Manifestation of quark-hadron duality via the Adler D-function

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Preface

To compare theoretical results and experimental data one often uses the concept of quark-hadron duality, which establishes a bridge between ***quarks and gluons***, a language of theoreticians, and real measurements with ***hadrons*** performed by experimentalists.

The idea of quark-hadron duality was formulated by Poggio, Quinn, and Weinberg (1976) as follows:

> Inclusive hadronic cross sections, once they are appropriately averaged over an energy interval, must approximately coincide with the corresponding quantities derived from the quark-gluon picture.

There are various areas of hadronic physics dealing with different manifestations of quark-hadron duality.

In the talk we concentrate on physical quantities and functions which are defined through the Drell R-ratio, $R(s) = \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$, and for considered quantities a corresponding interval of integration involves a low energy region (about 1 GeV and less) and therefore a standard perturbative QCD description can not be directly applied.

Clearly that it is fruitful to connect measured quantities with ``simplest'' theoretical objects. Some single-argument functions which are directly connected with experimentally measured quantities can play the role of these objects. The Adler D-function (Euclidean quantity) turned out to be a smooth function without traces of the resonance structure of the R-ratio is a convenient object for comparing theoretical results with experimental data, and one can expect that it will reflects more precisely the quark-hadron duality.

Outline

- Short historical overview
- Theoretical framework
- The Adler function: experiment and QCD model
- Results for *R*-related quantities
- *R*-*D* self-duality
- Example: hadronic contribution to the muon g-2
- Summary

Short historical review

In 1974 S. Adler [Phys. Rev. D 10] observed that whereas R(s) (the normalized cross-section for the process e+e- annihilation into hadrons) is measured in the timelike region the natural place to compare experiment with scaling predictions of various theories in QCD is in the spacelike region and consequently he suggested to use the function T(−s) which is the first derivative of R(s). In 1976 A. De Rujula and H. Georgi [Phys. Rev. D 13] used a modified version of this idea, defining D(s) = −sT(−s). [-s= >0 in the Euclidean region] The function $D(Q^2)$ has been revived under the name of the ``Adler function". The D-function have been considered in many works, among which are following

S. Peris, M. Perrottet, E. de Rafael, Matching long and short distances in large-N_c QCD, J. High Energy (1998).

S. Eidelman, F. Jegerlehner, A.I. Kataev, O. Veretin, Testing non-perturbative strong interaction effects via the Adler function, Phys. Lett. B (1999).

K. A. Milton, I. L. Solovtsov, O. P. Solovtsova, Adler function for light quarks in analytic perturbation theory, Phys. Rev. D (2001).

A.E. Dorokhov, Adler function and hadronic contribution to the muon g-2 in a nonlocal chiral quark model, Phys. Rev. D (2004).

A.L. Kataev, Is it possible to check urgently the 5-loop analytical results for the $e^+ e^-$ - annihilation Adler function? Phys. Lett. B (2008).

A.V. Nesterenko, On the low-energy behavior of the Adler function, Phys. Rev. D (2008).

F. Jegerlehner, The running fine structure constant via the Adler function, hep-ph:0807.4206 (2008).

P.A. Baikov, K.G. Chetyrkin, J. H. Kuhn, Hadronic Z and τ decays in order α_s^4, Phys. Rev. Lett. (2008).

B.A. Magradze, Testing the Concept of Quark-Hadron Duality with the ALEPH τ Decay Data, Few Body Syst. (2010).

T.Goecke, C.S.Fischer, R. Williams, Leading-order calculation of hadronic contributions to the muon g-2 using the Dyson-Schwinger approach, Phys. Lett. B (2011).

P.A. Baikov, K.G. Chetyrkin, J.H. Kuhn, J. Rittinger, Adler function, sum rules and Crewther relation of order $O(\alpha_s^4)$: the Singlet Case. Phys. Lett. B (2012).

