

Recent progress of automatic computation in HEP

Kiyoshi KATO (Kogakuin Univ.) and GRACE collaboration
June 28, 2013

The XXI International Workshop
High Energy Physics and Quantum Field Theory
Saint Petersburg Area, Russia

Automatic computation

- A software system to perform perturbative computation in **Quantum Field Theory** which is an important tool for **High Energy Physics** (→ QFTHEP)
- A solution to large scale computation which is beyond man-power calculation
 - multi-body final states
 - higher-order radiative corrections

systems

- Full systems or tools(e.g., LanHEP: from Lagrangian to Feynman rules, QGRAF:diagram generation)
- Tree level or loop level(radiative correction)
- Diagrammers vs. Unitarian
Use/Not use Feynman diagrams

Automatic computations and event generators, tree level

From Lagrangian to Feynman rules

- [LanHEP](#) (native symbolic language)
- [FeynRules](#) (Mathematica)

Tree level hard process integrand generator

- [CompHEP](#), (Squared amplitude, many models including MSSM), [CalcHEP](#) (SM, MSSM, + many)
- [Grace](#) (Helicity amplitudes, SM),
- [FDC](#) (Wang)
- [COMIX](#) (multi-leg tree level event generator, color recursive relation, many legs)
- [AMEGIC++](#) (parton level generator)
- [HELAC/Phegas](#) (dyson-schwinger)
- [O'Mega](#) (alpha algorithm)

General purpose mainly tree level (all decay, 2→2, 2→3, some 2→4, ISR, PS and Hadronisation)

- [Pythia++](#), [Ariadne](#), [MC++](#)
- [Herwig++](#)
- [Isajet](#)

General purpose event generators based on automatic integrand calculations

- [Whizard](#) (O'Mega)
- [Sherpa](#) (using Comix and AMEGIC++, CKKW, Pythia (or Pythia like))
- [Madgraph](#), [MadEvent](#)
- [Graceful](#) (based on Grace, interfaced to Pythia) in progress ...

Specific set of event generators

- [Gr@ppa](#) (Grace for matrix elements and Pythia/Herwig for parton shower/Hadronization)
- [AlpGen](#) (multi parton generation)

Parton shower matching

- [CKKW](#), [POWHEG](#), [MLM](#), Subtraction method (matrix element matching with shower MC)

Event database:

- MC database (CompHEP)
- [JETWEB](#)

Automatic computations and event generators, NLO,...

- [RESBOS](#) (Q_t resummation NLO/NNLO, no parton shower necessary) lepton distributions in Drell-Yan-like processes); inclusive with respect to hadronic radiation

NLO hard process

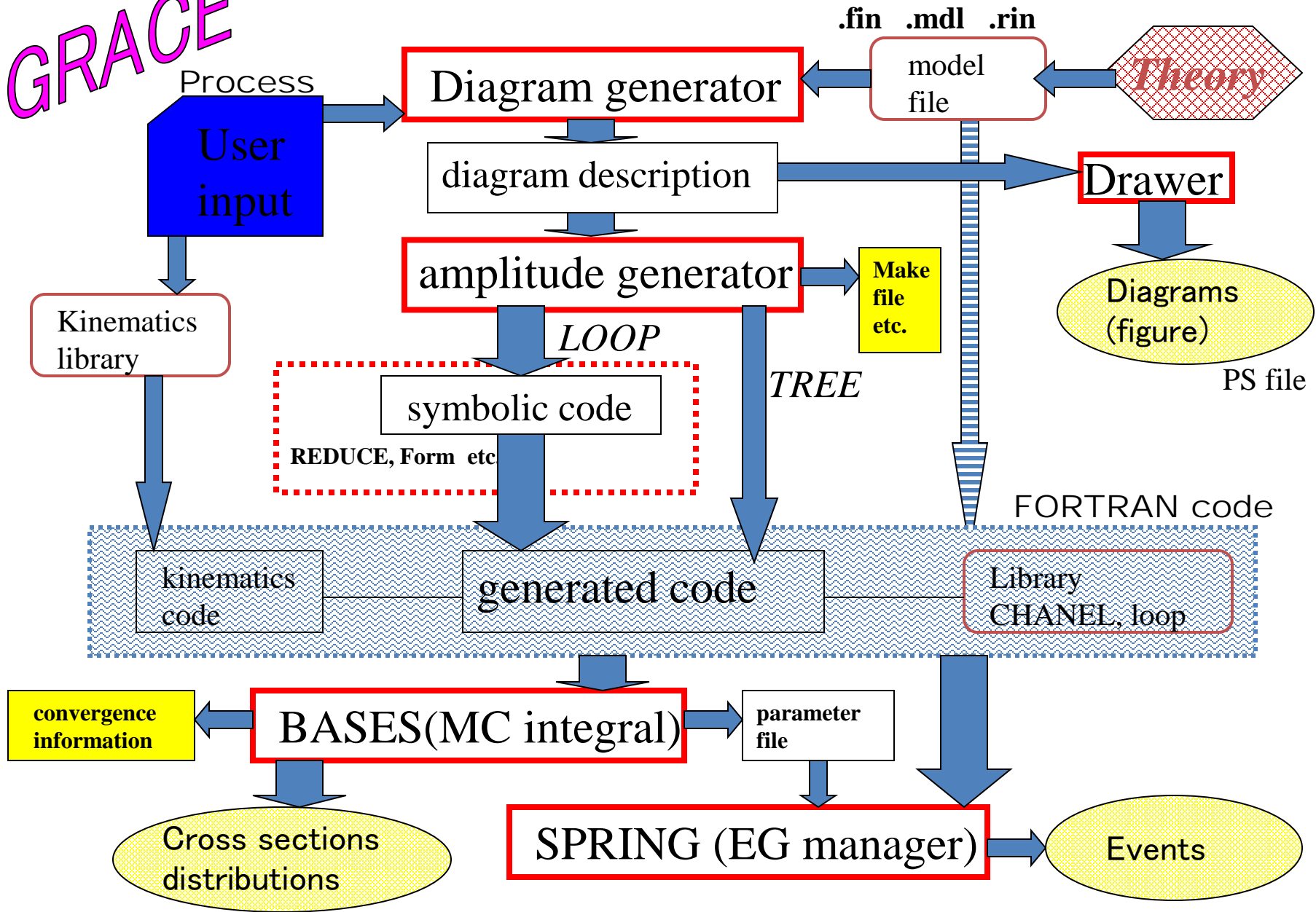
NLO event generators (specific processes)

- [MC@NLO](#) (Herwig + Subtraction methods: H, single Boson, single top, lepton pairs, H +W/Z, ...)
- [MCFM](#) (full spin correlation, Wbb, ...)
- [BlackHat](#) (one-loop, W/Z+3,4 jets)
- [NLOjet++](#) ([Catani-Seymour dipole subtraction method](#))
- [Rocket](#) (N-gluon amplitude, 2-q +N-g, W +2-q + N-g, W +4 q +Ng... N> 20)
- [CutTools](#) (one-loop amplitudes at integrand level)
- [JETRAD](#) (inclusive 1 and 2 jet production)
- JETI
- [FeynCalc](#) (Mathematica/Form Passarino-Veltman reduction of one-loop amplitudes to standard scalar integrals) FeynArt, FeynRules, ...
- [Grace one-loop](#)
- [FDG \(loop\)](#)
- [Golem](#) (one-loop amplitude up-to six legs)
- [Phox family JetPhox](#) (inclusive γ /hadron+jet, [Diphox](#) (hadron production of 2γ , hadron+ γ , hadron+hadron)

GRACE system

- Tree and 1-loop, SM and MSSM (and some extension)
Public version(in Web page) : tree-only
- Components:
 - diagram generator
 - amplitude/matrix element generator
 - libraries for kinematics, color, loop-integrals
 - Monte-Carlo integration in phase space
 - efficient event generation
- **This talk** : Recent results by GRACE group

GRACE



How do you believe the results?

System diagnosis

- Cancellation on UV divergence ($\frac{1}{\epsilon}$ is kept as a variable in the code)
- Cancellation of IR divergence (λ regularization)
- Cancellation of soft-photon cut k_C
- Non-linear gauge invariance

Change numerical precision

Analytical calculation(e.g., QED part)

Comparison with other study(*if any*)

non-Linear gauge

Gauge invariance check --- Powerful diagnosis

Loop integral --- numerator structure --- $\xi = 1$ (Feynman gauge) favorable

$$g^{\mu\nu} - (1 - \xi) \frac{p^\mu p^\nu}{p^2 - \xi M^2 + i\varepsilon}$$

Non-linear gauge fixing ---- $\xi = 1$ and free gauge parameters

$$L_{GF} = -\frac{1}{\xi_W} F^+ F^- - \frac{1}{2\xi_Z} (F^Z)^2 - \frac{1}{2\xi} (F^A)^2$$

$$F^\pm = \left(\partial^\mu \mp i e \tilde{\alpha} A^\mu \mp i \frac{e c_W}{s_W} \tilde{\beta} Z^\mu \right) W_\mu^\pm$$

$$+ \xi_W \left(M_W \chi^\pm + \frac{e}{2s_W} \tilde{\delta} H \chi^\pm \pm i \frac{e}{2s_W} \tilde{\kappa} \chi_3 \chi^\pm \right)$$

$$F^Z = \partial^\mu Z_\mu + \xi_Z \left(M_W \chi_3 + \frac{e}{2s_W c_W} \tilde{\varepsilon} H \chi_3 \right)$$

$$F^A = \partial^\mu A_\mu$$

$$\tilde{\alpha}, \tilde{\beta}, \tilde{\delta}, \tilde{\varepsilon}, \tilde{\kappa}$$

5 gauge parameters appear in vertices

$$e^+e^- \rightarrow t\bar{t}\gamma \quad e^+e^- \rightarrow e^+e^-\gamma$$

P.H. Khiem , J. Fujimoto, T. Ishikawa, T. Kaneko, K. Kato,

Y. Kurihara, Y. Shimizu, T. Ueda, J.A.M. Vermaseren, Y. Yasui
K.Tobimatsu, M.Igarashi, N.Nakazawa

Study for ILC ; top physics, background for new physics, Luminosity study, ...

Full electroweak 1-loop radiative correction

process $e^+e^- \rightarrow t\bar{t}\gamma$

$$e^+e^- \rightarrow t\bar{t}$$

J. Fujimoto and Y. Shimizu et al.,
 Mod. Phys. Lett. 3A, 581 (1988).
 J. Fleischer, A. Leike, T. Riemann et al.,
 Eur. Phys. J. C 31, 37 (2003).



