

# Searches for GeV-scale sterile neutrinos with CERN SPS proton beam

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# Phenomenological problems of the Standard Model

Gauge fields (interactions) –  $\gamma, W^\pm, Z, g$

Three generations of matter:  $L = \begin{pmatrix} v_L \\ e_L \end{pmatrix}, e_R; Q = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, d_R, u_R$

- Describes
  - ▶ all experiments dealing with electroweak and strong interactions
- Does not describe
  - ▶ Neutrino oscillations :
    - active neutrino masses
    - via mixing
  - ▶ Dark matter ( $\Omega_{DM}$ ) :
    - sterile neutrino as DM
  - ▶ Baryon asymmetry :
    - leptogenesis via sterile neutrino decays or oscillations
  - ▶ Sterile neutrinos explain the oscillations
  - ▶ and the cosmological problems

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# Sterile neutrinos: NEW ingredients

One of the optional physics beyond the SM:

sterile: new fermions uncharged under the SM gauge group

neutrino: explain observed oscillations by mixing with SM (active) neutrinos

Attractive features:

- possible to achieve within renormalizable theory
- only  $N = 2$  Majorana neutrinos needed
- baryon asymmetry via leptogenesis
- dark matter (with  $N \geq 3$  at least)
- light(?) sterile neutrinos might be responsible for neutrino anomalies...?

Disappointing feature:

Major part of parameter space is UNTESTABLE

# Active neutrino masses without new fields

Dimension-5 operator       $\Delta L = 2$

$$\mathcal{L}^{(5)} = \frac{\beta_L}{4\Lambda} F_{\alpha\beta} \bar{L}_\alpha \tilde{H} H^\dagger L_\beta^c + \text{h.c.}$$

$L_\alpha$  are SM leptonic doublets,  $\alpha = 1, 2, 3$ ,  $\tilde{H}_a = \varepsilon_{ab} H_b^*$ ,  $a, b = 1, 2$ ;      in a unitary gauge

$H^T = \begin{pmatrix} 0, (v+h)/\sqrt{2} \end{pmatrix}$  and

$$\mathcal{L}_{vv}^{(5)} = \frac{\beta_L v^2}{4\Lambda} \frac{F_{\alpha\beta}}{2} \bar{v}_\alpha v_\beta^c + \text{h.c.}$$

hence

$$\Lambda \sim 3 \times 10^{14} \text{ GeV} \times \beta_L \times \left( \frac{3 \times 10^{-3} \text{ eV}^2}{\Delta m_{\text{atm}}^2} \right)^{1/2}$$

The model has to be UV-completed at the neutrino scale  $\Lambda_\nu < \Lambda \dots$

What is beyond the neutrino scale  $\Lambda_\nu$  ?

Why neutrino scale, 1 eV, is so low ?



# Seesaw mechanism: $M_N \gg 1 \text{ eV}$ (Type I)

With  $m_{active} \lesssim 1 \text{ eV}$  we work in the seesaw (type I) regime:

$$\mathcal{L}_N = \bar{N}_I i\partial N_I - f_{\alpha I} \bar{L}_{\alpha} \tilde{H} N_I - \frac{M_{N_I}}{2} \bar{N}_I^c N_I + \text{h.c.}$$

When Higgs gains  $\langle H \rangle = v/\sqrt{2}$  we get in neutrino sector

$$\nu_N = v \frac{f_{\alpha I}}{\sqrt{2}} \bar{v}_{\alpha} N_I + \frac{M_{N_I}}{2} \bar{N}_I^c N_I + \text{h.c.} = \left( \bar{v}_1, \dots, \bar{N}_1^c, \dots \right) \begin{pmatrix} 0 & v \frac{\hat{f}}{\sqrt{2}} \\ v \frac{\hat{f}^{\dagger}}{\sqrt{2}} & \hat{M}_N \end{pmatrix} (v_1, \dots, N_1, \dots)^T$$

Then for  $M_N \gg \hat{M}^D = v \frac{\hat{f}}{\sqrt{2}}$  we find the eigenvalues:

seesaw at work

$$\simeq \hat{M}_N \quad \text{and} \quad \hat{M}^v = -\hat{M}^{D\dagger} \frac{1}{\hat{M}_N} \hat{M}^D \propto f^2 \frac{v^2}{M_N} \propto \theta^2 M_N \ll M_N$$

Mixings: flavor state  $v_{\alpha} = U_{\alpha i} v_i + \theta_{\alpha I} N_I$

$$\text{active-active mixing: } U^{\dagger} \hat{M}^v U = \text{diag}(m_1, m_2, m_3)$$

$$\text{active-sterile mixing: } \theta_{\alpha I} \propto \frac{(M^D)_{\alpha I}^{\dagger}}{M_N} = \hat{f}^{\dagger} \frac{v}{M_N} \ll 1$$

# Where is sterile neutrino scale?

eigenvalues:  $\hat{M}^v \simeq \hat{M}_N$  and  $\hat{M}^v = -\hat{M}^{D\dagger} \frac{1}{\hat{M}_N} \hat{M}^D \propto f^2 \frac{v^2}{M_N} \ll M_N$

SEESAW says nothing about the sterile neutrino scale  $M_N$  !

