Phenomenology of $SU(3)_L \times SU(3)_R \times SU(3)_C \times SO(3)_F$ and the Higgs Boson

Berthold Stech
University of Heidelberg

St. Petersburg, June 2013

Trinification

A little discussed extension of the Standard Model is to go from

$$SU(2)_L \times U(1) \times SU(3)_C$$

to

$$SU(3)_L \times SU(3)_R \times SU(3)_C$$

which is a maximal subgroup of E_6

some references to trinification: Y. Achiman, B.S. (1978-1979), A. de Rujula, H. Georgi, S.L. Glashow (1984), K.S. Babu, X.G. He, S.Pakvasa (1986), B.S. Phys. Rev. D 86, 055003 (2012)

3 gauge couplings can unify ⇒ Trinification is a GUT !

combine GUT with Flavor (Generation) Symmetry :

GUT × Flavor

with the consequence: all fermions, quarks and leptons, are in the same flavor representation.

This requirement leaves only very few models discussed in the literature. We take

$$Trinification \times SO(3)_F$$

all fermions are 3 vectors with respect to this flavor group.

Aim:

Construction of a relatively simple GUT× Flavor model in which all Higgs fields are flavor singlets and all flavon fields are GUT singlets . Few parameters.

This appears difficult if not impossible for SO(10) GUT's

but possible for the gauge groups

E_6 × flavor and Trinification × flavor !

Phys. Rev. D77, 076009 (2008) Z. Tavartkiladze, B.S.,
Fortschr. Phys. 58, No 7-9 (2010) 692 B.S.,
arXiv hep-ph 1012.6028 .B.S.

non supersymmetric model
favoured by simplicity

There are several excellent features of SUSY
but susy seems not relevant for TeV scale physics (LHC !)

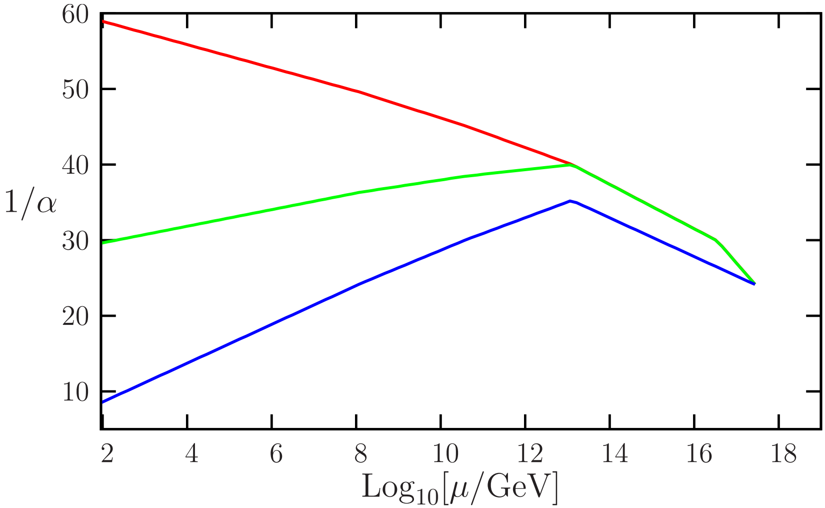
non supersymmetric models regain importance:

i) the quadratic divergencies causing the hierarchy problem affect vacuum expectation values and only indirectly particle masses. Vev's are not understood (cosmological constant !) and may have there origin at a very high scale.

ii) Vev's are due to tadpole diagrams which are momentum independent, can be subtracted and have then no influence on particle properties.

iii) the neutrino masses likely require a two step unification process which in non supersymmetric theories occur naturally by electroweak unification at $M_I \approx 2 \cdot 10^{13}$ GeV.

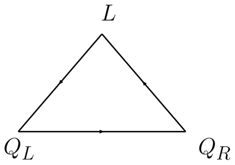
Concorde


60
50
40
30
20
10
$1/\alpha$
2 4 6 8 10 12 14 16 18
$\mathrm{Log}_{10}[\mu/\mathrm{GeV}]$

$$E6 \supset SU(3)_L \times SU(3)_R \times SU(3)_C$$

Single generation for fermions $\psi(27)$

$$\mathbf{27} = Q_L(x) + L(x) + Q_R(x)$$



$$(Q_L)^a_i = \begin{pmatrix} u^a \\ d^a \\ D^a \end{pmatrix}, \quad L^i_k = \begin{pmatrix} L^1_1 & E^- & e^- \\ E^+ & L^2_2 & \nu \\ e^+ & \hat{\nu} & L^3_3 \end{pmatrix}, \quad (Q_R)^k_a = \begin{pmatrix} \hat{u}_a, & \hat{d}_a, & \hat{D}_a \end{pmatrix},$$

$$Q_L(x) = (3, \quad 1, \quad \bar{3}), \quad L(x) = (\bar{3}, \quad 3, \quad 1), \quad Q_R(x) = (1, \quad \bar{3}, \quad 3)$$

