LAGRANGIAN ALTERNATIVE TO QCD STRING

Alisa Katanaeva
and Sergey Afonin

Saint-Petersburg State University

The XXI International Workshop
High Energy Physics and Quantum Field Theory
June 29, 2013

What methods to receive the spectra of radial excitations are frequently used in hadron spectroscopy?

1. The nonrelativistic potential models – their best feature is the description of heavy quark systems. They didn't always give correct answers for light mesons, however.
2. The hadron string models – the string is an approximation for the gluon flux-tube with nearly massless quarks at the ends.
   The semiclassical quantization of a thin flux-tube typically gives a linearly rising spectrum for masses squared:

$$\boxed{m_k^2 = a(k+b), \qquad k = 0, 1, 2, \dots} \tag{1}$$

What methods to receive the spectra of radial excitations are frequently used in hadron spectroscopy?

1. The nonrelativistic potential models – their best feature is the description of heavy quark systems. They didn't always give correct answers for light mesons, however.
2. The hadron string models – the string is an approximation for the gluon flux-tube with nearly massless quarks at the ends.
   The semiclassical quantization of a thin flux-tube typically gives a linearly rising spectrum for masses squared:

$$m_k^2 = a(k+b), \qquad k = 0, 1, 2, \dots \tag{1}$$

What methods to receive the spectra of radial excitations are frequently used in hadron spectroscopy?

1. The nonrelativistic potential models – their best feature is the description of heavy quark systems. They didn't always give correct answers for light mesons, however.
2. The hadron string models – the string is an approximation for the gluon flux-tube with nearly massless quarks at the ends.
   The semiclassical quantization of a thin flux-tube typically gives a linearly rising spectrum for masses squared:

$$m_k^2 = a(k + b), \qquad k = 0, 1, 2, \dots \tag{1}$$

Preliminary notes

I The usual models based on the QCD string possess a basic feature: The linear Regge recurrence, $m_J^2 \sim J$ (J denotes the hadron spin), emerges on the classical level.
The radial modes appear after quantization.

$\Rightarrow$ An idea appears to look for alternative dynamical approaches in which the spectrum (1) is realized on the classical level.

II The light non-strange mesons cluster near certain equidistant values of energy squared with repeating structure of the clusters.

$\Rightarrow$ The assumption of periodic structure of the gluon vacuum arises.

I The usual models based on the QCD string possess a basic feature: The linear Regge recurrence, $m_J^2 \sim J$ (J denotes the hadron spin), emerges on the classical level.
The radial modes appear after quantization.

$\Rightarrow$ An idea appears to look for alternative dynamical approaches in which the spectrum (1) is realized on the classical level.

II The light non-strange mesons cluster near certain equidistant values of energy squared with repeating structure of the clusters.

$\Rightarrow$ The assumption of periodic structure of the gluon vacuum arises.

I The usual models based on the QCD string possess a basic feature: The linear Regge recurrence, $m_J^2 \sim J$ (J denotes the hadron spin), emerges on the classical level.
The radial modes appear after quantization.

⇒ An idea appears to look for alternative dynamical approaches in which the spectrum (1) is realized on the classical level.

II The light non-strange mesons cluster near certain equidistant values of energy squared with repeating structure of the clusters.

⇒ The assumption of periodic structure of the gluon vacuum arises.

Preliminary notes

- I The usual models based on the QCD string possess a basic feature: The linear Regge recurrence, $m_J^2 \sim J$ (J denotes the hadron spin), emerges on the classical level.
  The radial modes appear after quantization.
- ⇒ An idea appears to look for alternative dynamical approaches in which the spectrum (1) is realized on the classical level.
- II The light non-strange mesons cluster near certain equidistant values of energy squared with repeating structure of the clusters.
- ⇒ The assumption of periodic structure of the gluon vacuum arises.

Consider the following effective field theory for a real scalar field in four dimensions

$$\boxed{\mathcal{L} = \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi + \frac{\mu^4}{2\lambda}\left[\cos\left(\frac{\lambda}{\mu^2}\varphi^2 - 2\pi b\right) - \cos(2\pi b)\right],} \qquad (2)$$

where μ has the dimension of mass and the parameters λ and b are dimensionless.

