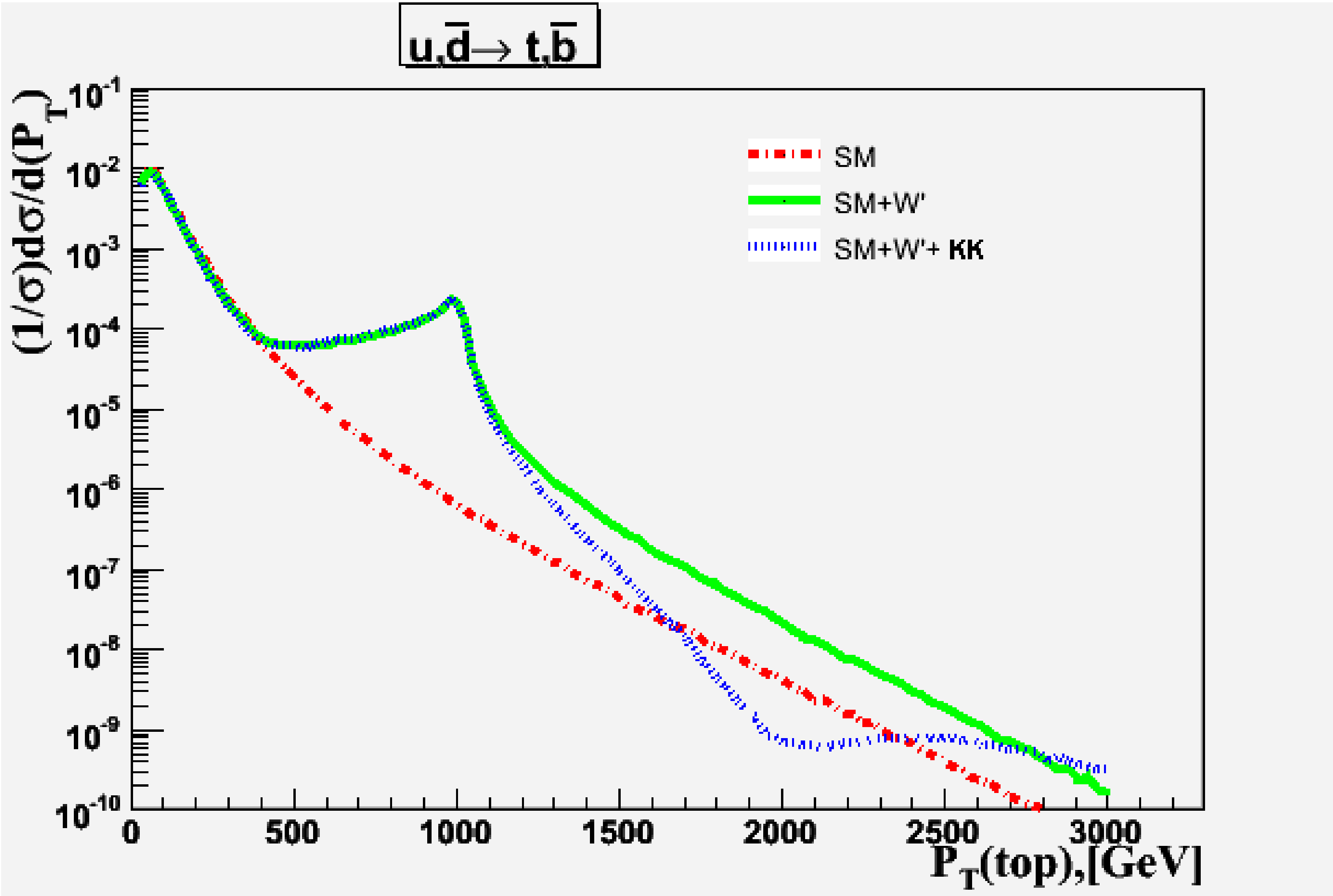
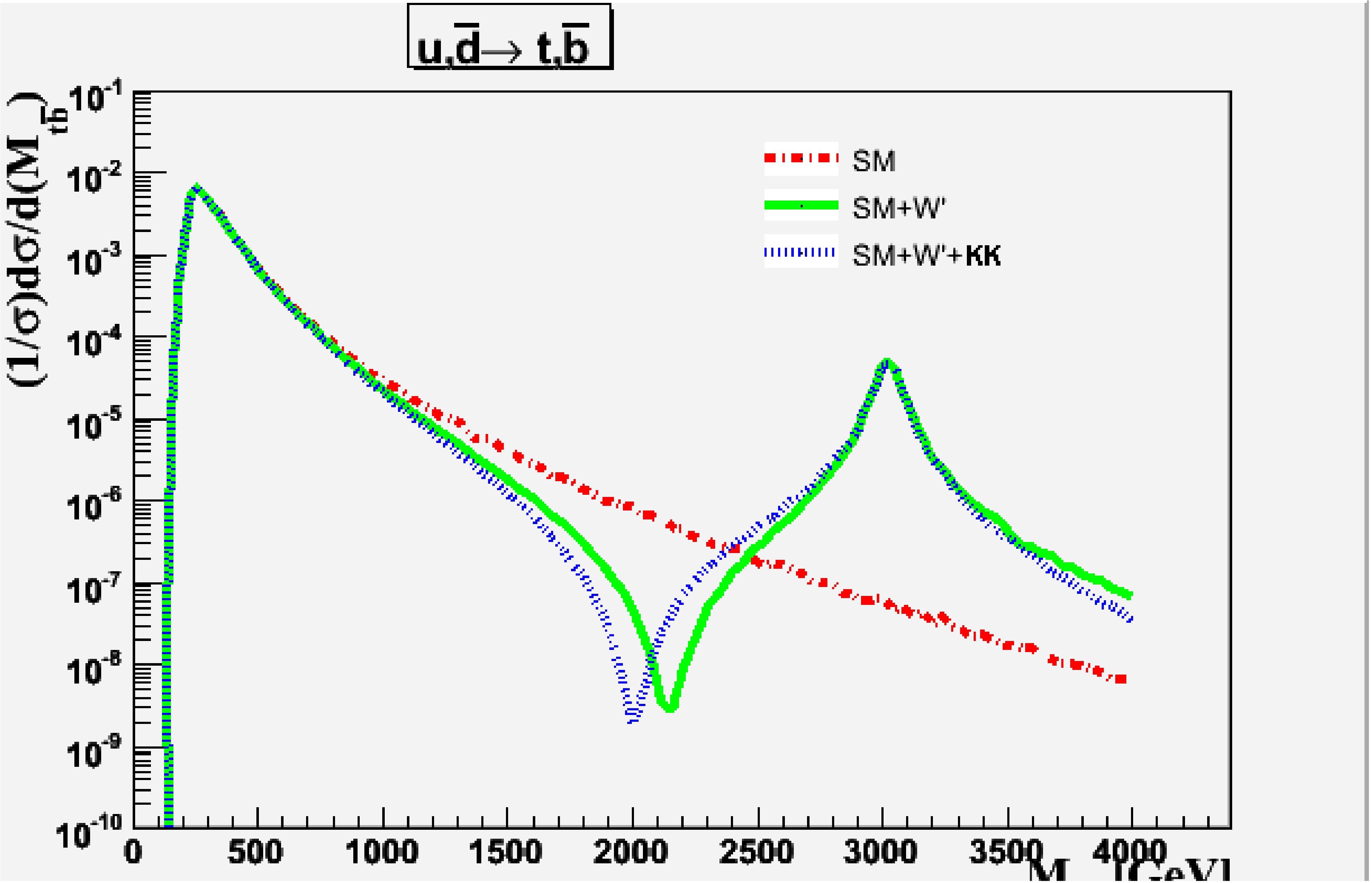


$$M_{W'} = 3 \text{ TeV}, \qquad M_{W'\_sum} = 4.2 \text{ TeV}, \qquad \Gamma_{W'} = 98.7 \text{ GeV}$$