Theoretical framework

The main object in description of hadronic part of many physical processes is a correlator $\Pi(q^2)$

$$\Pi_{\mu\nu}(q^2) = i\int d^4x\, e^{iqx} < 0 | T V_\mu(x) V_\nu(0)^+ | 0 >$$

$$= (q_\mu q_\nu - g_{\mu\nu} q^2)\Pi(q^2)\ , \quad V_{ij}^\mu = \bar{\psi}_j \gamma^\mu \psi_i$$

$$D(Q^2) = -Q^2 \frac{d\Pi(-Q^2)}{dQ^2}\ , \quad Q^2 = -q^2 > 0$$

[in Euclidean (spacelike) region]

The integral representation for the D-function is given in terms of the the discontinuity of the correlator across the cut

$$D(Q^2) = Q^2 \int_0^\infty \frac{ds}{(s+Q^2)^2} R(s)$$

$$R(s) = \frac{1}{\pi} \operatorname{Im} \Pi(s)$$

[in Minkowskian (timelike) region]

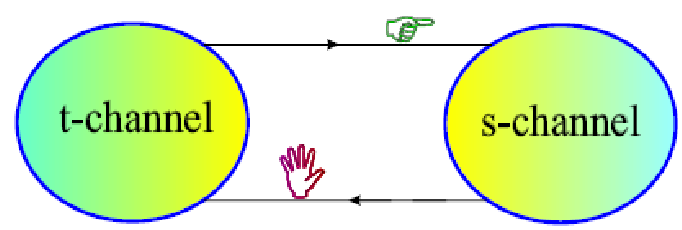
This representation defines the Adler function is an analytic function in the complex Q^2 plane with a cut along the negative real axis.

Relations between R and D function

To parameterize R(s) in terms of QCD parameters a procedure of analytic continuation from Euclidean (s-channel) to Minkowskian (t-channel) region is required.

$$D(Q^2) = Q^2 \int_0^\infty \frac{ds}{(s+Q^2)^2} R(s)$$

$$R(s) = -\frac{1}{2\pi i} \int_{s-i\varepsilon}^{s+i\varepsilon} \frac{dz}{z} D(-z)$$



Minkowskian $\Longleftrightarrow$ Euclidean

$$D \propto 1 + d\,, \qquad R \propto 1 + r$$

$$d(Q^2) = Q^2 \int_0^\infty \frac{ds}{(s+Q^2)^2}\ r(s)\,, \qquad r(s) = -\frac{1}{2\pi i} \int_{s-i\epsilon}^{s+i\epsilon} \frac{dz}{z}\ d(-z)$$

$$d(Q^2, RS) = a(Q^2, RS)\left[1 + d_1(RS)\, a(Q^2, RS) + d_2\,(RS) a^2(Q^2, RS) + \ d_3\,(RS) a^3(Q^2, RS)\right] \qquad a \equiv \alpha_s/\pi$$

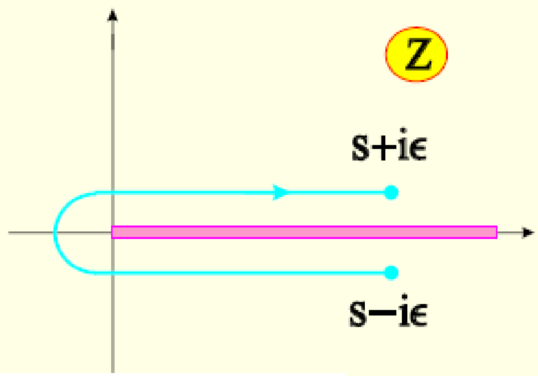
$$d_1^{\overline{\rm MS}} = 1.9857 - 0.1153\, n_f\,, \quad d_1^{\overline{\rm MS}} = 1.6398 \qquad (n_f = 3)$$

$$d_2^{\overline{\rm MS}} = 6.3710\,, \qquad d_3^{\overline{\rm MS}} = 49.08$$

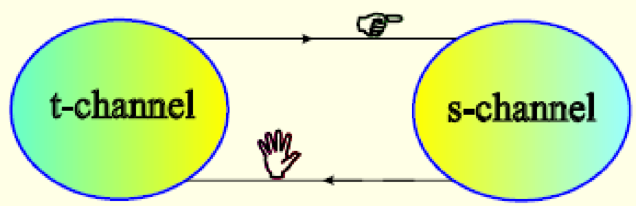
Baikov, Chetyrkin, Kuhn, RPL (2008)

Running coupling in the timelike region

$$\alpha_{\mathrm{E}}(Q^2) = Q^2 \int_0^\infty \frac{d\,s}{(s+Q^2)^2}\,\alpha_{\mathrm{M}}(s), \quad \alpha_{\mathrm{M}}(s) = -\frac{1}{2\pi \mathrm{i}} \int_{s-\mathrm{i}\,\epsilon}^{s+\mathrm{i}\epsilon} \frac{d\,z}{z}\,\alpha_{\mathrm{E}}(-z).$$



Contour of integration.