P. H. Kiem et al.,
 Eur. Phys. J. C 73, 2400(2013)

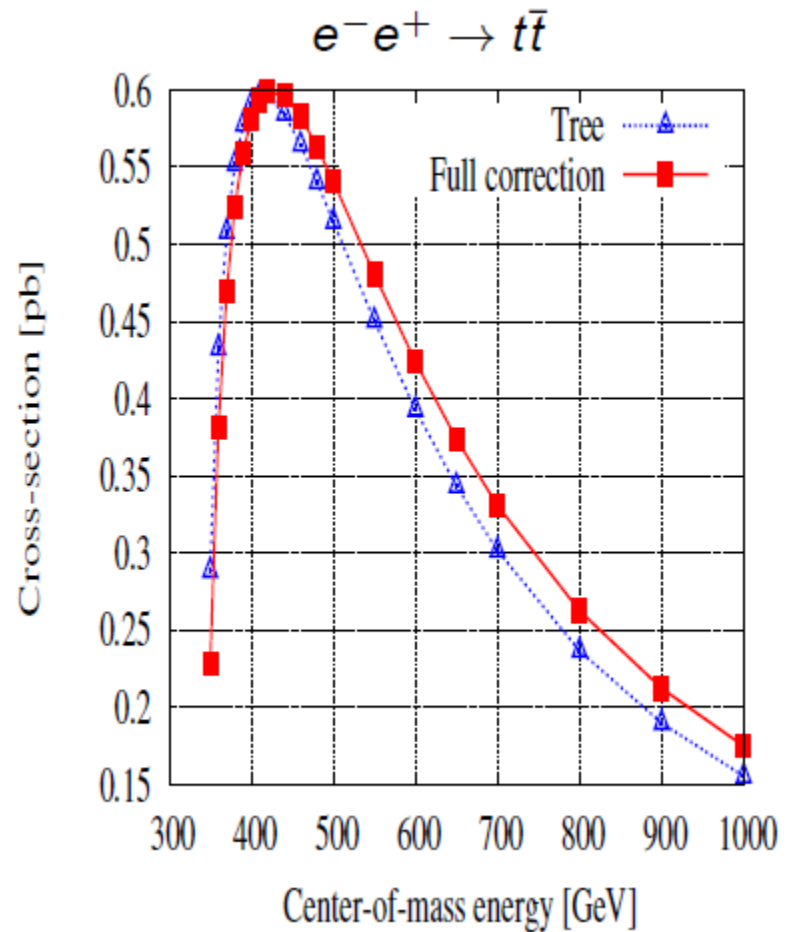
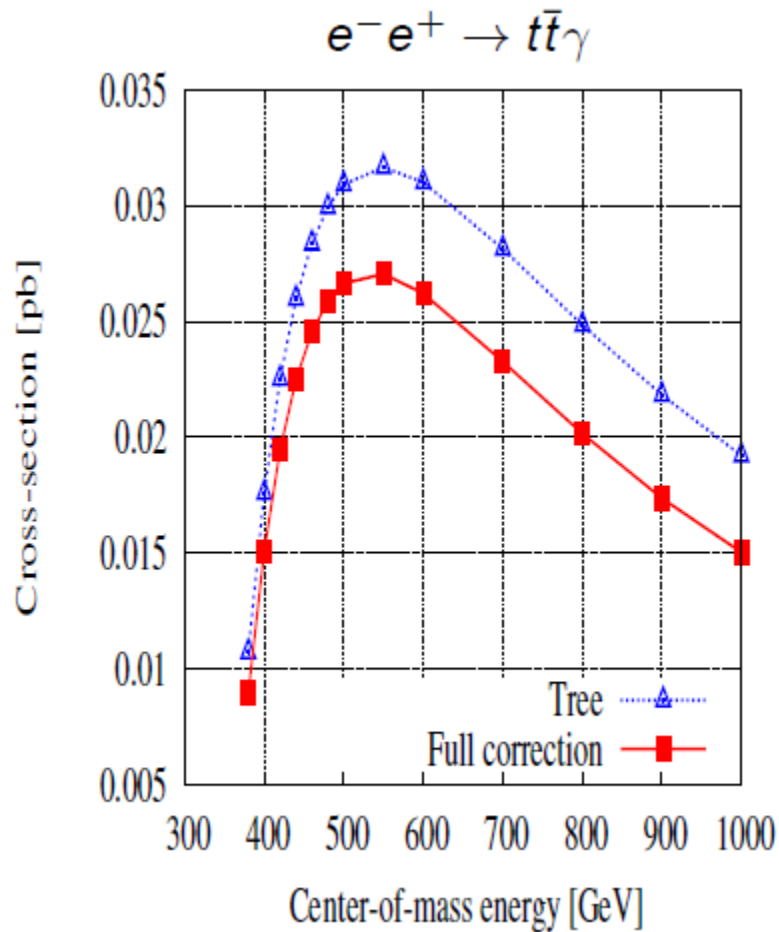
Confirmed the results
 for $e^+e^- \rightarrow t\bar{t}$ also

$$E_\gamma > 10\text{GeV}, \quad 10^\circ < \theta_\gamma < 170^\circ$$

No. of graphs	tree	1-loop
$e^+e^- \rightarrow t\bar{t}$	4	150
$e^+e^- \rightarrow t\bar{t}\gamma$	16	1814
$e^+e^- \rightarrow t\bar{t}\gamma\gamma$	80	

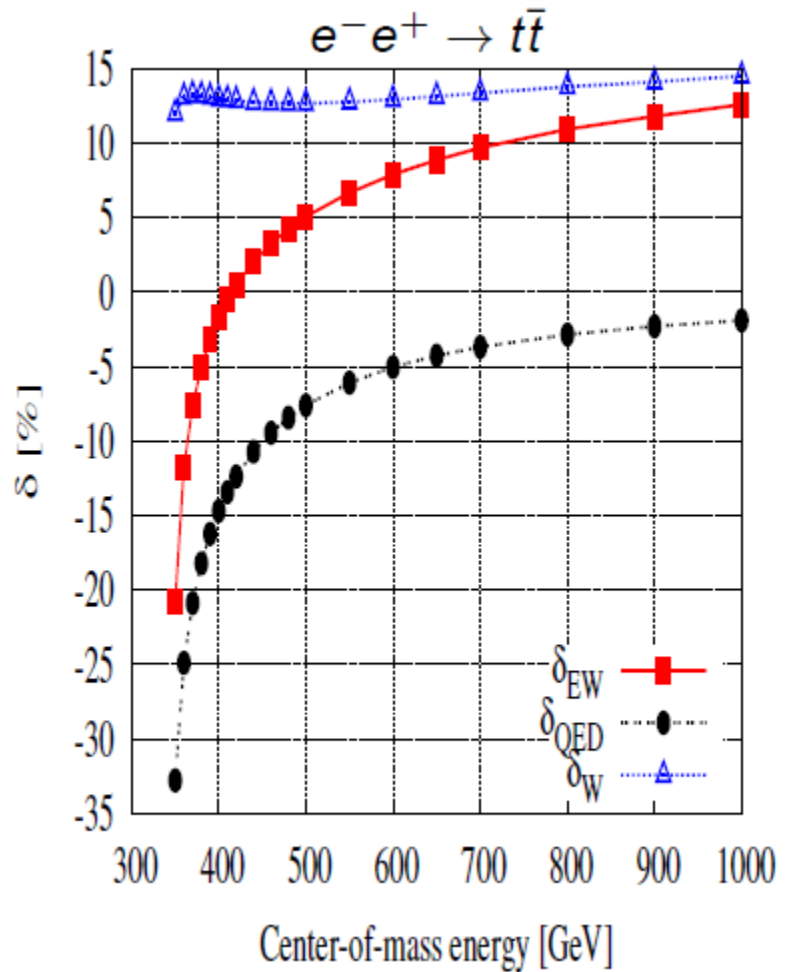
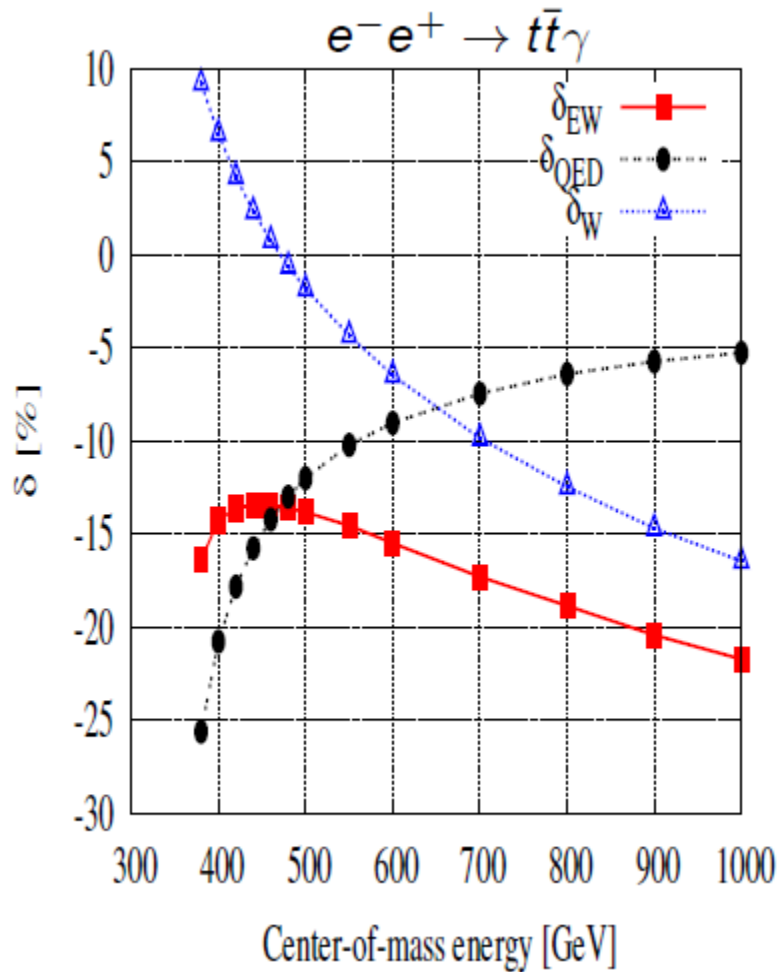
Numbers are in non-linear
 gauge fixing model.