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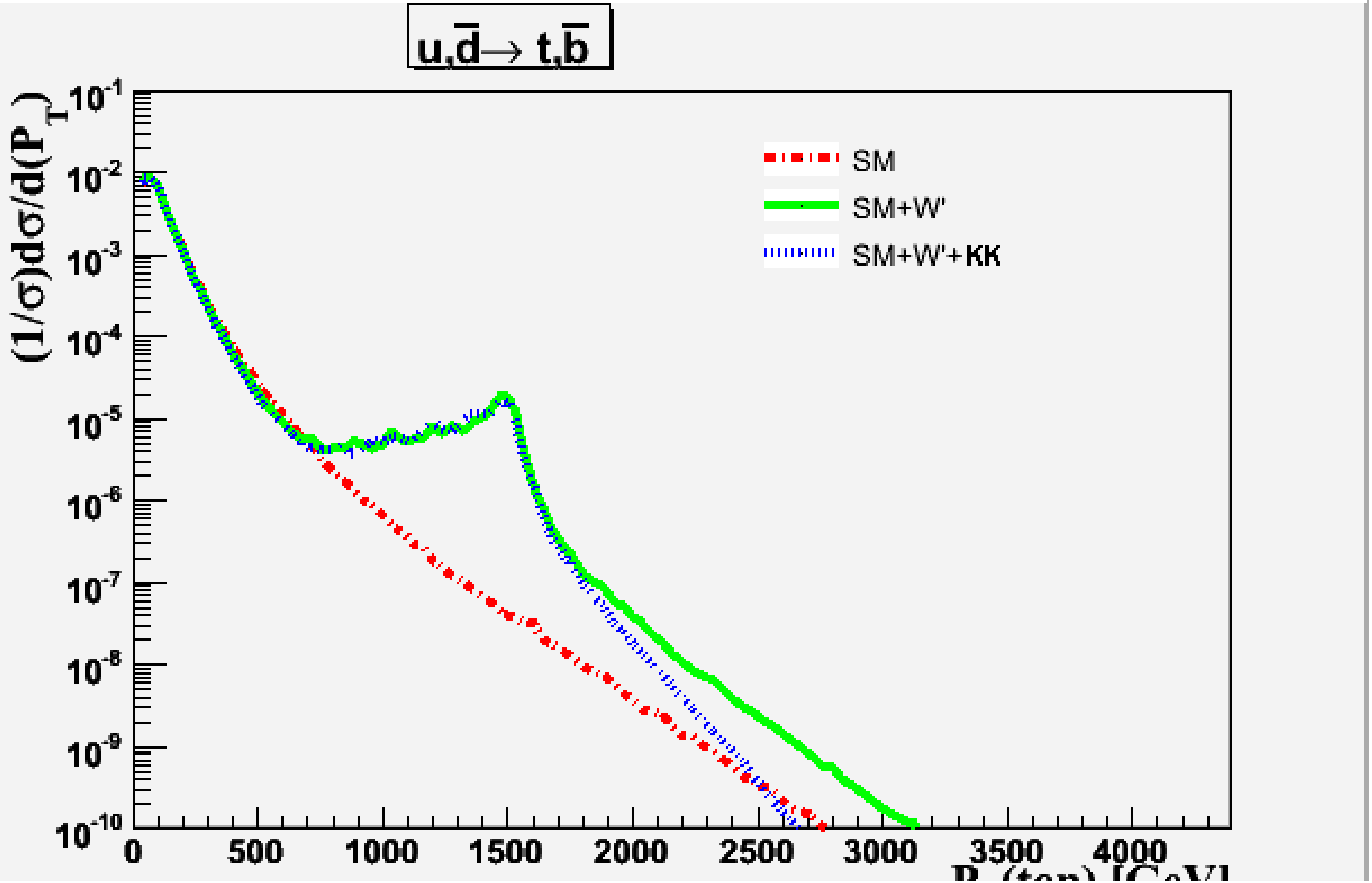
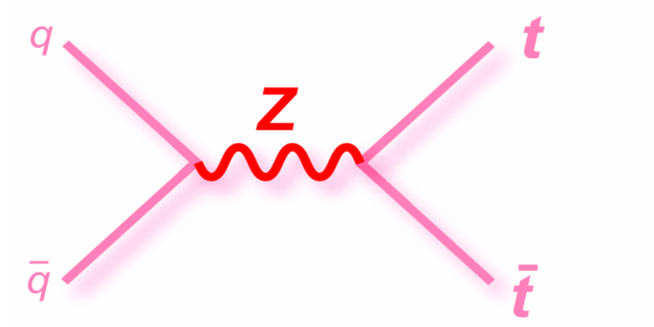
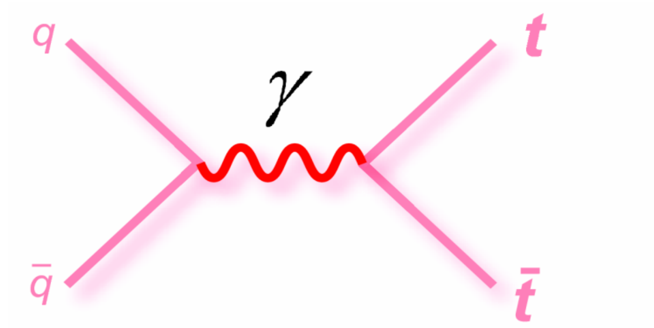
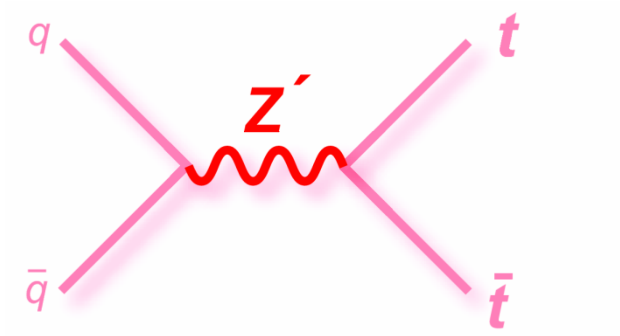
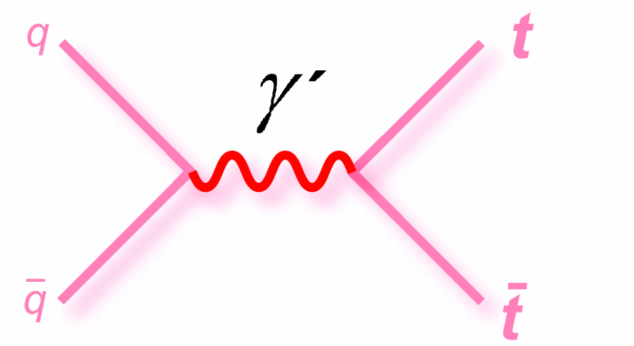