- mixing: $d \leftrightarrow D$ $\quad \mathcal{U}_L$-spin, $\quad \hat{d} \leftrightarrow \hat{D}$ $\quad \mathcal{U}_R$-spin

B. S., Z. Tavartkiladze, Phys. Rev. D 77 (2008) 076009

Higgs fields H , $\tilde{H}$

2 Higgs fields $(3^*, 3, 1)$ break the trinification group down to the standard model :

$$\langle H\rangle = \begin{pmatrix} v_1 & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & M_I \end{pmatrix} \qquad \langle \tilde{H}\rangle \simeq \begin{pmatrix} v_2 & 0 & 0 \\ 0 & b_2 & b_3 \\ 0 & M_R & M_3 \end{pmatrix}$$

$M_I \neq 0$ gives $SU(2)_L \times SU(2)_R \times U(1)$,
$M_I \neq 0$ and $M_R \neq 0$ leads to $SU(2)_L \times U(1)_Y \times SU(3)_C$
With $v_1 \neq 0$, $v_2 \neq 0$ only $U(1)_e$ remains.

$m_t = g_t v_1,\ m_b = g_b b,\ M_D = M_I, \quad v_1^2 + v_2^2 \simeq (174\ \text{GeV})^2$

$\langle H\rangle$ is diagonal, $\tilde{H}$ fields *not* directly coupled to fermions.

parity like quantum number

Potentials

ol' man river

The Higgs fields H , $\tilde{H}$ are two $(3_L^*, 3_R)$ matrix fields formed from $2 \times 18 = 36$ scalar fields

?? Can one construct a Potential for 36 scalar fields leading to the hierarchy $M_I, M_R \gg m_t, m_b$??

This potential should provide for spontaneous symmetry breaking giving $36 - 15 = 21$ massive Higgs particles and 15 Goldstone bosons eaten up by W^+, W^-, Z and 12 heavy vector bosons!

A strict treatment is complicate. We use a phenomenological ansatz. Only vevs, no explicit mass terms.

we start with the simplest 4 invariants

$$V_0 = c_1 J_1 + c_2 J_2 + c_3 J_3 + c_4 J_4$$

$$J_1 = (Tr[H^\dagger \cdot H])^2, \quad J_2 = Tr[H^\dagger \cdot H \cdot H^\dagger \cdot H],$$
$$J_3 = (Tr[\tilde{H}^\dagger \cdot \tilde{H}])^2, \quad J_4 = Tr[\tilde{H}^\dagger \cdot \tilde{H} \cdot \tilde{H}^\dagger \cdot \tilde{H}]$$

vevs: $\langle J_1 \rangle = (M^2 + v_1^2 + b^2)^2$, $\langle J_2 \rangle = (M^4 + v_1^4 + b^4)$

$\langle J_3 \rangle = (M^2 + v_2^2 + b_3^2)^2$, $\langle J_4 \rangle = M^4 + v_2^4 + b_3^4$

for $M_I = M_R = M$ and $M_3 = b_2 = 0$ (most symmetric case)

no linear combination of the 4 invariants produces these vevs

The potential needs a logarithmic dependence on invariants (Coleman- Weinberg type). We take

$$V = \frac{1}{k + \log[\frac{J_1 J_2 J_3 J_4}{\langle J_1 \rangle \langle J_2 \rangle \langle J_3 \rangle \langle J_4 \rangle}]} V_0$$

requirement : derivatives with respect to all 36 scalar fields have to vanish at the proposed minimum:
$H_1^1 = v_1,\ H_2^2 = b,\ H_3^3 = M,\ \tilde{H}_1^1 = v_2^2$ etc

result: $k = 4$ and for $c_2,\ c_3\ ,\ c_4$

$$c_1 \frac{\left(b^2 + M^2 + v_1^2\right)^2}{b^4 + M^4 + v_1^4},\ \ c_1 \frac{\left(b^2 + M^2 + v_1^2\right)^2}{\left(b_3^2 + v_2^2 + M^2\right)^2},\ \ c_1 \frac{\left(b^2 + M^2 + v_1^2\right)^2}{b_3^4 + v_2^4 + M^4}.$$

Obviously, one has $c_2 = c_3 = c_4 = c_1$ in the large M limit.

$$b^2 + b_3^2 + v_1^2 + v_2^2 = (174\ \text{GeV})^2 = \frac{1}{2}(246\ \text{GeV})^2$$

for $v_1 = v_2$ and $b, b_3 \ll v_1$ one finds $v_1 \simeq 123$ GeV.