Total cross section



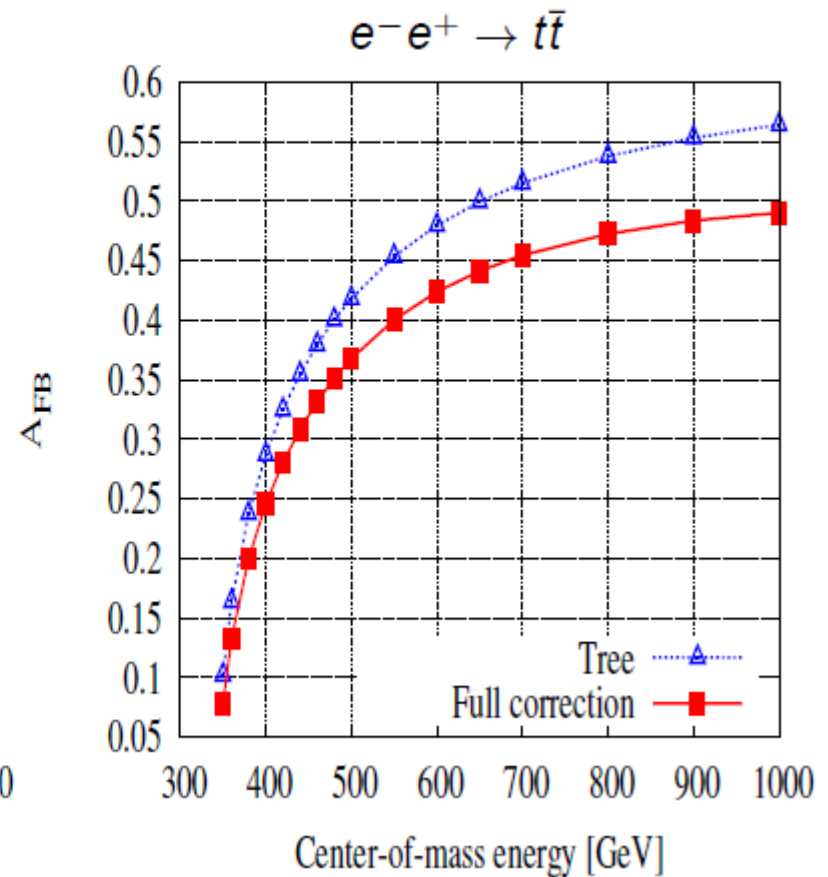
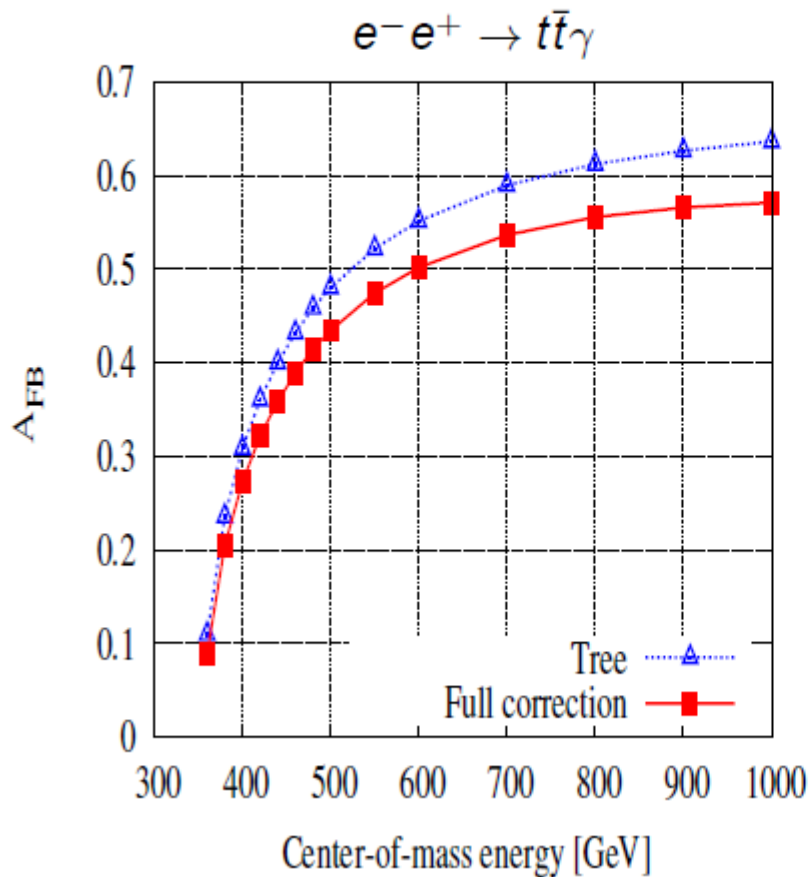
$$E_\gamma > 10\text{GeV}, \quad 10^\circ < \theta_\gamma < 170^\circ$$

Radiative corrections



Forward-backward asymmetry of t

$$A_{FB} = \frac{\sigma(0^\circ \leq \theta_t \leq 90^\circ) - \sigma(90^\circ \leq \theta_t \leq 180^\circ)}{\sigma(0^\circ \leq \theta_t \leq 90^\circ) + \sigma(90^\circ \leq \theta_t \leq 180^\circ)}$$



process $e^+ e^- \rightarrow e^+ e^- \gamma$

One-loop electroweak corrections to $e^+ e^- \rightarrow e^+ e^-$:

J. Fujimoto et al., *Prog. Theor. Phys. Supplement* (1990) 100.

M. Bohm et al., *Nuclear Physics B304* (1988) 687-711

Tree level calculation of $e^+ e^- \rightarrow e^+ e^- \gamma$:

Y. Shimizu et al., *Comp.Phys. Commun.* 55, 337-385(1989).

Two-loop QED correction to $e^+ e^- \rightarrow e^+ e^-$:

A.A. Penin: *Phys.Rev.* D72 (2005) 051301; *Phys. Rev.* D74 (2006) 019901.



One-loop electroweak correction to $e^+ e^- \rightarrow e^+ e^- \gamma$

P. H. Kiem et al.,
2013, JPS meeting (Hiroshima)

Tree 32
1-loop 3456

Calculation

Against numerical instability and large scale computation ...

Use LoopTools for evaluating scalar one-loop integrals.

Heavy computation... 5,6-point functions with reduction

Use MPI(parallel computing) in quadruple precision calculation.

Use dynamic link method with ifort.

Cuts

$$e^+, e^-, \gamma \quad E > 10\text{GeV} \quad 10^\circ < \theta_{\text{Beam}} < 170^\circ$$

$$\angle(e^+, e^-) > 10^\circ \quad \text{Can be relaxed}$$

$$\angle(\gamma, e^+), \angle(\gamma, e^-) > 10^\circ$$

Large cancelation

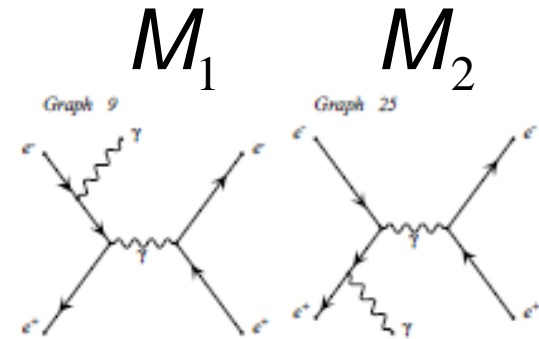
External photon
Polarization sum

$$(1) \quad \langle \varepsilon^\mu \varepsilon^\nu \rangle = -g^{\mu\nu}$$

$$(2) \quad \langle \varepsilon^\mu \varepsilon^\nu \rangle = -g^{\mu\nu} + \frac{q^\mu n^\nu + n^\mu q^\nu}{qn} - n^2 \frac{q^\mu q^\nu}{(qn)^2}$$

(1)

(2)



produced by GRACEFYG

Code : lengthy
Calculation: stable

$\mathcal{M}_1^2 + \mathcal{M}_2^2$	$0.1116212357E+13$	$0.3644158264E+02$
$2\mathcal{M}_1^* \mathcal{M}_2$	$-0.1116212356E+13$	$0.1546482734E+03$
$ \mathcal{M}_1 + \mathcal{M}_2 ^2$	$0.1910871582E+03$	$0.1910898560E+03$

Diagnosis(1) UV divergence

At a phase
space point

1. C_{UV} independence of the result.	
C_{UV}	$2\mathcal{R}(\mathcal{M}_{Tree}^+ \mathcal{M}_{Loop}) + \text{soft contribution.}$
0	-0.392635564863145913546569687849770
10^2	-0.392635564863145913546569687849752
10^4	-0.392635564863145913546569687849529

Diagnosis(2) IR divergence

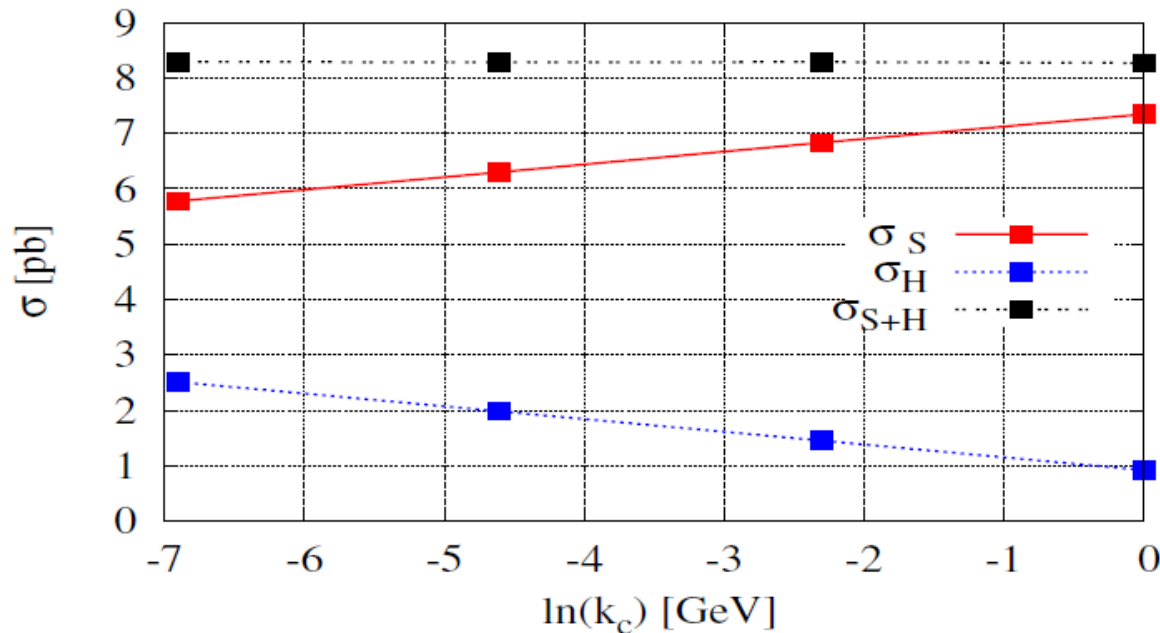
2. λ independence of the result.	
λ [GeV]	$2\mathcal{R}(\mathcal{M}_{Tree}^+ \mathcal{M}_{Loop}) + \text{soft contribution.}$
10^{-17}	-0.392635564863145965951753297380674
10^{-19}	-0.392635564863145910496372637287657
10^{-21}	-0.392635564863145913546569687849770
10^{-23}	-0.392635564863145917181820668621704
10^{-25}	-0.392635564863145920823132105718981