$t\bar{t}$ production

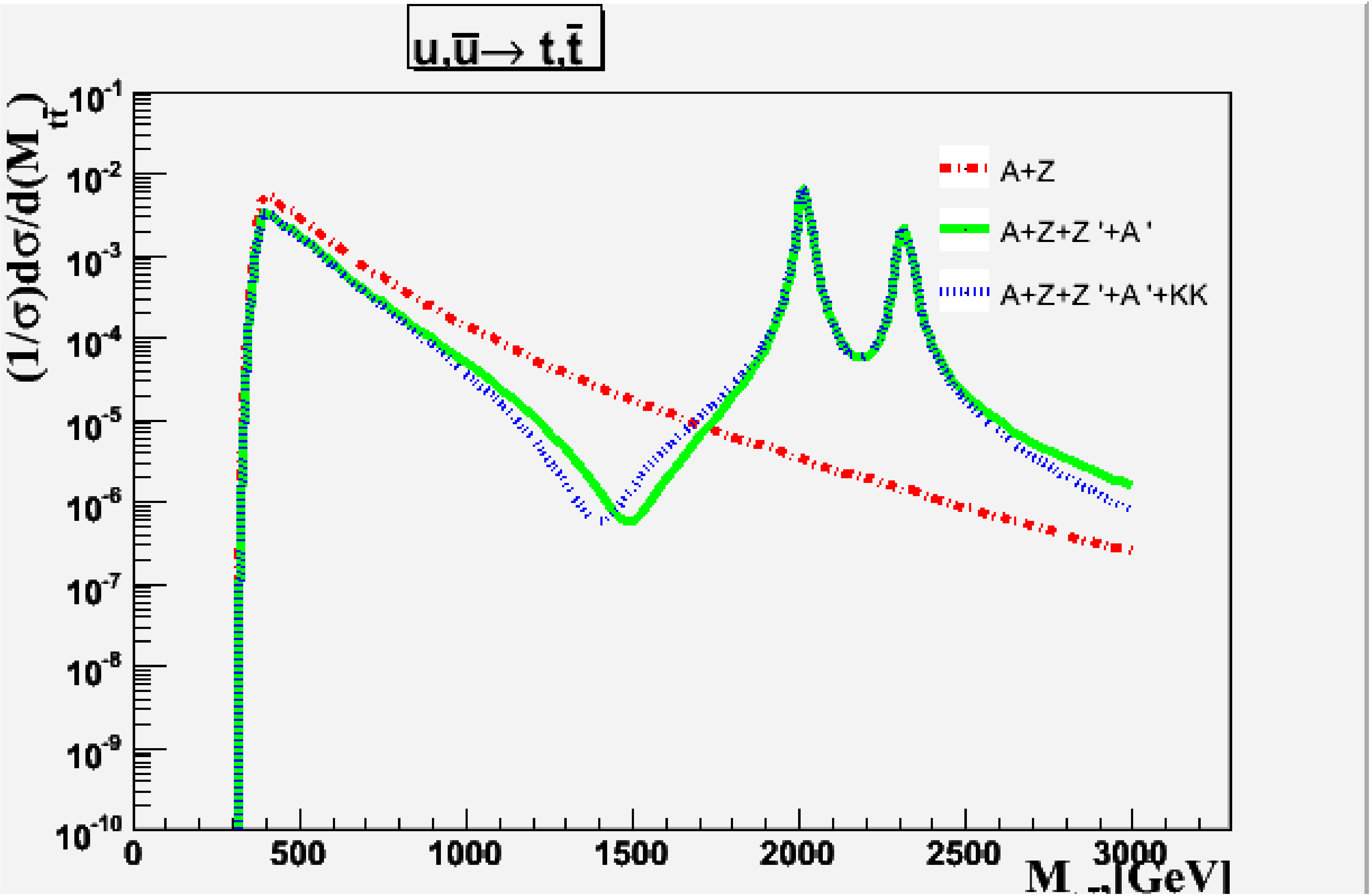








$M_{Z'} = 2.3,\ \ M_{Z'\_sum} = 3.2,\ \Gamma_{Z'} = 0.026,\ M_{\gamma'} = 2,\ M_{\gamma'\_sum} = 2.8,\ \Gamma_{\gamma'} = 0.021$ (TeV)