The potential V is fully invariant and provides for the spontaneous symmetry breaking to $U(1) \times U(1)_e$.
The 36×36 matrix of second derivatives

$$M^2_{ab} = \frac{1}{2} \frac{\partial^2 V}{\partial h_a \partial h_b}$$

has still a high symmetry because the invariants used so far do not connect H with $\tilde{H}$. There are 4 massive states, 15 Goldstone bosons and 17 still massless states. To second order in v_1 and v_2 the masses obtained are

$$m_1^2 = c_1(v_1^2 + b^2), \;\; m_2^2 = c_1(v_2^2 + b_3^2) \quad \text{light}$$

$$m_3^2 = c_1(4M^2 + 5v_1^2 + 5b^2), \;\; m_4^2 = c_1(4M^2 + 5v_2^2 + 5b_3^2) \quad \text{heavy}$$

.

The Higgs

for $v_1 = v_2$ one obtains $v_1 = v_2 \simeq \frac{1}{\sqrt{2}} 174 = 123$ GeV

$$m^2_{Higgs} = m^2_1 = m^2_2 = c_1 v^2_1 \simeq (125 \text{ GeV})^2 \quad \text{for} \quad c_1 \simeq 1.04$$

$c_1 \simeq 1$ (or $c_1 \simeq \frac{1}{2}$ for $v_2 = 0$) appears natural (predicted 2010 and 2012)
Input – Eigenvalues $\rightarrow 0$ for $M \rightarrow \infty$

Is the Higgs a Twin ??

2 degenerate states: a combination of normal and fermiophobic Higgs fields ?

seems a possibility, but the necessary additional invariants and states not yet considered can remove the degeneracy.
arXiv hep-ph 1303.6931 B.S.

Inclusion of more invariants for V:

$J_1, ...J_9, \qquad J_5 = Tr[H^\dagger \cdot \tilde{H} \cdot \tilde{H}^\dagger \cdot H]$ etc

The 36 first derivatives at the chosen minimum fixes the coefficients in terms of 3 parameters

result: Eigenvalues of the 36×36 mass matrix :

15 would be Goldstone particles

00 All new masses can be taken to be $m_i^2 \gg m_{Higgs}^2,$

the first higher ones are charged $(\pm)$. The degeneracy of the

Higgs (125 GeV) can be kept or removed.

it depends on the vev's of $\tilde{H}$

V_{eff} with shifted fields

$$H_1^1 \to v + H_1^1, \quad H_3^3 \to M + H_3^3 \quad \text{etc}$$

power expansion in M neglecting inverse powers of M

V becomes polynomial with field configurations up to 4th powers only $(h_3^3 = Re[H_3^3], ...)$

$$V_{eff} \Rightarrow 4(h_3^3)^2 M^2 + 4\, h_1^1\, h_3^3\, M + 2(h_1^1)^2\, v_1^2 + ...$$

$$+\, O(H^3) + O(H^4) + O(\tilde{H}^3) + O(\tilde{H}^4)$$

replaces the standard model in presence of a huge hierarchy

symmetry breaking properties remain unchanged

valid at low scales, renormalizable

Trinification $\times$Flavor

Flavor $= SO(3)_F$

$\Phi_{\alpha\beta}$: scalar flavon fields (GUT singlets)

$\frac{\langle\Phi_{\alpha\beta}\rangle}{M}$ = coupling matrix $\quad \alpha, \beta = 1, 2, 3$

$$\mathcal{L}_Y^{eff} = \frac{1}{M}\langle\Phi_{\alpha\beta}\rangle \left(\psi^{\alpha T} H \psi^{\beta}\right) + \ldots\ldots$$

$\Rightarrow$ effective interaction to be understood on a deeper level

Flavor SO(3)

- The scalar flavon fields are represented by the hermitian matrix field $\Phi_{\alpha\beta}(x)$:

  $\Phi_{\alpha\beta}(x) = \chi_{\alpha\beta}$ **(symmetric)** $+i\,\xi_{\alpha\beta}$ **(antisymmetric)**

  $$\chi_{\alpha\beta} \sim \text{"1"} + \text{"5"} \qquad\qquad \xi_{\alpha\beta} \sim \text{"3"}$$