LO PT coupling

$$\alpha_{PT}^{E}(z) = \frac{1}{\beta_0}\frac{1}{\ln(-z)}\;, \quad \alpha_{PT}^{E}(z) \neq Q^2 \int_0^\infty \frac{ds}{(s-z)^2}\,\alpha_{PT}^{M}(s)$$

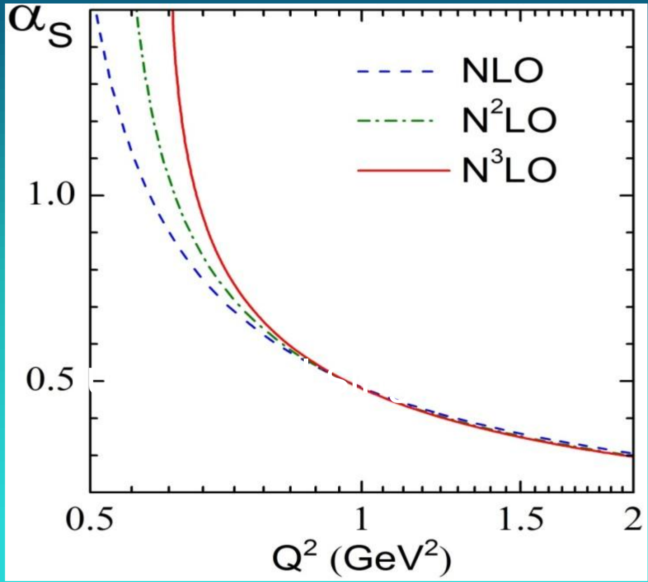
The perturbative approximation, in which the running coupling with unphysical singularities is used, breaks this connection between space and timelike quantities.

K.Milton, O.Solovtsova,
Phys. Rev. D 57 (1998) 5402.

Higher-loop PT orders not resolve this problem

$$\alpha_s^{(4)}(L) = \frac{1}{\beta_0 L} - \frac{1}{\beta_0^3 L^2}\beta_1 \ln L + \frac{1}{\beta_0^3 L^3}\left(\frac{\beta_1^2}{\beta_0^2}(\ln^2 L - \ln L - 1) + \frac{\beta_2}{\beta_0}\right) + \frac{1}{\beta_0^4 L^4}\left[\frac{\beta_1^3}{\beta_0^3}\left(-\ln^3 L + \frac{5}{2}\ln^2 L + 2\ln L - \frac{1}{2}\right) - 3\frac{\beta_1\beta_2}{\beta_0^2}\ln L - \frac{\beta_3}{2\beta_0}\right],$$

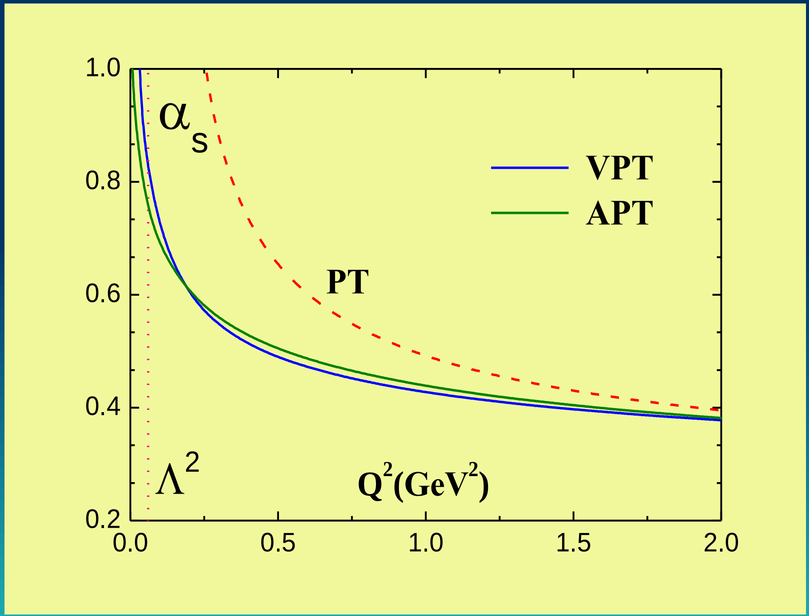
We concentrate on cases when a process can be described in terms of Minkowskian or Euclidean variables equivalently.
The simplest case -> running coupling
other case -> hadronic tau decays
one more case -> hadronic contribution to the lepton anomalous magnetic moment (AMM) and so on,



The approach used must maintains the required analytic properties.
The perturbative approximation, in which the running coupling with unphysical singularities is used, breaks the connection between space and timelike quantities.