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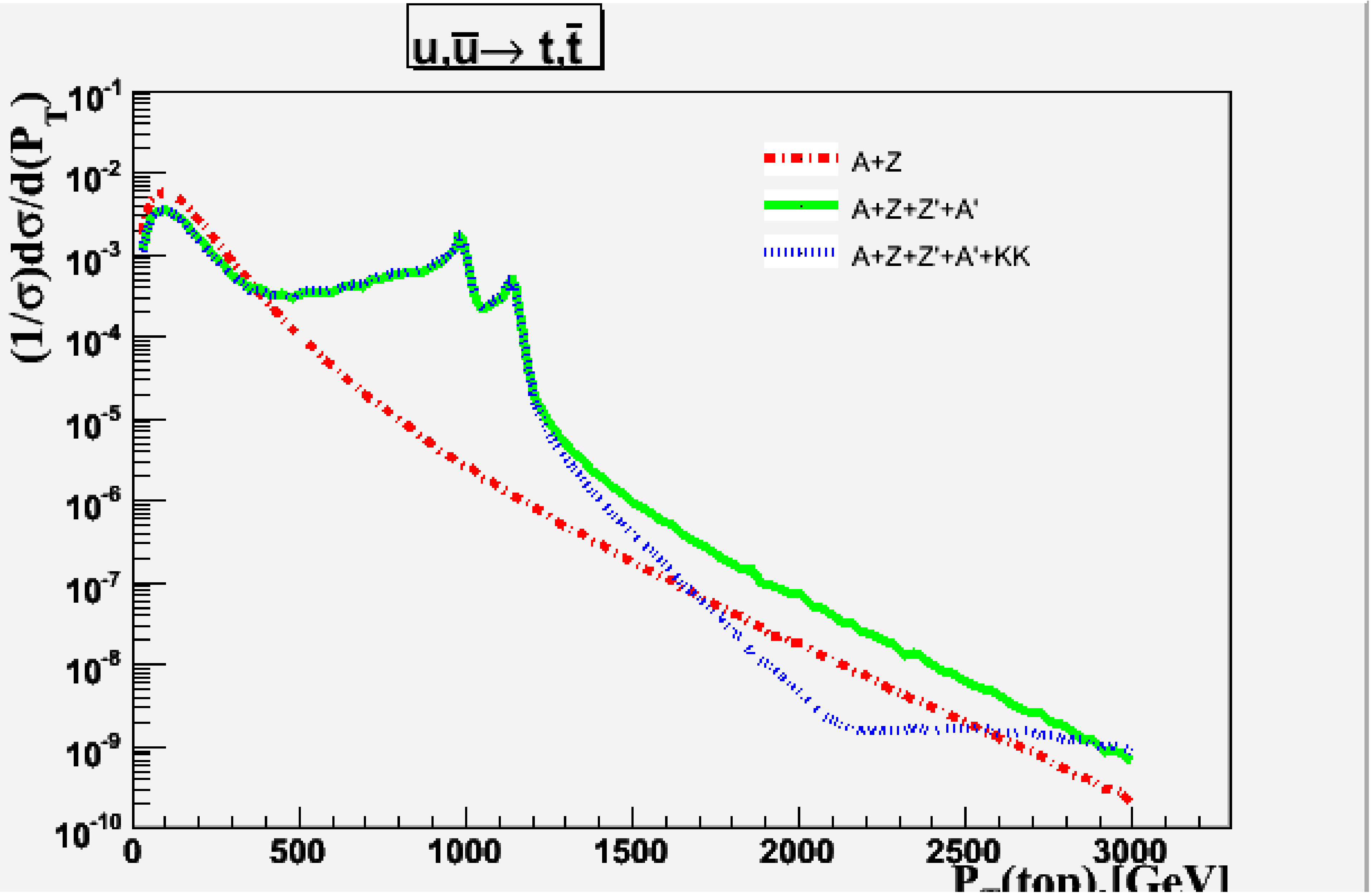
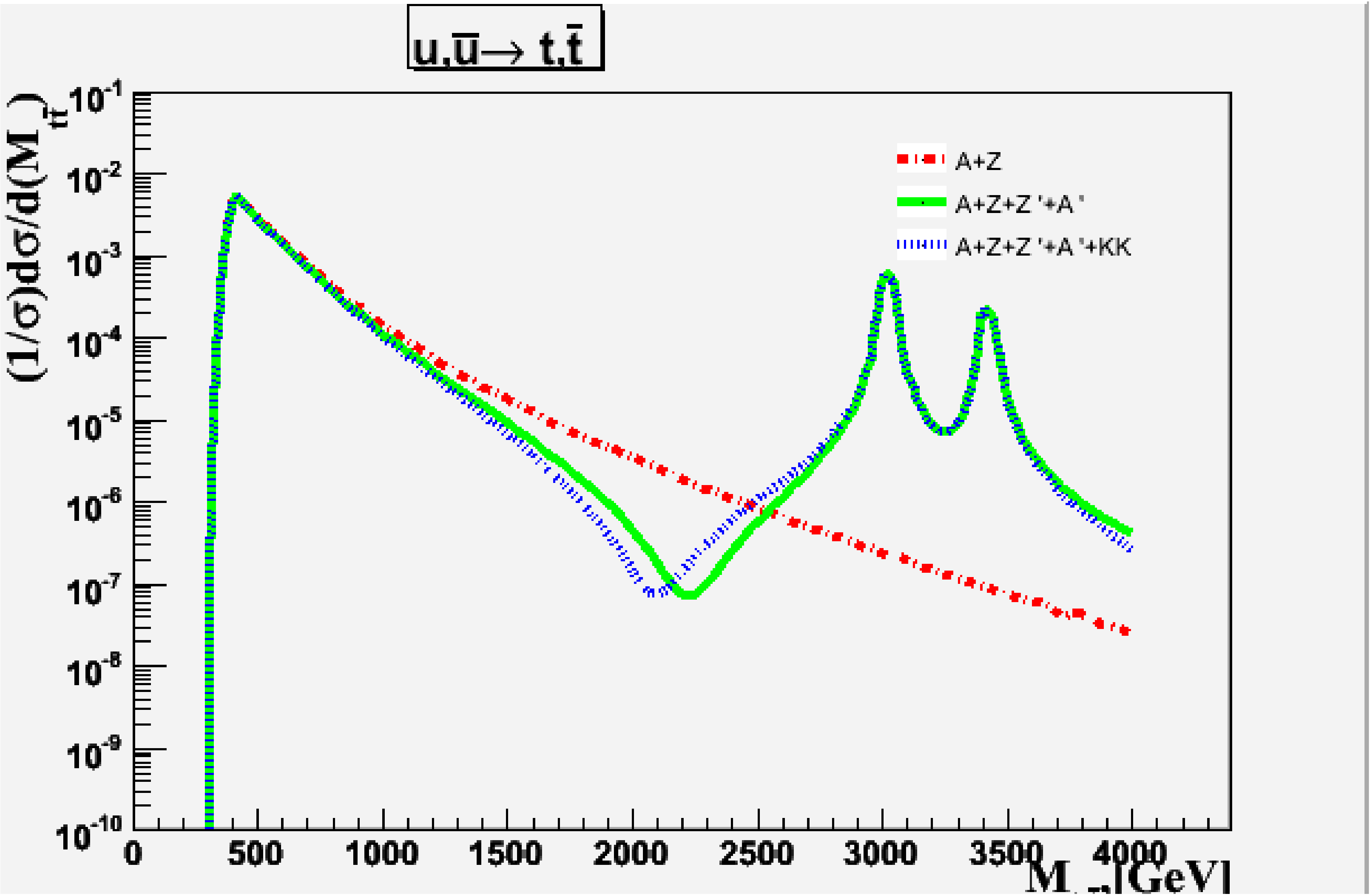


$M_{Z'} = 3.4,\ M_{Z'\_sum} = 4.8,\ \Gamma_{Z'} = 0.034,\ M_{\gamma'} = 3,\ M_{\gamma'\_sum} = 4.2,\ \Gamma_{\gamma'} = 0.031$ (TeV)



