Reaction-diffusion approach in soft diffraction

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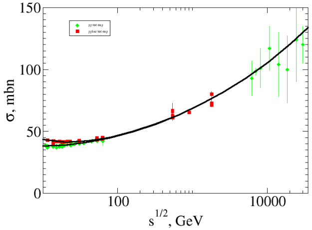
Power-like contributions to the amplitude



PDG fit:

$$\sigma_{tot}^{pp(\bar{p})} = 18.3 s^{0.095} + 60.1 s^{-0.34} \pm 32.8 s^{-0.55}$$

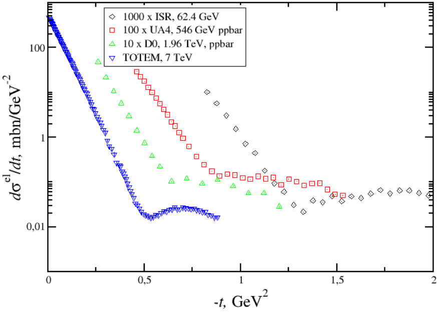
Optical theorem:

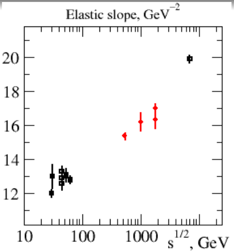
$$\sigma_{tot} = \frac{1}{s} 2\Im A_{el}(q=0) \equiv 2\Im M_{el}(q=0)$$

Indication:

- High energy elastic scattering goes via quasiparticle ("Reggeon") exchanges with powerlike asymptotic in c.m.energy.
- Leading contirbution – Pomeron, $M_{\mathbb{P}} \sim s^{\Delta} \sim e^{\Delta y}$, ($\Delta > 0$ $y = \ln s$ – overall rapidity)

Elastic scattering – shrinkage of diffractive cone





$$\frac{d\sigma_{el}}{dt} = \frac{1}{2\pi}|M(s,t)|^2$$

$$B \equiv -\frac{d}{dt}\ln\frac{d\sigma^{el}}{dt}\Big|_{t=0}$$

$$M(s = e^y, t) \sim \exp[\Delta y - (R^2 + \alpha' y)t]$$

- implies reasonable assumptions about the analytic properties of $T(s,t)$

Caveat:

The power-like behaviour violates unitarity bound ($\sigma_{tot} \lesssim C\ln^2 s$).

Impact parameter representation

Fourier transform:

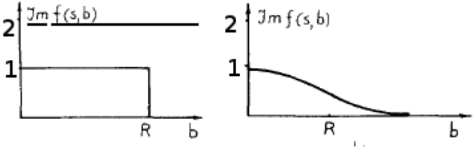
$f(Y, \mathbf{b}) = \frac{1}{(2\pi)^2} \int d^2 q \, e^{-i\mathbf{q}\mathbf{b}} M(Y, \mathbf{q})$.

- $\sigma^{\rm el} = \int \frac{d^2 q}{(2\pi)^2} \, |M(Y, \mathbf{q})|^2 = \int d^2 b \, |f(Y, \mathbf{b})|^2$.
- $\sigma^{\rm tot}(Y) = 2 \, \Im M(Y, \mathbf{q} = 0) = 2 \int d^2 b \, \Im f(Y, \mathbf{b})$,

<u>**Definition**</u> $\sigma^{\rm inel}(b) \equiv 2\Im f(\mathbf{b}) - |f(\mathbf{b})|^2$
Unitarity constraint: $0 < \Im f(b) < 2 \quad \Rightarrow \quad 0 \leq \sigma^{\rm inel}(b) \leq 1$

Interpretation: $\sigma^{\rm inel}(b) \equiv$ ***probability of inelastic interaction***

Geometrical models



Unitarity limit: $f(b) = 2\theta(R-b) \Rightarrow \sigma^{\text{inel}}(b) = 0.$
Black disk limit: $f(b) = \theta(R-b) \Rightarrow \sigma^{\text{inel}}(b) = \theta(R-b),\ \sigma^{\text{el}} = 1/2\sigma^{\text{tot}}.$