Diagnosis(3) non-linear gauge invariance

At a phase space point

3. Gauge invariance of the result.	
$(\vec{\alpha}, \vec{\beta}, \vec{\delta}, \vec{\epsilon}, \vec{\kappa})$	$2\mathcal{R}(\mathcal{M}_{Tree}^+ \mathcal{M}_{Loop}) + \text{soft contribution.}$
$(1,2,3,4,5) \times 10^{-1}$	-0.392635564863145913554033322734
$(1,2,3,4,5) \times 10^{-2}$	-0.392635564863145913547492915486
$(1,2,3,4,5) \times 10^{-3}$	-0.392635564863145913546838874761
$(1,2,3,4,5) \times 10^{-4}$	-0.392635564863145913546773470689
$(0,0,0,0,0) \times 10^0$	-0.392635564863145913546569687849

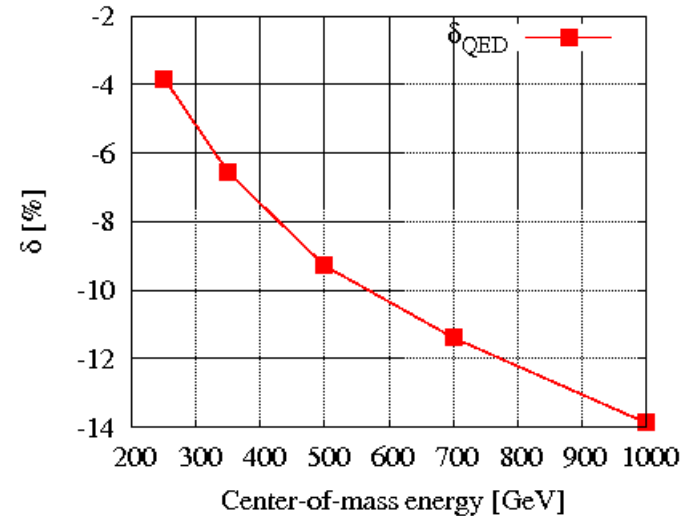
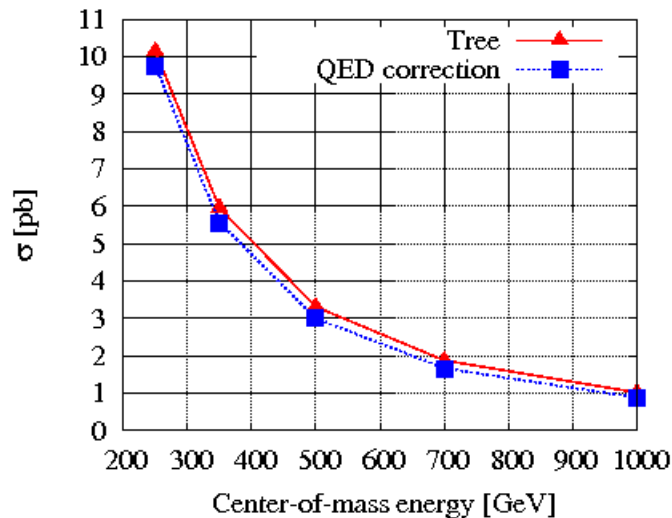
Diagnosis(4) k_c independence



Integrated in phase space

FULL QED corrections

partial comparison with analytical formula(Igarashi, Tobimatsu)

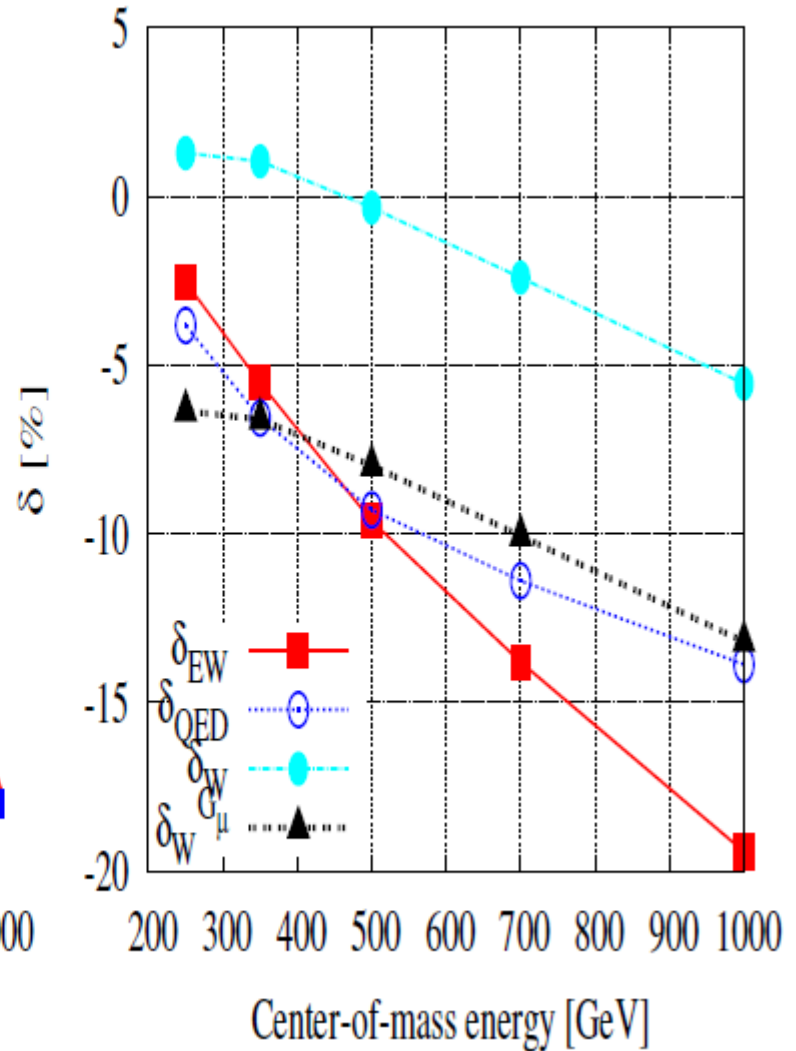
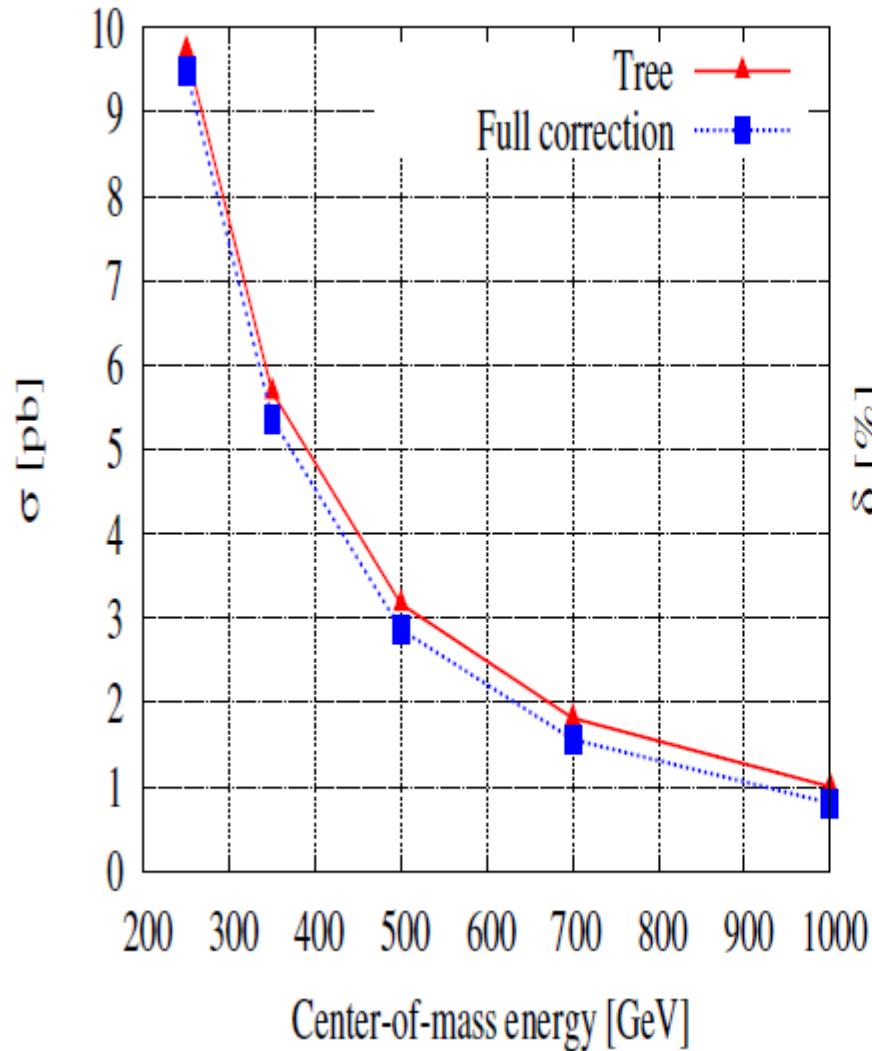


Amplitude	GRACE-Loop	IGARASHI et al
$2\mathcal{R}(\mathcal{M}_{Tree}^+ \mathcal{M}_{Loop})$	6.400328327×10^3	6.400328334×10^3



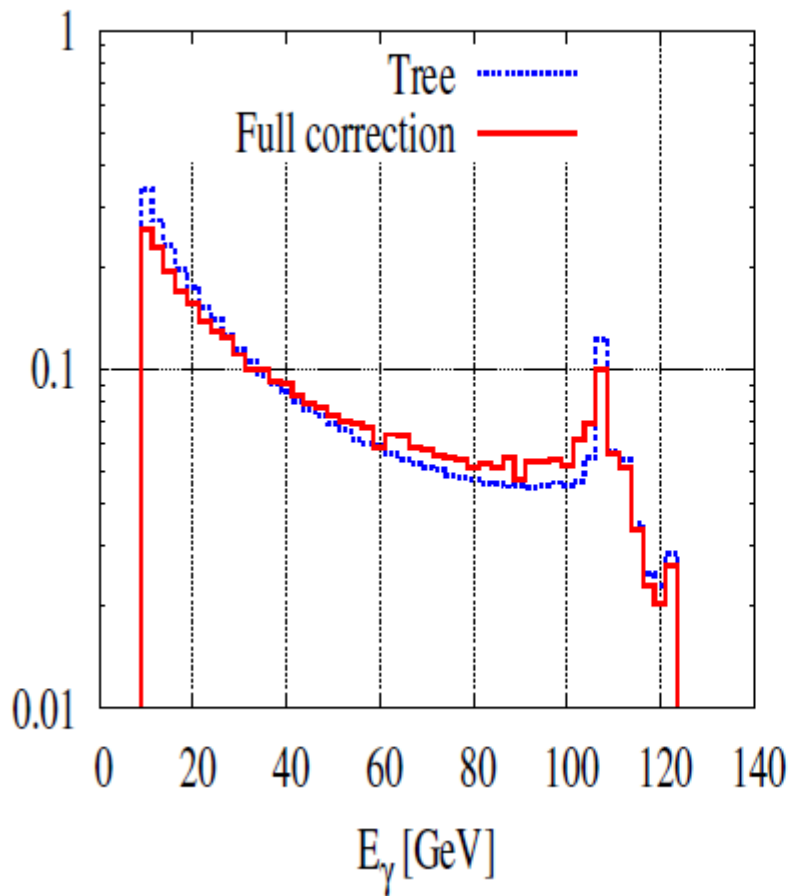
Analytic formula for full QED correction ... in progress
Compact and efficient code can be developed.