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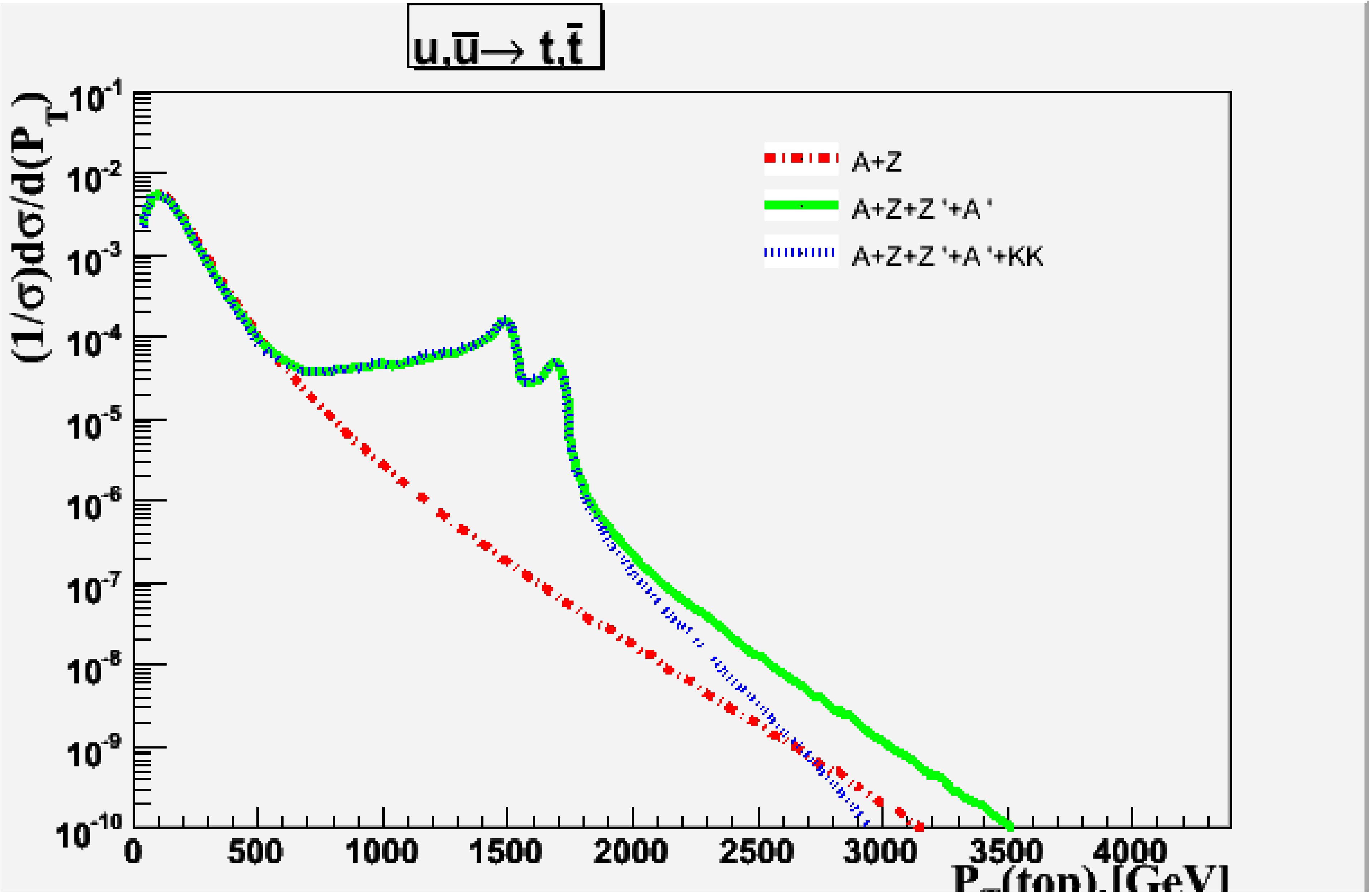
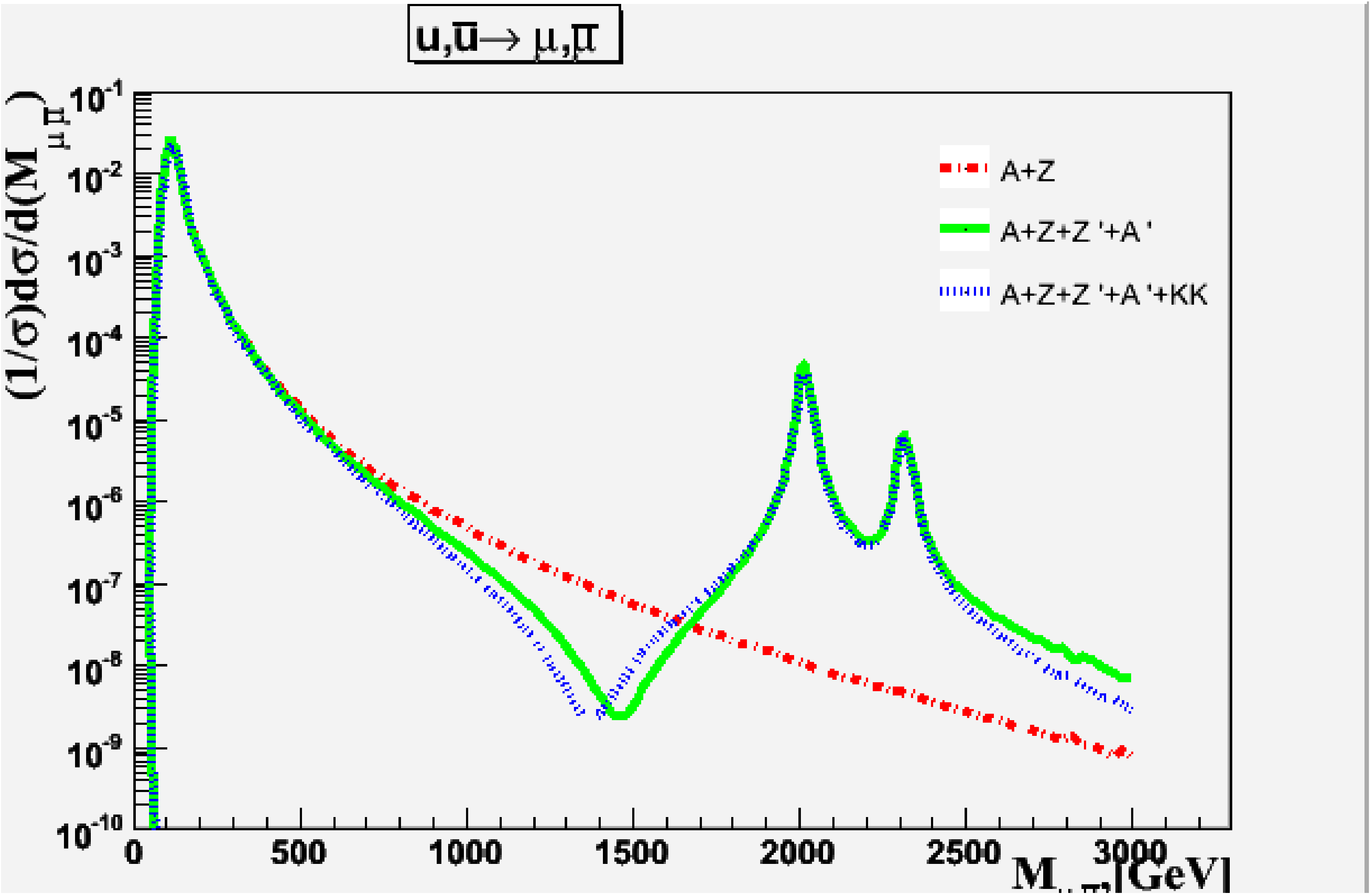


- Drell-Yan processes

$$u\bar{u} \to Z, \gamma \to \mu^+ \mu^-$$

$$u\bar{u} \to Z', \gamma' \to \mu^+ \mu^-$$

$M_{Z'} = 2.3,\ \ M_{Z'\_sum} = 3.2,\ \Gamma_{Z'} = 0.026,\ M_{\gamma'} = 2,\ M_{\gamma'\_sum} = 2.8,\ \Gamma_{\gamma'} = 0.021$ (TeV)