The VPT /or APT approach (free from unphysical singularities) preserves the correct analytic properties and leads to a self-consistent definition of analytic continuation.

At small enough Q^2 the properties of the VPT/ APT expansion become considerably different from the PT power series.



Variation Perturbation Theory (VPT):

Sissakian, Solovtsov, «Variational expansions in quantum chromodynamics», Phys. Part. Nucl. 30, 461-487 (1999)].

Analytic perturbation theory (APT)

leads to very close result.

[D.V. Shirkov, I.L.Solovtsov, «Analytic QCD running coupling with finite IR behavior and universal alpha_s(0) value», Phys. Rev. Lett. 79 (1997) 1209; Theor. Math. Phys. 150 (2007) 132.]

On the method

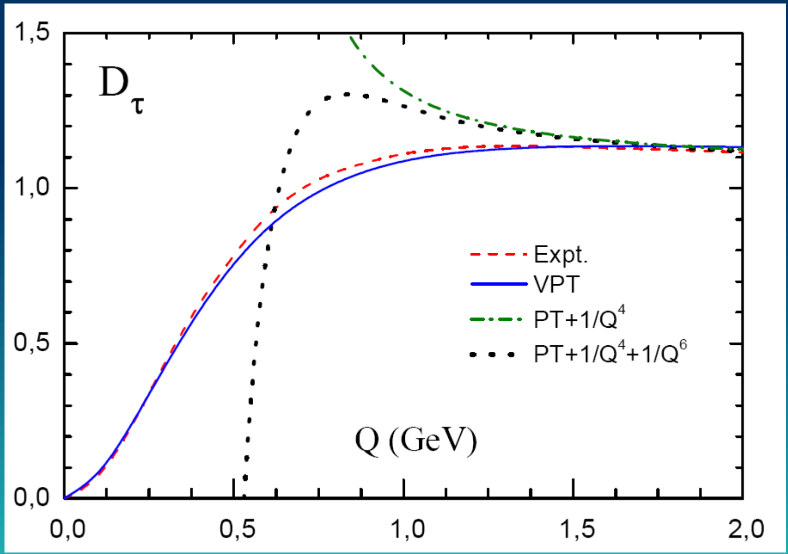
- The method based on the idea of variational perturbation theory (VPT) which combines an optimization procedure of variational type with a regular method of calculating corrections. In the case of QCD the non-perturbative expansion parameter, a, obeys an equation whose solutions are always smaller than unity for any value of the original coupling constant **[Solovtsov, PLB 327 and 340 (1994)].**

An important feature of this approach is the fact that for sufficiently small value of the running coupling the a -expansion reproduces the standard perturbative expansion, and, therefore, the perturbative high-energy physics is preserved.

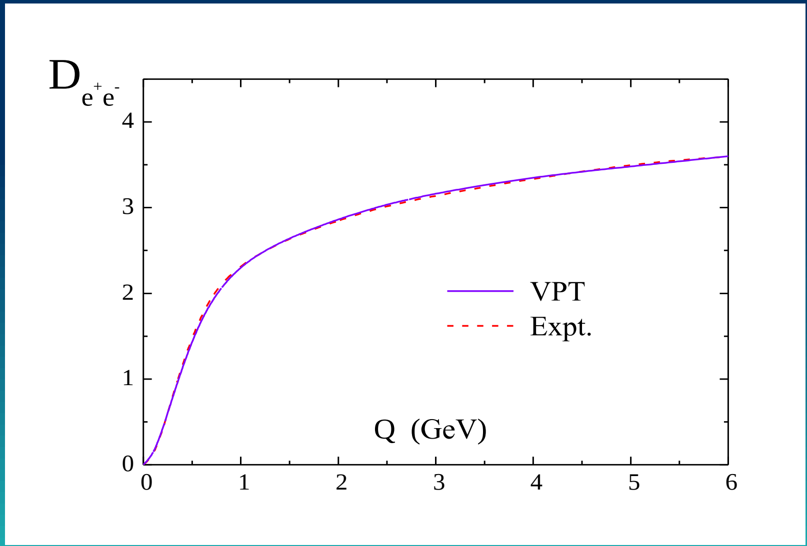
$$\lambda = \frac{g^2}{(4\pi)^2} = \frac{\alpha_s}{4\pi} = \frac{1}{C}\frac{a^2}{(1-a)^3}, \quad 0 \le a < 1$$