- The 3×3 coupling matrices in front of the Higgs fields are then obtained from the VEV's of Φ

$$\mathcal{L}_Y^{eff} = \frac{\langle \chi_{\alpha\beta} \rangle}{M} (\psi^{\alpha T} H \psi^{\beta}) + i \frac{\langle \xi_{\alpha\beta} \rangle}{M'} (\psi^{\alpha T} H_A \psi^{\beta})$$

Phenomenology

The coupling matrix $G = \frac{\langle\chi\rangle}{M_I}$ determines the mass hierarchy

$$G = \frac{\langle\chi\rangle}{M} = \begin{pmatrix} m_u & 0 & \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} \frac{1}{m_t} = \begin{pmatrix} \sigma^4 & 0 & 0 \\ 0 & \sigma^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}_{\text{at } \mu = M_I}$$

$\sigma = 0.050 \;\Rightarrow$ correct up quark mass ratios

The coupling matrix $A = i\frac{\langle\xi\rangle}{M'}$ describes particle mixings.
It is antisymmetric and hermitian, 1 real parameter:

$$A = i\frac{\langle\xi\rangle}{M'} = i\begin{pmatrix} 0 & \sigma & -\sigma \\ -\sigma & 0 & 1/2 \\ \sigma & -1/2 & 0 \end{pmatrix}$$

The generation matrices $G_{\alpha\beta}$ and $A_{\alpha\beta}$ appear in the effective Yukawa interaction

$$\mathcal{L}_Y^{\text{eff}} = G_{\alpha\beta}(\psi^{\alpha T} H \psi^{\beta}) + A_{\alpha\beta}(\psi^{\alpha T} H_A \psi^{\beta}) + \frac{(G^2)_{\alpha\beta}}{M_N}\Big((\psi^{\alpha T} H^{\dagger})_1 (\tilde{H}^{\dagger}\psi^{\beta})_1\Big) + h.c.$$

- the 3_{rd} term gives masses to the heavy leptons
- hierarchy of masses and the mixings of all fermions are now fully determined
- The A term is CP odd → unique ~~CP~~

The Masses and Mixings of all 3×27 fermions are obtained from the mass matrix

$$M_{ij}^{\alpha\beta} = G_{\alpha\beta}\langle H\rangle_{ij} + A_{\alpha\beta}\langle H_A\rangle_{ij} + (G^2)_{\alpha\beta}\langle \tilde{H}^\dagger\rangle_i \frac{1}{M_N}\langle \tilde{H}^\dagger\rangle_j$$

$$\langle H\rangle = \begin{pmatrix} v_1 & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & M_I \end{pmatrix} \qquad \langle \tilde{H}\rangle = \begin{pmatrix} v_2 & 0 & 0 \\ 0 & b_2 & b_3 \\ 0 & M_R & M_3 \end{pmatrix}$$

$$\langle H_A\rangle_{\text{quarks}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & f_2^2 & f_3^2 \\ 0 & f_2^3 & f_3^3 \end{pmatrix} \qquad \langle H_A\rangle_{\text{leptons}} = \text{similar to quarks}$$

$$m_D = m_L = M_I$$

Mass Matrices

Up quark matrix $\quad M_u = G\, m_t$

Down quark matrix

$$\begin{array}{cc} & \begin{array}{cc} \hat{d} & \hat{D} \end{array} \\ M_{d,D} = \begin{array}{c} d \\ D \end{array} & \begin{pmatrix} m_b^0 G + f_2^2 A\ , & f_3^2 A \\ f_2^3 A\ , & M_I G \end{pmatrix} \end{array}$$

$$\Rightarrow \quad M_d = m_b^0 G + f_2^2 A - f_0\, A G^{-1} A$$

$$\langle H_A \rangle \sim (\bar{3}, 3, 1), \quad f_j^i = \langle H_A \rangle_j^i : \qquad f_2^3 = \sigma^3\, x_g\, M_I, \quad f_0 = f_3^2\, f_2^3\ /\ M_I$$

This gives a very good fit for the *CKM* unitarity triangle

Results

- $m_u = G\, m_t$
- $m_d \simeq G\, m_b + A\, f_2^2 - f_0\, A \cdot G^{-1} \cdot A$ small mixing and $\cancel{CP}$
- $m_\nu \simeq \frac{v_1^2}{M_I}\left(\kappa\, \mathbb{1} + \frac{f_2^3}{M_I}(G^{-1} \cdot A - A \cdot G^{-1})\right)$ large mixing and $\cancel{CP}$

  observables:
  quarks → 10, charged leptons → 3, light neutrinos → 9,
  heavy fermions → 15 $22 + 15 = 37$