There are restriction on the value of parameter b: the interval $0 \leq b < 1$ covers all nontrivial cases.

The parameter λ plays the role of coupling constant. In the weak coupling regime, $|\lambda| \ll 1$, the Lagrangian (2) reduces to the scalar theory $\lambda\varphi^4$:

$$\mathcal{L}_{|\lambda|\ll 1} = \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi + \frac{1}{2}\mu^2\sin(2\pi b)\varphi^2 - \frac{\lambda}{4}\cos(2\pi b)\varphi^4 + \mathcal{O}(\lambda^2\varphi^6).$$

The potential in (2) is minimized on the constant field configurations:

$$\langle\varphi\rangle_k = \sqrt{\frac{2\pi\mu^2}{\lambda}(k+b)}, \qquad k = 0, 1, 2, \ldots \tag{3}$$

Consider small perturbations around the vacua (3):

$$\varphi = \langle\varphi\rangle_k + \sigma(x).$$

The quadratic part of the Lagrangian for σ reads:

$$\mathcal{L}_\sigma^{(2)} = \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \lambda\langle\varphi\rangle_k^2\sigma^2.$$

The spectrum of excitations has the Regge-like form (1):

$$M_{\sigma,k}^2 = 2\lambda\langle\varphi\rangle_k^2 = 4\pi\mu^2(k+b), \qquad k = 0, 1, 2, \ldots$$

The most close analogue to the sine-Gordon model is provided by the following modification of the Lagrangian (2):

$$\tilde{\mathcal{L}} = \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi + \frac{\mu^6}{\lambda^2\varphi^2}\left[\cos\left(\frac{\lambda}{\mu^2}\varphi^2\right) - 1\right]. \tag{4}$$

The minima of effective potentials in (4) and in (2) at $b = 0$ coincide. As $\lambda \to 0$ we obtain the theory:

$$\tilde{\mathcal{L}} = \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{1}{2}\mu^2\varphi^2 + \frac{1}{4!}\frac{\lambda^2}{\mu^2}\varphi^6 + \dots$$

The equation of motion for the field φ in the static case has a soliton-like solution:

$$\boxed{\varphi = \pm 2\sqrt{\frac{\mu^2}{\lambda}\arctan e^{2\mu(x-x_0)}}} \tag{5}$$

Consider now a $U(1)$ gauge model of the Nielsen-Olesen type. The Lagrangian reads:

$$\mathcal{L} = D_\mu\varphi(D^\mu\varphi)^* - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \\ + \frac{\mu^4}{2\lambda}\left[\cos\left(\frac{2\lambda}{\mu^2}\varphi\varphi^* - 2\pi b\right) - \cos(2\pi b)\right], \quad (6)$$

where

$$\varphi = \frac{\varphi_1 + i\varphi_2}{\sqrt{2}}, \quad D_\mu\varphi = \partial_\mu\varphi - ieA_\mu\varphi, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu.$$

Vacua for
- the scalar field: $\langle\varphi\rangle_k = \sqrt{\frac{2\pi\mu^2}{\lambda}(k+b)}$
- the vector field: 0

Fluctuations above the vacua for
- the scalar field: $\varphi = \frac{\langle\varphi\rangle_k + \sigma + i\eta}{\sqrt{2}}$
- the gauge field: A_μ

The standard change of the field variables:

$$V_\mu = A_\mu - \frac{1}{e\langle\varphi\rangle_k}\partial_\mu\eta(x), \quad \varphi = \frac{\langle\varphi\rangle_k + \sigma(x)}{2}\exp\left(\frac{i\eta(x)}{\langle\varphi\rangle_k}\right). \quad (7)$$