The data suggest:

- shrinkage of diffracitve cone $\Rightarrow$ growing size
- presence of dip $\Rightarrow$ deviations from Gaussian profile shape
- The inelastic profile in the center is close to the upper limit (e.g. $\sigma^{\text{inel}}(b) = 0.94$ at $\sqrt{s} = 53$ GeV)

Inelastic diffraction – a special case of inelastic event

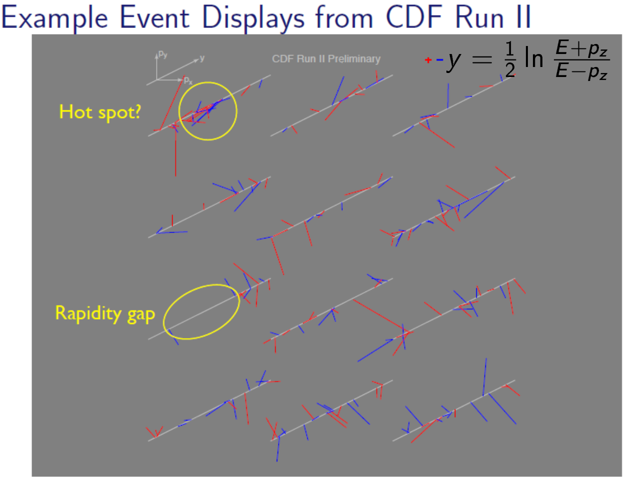
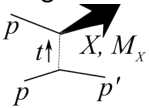


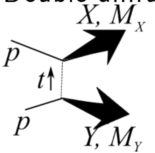
Illustration: talk by Chris Quigg at Spaatind'2012

Soft inelastic diffraction

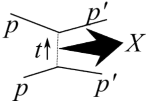
- Single diffraction

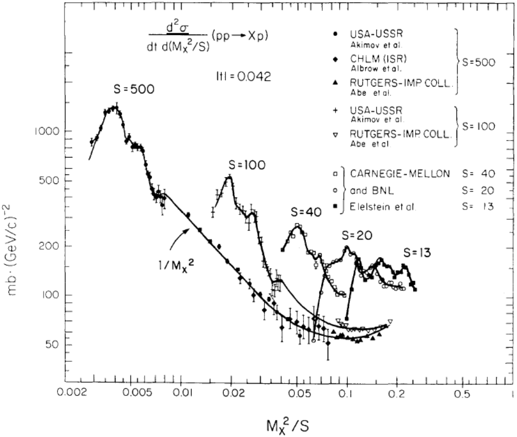


- Double diffraction



- Central diffraction





$\Delta y_{\rm gap} = \ln s/M_X^2$ – rapidity gap

$\sigma_{SD} \sim 10$ mbn @ 7TeV [TOTEM preliminary @ Trento pA workshop 05/2013]

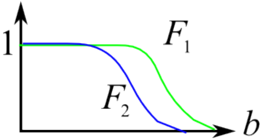
Multichannel approach (Good, Walker '60):

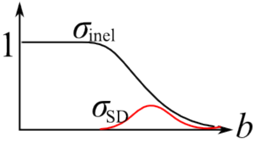
$$|p\rangle = \alpha_1|1\rangle + \alpha_2|2\rangle; \quad \alpha_1^2 + \alpha_2^2 = 1$$

$|1\rangle$, $|2\rangle$ – scattering eigenstates (amplitudes $iF_1(y,b)$ and $iF_2(y,b)$)

$$\sigma^{\rm tot} = 2\int d^2b[\alpha_1^2F_1(b)+\alpha_2^2F_2(b)]; \quad \sigma^{\rm el} = \int d^2b[\alpha_1^2F_1(b)+\alpha_2^2F_2(b)]^2$$

$$\sigma^{\rm SD} = \int d^2b\,[\alpha_1\alpha_2(F_1(b)-F_2(b))]^2$$





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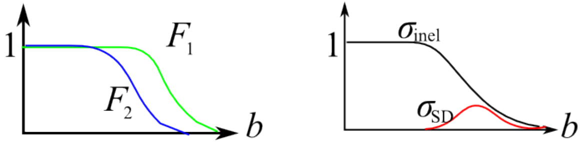


Lessons from the example

1: Has a peripheral nature

2: Black disc limit for the elastic amplitude implies $\sigma_{diffr} \sim \ln s$: (growing ring).