Full EW corrections

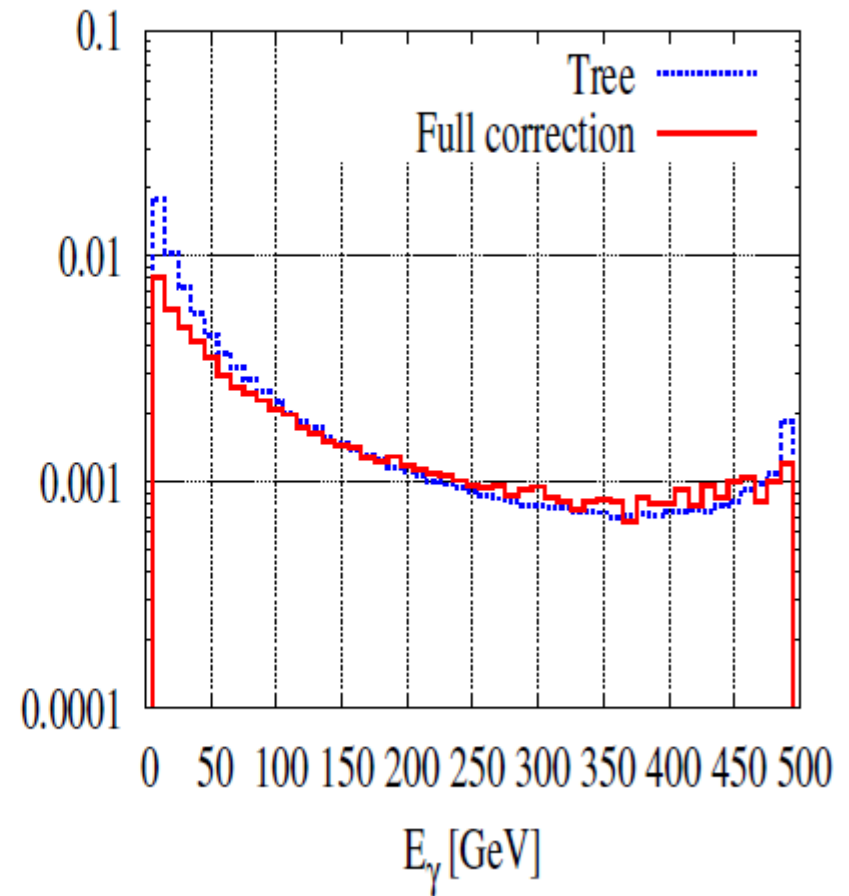


photon energy distribution

$d\sigma/dE_\gamma$ [pb] at 250 GeV.

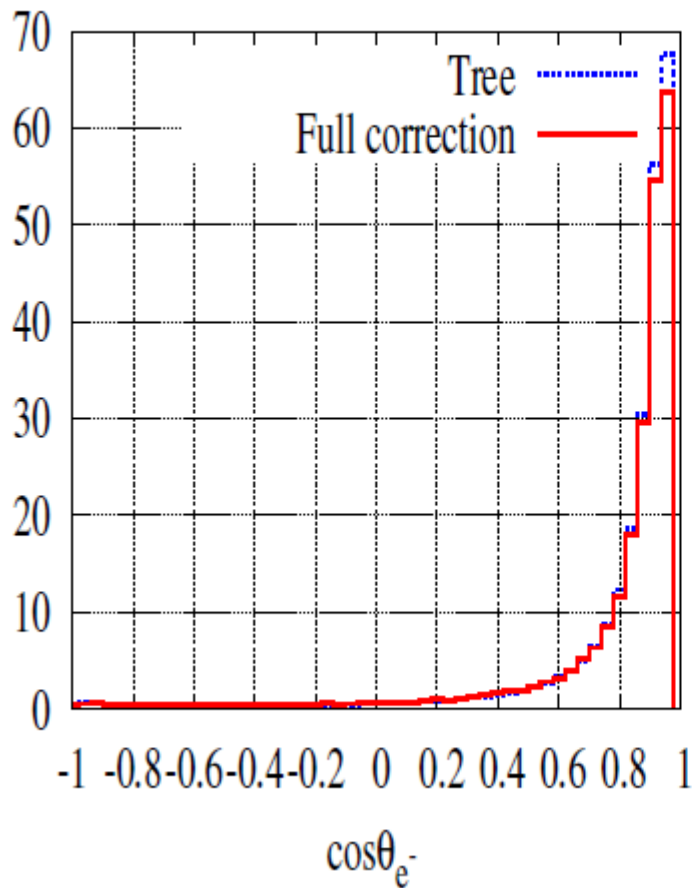


$d\sigma/dE_\gamma$ [pb] at 1 TeV.

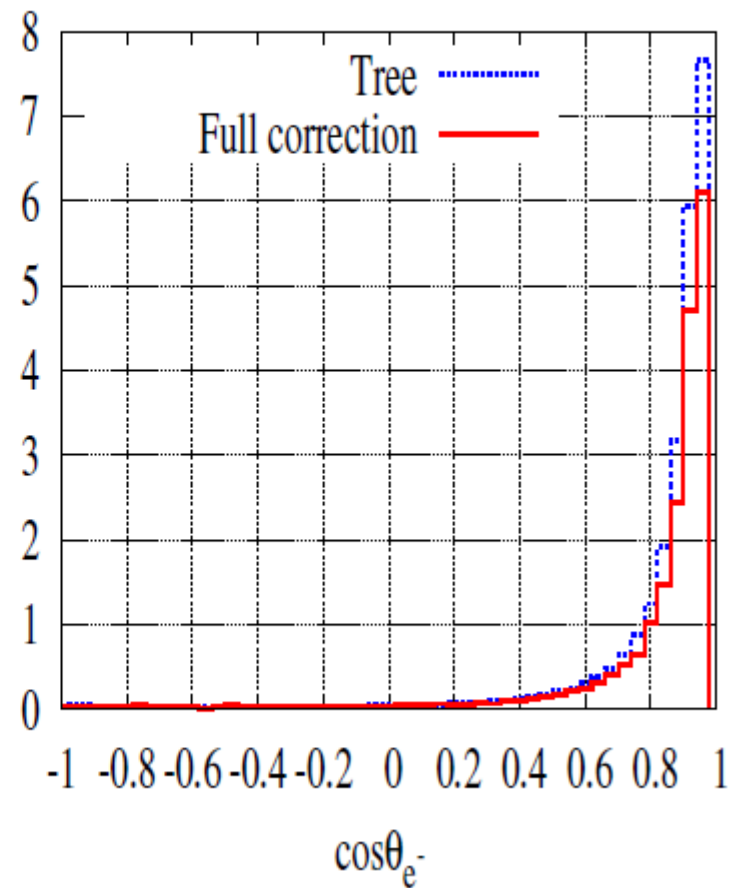


electron angular distribution

$d\sigma/d \cos\theta_{e^-}$ [pb] at 250 GeV.



$d\sigma/d \cos\theta_{e^-}$ [pb] at 1 TeV.



muon $g-2$, 2-loop EW

N.Nakazawa, T.Ishikawa, Y.Yasui

Toward 2-loop automation

- System development → started complete 2-loop EW muon $g-2$ project
- 2-loop integrals ▪ ▪ ▪ DCM (QFTHEP Sochi, 2011)

Present Status of the Experiment and Theory

BNL E821 (2002,2004,2006)

$$a_{\text{exp}}(\mu^+) = 11\,659\,204(6)(5) \times 10^{-10}$$

$$a_{\text{exp}}(\mu^-) = 11\,659\,215(8)(3) \times 10^{-10}$$

Assuming CPT –invariance,

$$a_{\text{exp}}(\mu) = 11\,659\,208.9(5.4)(3.3) \times 10^{-10}$$

first error --- statistical
second error --- systematic

$$\text{QED-Theory : } a_{\text{QED}}(\mu) = 11\,658\,471.809(0.015) \times 10^{-10}$$

up to 5-th loop

$$\begin{aligned} \text{ELWK-theory: } a_{\text{ELWK}}(\mu) &= 19.48 \times 10^{-10} - 4.07(0.1)(0.18) \times 10^{-10} \\ &\quad \text{1-loop} \qquad \qquad \qquad \text{2-loop} \\ &= 15.4(0.1)(0.2) \times 10^{-10} \end{aligned}$$

$$\text{Hadronic: } a_{\text{(HAD)}}(\mu) = [701.5(4.2)(1.9)(0.3) + 0.7(2.6)] \times 10^{-10}$$

$$\text{Total(SM): } a_{\text{(SM)}}(\mu) = 11\,659\,180.2(0.2)4.2(2.6) \times 10^{-10}$$