- Summation of threshold singularities was performed **[Milton-Solovtsov, Mod.Phys.Lett. A16 (2001) 2213].**
- Nonperturbative character of the light quark masses was taken into account **[Milton-Solovtsov, OS, Mod. Phys.Lett. A21 (2006) 1355].**

Behaviour of Adler D-function



Expt. and PT + OPE curves from the paper S. Peris, M. Perrottet, and E. de Rafael, JHEP 05, 011 (1998)



Data from the paper y S. Eidelman, F. Jegerlehner, A.I. Kataev, O. Veretin, Phys. Lett. B (1999)

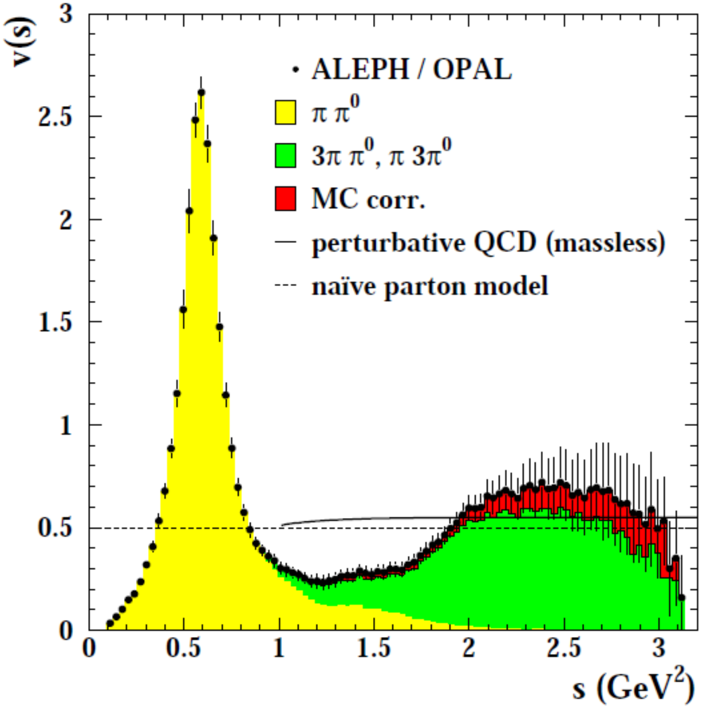
The experimental *D*-function (**dashed curve**) turned out to be a smooth and monotone function without traces of the resonance structure.

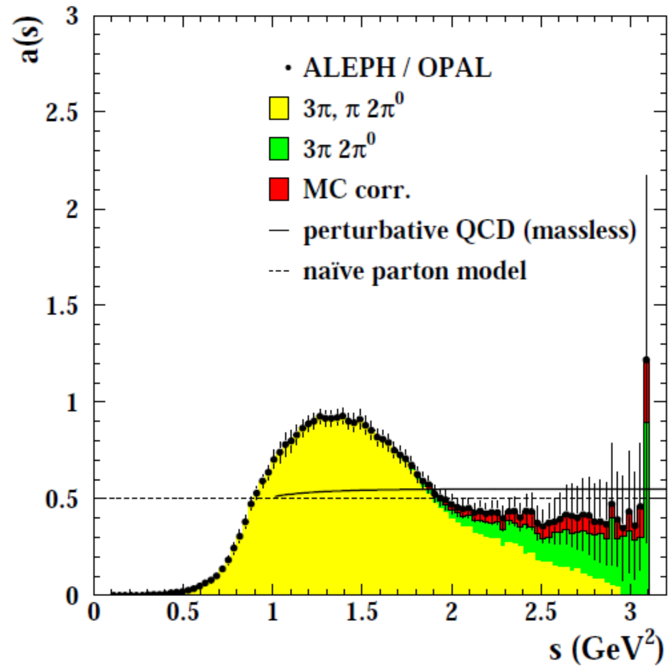
The theoretical approach VPT which we used in this study describes experimental curves rather well (**solid line**) for the whole interval including the infrared region.