In terms of the variables (7), the quadratic part of the Lagrangian (6) is cast into the canonical form:

$$\mathcal{L}^{(2)}_{\sigma,V} = \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}M_\sigma^2\sigma^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}M_V^2 V_\mu V^\mu + \text{const},$$

where

$$\boxed{M^2_{\sigma,k} = 2\lambda\langle\varphi\rangle_k^2, \qquad k = 0, 1, 2\ldots}$$

$$\boxed{M^2_{V,k} = e^2\langle\varphi\rangle_k^2, \qquad k = 0, 1, 2\ldots}$$

The equations of motion for the Lagrangian (6) are:

$$D^{\mu} D_{\mu}\varphi = -\mu^2 \varphi \sin\left(\frac{2\lambda}{\mu^2}\varphi\varphi^* - 2\pi b\right), \quad (8)$$

$$\partial^{\nu} F_{\mu\nu} = ie(\varphi^* \partial_{\mu}\varphi - \varphi \partial_{\mu}\varphi^*) + 2e^2 A_{\mu}\varphi\varphi^*. \quad (9)$$

We will consider the static case, with the gauge choice $A_0 = 0$. Following the paper H. B. Nielsen and P. Olesen, Nucl. Phys. B **61**, 45 (1973) we look for a cylindrically symmetric solution, with axis along the z-direction. The corresponding ansatz is:

$$\boxed{\vec{A}(\vec{r}) = \frac{\vec{r} \times \vec{e}_z}{r} A(r), \qquad \varphi(\vec{r}) = \chi(r) e^{in\theta},} \quad (10)$$

where $\vec{e}_z$ denotes a unit vector along the z-direction and n is an integer.
In addition, it is assumed that $A_{\theta} = A(r)$ and $A_r = A_z = 0$.

The flux is given by $\Phi(r) = 2\pi r A(r)$ so that the magnetic field is:

$$B(r) = \frac{1}{2\pi r}\frac{d}{dr}\Phi(r) = \frac{1}{r}\frac{d}{dr}(rA(r)).$$

Inserting ansatz to the equations of motion we arrive at

$$-\frac{1}{r}\frac{d}{dr}\left(r\frac{d}{dr}\chi(r)\right) + \left[\left(\frac{n}{r} - eA\right)^2 + \mu^2 \sin\left(\frac{2\lambda}{\mu^2}\chi^2 - 2\pi b\right)\right]\chi = 0, \quad (11)$$

$$-\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}(rA(r))\right] + 2\chi^2\left(Ae^2 - \frac{ne}{r}\right) = 0. \quad (12)$$

We are going to find a solution with asymptotic behavior on the spacial infinity of the type $\chi \simeq$ const, then:

$$A_k(r) = \frac{n}{er} + \frac{C}{e}K_1(e\langle\varphi\rangle_k r) = \frac{n}{er} + \frac{C}{e}\sqrt{\frac{\pi}{2e\langle\varphi\rangle_k r}}\, e^{-e\langle\varphi\rangle_k r} + \dots, \quad (13)$$

where K_ν is the modified Bessel function of the second kind.

Then the magnetic field is

$$B_k = -C\langle\varphi\rangle_k r K_0\left(e\langle\varphi\rangle_k r\right) = -C\sqrt{\frac{\pi\langle\varphi\rangle_k}{2er}}\, e^{-e\langle\varphi\rangle_k r} + \dots$$

Substituting the asymptotics for $A_k(r)$ into the equation (11) we obtain the approximate solution

$$\chi_k \simeq \langle\varphi\rangle_k/\sqrt{2},$$

which defines the characteristic length $\Lambda_k = (e\chi_k)^{-1}$.

- Each vacuum possesses not only its own spectrum but also its own vortex.
- The vortex-lines have a flux-tube structure and move according to the equation of motion of the Nambu dual string.
- The energy density of the vortex-line in the k-th vacuum $\sim \langle\varphi\rangle_k^2$.

Then the magnetic field is

$$B_k = -C\langle\varphi\rangle_k r K_0\left(e\langle\varphi\rangle_k r\right) = -C\sqrt{\frac{\pi\langle\varphi\rangle_k}{2er}}\, e^{-e\langle\varphi\rangle_k r} + \dots$$

Substituting the asymptotics for $A_k(r)$ into the equation (11) we obtain the approximate solution

$$\chi_k \simeq \langle\varphi\rangle_k/\sqrt{2},$$

which defines the characteristic length $\Lambda_k = (e\chi_k)^{-1}$.