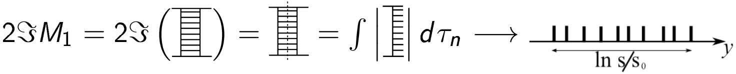
This holds also for large-M^2 diffraction which however has a different origin.

Contributions to σ_{tot} from inelastic cuts

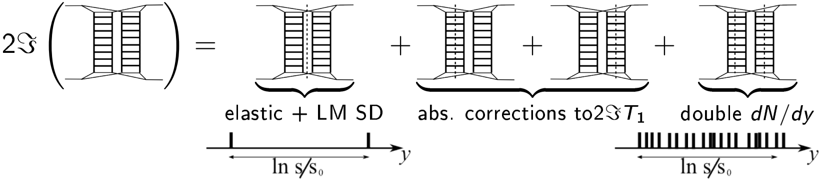
Contributions to imaginary part (Cutkosky rules):

- Cut the diagram for the elastic scattering amplitude
- Put cut lines on the mass shell, integrate over the phase space

Single "ladder" exchange – uniform rapidity distribution



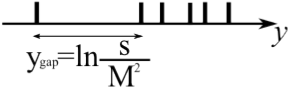
Double "ladder"

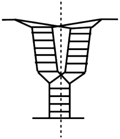


High-M^2 diffraction

Rapidity gaps – splitting of the “ladder”:

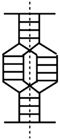
Single diffraction dissociation





+ abs. corrections

Double diffraction dissociation



+ abs. corrections

- Motivates the effective theory of the Pomeron (Reggeon) exchanges and interactions
- Multy-$\mathbb{P}$ exchantes, enhanced & loop graphs
  - Tame the growth, restore *s*-channel unitarity
  - Give inelastic contributions with rapidity gaps

Account of *all* enhanced graphs is an untrivial task

Systematic account of enhanced graphs – RFT

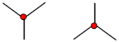
The elastic amplitude $iT \equiv A/(8\pi s)$ is factorized:

$$T = \sum_{n,m} V_n \otimes G_{nm} \otimes V_m$$

G_{mn} – process independent, obtained within 2D+1 field theory (only $\mathbb{P}$):

$$\mathcal{L} = \frac{1}{2}\phi^\dagger(\overleftarrow{\partial_y} - \overrightarrow{\partial_y})\phi - \alpha'(\nabla_\mathbf{b}\phi^\dagger)(\nabla_\mathbf{b}\phi) + \Delta\phi^\dagger\phi + \mathcal{L}_{int}.$$

Minimal choice (classic): $\mathcal{L}_{int} = i\, r_{3P}\phi^\dagger\phi(\phi^\dagger + \phi)$



Infinite $\sharp$ of vertices [KMR, Ostapchenko, MP+ABK]: $r_{mn}\phi^m{\phi^\dagger}^n$

Fine tuning of the vertices, some contributions neglected

Systematic account of enhanced graphs – RFT

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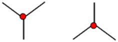
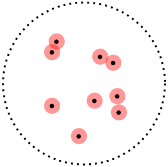


Infinite $\sharp$ of vertices [KMR, Ostapchenko, MP+ABK]: $r_{mn}\phi^m{\phi^\dagger}^n$

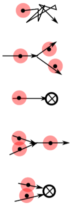
"Almost minimal": $i\, r_{3P}\phi^\dagger\phi(\phi^\dagger + \phi) + \chi{\phi^\dagger}^2\phi^2$

the reaction-diffusion approach is applicable for numerical computation of all-loop Green functions. [Grassberger'78; Boreskov'01]

The reaction-diffusion (stochastic) approach.



