Gravity effects on the spectrum of scalar states on a thick brane

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QFTHEP-2013

Introductory notes

Brane-world models - popular models for possible BSM physics (for example for solving hierarchy problem without SUSY)
Thick brane models - brane is a domain wall generated dynamically by interaction of the fields in the bulk

Influence of gravity on (de)localization of matter fields on a brane

It was shown that gravity can nonperturbatively change the spectrum of localized states

- M.Giovannini, 2001, 2003
- A.Andrianov, L.Vecchi, 2007

} Disappearance of translational Goldstone

The reason is mixing of gravitational and matter degrees of freedom due to the symmetry under coordinate transformations
This can lead to low energy behavior different from the model neglecting gravity from start

Formulation of the model

Let's consider five-dimensional model with two interacting scalar fields minimally coupled to gravity in the bulk

$$X^A = (x^\mu, y), x^\mu = (x^0, x^1, x^2, x^3), \eta^{AB} = diag(+,-,-,-,-)$$

$$S = \int d^5X \sqrt{|g|} \left\{ -\frac{1}{2} M_*^3 R + Z \left(\frac{1}{2} \partial_A \Phi \partial^A \Phi + \frac{1}{2} \partial_A H \partial^A H - V(\Phi, H) \right) \right\}$$

If for example one scalar field form kink and another form bell-curve we can couple matter fields to them in a way to produce massive localized states in low-energy theory.

Background solutions

$\Phi = \Phi(y), H = H(y)$ - we restrict ourselves to background solutions not breaking 4D Lorentz invariance
Conformally flat ansatz for background metrics in gaussian coordinates (important for perturbation theory of gravitational constant)

$$g_{AB}dx^A dx^B = e^{-2\rho(y)} dx^\mu dx^\nu - dy^2$$

Equations on classical backgrounds

$$\rho'' = \frac{Z}{3M_*^3}(\Phi'^2 + H'^2), \quad \frac{2Z}{3M_*^3} V(\Phi, H) = \rho'' - 4(\rho')^2,$$
$$\Phi'' - 4\rho'\Phi' = \frac{\partial V}{\partial \Phi}, \quad H'' - 4\rho' H' = \frac{\partial V}{\partial H}.$$

In the limit of turned off gravity $\frac{Z}{M_*} \to 0$ the same equations as in the model without gravity

Small fluctuations around the background metric

Let's define the fluctuations of the metric h_{AB} and the scalar fields ϕ, χ on the background solutions of the equation of motion.

$$g_{\mu\nu}(X) = e^{-2\rho}(\eta_{\mu\nu} + h_{\mu\nu}(X)), g_{5\nu} = e^{-2\rho} v_\mu(X), g_{55} = -1 + S(X)$$

$$\Phi(X) = \Phi(y) + \phi(X), H(X) = H(y) + \chi(X),$$

A lot of redundant degrees of freedom! Separate the longitudinal and transverse components

$$h_{\mu\nu} = b_{\mu\nu} + \partial_\mu F_\nu + \partial_\nu F_\mu + \partial_\mu \partial_\nu E + \eta_{\mu\nu}\psi, \qquad v_\mu = v_\mu^\perp + \partial_\mu \eta,$$

$$\partial^\mu b_{\mu\nu} = 0, \qquad \partial_\mu F^\mu = 0, \qquad \partial^\mu v_\mu^\perp = 0$$

Then **up to quadratic order** different spins are separated from each other. We are interested in scalar sector.

Gauge invariance

Symmetry under coordinate transformations generates gauge symmetry changing graviscalars

$$\eta \to \eta - \zeta_5 - C', \quad E \to E - 2C, \quad \psi \to \psi + 2\rho'\zeta_5, \quad S \to S - 2\zeta_5',$$

and scalars

$$\phi \to \phi + \Phi'\zeta_5, \chi \to \chi + H'\zeta_5$$

We can rewrite our model in gauge invariant variables

$$\check{\eta} = E' - 2\eta + \frac{1}{\rho'}\psi, \quad \check{S} = S - \frac{1}{\rho'}\psi' + \frac{\rho''}{(\rho')^2}\psi$$

$$\check{\phi} = \phi + \frac{\Phi'}{2\rho'}\psi, \quad \check{\chi} = \chi + \frac{H'}{2\rho'}\psi$$

Note that we have to mix gravitational and matter degrees of freedom
$\check{\eta}$ - lagrange multiplier for constraint $\rho'\check{S} = \frac{2Z}{3M_*^3}(\Phi'\check{\phi} + H'\check{\chi})$
Only two independent variables $\check{\phi}$ and $\check{\chi}$