- Each vacuum possesses not only its own spectrum but also its own vortex.
- The vortex-lines have a flux-tube structure and move according to the equation of motion of the Nambu dual string.
- The energy density of the vortex-line in the k-th vacuum $\sim \langle\varphi\rangle_k^2$.

From the phenomenological point of view, the model under consideration predicts a linear Regge-like spectrum for radially excited scalar and vector states.

In the general situation, the slopes of radial trajectories for these two types of particles are different.

However: the slope is universal in the hadron string models because it is determined by the gluodynamics there.

The universality of slopes: $e^2 = 2\lambda$.

$\Rightarrow$ This entails a complete degeneracy of the scalar and vector states.

The scalar field $\varphi \Leftrightarrow$ the scalar isoscalar particles $\Leftrightarrow f_0$-mesons

The vector field $A_\mu \Leftrightarrow$ the vector isoscalar particles $\Leftrightarrow \omega$-mesons

$\Rightarrow$ For experimental proof see Fig. 1.

From the phenomenological point of view, the model under consideration predicts a linear Regge-like spectrum for radially excited scalar and vector states.

In the general situation, the slopes of radial trajectories for these two types of particles are different.

However: the slope is universal in the hadron string models because it is determined by the gluodynamics there.

The universality of slopes: $e^2 = 2\lambda$.

$\Rightarrow$ This entails a complete degeneracy of the scalar and vector states.

The scalar field $\varphi \Leftrightarrow$ the scalar isoscalar particles $\Leftrightarrow f_0$-mesons

The vector field $A_\mu \Leftrightarrow$ the vector isoscalar particles $\Leftrightarrow \omega$-mesons

$\Rightarrow$ For experimental proof see Fig. 1.

From the phenomenological point of view, the model under consideration predicts a linear Regge-like spectrum for radially excited scalar and vector states.

In the general situation, the slopes of radial trajectories for these two types of particles are different.

However: the slope is universal in the hadron string models because it is determined by the gluodynamics there.

The universality of slopes: $e^2 = 2\lambda$.

$\Rightarrow$ This entails a complete degeneracy of the scalar and vector states.

The scalar field $\varphi \Leftrightarrow$ the scalar isoscalar particles $\Leftrightarrow f_0$-mesons

The vector field $A_\mu \Leftrightarrow$ the vector isoscalar particles $\Leftrightarrow \omega$-mesons

$\Rightarrow$ For experimental proof see Fig. 1.

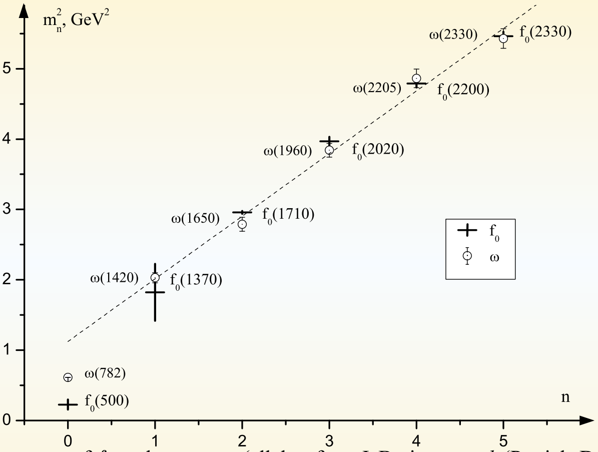


FIG. 1. The spectrum of f_0 and ω mesons (all data from J. Beringer *et al.* (Particle Data Group), Phys. Rev. D **86**, 010001 (2012)). The vertical size displays the experimental uncertainty. Other states f_0 reported by the PD are assumed to belong to the $s\bar{s}$-trajectory. The state $\omega(2290)$ is identified with the $\omega(2330)$-meson.

The way to keep the universal slope but remove the degeneracy:

$$\tilde{\mathcal{L}}_v = \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi + \frac{\mu^4}{2\lambda}\left[\cos\left(\frac{\lambda}{\mu^2}\varphi^2 - 2\pi b\right) - \cos(2\pi b)\right] - \\ - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(m_v^2 + e^2\varphi^2)A_\mu A^\mu$$

The models discussed above should be viewed as effective models for QCD in the large-N_c limit (because the number of resonances is infinite and the zero-width approximation is implied).