Consider a system of classic "partons" in the transverse plane with:



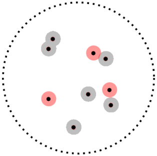
- Diffusion (chaotical movement) D;
- Splitting (λ – prob. per unit time)
- Death (m_1)
- Fusion ($\sigma_\nu \equiv \int d^2 b\, p_\nu(b)$)
- Annihilation ($\sigma_{m_2} \equiv \int d^2 b\, p_{m_2}(b)$)

Parton number and positions are described in terms of

$$\boxed{\textbf{probability densities}\ \rho_N(y, \mathcal{B}_N)\ (N = 0, 1, ...; \mathcal{B}_N \equiv \{b_1, \ldots, b_N\})}$$

with normalization $p_N(y) \equiv \frac{1}{N!} \int \rho_N(y, \mathcal{B}_N) \prod d\mathcal{B}_N; \quad \sum_0^\infty p_N = 1.$

S-parton inclusive distributions:



$$f_s(y;\mathcal{Z}_s)=\sum_N\frac{1}{(N-s)!}\int d\mathcal{B}_N\ \rho_N(y;\mathcal{B}_N)\prod_{i=1}^{s}\delta(\mathbf{z}_i-\mathbf{b}_i);$$

$\int d\mathcal{Z}_s f_s(y;\mathcal{Z}_s)=\sum\frac{N!}{(N-s)!}\,p_N(y)\equiv\mu_s(y)$. – factorial moments.

Example: Start with a single parton with only diffusion and splitting allowed.

$$f_1^{1\ \text{parton}}(y,b)=\frac{\exp(\lambda y)\exp(-b^2/4Dy)}{4\pi Dy}.$$

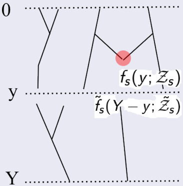
– the bare Pomeron propagator in b-representation.

The set of evolution equations for $f_s(\mathcal{Z}_s)$, $(s=1,\ldots)$ coincides with the set of equations for the Green functions of the RFT.

The amplitude.

To compute the RFT elastic amplitude:

- Hadron–$n\mathbb{P}$ vertices $\Rightarrow$ distribution of "partons" at $y=0$ evolution time: $\tilde{f}_s(0,\mathcal{Z}_s) = N_s(\mathcal{Z}_s)/\epsilon^{s/2}$
- MC evolution $\Rightarrow$ set of $f_s(y,\mathcal{B}_s)$ ($\tilde{f}_s(y,\tilde{\mathcal{B}}_s)$) for the projectile (target)
- With some narrow $g(b)$, $\int g(b) d^2b \equiv \epsilon$:



$$T(Y,b) \equiv \Im M(Y,b) = \langle A|T|\tilde{A}\rangle =$$

$$= \sum_{s=1}^{\infty} \frac{(-1)^{s-1}}{s!} \int d\mathcal{Z}_s d\tilde{\mathcal{Z}}_s f_s(y;\mathcal{Z}_s) \tilde{f}_s(Y-y;\tilde{\mathcal{Z}}_s) \prod_{i=1}^{s} g(\mathbf{z}_i - \tilde{\mathbf{z}}_i - \mathbf{b}).$$

T **does not depend on the linkage point** y ("boost invariance") **if**

$$\lambda \int g(b) d^2b = \int p_{m_2}(b) d^2b + \frac{1}{2}\int p_\nu(b) d^2b \ ,$$

$\Leftrightarrow$ equality of fusion and splitting vertices in the RFT.

We use the simplest form of $g(b)$, $p_{m_2}(b)$ and $p_\nu(b)$:

$$p_{m_2}(\mathbf{b}) = m_2\ \theta(a - |\mathbf{b}|); \quad p_\nu(\mathbf{b}) = \nu\ \theta(a - |\mathbf{b}|);$$

$$g(\mathbf{b}) = \theta(a - |\mathbf{b}|);.$$

with a – some small scale; $\epsilon \equiv \pi a^2$.