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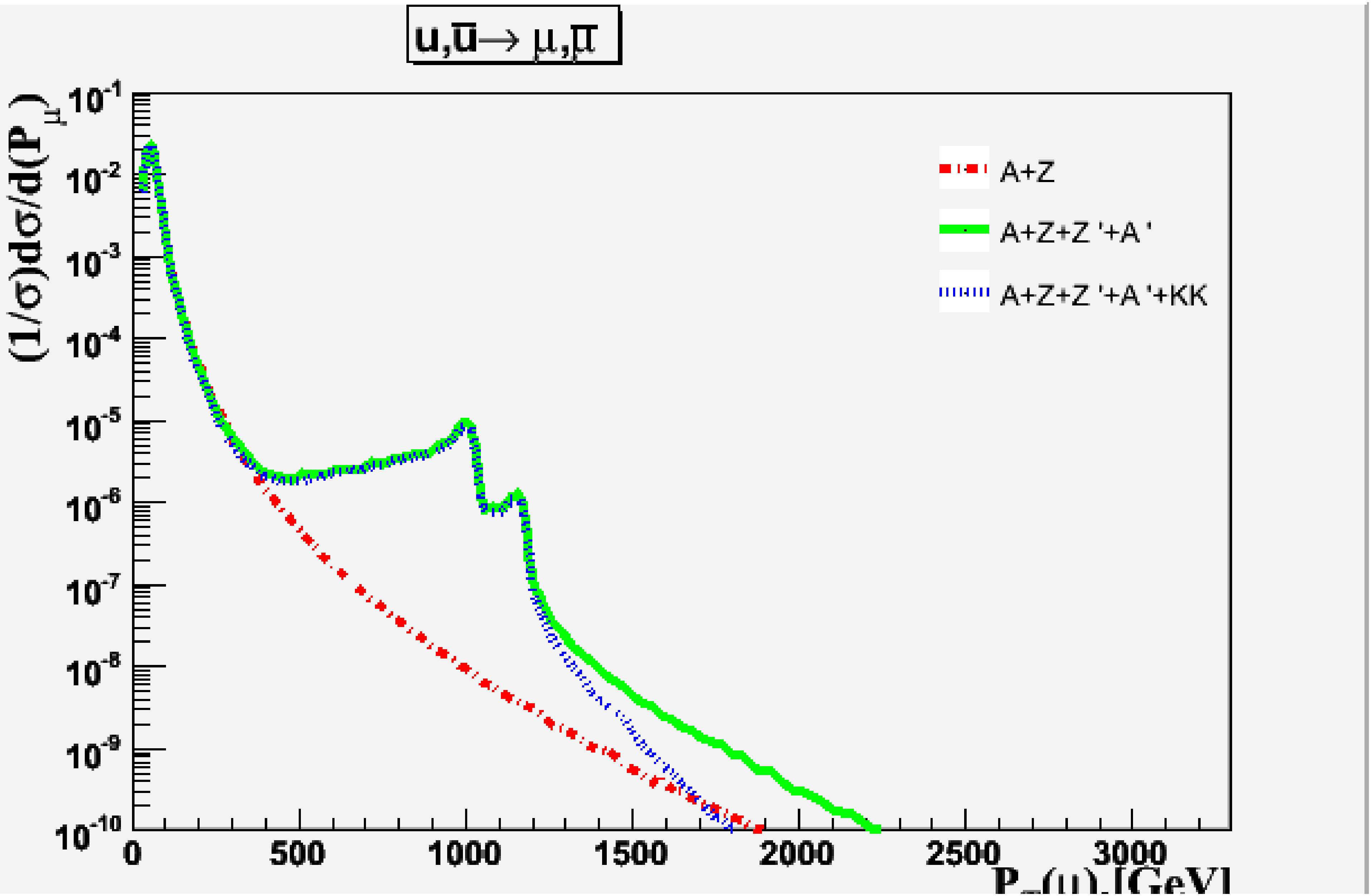