T. Gribouk and A. Czarnecki,
Phys. Rev. D 72, 053016
(2005)

S. Heinemeyer, D.
Stöckinger, G. Weiglein,
Nucl. Phys. B 699 (2004)
103–123

$$\Delta = (\text{exp}) - (\text{SM})$$

$$= 28.7 \times 10^{-10}$$

strategy

1. Draw the diagrams and Corresponding Feynman Amplitudes (in Non-linear gauge)

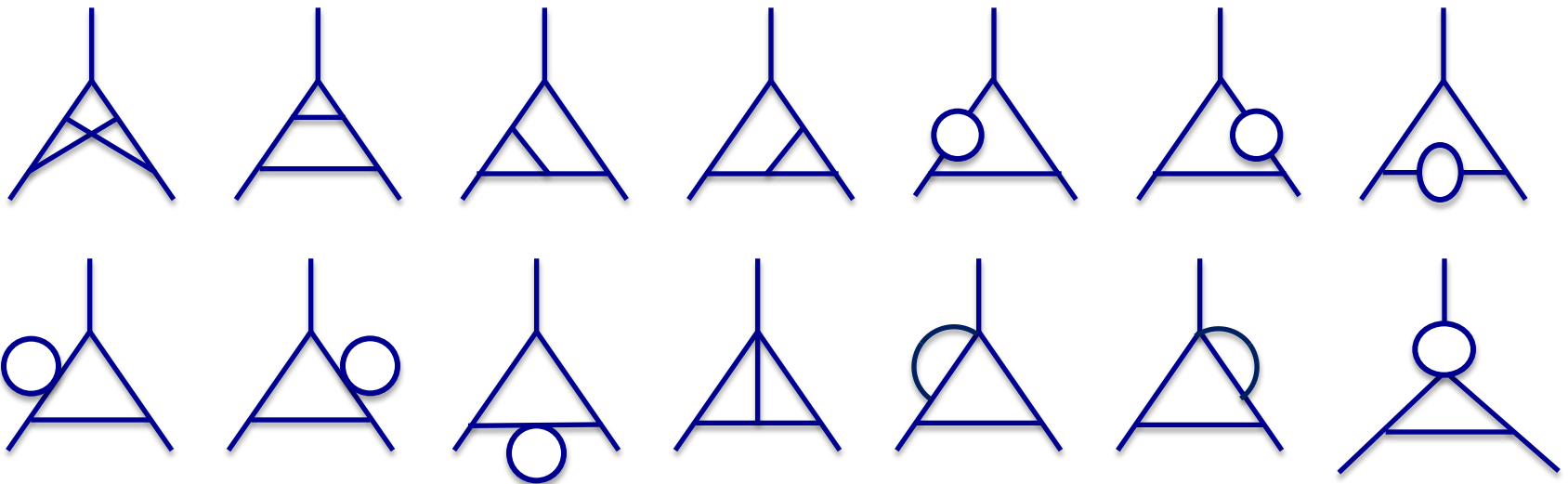
1780 : 2-loop proper diagrams

70 : 1-loop proper diagrams with a 1-loop counter term

2. Classify the diagrams according to **Topology** types

14 types for 2-loop proper diagrams

6 types for diagrams with 1-loop C.T.



calculation

denominator

$$\prod_{j=1}^6 \frac{1}{(p_j^2 - m_j^2)} = \Gamma(6) \int \prod_{j=1}^6 \frac{\delta(1 - \sum z) dz_j}{[z_j (p_j^2 - m_j^2)]^6}, \quad p_j = \sum_{s=1}^2 \eta_j(s) \ell_s + q_j$$

After loop integral

$$F = \int \prod dz_j \frac{1}{(4\pi)^n} \frac{\Gamma(6-n)}{(\det U)^{n/2}} \frac{1}{(V - i\varepsilon)^{6-n}}$$

numerator

$$\frac{p_j^\mu}{(p_j^2 - m_j^2)} = D_j^\mu \frac{1}{(p_j^2 - m_j^2)} = \frac{1}{2} \int_{m_j^2}^{\infty} dm_j^2 \frac{\partial}{\partial q_j^\mu} \frac{1}{(p_j^2 - m_j^2)}$$

g-2 projection operation

$$\text{Proj}(\mu) = \frac{1}{4} (\gamma \cdot p - \frac{1}{2} \gamma \cdot q + m) (m \gamma_\mu \cdot p^2 - (m^2 + \frac{q^2}{2}) p_\mu) (\gamma \cdot p + \frac{1}{2} \gamma \cdot q + m)$$

$$F_2^{(4)}(0) = \lim_{q^2 \rightarrow 0} \frac{m}{p^4 q^2} \text{Tr}(\Gamma_\mu \cdot \text{Proj}(\mu))$$

$q_\mu \Rightarrow$ photon momentum

$(p - q/2)_\mu \Rightarrow \mu^-$ momentum

$(p + q/2)_\mu \Rightarrow \mu^+$ momentum

Ultraviolet divergence appear as $\frac{1}{\varepsilon}$ pole

Infrared singularity is regularized by fictitious mass λ

To get the Fortran Source, all the symbolic manipulations are done by **FORM**

To get the numerical value, we use the adoptive MC system **BASES**

Status

UV cancelation OK

Non-linear gauge invariance ... in progress

Final results ... to appear soon.

Radiative corrections for MSSM

- GRACE includes MSSM model.
- Study decay channels, production processes

After LHC search and 'Higgs' discovery, window for MSSM turns narrower.

Relatively light stop scenario still seems to work.

T.Kon, Y.Kouda, M.Jimbo, T.Ishikawa, Y.Kurihara. M.Kuroda, K.Kato

Renormalization, Nonlinear gauge extension

T.Kon, M.Jimbo, T.Ishikawa, M.Kuroda

light \tilde{t}_1 scenario

MSSM 126GeV Higgs ----- Sparticles are heavy? O(1TeV)

300 GeV stop ----- possibly observable in LHC/ILC

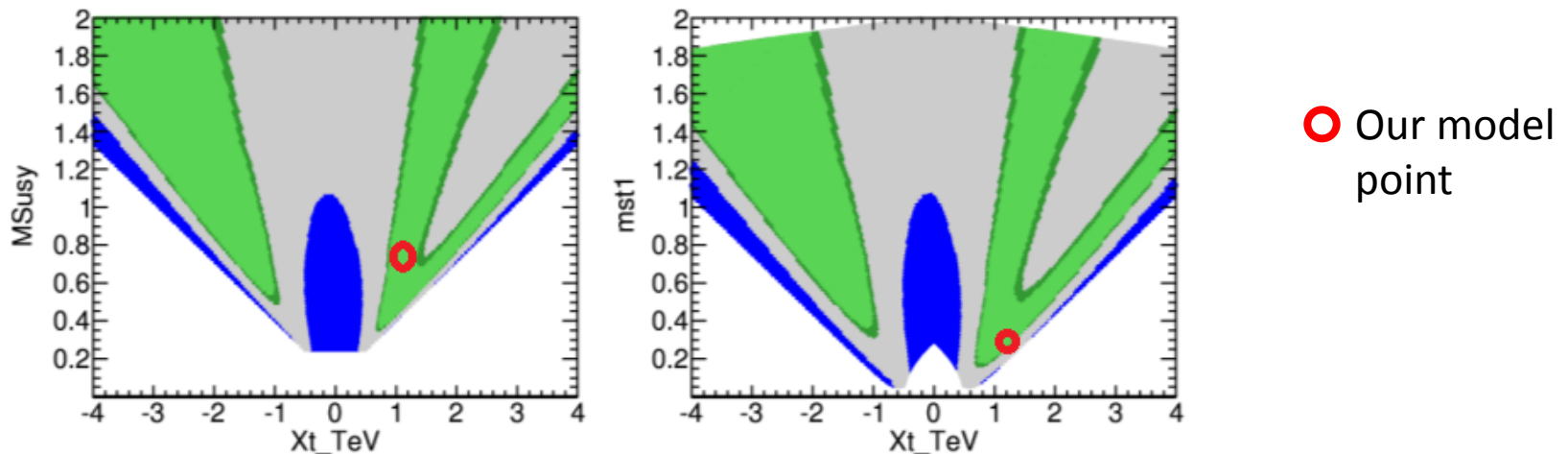
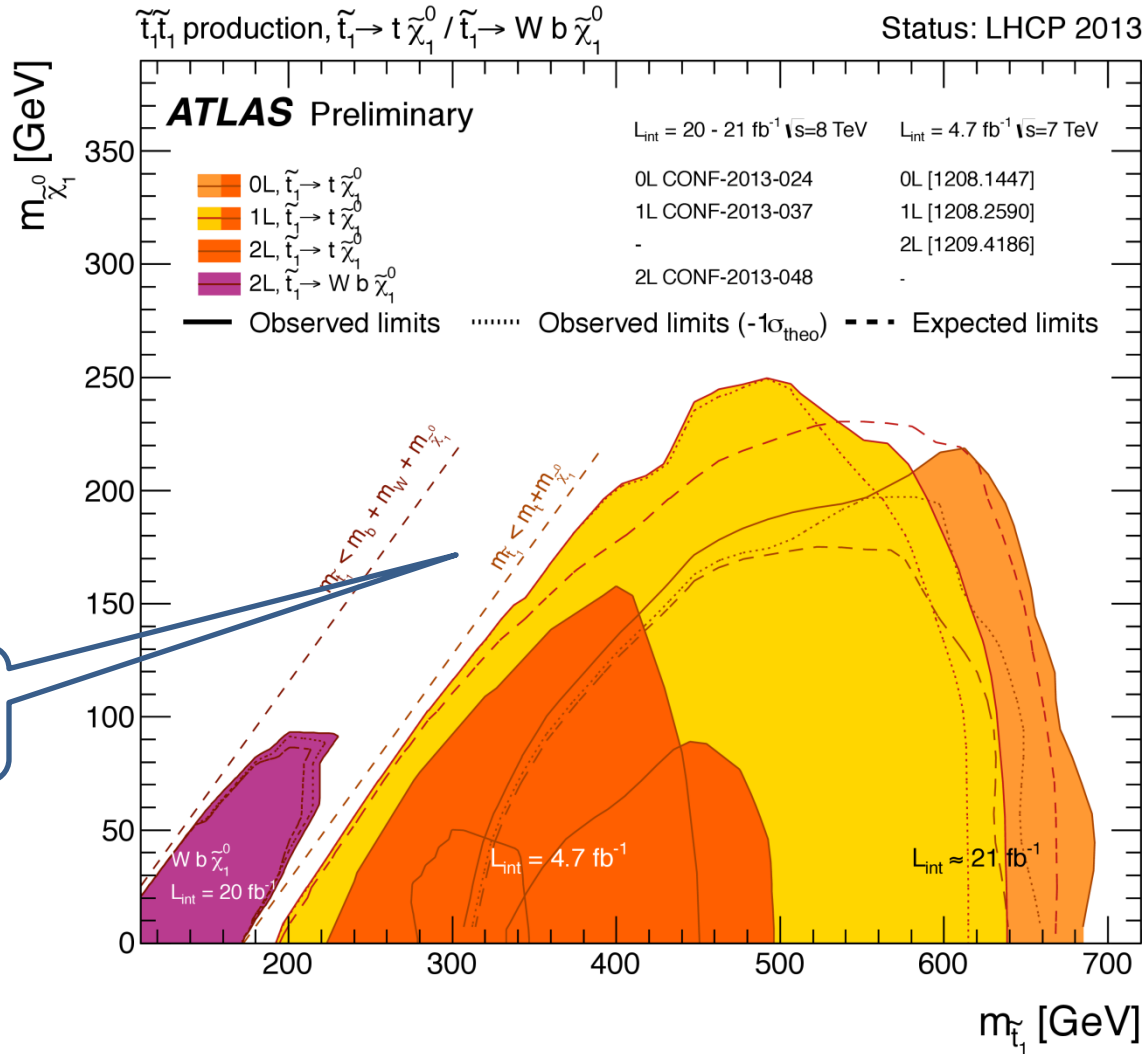


Fig. 15 Scalar top masses in the m_h^{max} scenario (with M_{SUSY} and X_t free) that yield $M_h \sim 125.5$ GeV (green area), LEP excluded regions are shown in blue. Left: X_t - M_{SUSY} plane, right: X_t - $m_{\tilde{t}_1}$ plane [7].