Equations on fluctuations

Representing fields as a sum of excitations with definite 4D mass we obtain equations

$$(-\partial_y^2 + \partial^2 V + \hat{\mathcal{M}}_{NP} - 2\rho'' + 4(\rho')^2) \begin{pmatrix} \phi^{(m)} \\ \chi^{(m)} \end{pmatrix} = e^{2\rho} m^2 \begin{pmatrix} \phi^{(m)} \\ \chi^{(m)} \end{pmatrix}$$

$$\hat{\mathcal{M}}_{NP} = \frac{2Z}{3M_*^3} \left[\frac{\rho'' + 4(\rho')^2}{(\rho')^2} \begin{pmatrix} (\Phi')^2 & \Phi' H' \\ \Phi' H' & (H')^2 \end{pmatrix} - \frac{1}{\rho'} \partial_y \begin{pmatrix} (\Phi')^2 & \Phi' H' \\ \Phi' H' & (H')^2 \end{pmatrix} \right],$$

In the limit of turned off gravity $\frac{Z}{M_*} \to 0$ the term $\hat{\mathcal{M}}_{NP}$ doesn't disappear completely and introduces the discontinuity

Theory with quartic potential

Can be induced by five-dimensional fermions (automatically producing mechanism for massive localized fermions)

$$S_{eff} = \frac{1}{2}M_*^3 \int d^5X \Big\{ -R + 2\lambda + \frac{3\kappa}{M^2}\Big(\partial_A\Phi\partial^A\Phi + \partial_A H\partial^A H + \\ +2M^2\Phi^2 + 2\Delta_H H^2 - (\Phi^2 + H^2)^2\Big)\Big\}$$

Assume that $\kappa = \frac{ZM^2}{3M_*^3}$ is small

$$\Phi'' = -2M^2\Phi + 4\rho'\Phi' + 2\Phi^3 + 6H^2\Phi,$$

$$H'' = -2\Delta_H H + 4\rho' H' + 2H^3 + 6\Phi^2 H,$$

$$\rho'' = \frac{\kappa}{M^2}\Big((\Phi')^2 + (H')^2\Big)$$

$$\lambda = -6(\rho')^2 + \frac{3\kappa}{2M^2}\Big\{(\Phi')^2 + (H')^2 + 2M^2\Phi^2 + 2\Delta_H H^2 - (\Phi^2 + H^2)^2 - M^4\Big\}$$

Background solutions

Let's assume $|\rho'|/M = O(\kappa)$ then we can neglect its contribution in the zeroth order

Massless phase

$$\Phi = M \tanh My + O(\kappa), H = 0$$

Massive phase (transition to massless phase at $\mu = 0$)

$$\Phi = M \tanh \tau + O(\kappa), H = \frac{\mu}{\cosh \tau} + O(\kappa),$$

$$\rho = \frac{2\kappa}{3} \ln \cosh \tau + \frac{\kappa}{6} \tanh^2 \tau + O(\kappa^2)$$

$$\tau \equiv M\beta y, \quad \beta = \sqrt{1 - \frac{\mu^2}{M^2}} + O(\kappa), \quad \Delta_H = \frac{M^2 + \mu^2}{2} + O(\kappa)$$

Background solutions: next order of κ

Treating both κ and μ/M as perturbation parameters. The choice of gaussian coordinates is important.

$$\begin{aligned}\Phi &= M\tanh\tau - M\kappa\frac{2}{9}\frac{\tanh\tau}{\cosh^2\tau} - \\ &-M\kappa\left(\frac{\mu}{M}\right)^2\left[\frac{\tau}{\cosh^2\tau}\left(\frac{26}{27}+\frac{2}{27}C^H_{1,0}+\frac{4}{27}\ln 2-\frac{1}{2}\left(\frac{1}{\beta^2}\right)_{1,1}\right)+\right.\\ &\left.+\frac{2}{9}\frac{\tanh\tau}{\cosh^2\tau}+\frac{1}{27}\frac{\mathrm{Li}_2(-e^{-2\tau})-\mathrm{Li}_2(-e^{2\tau})}{\cosh^2\tau}\right]+O\left(\kappa\left(\frac{\mu}{M}\right)^4\right)\end{aligned}$$

$$H=\frac{\mu}{\cosh\tau}+M\kappa\left(\frac{\mu}{M}\right)^3\frac{2}{27\cosh\tau}\left(C^H_{1,0}-2\log\cosh\tau+3\tanh^2\tau\right)+O\left(\kappa\left(\frac{\mu}{M}\right)^5\right)$$

where $C^H_{1,0}=-7-2\ln 2$

Near critical point

$$\beta = 1-\frac{2}{3}\kappa+O(\kappa^2),\quad \Delta^{(c)}_H = M^2\left(\frac{1}{2}-\frac{22}{27}\kappa+O(\kappa^2)\right)$$

Spectrum in scalar sector

If we neglect gravity from start - two localized states

- massless Goldstone boson from translational symmetry breaking

$$\phi = \Phi', \chi = H'$$

- light massive Higgs-like boson

$$\chi = \frac{1}{\cosh \tau} + O\left(\frac{\mu^2}{M^2}\right), \phi = O\left(\frac{\mu}{M}\right)$$

with mass $m^2 = 2\mu^2 + O(\mu^4/M^2)$
(A. & V.Andrianovs, P.Giacconi, R. Soldati,2003)