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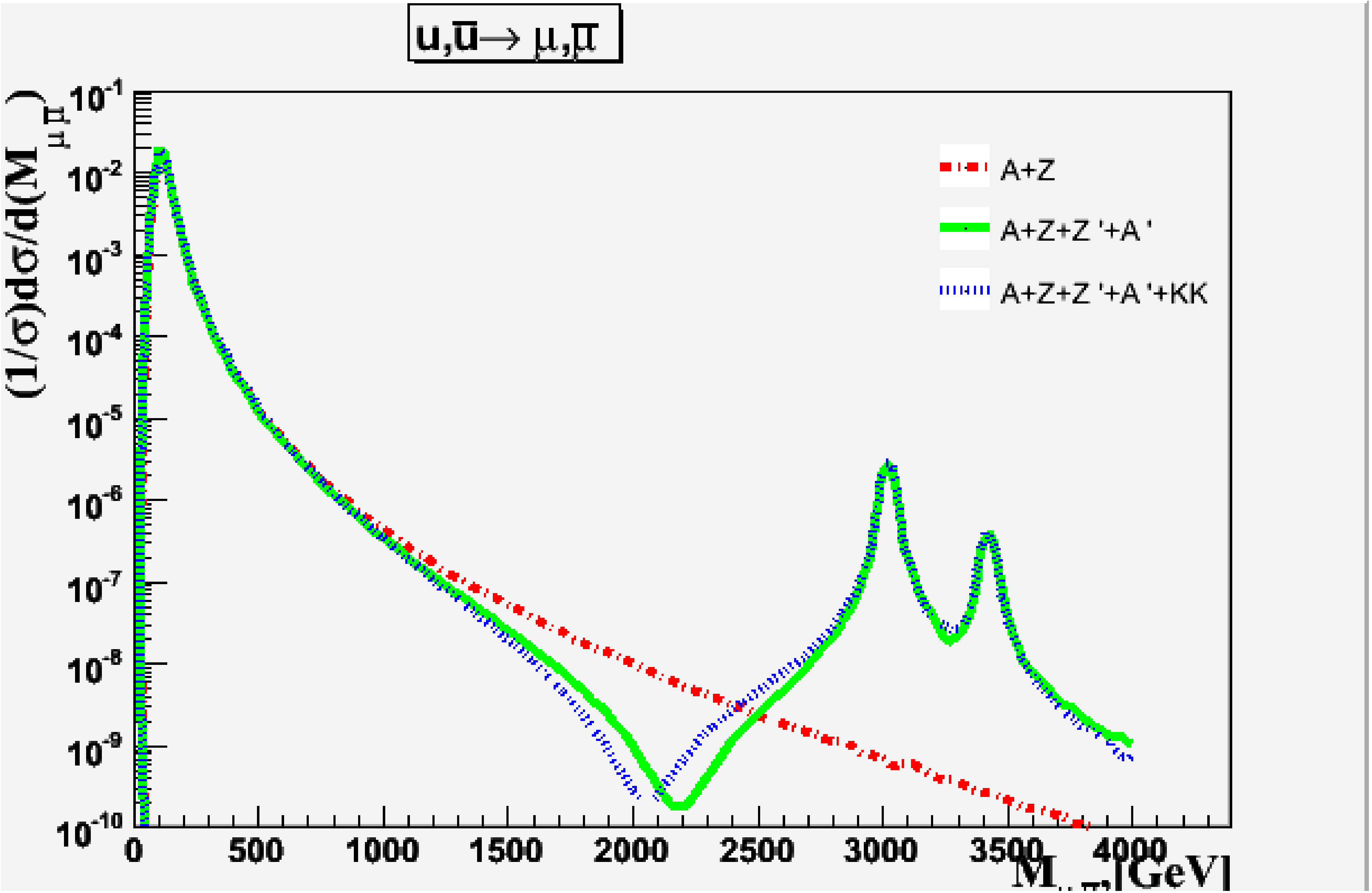
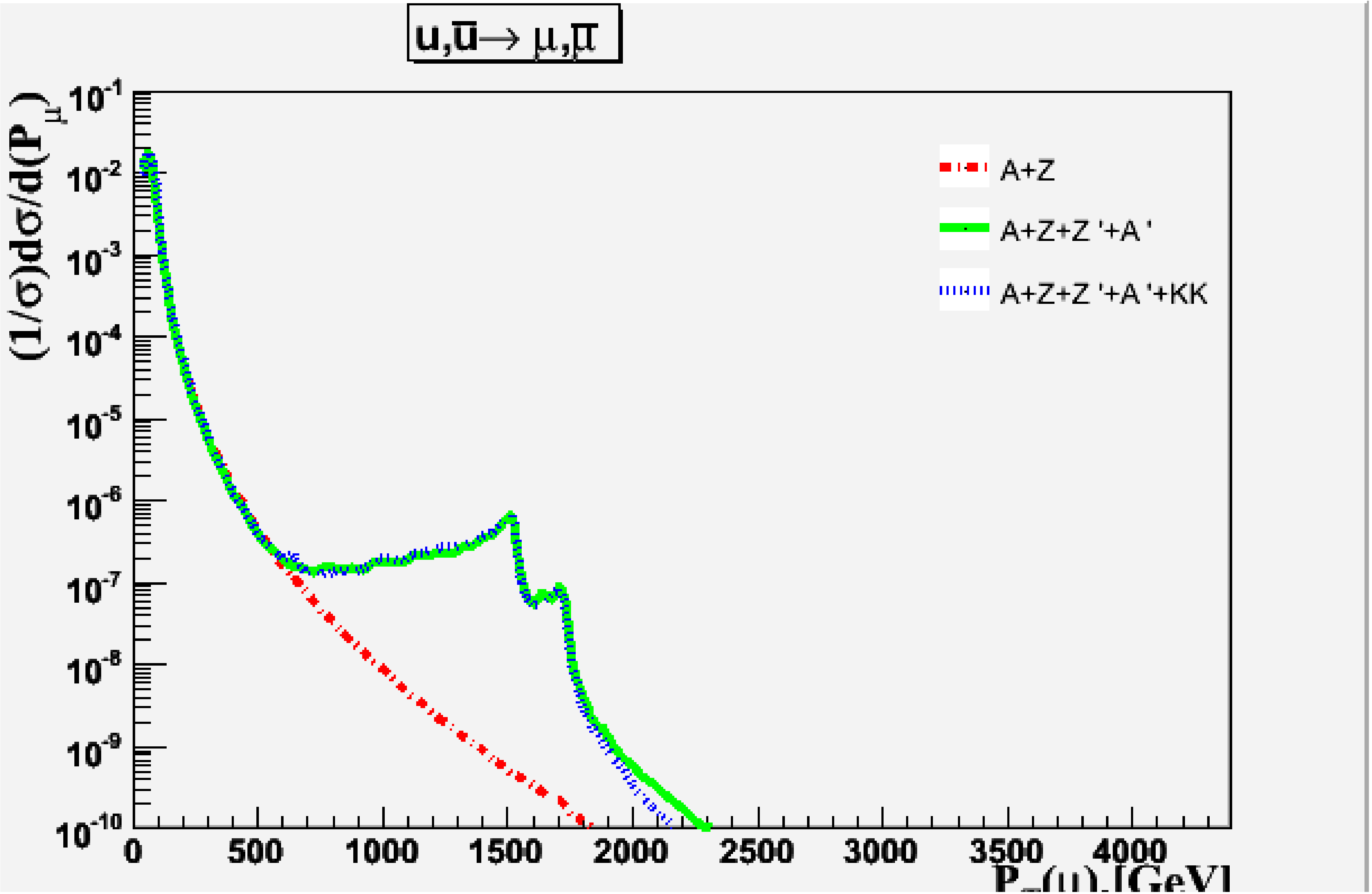
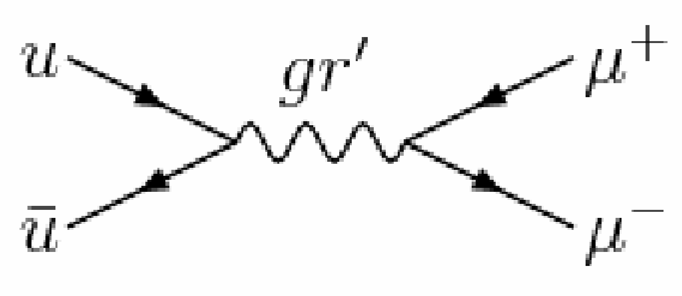
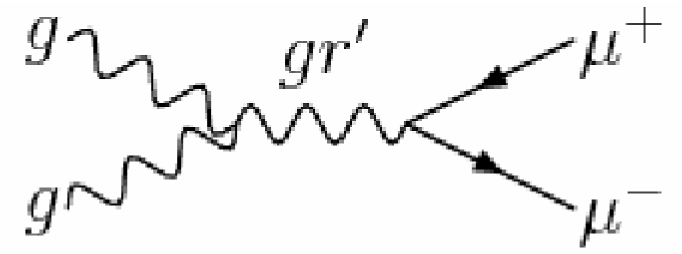


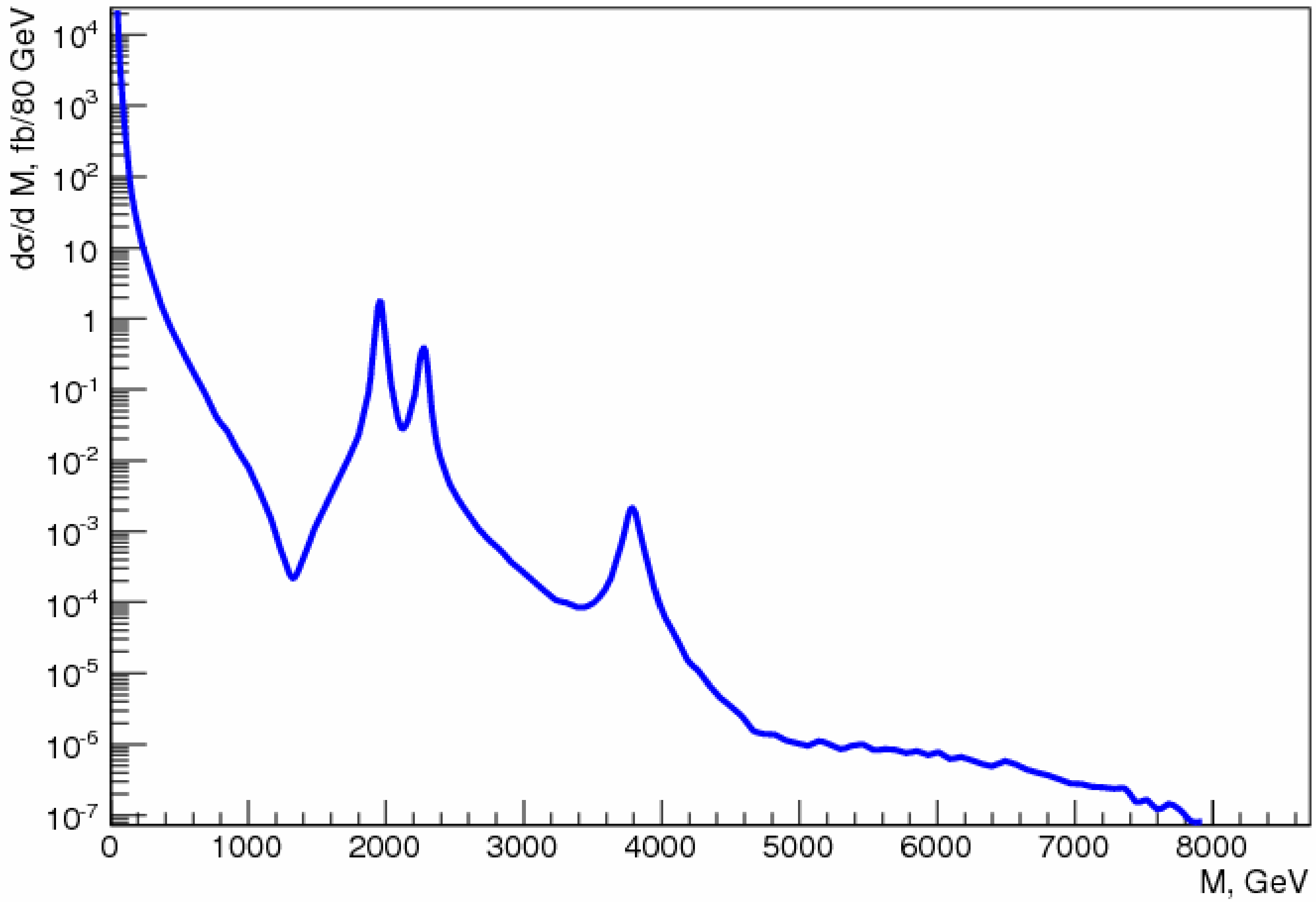
$M_{Z'} = 3.4,\ M_{Z'\_sum} = 4.8,\ \Gamma_{Z'} = 0.034,\ M_{\gamma'} = 3,\ M_{\gamma'\_sum} = 4.2,\ \Gamma_{\gamma'} = 0.031$ (TeV)