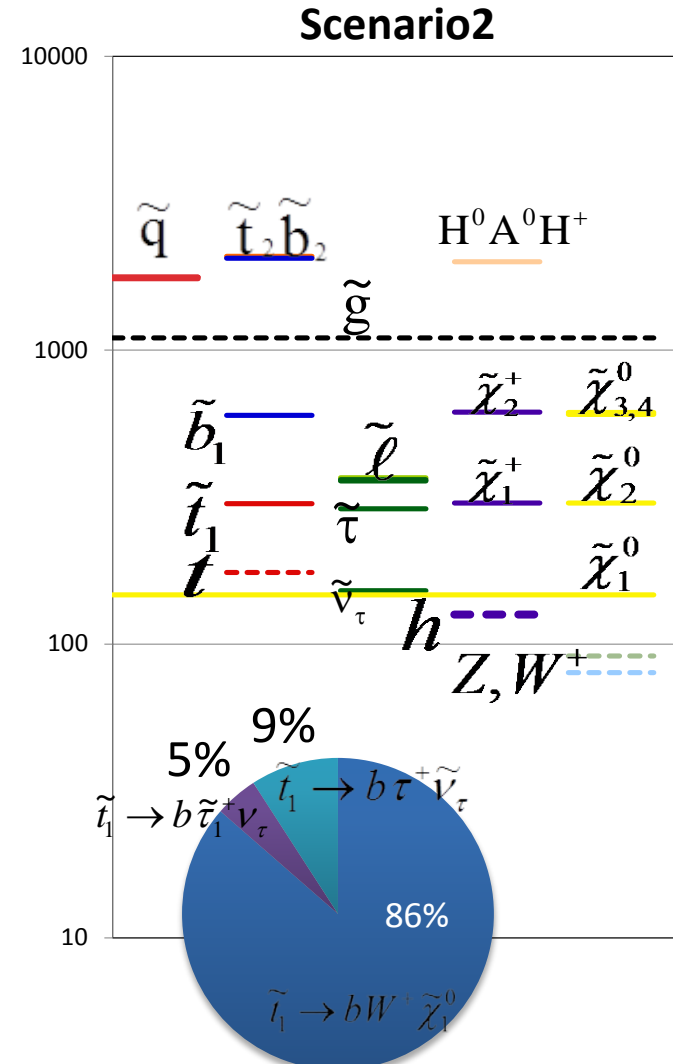
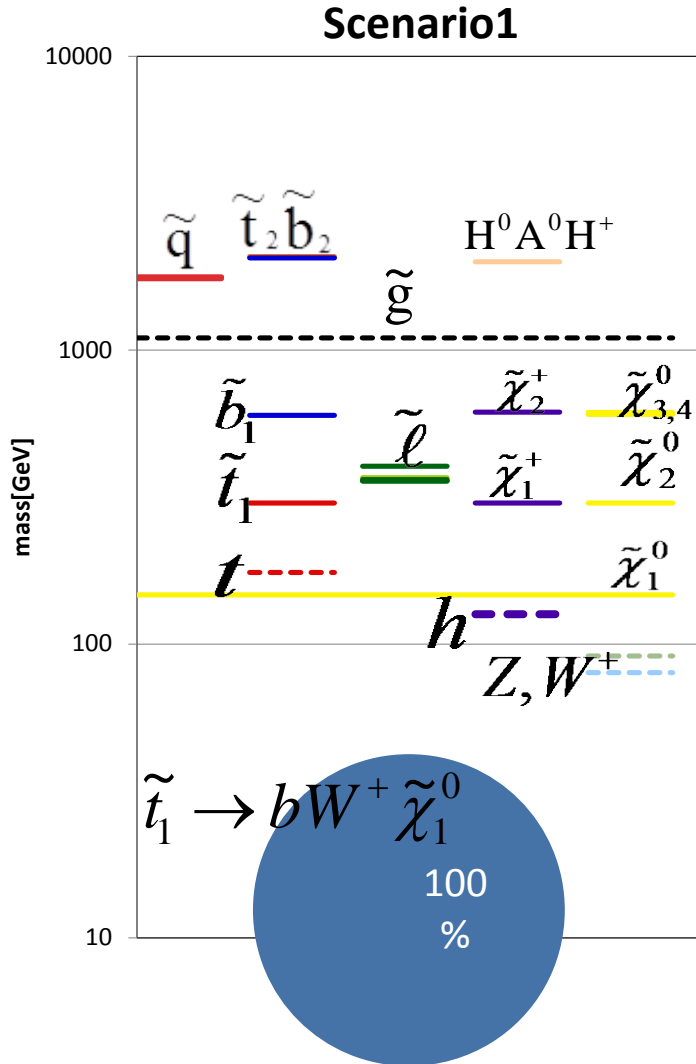
(7) S. Heinemeyer, O. Stål and G. Weiglein, *Phys. Lett. B* **710** (2012) 201 [arXiv:1112.302[hep-ph]];

Recent Results of Light Stop Search at ATLAS



ATLAS-CONF-2013-024
 ATLAS-CONF-2013-048
 ATLAS-CONF-2013-053

Light stop models



scenario1,2 : $\tan\beta=30$ $\mu=600\text{GeV}$

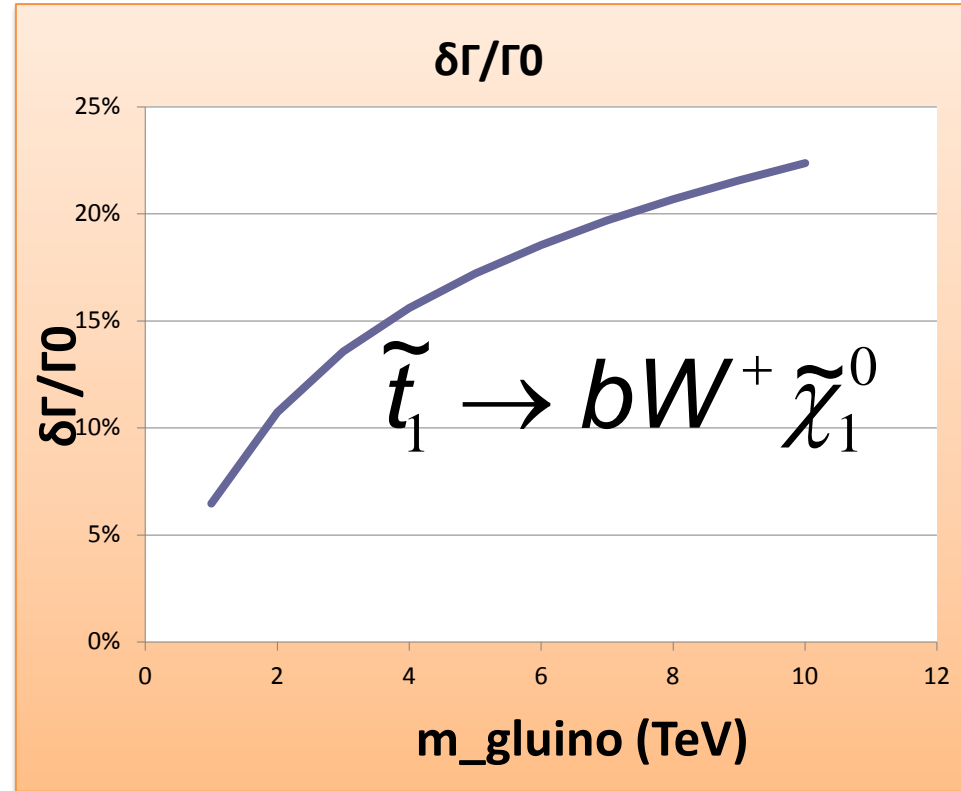
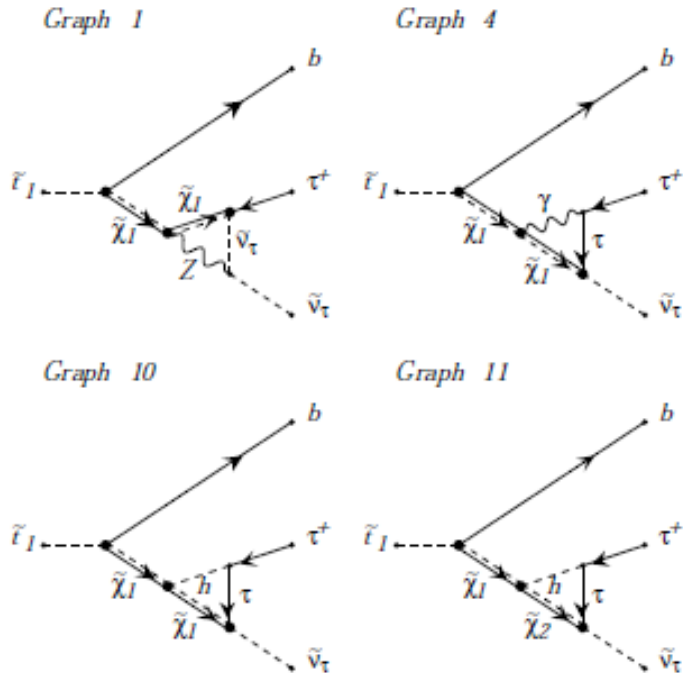
1-loop R.C. to stop-1 decay