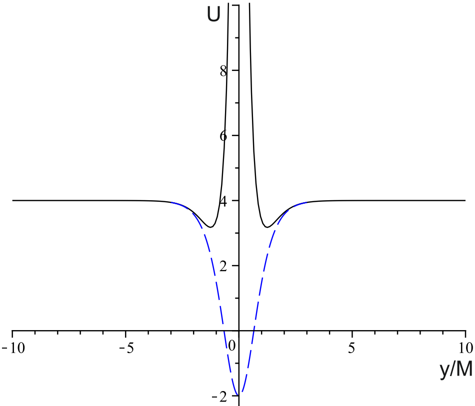
Let's consider now what is spectrum if we account for nonperturbative corrections

Critical point: ϕ-channel

At critical point $H = 0$, ϕ and χ channels decouple. In the limit $\kappa \to 0$

$$\left(-\partial_\tau + \frac{\rho''_{,1}}{\rho'_{,1}} - \frac{\Phi''}{\Phi'}\right)\left(\partial_\tau + \frac{\rho''_{,1}}{\rho'_{,1}} - \frac{\Phi''}{\Phi'}\right)\phi = \left(\frac{m^2}{M^2\beta^2}\right)\phi$$

The centrifugal barrier $U \sim 2/\tau^2$ - neither zero-mode nor resonant states exists (A.Andrianov, L.Vecchi,2007)



Critical point: χ-channel

At critical point $H = 0$, ϕ and χ channels decouple

χ-channel in the limit $\kappa \to 0$ smoothly reproduces the model without gravity

$$\Big(-\partial_\tau + \tanh\tau\Big)\Big(\partial_\tau + \tanh\tau\Big)\chi = \Big(\frac{m^2}{M^2\beta^2}\Big)\chi$$

There is a zero-mode as in the model without gravity. It's still zero-mode if we take into account the next order in κ corrections

$$\chi_0 = (1 + C^{\chi}_{0,1}\kappa)\frac{1}{\cosh\tau} + \kappa\frac{1}{2}\frac{4\cosh^2\tau\ln(2\cosh\tau) + 2\ln 2 + 3}{\cosh\tau(2\cosh^2\tau + 1)} + O(\kappa^2)$$

The mass of the Higgs-like boson

Treating both κ and μ/M as small perturbation parameters we can find what mass gains the mode in the χ-channel. Our choice of gauge invariant variables is appropriate for perturbation expansion.

$$m^2 = M^2 \sum_{n,k}^{\infty} \kappa^n \left(\frac{\mu}{M}\right)^k (m^2)_{n,k}$$

Leading order is the same as in the model without gravity

$$(m^2)_{0,1} = 2$$

NLO however is different:

$$(m^2)_{0,2} = -128\sqrt{3}\,\mathrm{arctanh}\frac{\sqrt{3}}{3} + 146 + \frac{4}{3}\ln 2 \cdot (1 + \ln 2) - \frac{\pi^2}{9} \approx +0.4817$$

Whereas analogous computation for similar model without gravity gives,

$$(m^2)^{NG}_{0,2} = -\frac{130442}{121275} \approx -1.0756$$

Estimates of the characteristic scales of the model

Low-energy effective theory (including only the lightest states)

$$\mathcal{L}_{low} = \partial_\mu h \partial^\mu h - m_h^2 h^2 + \bar{\psi}\Big(i\partial\!\!\!/ - m_f - g_f h\Big)\psi,$$

Assume that h is the observed Higgs-like boson. Then phenomenological λ of Higgs potential is related to the scales of the model in the following way

$$M = \sqrt{3\sqrt{\lambda}\; k M_P}; \quad \kappa = \frac{1}{2\sqrt{\lambda}} \frac{M}{M_P}$$

where M_P is a 4D Planck mass, k is a AdS space curvature $k > 0.004eV$ (Adelbeger et al, 2009)

$$M > 3.5\,TeV; \quad M_* > 3 \cdot 10^8\,GeV; \quad \kappa > 2 \cdot 10^{-15}.$$

Conclusions

The mixing of the gravitational and scalar degrees of freedom drastically changes the spectrum in the scalar sector

- The model is formulated in the gauge invariant variables
- The phase with nonzero H can be studied with help of perturbation theory from critical point. The choice of the gauge invariant variables and gaussian coordinates is important for good perturbation expansions
- The centrifugal potential leads to the absence of the massless Goldstone mode associated with translational symmetry breaking
- Though in the leading order the mass of the Higgs-like boson happens to be the same as in the model neglecting gravity from start, the difference appears already in the next to leading order

Thank you for your attention!