Note that any finite order of the operator product expansion (OPE) fails to describe the infrared tail of the *D*-function (dotted curve) .

These Figs from the paper Sissakian-Solovtsov, OS, A Nonperturbative a -Expansion Technique and the Adler D –function, JETP Letts. (2001).

ALEPH / OPAL spectral functions from hadronic τ decays

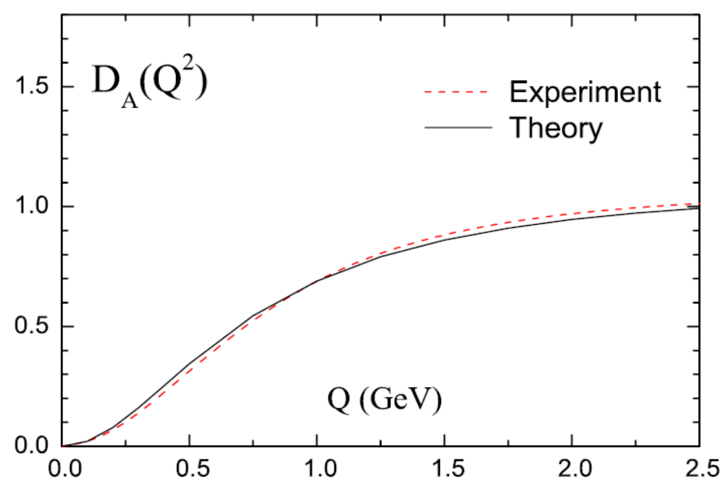




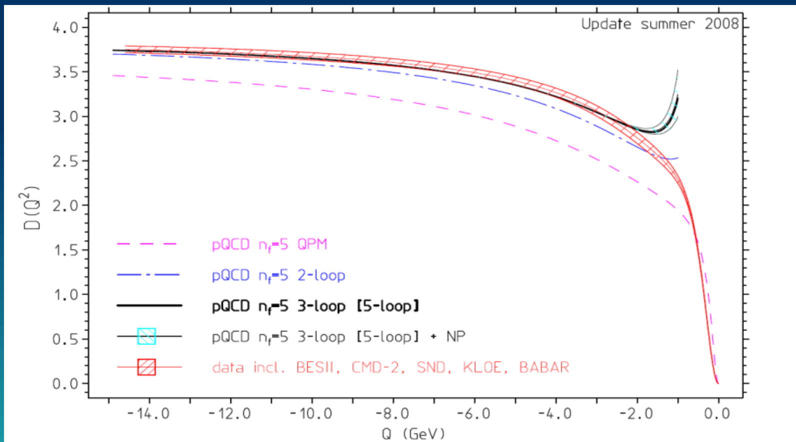
$$v(s)/a(s) \equiv \frac{1}{2} R_{V/A}(s)$$

$$D_{V/A}^{\exp/theor}(Q^2) = Q^2 \int_0^\infty \frac{R_{V/A}^{\exp/theor}(s)}{(s+Q^2)^2} ds$$

$$R_{V/A}^{\exp/theor}(s) = R_{V/A}^{\exp/theor}(s)\theta(s_0 - s) + R_{V/A}^{theor}(s)\theta(s - s_0)$$



The Adler function from $e^+e^- \to$ hadrons data (update)