RFT	stochastic model
Rapidity y	Evolution time y
Slope α'	Diffusion coefficient D
$\Delta = \alpha(0) - 1$	$\lambda - m_1$
Splitting vertex r_{3P}	$\lambda\sqrt{\epsilon}$
Fusion vertex r_{3P}	$(m_2 + \frac{1}{2}\nu)\sqrt{\epsilon}$
Quartic coupling χ	$\frac{1}{2}(m_2 + \nu)\epsilon$

Few things to note:

Boost invariance ($\lambda = m_2 + \frac{\nu}{2}$) $\Leftrightarrow$ equality of fusion and splitting vertices

The $2 \to 2$ vertex cannot be set to zero ($m_2, \nu > 0$).

Calculation method – elastic amplitude

Convenient choice – set the linkage point to target rapidity:

- $\tilde{f}_s(y = 0, \mathcal{Z}_s) = N_s(\mathcal{Z}_s)/\epsilon^{s/2}$
- for a given realization via MC evolution
  $$f_s^{\mathrm{sample}}(Y, \mathcal{Z}_s) = \sum_{\{\hat{\mathbf{x}}_{i_1},..,\hat{\mathbf{x}}_{i_s}\} \in \hat{\mathcal{X}}_N} \delta(\mathbf{z}_1 - \hat{\mathbf{x}}_{i_1}) \ldots \delta(\mathbf{z}_s - \hat{\mathbf{x}}_{i_s})$$
- $T^{el}_{\mathrm{sample}} = \sum_{s=1}^{N} (-1)^{s-1} \tilde{\mu}_s \epsilon^s \sum_{i_1 < i_2 \ldots < i_s} \tilde{p}_s(\hat{\mathbf{x}}_{i_1} - \mathbf{b}, \ldots, \hat{\mathbf{x}}_{i_s} - \mathbf{b}).$

Calculation method – the SD cut

For the SD cut substituting "event-by-event Green functions" gives

$$T^{SD}_{\rm sample} = 2\,T^{el}_{\rm sample} - T'_{\rm sample}$$

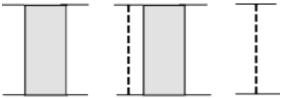
$T'_{\rm sample}$ is computed the same way as $T^{el}_{\rm sample}$ with two distinctions:

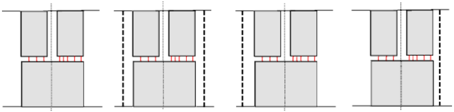
- Not one, but two sets from the projectile side
- which are evolved independently until the Δy_{gap} and then combined into a single one

Resumé: The elastic scattering amplitude and its SD cut are computed within the same numerical framework.

Model parameters

- Two-channel eikonal p–$n\mathbb{P}$ vertices to incorporate low-M^2 diffraction
- Account the secondary Reggeons contribution to the lowest order
- Real part of the Pomeron exchange amplitude evaluated via Gribov–Migdal relation
- Neglect central diffraction in calculation of SD cross sections (CD contribution is accounted twice in calculation of 2-side SD, the extra contribution should have been subtracted).





Model parameters

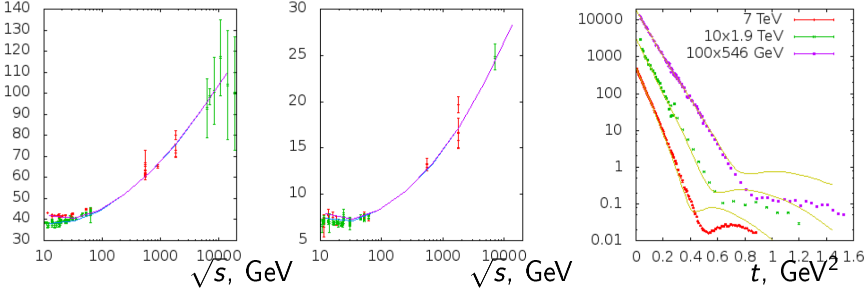
$r_{3\mathbb{P}}$ – fixed from [Kaidalov'79]
a – regularization scale
$1+\Delta$ – bare Pomeron intercept
α' – Pomeron slope
$|p\rangle = \beta_1|1\rangle + \beta_2|2\rangle; \quad |\beta_1|^2 \equiv C_1;\ |\beta_2|^2 \equiv C_2 = 1 - C_1.$
$\mathbb{P}$ couplings to $|1\rangle$ and $|2\rangle$: $g_{1/2} = g_0(1 \pm \eta)$
R_1, R_2 – size of the p–$\mathbb{P}$ vertex (Gaussian)
Strategy:

1. Eikonal fit to σ_{tot}, σ_{el}, $d\sigma_{\mathrm{el}}/dt$ keeping low-M^2 $\sigma_{SD} \approx 1.5$mbn at $\sqrt{s} = 35\,GeV/c$
2. All-loop fit to σ_{tot}, σ_{el}, $d\sigma_{\mathrm{el}}/dt$ starting with parameter set from [1]
3. Calculation of diffractive cross sections with parameters obtained at [2]

Calculation results

Total and elastic cross sections:

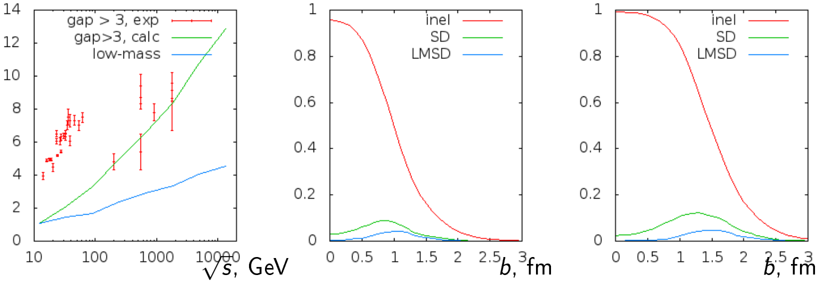
$\sigma_{tot}(\sqrt{s})$, mbn $\quad$ $\sigma_{el}(\sqrt{s})$, mbn $\quad$ $\frac{d\sigma_{el}}{dt}(t)$, mbn GeV^{-2}



$\Delta = 0.19$; $\alpha' = 0.236$ GeV^{-2};
$C_1 = 0.1$, $C_2 = 1 - C_1 = 0.9$; $R_1 = 0.51$ GeV^{-1}; $R_2 = 2.8$ GeV^{-1}; $g_1 = 46.7$ GeV^{-1}; $g_2 = 11.7$ GeV^{-1};
$r_{3P} = 0.087$ GeV^{-1} [Kaidalov'79].

Single diffraction

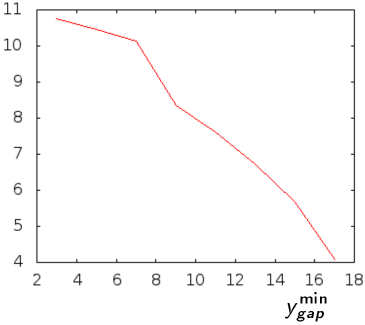
$\sigma_{SD}(\sqrt{s})$, mbn Profile, $\sqrt{s} = 240$ GeV/c Profile, $\sqrt{s} = 13.5$ TeV/c

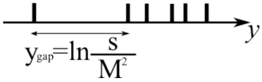


M^2 (rapidity gap) dependence, preliminary

Single-diffractive cross section as a function of $y_{\rm gap}^{min}$, $\sqrt{s} = 5\,TeV$:
(linear behaviour corresponds to $1/M_X^2$-scaling of $d\sigma/dM^2$)