Complementarity to the usual effective field theories for the strong interactions (the sigma-models, chiral perturbation theory, *etc*):

- The input parameters (first of all the bare masses) originate from integration of the low-energy degrees of freedom below 1 GeV.
- A phenomenological support: the linear ansatz (1) for the radial meson spectrum is a reasonable approximation for the excited states, while the ground ones below 1 GeV deviate substantially from the linear trajectory. For example, see Fig. 1.

The way to keep the universal slope but remove the degeneracy:

$$\tilde{\mathcal{L}}_v = \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi + \frac{\mu^4}{2\lambda}\left[\cos\left(\frac{\lambda}{\mu^2}\varphi^2 - 2\pi b\right) - \cos(2\pi b)\right] - \\ - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(m_v^2 + e^2\varphi^2)A_\mu A^\mu$$

The models discussed above should be viewed as effective models for QCD in the large-N_c limit (because the number of resonances is infinite and the zero-width approximation is implied).
Complementarity to the usual effective field theories for the strong interactions (the sigma-models, chiral perturbation theory, *etc*):

- The input parameters (first of all the bare masses) originate from integration of the low-energy degrees of freedom below 1 GeV.

- A phenomenological support: the linear ansatz (1) for the radial meson spectrum is a reasonable approximation for the excited states, while the ground ones below 1 GeV deviate substantially from the linear trajectory. For example, see Fig. 1.

QCD in the large-N_c limit represents (assuming that confinement persists in this limit) a theory of an infinite number of narrow and stable non-interacting mesons and glueballs which should appear in the classical effective action.

It is possible to interpret the field φ and its periodic potential as an effective model for the non-perturbative gluon vacuum in QCD.

All quarkonia in the sector of light quarks get masses mainly through interaction with the vacuum field φ.

The simplest Lagrangian for a scalar quarkonium f is:

$$\tilde{\mathcal{L}}_s = \frac{1}{2}\partial_\mu f \partial^\mu f - \frac{1}{2}(m_s^2 + g_s^2\varphi^2)f^2 + \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi + \\ + \frac{\mu^4}{2\lambda}\left[\cos\left(\frac{\lambda}{\mu^2}\varphi^2 - 2\pi b\right) - \cos(2\pi b)\right],$$

- The spectrum of the scalar glueballs has the Regge-like form.
- The bare mass m_s must depend on quantum numbers.
- Universality of couplings: $g_s^2 \simeq e^2 \simeq 2\lambda$.

QCD in the large-N_c limit represents (assuming that confinement persists in this limit) a theory of an infinite number of narrow and stable non-interacting mesons and glueballs which should appear in the classical effective action.

It is possible to interpret the field φ and its periodic potential as an effective model for the non-perturbative gluon vacuum in QCD.

All quarkonia in the sector of light quarks get masses mainly through interaction with the vacuum field φ.

The simplest Lagrangian for a scalar quarkonium f is:

$$\tilde{\mathcal{L}}_s = \frac{1}{2}\partial_\mu f \partial^\mu f - \frac{1}{2}(m_s^2 + g_s^2\varphi^2)f^2 + \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi + \\ + \frac{\mu^4}{2\lambda}\left[\cos\left(\frac{\lambda}{\mu^2}\varphi^2 - 2\pi b\right) - \cos(2\pi b)\right],$$

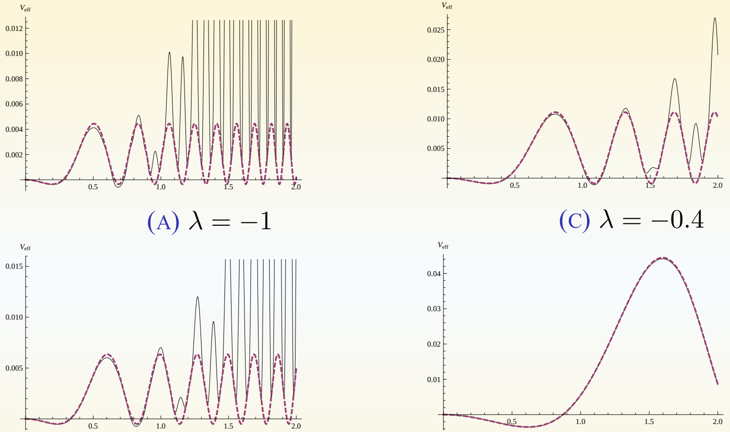
- The spectrum of the scalar glueballs has the Regge-like form.
- The bare mass m_s must depend on quantum numbers.
- Universality of couplings: $g_s^2 \simeq e^2 \simeq 2\lambda$.