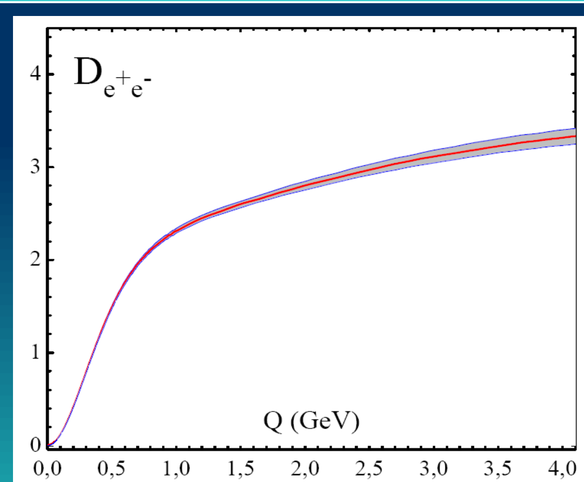
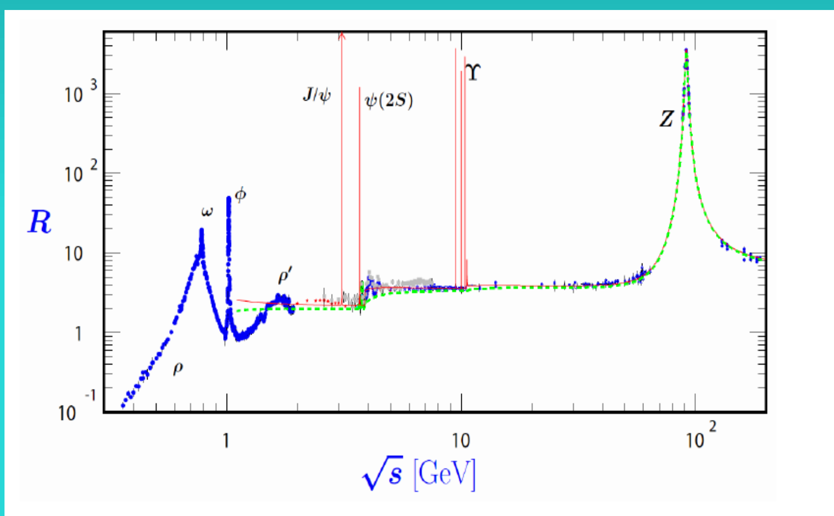


Figure from F. Jegerlehner

Data from F. Jegerlehner (2008)
Experimental errors are small.



The Adler function provides a way of comparing theoretical predictions from QCD with timelike experimental e+e− → hadrons data.

R-related quantities

- R_τ^V – inclusive τ-decay characteristic in the vector channel;
- D – light Adler function;
- smeared R_Δ-function;
- $a_\mu^{\rm had}$ – hadronic contribution to the anomalous magnetic moment of the muon;
- $\Delta\alpha_{\rm had}^{(5)}$ – hadronic contribution to the fine structure constant;
- model incorporates the lower condensates and describes well ρ-meson parameters.

To this list we add $a_e^{\rm had}$ and $a_\tau^{\rm had}$

A common feature of all these quantities and functions is that they are defined through the function R(s) integrated with some other function.

$$R_\tau = \frac{2}{\pi} \int_0^{M_\tau^2} \frac{ds}{M_\tau^2} \left(1 - \frac{s}{M_\tau^2}\right)^2 \left(1 + 2\frac{s}{M_\tau^2}\right) \mathrm{Im}\Pi(s)$$

All these quantities and functions include an infrared region as a part of the interval of integration and, therefore, they cannot be directly calculated within perturbative QCD.

Hadronic contribution to the anomalous magnetic moment of the leptons (in the leading order in the electromagnetic coupling constant)

$$a_l^{had} = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty \frac{ds}{s} K_l(s) R(s) \;, \quad l = \mu,\ e,\ \tau$$

K(s) - known QED kernel

The largest uncertainties in the SM prediction comes from hadronic parts.

The discrepancy between experiment and theory for the muon;s anomaly ~ 3 σ.

Some numerical result

In the case of the muon: $a_\mu^{had} = (694.9 \pm 3.7) \times 10^{-10}$ [one of set expt. result 2012]

$(702 \pm 16) \times 10^{-10}$ [our]

electron's anomaly: $a_e^{had} = (1.678 \pm 0.014) \times 10^{-12}$ [Nomura, Teubner 2013]

$(1.64 \pm 0.07) \times 10^{-12}$[our] (NP ~ 0.08×10^{-12}, arXiv:1208.6583)

tau lepton: $a_\tau^{had} = (3.38 \pm 0.04) \times 10^{-6}$ [Passera'07, Nomura'2012]

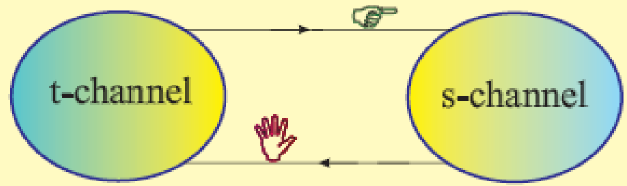
$(3.28 \pm 0.05) \times 10^{-6}$ [our]

$\Delta\alpha_{had}^{(5)}(M_Z^2) = (276.26 \pm 1.38) \times 10^{-4}$ [Hagivara et al. 2011] , $(279.9 \pm 4.0) \times 10^{-4}$ [our].