$\sigma_{SD}(y_{gap}^{min})$, mbn



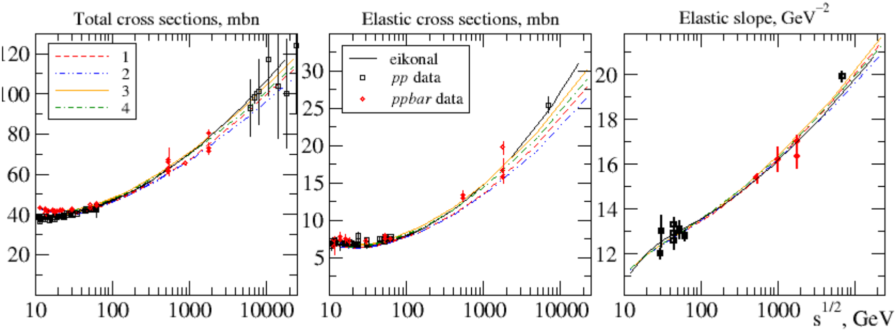


$$\frac{d}{dy_{gap}^{min}} = -M_X^2 \frac{d}{dM_X^2}$$

Conclusions

- Total, elastic and single diffractive cross sections are computed in RFT within the same numerical framework to all orders in the number of loops;
- A satisfactory description on total and elastic cross sections is obtained within the all-loop framework;
- The single diffractive cross sections energy behaviour is compatible with logarithmic growth.

Backup – scale and 4$\mathbb{P}$ vertex dependence

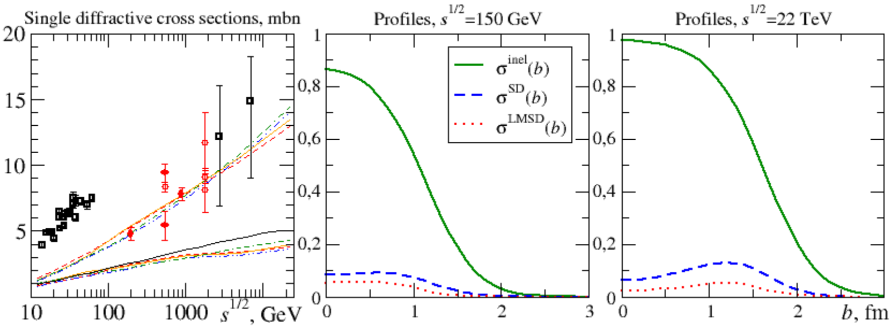


$\chi_3 > \chi_1 = \chi_4 > \chi_2$; $a_1 = a_2 = 0.018$ fm; $a_3 = a_4 = 0.036$ fm. $C_1 = C_2 = 0.5$, $\eta = 0.55$.
$\Delta = 0.195$; $\alpha' = 0.154$ GeV^{-2}; $R^2 = 3.62$ GeV^{-2}; $g_0 = 4.7$ GeV^{-1}; $r_{3P} = 0.087$ GeV^{-1} [Kaidalov'79].

Fits with $C_1 = C_2 = 0.5$ and $R_1 = R_2$. Much worse description of $\frac{d\sigma_{el}}{dt}$ at larger t compared to fits with $C_1 \neq C_2$ and $R_1 \neq R_2$ (though still a nice fit of slope B)

Backup – scale and 4$\mathbb{P}$ vertex dependence

Inelastic and diffractive profiles



Backup – secondary trajectories

pp: $\Im f_{pp}(b) = \Im A_P(b) + [\Im A_+(b) + \Im A_-(b)]\,[1 - \Im A_P(b)]$

$\Re f_{pp}(b) = \left[\Re A_{R_+} + ReA_{R_-}\right][1 - \Im A_P(b)]$

pp: $\Im f_{pp}(b) = \Im A_P(b) + [\Im A_+(b) - \Im A_-(b)]\,[1 - \Im A_P(b)]$

$\Re f_{pp}(b) = \left[\Re A_{R_+} - ReA_{R_-}\right][1 - \Im A_P(b)]$

pp SD:

$$f_{pp}^{\mathrm{Diff}}(b) = f_{pp}^{\mathrm{Diff}}(b)\Big|_{\mathbb{P}\mathrm{only}}\left[1 + |A_{R_+}(b) + A_{R_-}(b)|^2 - 2\Im(A_{R_+}(b) + A_{R_-}(b))\right]$$

$$A_\pm(y,b) = \eta_\pm \beta_\pm^2 \frac{\exp(\Delta_\pm y)}{2\alpha'_\pm y + 2R_\pm^2} \exp\left(-\frac{b^2}{4(\alpha'_\pm y + R_\pm^2)}\right)$$

$$\eta_\pm = \pm i - \frac{1 \pm \cos \pi\alpha_\pm(0)}{\sin \pi\alpha_\pm(0)}$$