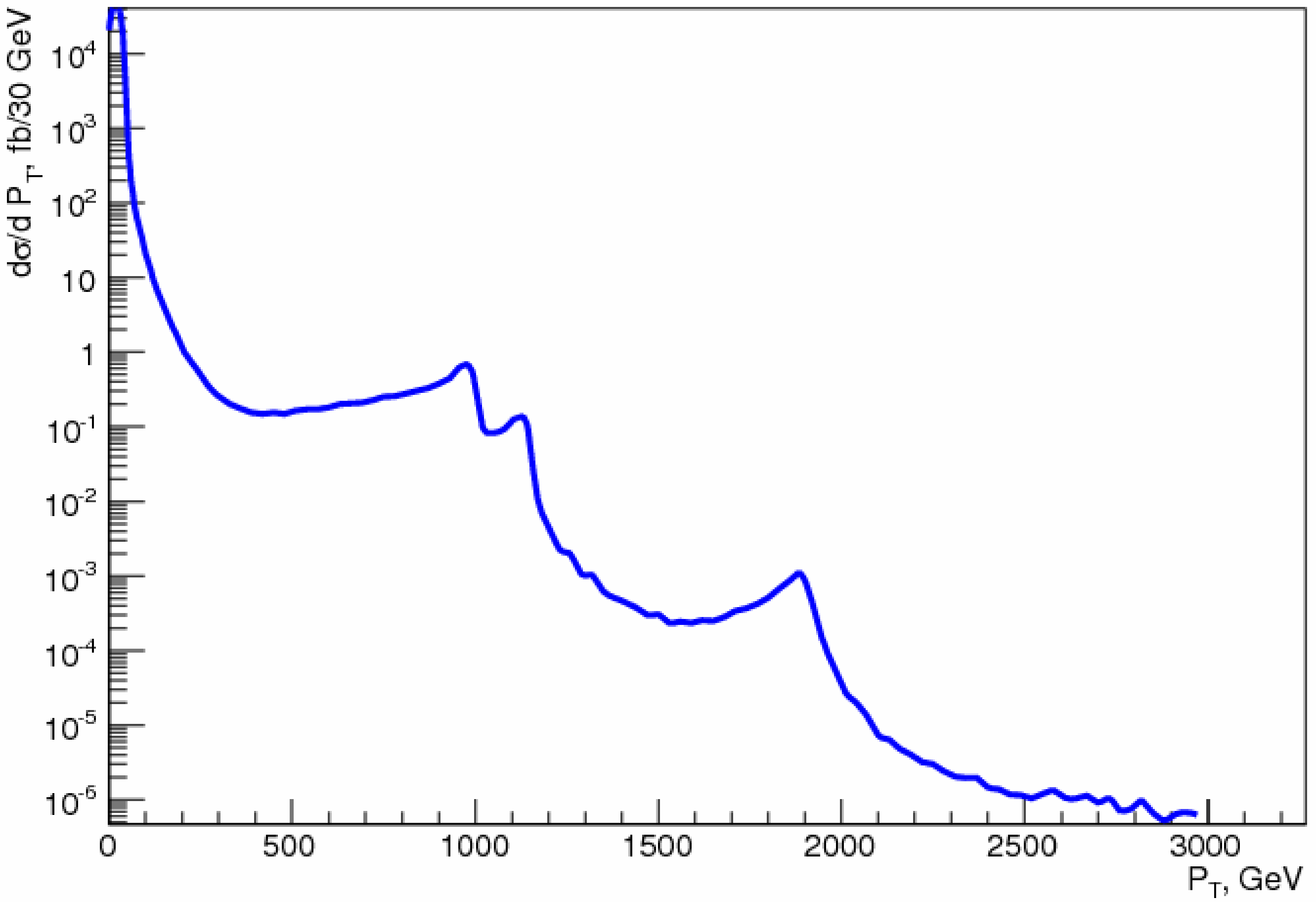

u
$\bar{u}$
gr'
μ^+
μ^-


g
g
gr'
μ^+
μ^-

$$pp \to \mu^+\mu^-, \quad M_{\gamma'} = 2\,TeV, \quad M_{Z'} = 2,3\,TeV, \quad M_{gr'} = 3.83TeV$$



$$pp \to \mu^+\mu^-, \quad M_{\gamma'} = 2\,TeV, \quad M_{Z'} = 2,3\,TeV, \quad M_{gr'} = 3.83\,TeV$$



6 Conclusion

- In higher-dimensional Einstein gravity it is impossible to find braneworld solutions with more that two extra dimensions.

- The stabilized Randall-Sundrum model is phenomeno-logically acceptable. If the values of the fundamental parameters lie in the TeV energy range, then the effects due to the massive modes can be observed in collider experiments.

- The Tevatron data give the estimate

$$\Lambda_\pi > 0.61 \cdot \xi^{1/4}\, TeV.$$

- If the SM gauge bosons can propagate in the bulk, for the fermions of the SM there arise low energy four-fermion interactions of the Fermi type.

- The effective contact interaction induced by the infinite towers of gravitons or KK excitations of the SM particles should be taken into account also in the case, where the centre of mass energy is above the production threshold of the first mode.

- An observation of the interference between the first KK graviton or SM particle resonance and the rest of its KK tower should be considered as a strong argument in favour of the extra dimensions hypothesis.

Thank you!