$$\tilde{t}_1 \rightarrow b\tau^+\tilde{\nu}_\tau$$

tree 2 diagrams

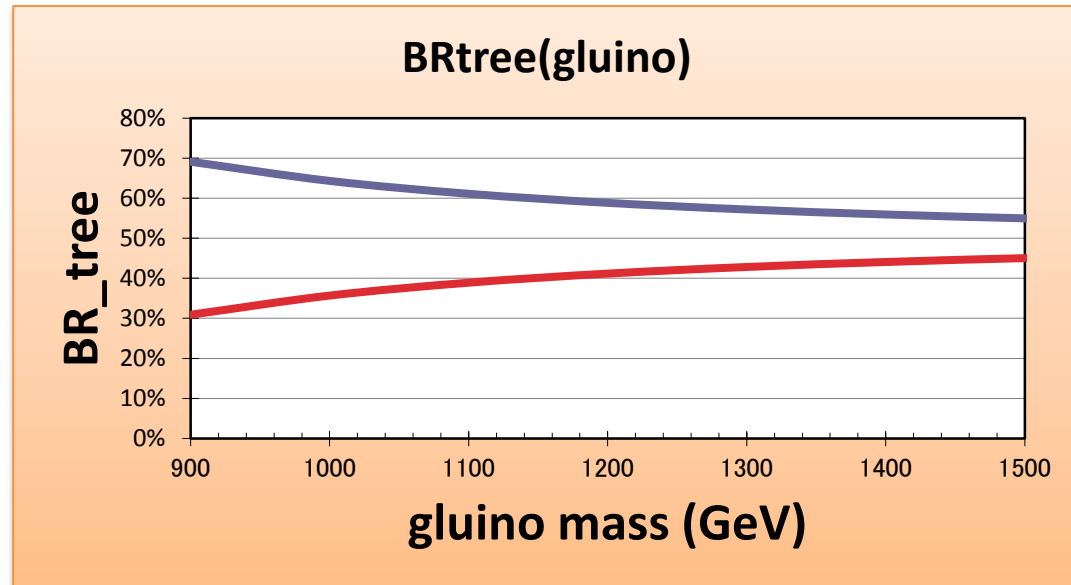
QCD 1-loop 12 diagrams

ELW 1-loop 490 diagrams



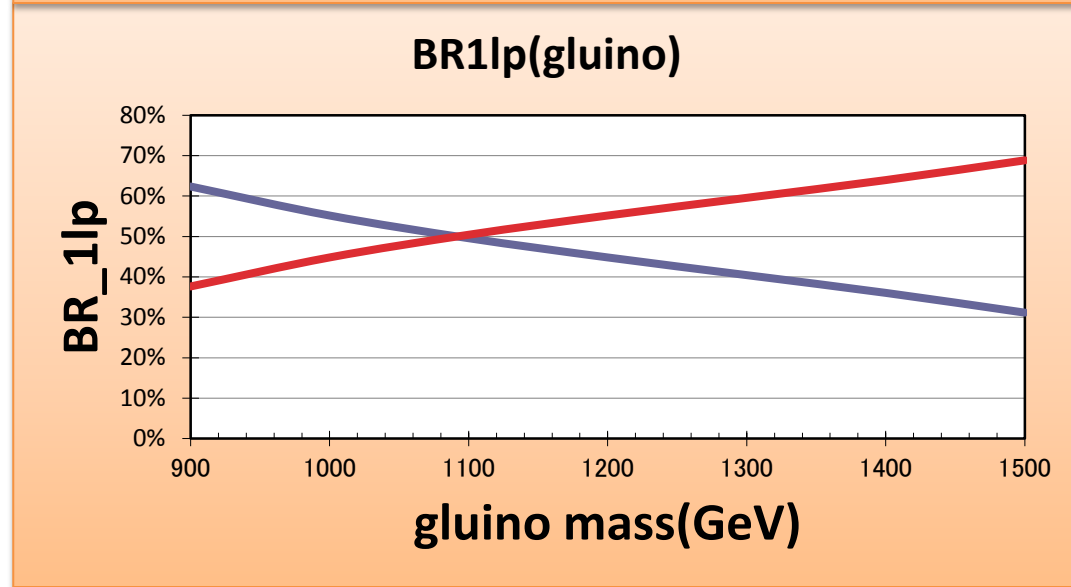
gluino mass vs gluino decay width

Tree



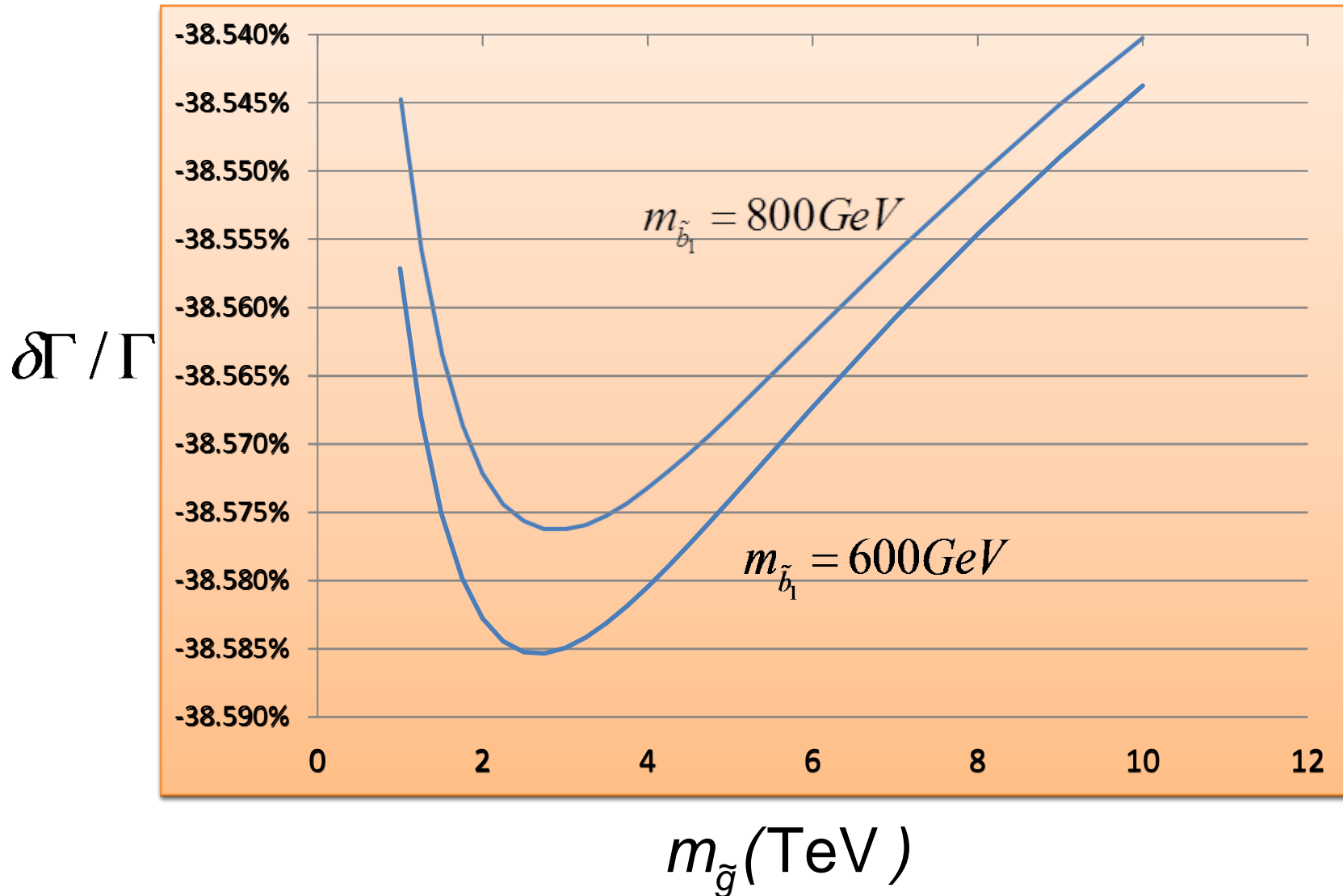
$t\tilde{t}_1$
 $b\tilde{b}_1$

1Loop

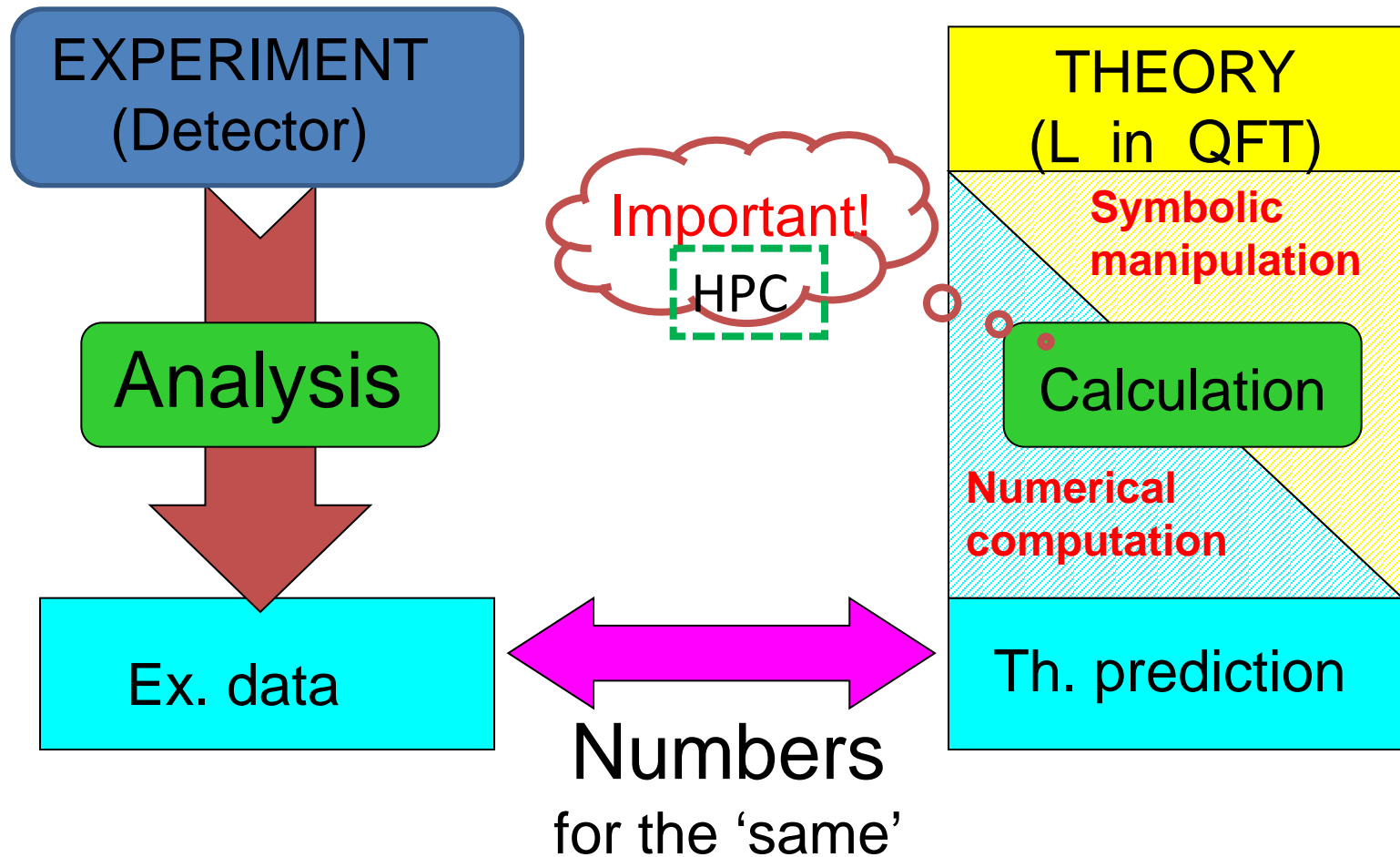


$b\tilde{b}_1$
 $t\tilde{t}_1$

$H \rightarrow bb$ mode, gluino mass dep.



Summary



GRACE system is developed to perform large scale calculation in HEP automatically.

It successfully calculates

$$e^+ e^- \rightarrow t\bar{t}\gamma, e^+ e^- \gamma$$

radiative corrections to MSSM particles

and extends its capability further.

2-loop automation can be possible in the future.

THANK YOU !