- 9 parameters: (besides the gauge couplings)

  $m_t,\ m_b,\ m_\tau, f_2^2,\ f_0,\ f_2^3,\ \kappa,\ \sigma,\ 1/2$

- heavy neutrino masses: very large hierarchy

Neutrinos

- with x_g from $\langle H_A \rangle$ the 3×3 neutrino matrix is

$$m_\nu \simeq \frac{v_1^2}{M_I} \begin{pmatrix} \kappa & \mathrm{i}\ \mathrm{x_g} & -\mathrm{i}\ \mathrm{x_g} \\ \mathrm{i}\ \mathrm{x_g} & \kappa & \mathrm{i}\ \mathrm{x_g}\frac{\sigma}{2} \\ -\mathrm{i}\ \mathrm{x_g} & \mathrm{i}\ \mathrm{x_g}\frac{\sigma}{2} & \kappa \end{pmatrix}$$

- Eigenvalues of $m_\nu m_\nu^\dagger$:

$$(m_2)^2 \simeq \left(\kappa^2 + 2\, x_g^2 + \frac{\sigma\, x_g^2}{\sqrt{2}}\right) \frac{v_1^4}{M_I^2}\ ,$$

$$(m_1)^2 \simeq \left(\kappa^2 + 2\, x_g^2 - \frac{\sigma\, x_g^2}{\sqrt{2}}\right) \frac{v_1^4}{M_I^2}\ ,$$

$$(m_3)^2 \simeq \kappa^2\, \frac{v_1^4}{M_I^2}$$

i) Inverted hierarchy

ii) degeneracy in the no mixing limit $x_g \to 0$

iii) $R = (m_2^2 - m_1^2)/(m_2^2 - m_3^2) \simeq \sigma/\sqrt{2} \simeq 0.035$ exp: 0.032

iv) x_g fixed from Δm^2_{atm}

$$\begin{aligned} x_g \frac{v_1^2}{M_I} &\simeq \frac{1}{\sqrt{2}} \sqrt{\Delta m^2_{\text{atm}}} \simeq 0.034\ eV\ , \quad \Rightarrow \ x_g \approx 0.04 \\ \kappa &= m_3 \frac{x_g}{0.034}. \end{aligned}$$

v) M_I together with σ
fixes masses of all heavy fermions

vi) tiny increase of $(m_\nu)_{33}$ element by $\simeq 1.01$ gives correct neutrino mixing pattern.

Neutrino properties

example : setting $m_3 = 0.07$ and fixing $m_{3,3} = 1.007\, m_3$
one finds

- masses:
  $m_2 = 0.08542, \quad m_1 = 0.08487, \quad m_3 = 0.07025$ eV
- mixing angles:
  $\Theta_{12} \simeq 33°, \quad \Theta_{23} \simeq 50°, \qquad \Theta_{13} \simeq 3.7°$ not good
  sensitive to charged lepton matrix ("Cabibbo" angle too small)
- $\cancel{CP}$: $\delta_\nu \simeq 70°$ Majorana phases: $\simeq (-26°, \; -96°)$
- Neutrinoless double β decay parameter:
  $|m_{\beta\beta}| = 0.07$ eV $\quad |m_{\beta\beta}|$ scales with m_3

Summary

$$E_6 \times Flavor \supset \ Trinification \times SO(3)_F$$

- The effective Yukawa interaction at the weak scale has a simple form: only flavor singlet Higgs fields and GUT singlet flavon fields.
- $M_I \simeq 2 \cdot 10^{13}$ GeV, the meeting point of g_1 and g_2, fixes the mass scales of light and heavy neutrinos and new physics.
- phenom. Higgs potential $\Rightarrow$ desired spont. symmetry breaking, 15 w. b. Goldstones, 1 or 2 Higgs of ≈ 125 GeV and heavier (partly degenerate) scalars. Some scalars are fermiophob , some have tiny decay widths.

- mixing of generations and $\not{CP}$ is combined with the mixing of standard model states with heavy particles.
- The known quark and charged lepton properties determine to some extent the neutrino properties.
  Predictions for hierarchy, $\not{CP}$, Majorana phases as well as the mass parameter for $O\nu\beta\beta$ decays.
- few symmetry breaking parameters allow for a fit of all fermion masses and mixings !

it's fun to work on non fashionable models!