Good agreement for all considered quantities has been obtained.
The question : Wy?

In more general form:

$$Q_M = \int_0^\infty \frac{ds}{s}\, M(s) R(s),$$



Minkowskian $\Longleftrightarrow$ Euclidean

$$Q_E = \int_0^\infty \frac{dt}{t}\, E(t) D(t),$$

What we can say about connection between kernels M(s) and E(t) to get $\boxed{Q_M = Q_E}$? (R-D self-duality)

$$R(s) = \frac{1}{2\pi \mathrm{i}} \left[\Pi(s + \mathrm{i}\,\epsilon) - \Pi(s - \mathrm{i}\,\epsilon)\right] = -\frac{1}{2\pi \mathrm{i}} \int_{s-\mathrm{i}\,\epsilon}^{s+\mathrm{i}\,\epsilon} \frac{d\,z}{z}\, D(-z).$$

$$D(t) = -t\, \frac{d\,\Pi(-t)}{d\,t} = t \int_0^\infty ds\, \frac{R(s)}{(s+t)^2}$$

$$Q_E = \int_0^\infty \frac{dt}{t} E(t)\, t \int_0^\infty ds \frac{R(s)}{(s+t)^2} = \int_0^\infty \frac{ds}{s} \left[s \int_0^\infty dt \frac{E(t)}{(s+t)^2} \right] R(s).$$

Compare with

$$Q_M = \int_0^\infty \frac{ds}{s} M(s) R(s),$$

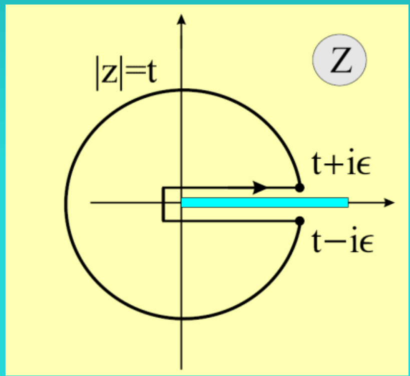
Answer

$$M(s) = s \int_0^\infty dt \frac{E(t)}{(s+t)^2}.$$

M(s) should be an analytic function with a cut along the negative real axis.

Inverse relation reads

$$E(t) = -\frac{1}{2\pi \mathrm{i}} \int_{t-\mathrm{i}\,\epsilon}^{t+\mathrm{i}\,\epsilon} \frac{d\,z}{z} M(-z).$$



$$E(t) = \frac{1}{2\pi \mathrm{i}} \oint_{|z|=t} \frac{d\,z}{z} M(-z).$$

The integration contour lies in the region of analyticity of the integrand and encircles the cut of M(-z) on the positive real z axis.

$a_\mu^{\rm had}$ - example

R-D self-duality presentations

$$a_\mu^{had} = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty \frac{ds}{s} K(s) R(s)\,, \quad a_\mu^{had} = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty \frac{dt}{t} E(t) D(t)$$

$$M(s) \equiv K(s) = = \int_0^1 dx \frac{x^2}{x^2 + (1-x)s/m_\mu^2}$$

$$\Rightarrow$$

$$E(t) = \frac{1}{2}\left[\frac{\sqrt{1+4m^2/t}-1}{\sqrt{1+4m^2/t}+1}\right]^2,$$

$$\eta = \frac{1-v}{1+v}, \qquad v = \sqrt{1-\frac{4m^2}{s}}.$$

The expressions in terms of R(s) and D(Q^2) are equivalent. If, however, one uses a method that does not maintain the required analytic properties these expressions will no longer are equivalent and will imply different results.

Answer on the question about a simultaneous good agreement of various QCD observables is:

An approach used is support required analytic properties and gives good description of the D-function **down to low energy scale.**

Summary

We have analyzed various physical quantities and functions generated by *R(s)* based on the nonperturbative VPT-method (Adler functions, hadronic contributions to anomalous magnetic moments of leptons and so on).

It was showed that the method allows us to describe these quantities well down to low energy scale.

We investigate the reason of such good agreement and as a result we formulate a criterion which we called as the R-D self-duality.

$$Q_M = \int_0^\infty \frac{dt}{t} E(t) D(t) \equiv Q_E$$

Thanks for your attention !