The effective potential for the theory of one scalar field has the following expansion in the Planck constant:

$$V_{eff}(\varphi_{cl}) = -\frac{\mu^4}{2\lambda}\left(\cos\left(\frac{\lambda}{\mu^2}\varphi_{cl}^2 - 2\pi b\right) - \cos(2\pi b)\right) + \\ + \hbar\frac{(V''(\varphi_{cl}))^2}{64\pi^2}\left(-\frac{3}{2} + \ln\frac{V''(\varphi_{cl})}{\mathcal{M}^2}\right) + O(\hbar^2),$$

where the second derivative reads

$$V''(\varphi_{cl}) = \mu^2\left(2\frac{\lambda}{\mu^2}\varphi_{cl}^2\cos\left(2\pi b - \frac{\lambda}{\mu^2}\varphi_{cl}^2\right) - \sin\left(2\pi b - \frac{\lambda}{\mu^2}\varphi_{cl}^2\right)\right).$$



(A) $\lambda = -1$

(C) $\lambda = -0.4$

(B) $\lambda = -0.7$

(D) $\lambda = -0.1$

FIG. 2. The effective potential (solid line) and the classical potential (dashed line) at different values of λ. The parameters are taken from the trajectory of f_0-mesons in Fig. 1: $\mu^2 = 0.069$ GeV^2, $b = 1.41$, $\mathcal{M} = 0.5$ GeV.

Constructive results:

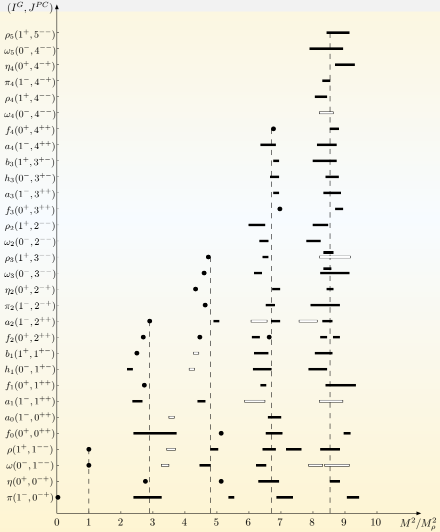
- The models have been built in which the Regge-like form of the spectrum of the radially excited states appears on the classical level.
- Each model discussed has an infinite number of non-equivalent vacua. The theory with abelian gauge group has the Nielsen — Olesen vortex solution in each vacuum.

Corollaries:

- Interpretation of the scalar model as an effective model for the non-perturbative gluon vacuum in QCD yields the Regge-like form for the spectrum of the scalar glueballs.
- The analytical expression for the effective potential of scalar theory with the first quantum correction has been received. As it was plotted, the effect of quantum doubling was noticed, which has also a phenomenological correspondence.

Thank you for your attention!

Backup: the figure from review S. S. Afonin, Mod. Phys. Lett. A **22**, 1359 (2007)



The predicted phenomenon of quantum doubling is difficult to observe because of overlapping decay widths of highly excited resonances. However, the case of f_2-mesons demonstrates such a phenomenon.

In terms of the quark model, the doubling can have a simple explanation. For instance, the vector mesons can have $L = 0$ and $L = 2$ and these states are nearly degenerate due to the mass dependence $M_{n,L} \sim n + L$. The degeneracy here takes place for the radial number $n \geq 2$, *i.e.* beginning from the third minimum as in Fig. 2.

In the case of the scalar mesons, the source of doubling can arise from the mixing with the $s\bar{s}$ scalar states (s denotes the strange quark). The scalar mesons with the dominant $s\bar{s}$-component are not displayed here.