Unitarity:  $f \lesssim 1 \implies M_N \lesssim 3 \times 10^{14} \text{ GeV} \times \left( \frac{3 \cdot 10^{-3} \text{ eV}^2}{\Delta m_{atm}^2} \right)^{1/2} \xrightarrow{\text{blue arrow}} \Lambda \text{ in } (LH)^2/\Lambda$

Integrating out sterile neutrinos get dim-5 operator  $-f_{\alpha I} \bar{L}_{\alpha} \tilde{H} N_I - \frac{M_{N_L}}{2} \bar{N}_I^c N_I \rightarrow f^2 (LH)^2 / M_N$

SM Higgs without NP at EW-scale favors sterile neutrinos at EW-scale (or below) !

- Majorana mass violates scale-invariance  $\implies$  finite corrections  $\delta m_h^2 \propto f^2 M_N^2$
- Scale invariance helps to abandon infinite corrections  $\delta m_h^2 \propto f^2 \Lambda^2 M_N^2$
- In SM scale invariance is broken by the Higgs mass and running of coupling constants  
 $T_{\mu}^{\mu} \propto \beta(\alpha) \times \hat{O} + (m_h^2 + \alpha \Lambda^2) \times h^2 \implies$  quadratic divergences are irrelevant

W.Bardeen (1995)

# Sterile neutrinos: $M_N$ , violate lepton symmetry

Most general renormalizable with  $2(3\dots)$  right-handed neutrinos  $N_i$  ?

$$\mathcal{L}_N = \bar{N}_I i\partial N_I - f_{\alpha I} \bar{L}_{\alpha} \tilde{H} N_I - \frac{M_{N_I}}{2} \bar{N}_I^c N_I + \text{h.c.}$$

## Parameters to be determined from experiments

9(7): active neutrino sector

2

$\Delta m_{ij}^2$ : oscillation

experiments

3  $\theta_{ij}$ : oscillation experiments

1 CP-phase: oscillation experiments

2(1) Majorana phases: 0 $\nu ee$ , 0 $\nu \mu \mu$

1(0)  $m_\nu$ :  ${}^3\text{H} \rightarrow {}^3\text{He} + e + \bar{\nu}_e$ , cosmology, ...

11:  $N = 2$  sterile neutrinos  
(works if  $m_\nu = 0$  !!)

2: Majorana masses  $M_{N_I}$

9: New Yukawa couplings  $f_{\alpha I}$

which form

2: Dirac masses  $M^D = f(H)$

3+1: mixing angles

2+1: CP-violating phases

4 new parameters in total  
help with leptogenesis

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both BAU and DM are possible

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oscillation

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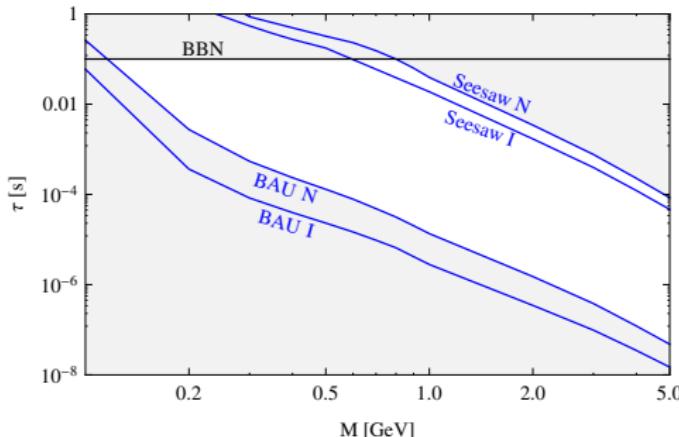
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# $\nu$ MSM: 2 GeV-scale & 1 keV-scale neutrinos

T.Asaka, S.Blanchet, M.Shaposhnikov (2005)

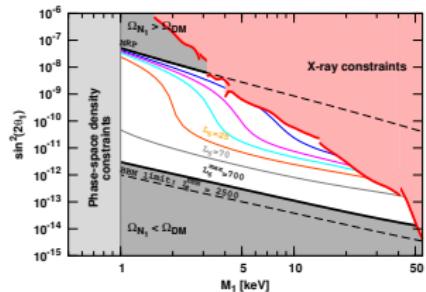
## 2 GeV-scale seesaw neutrinos

- give masses and mixing to active neutrinos
  - violate CP
  - out-of-equilibrium oscillations  $\nu_a \leftrightarrow N_{2,3}$
- need very small mixing  $\theta_{\alpha l}^2 \ll 1$   
in the early Universe redistribute lepton charge  
need degeneracy:  $\Delta M_{N_{2,3}} \ll M_{N_2}, M_{N_3}$



DM: 1-50 keV  
mixing with active neutrinos:

- $\tau_{N_1} > \tau_U \rightarrow$   
little contribution to  $m_\nu$
- signature in X-rays  
 $N_1 \rightarrow \gamma \nu_a$
- produced in early  
Universe in plasma  
needs strong fine-tuning



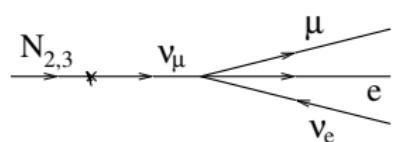
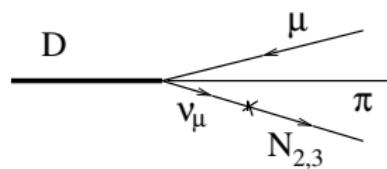
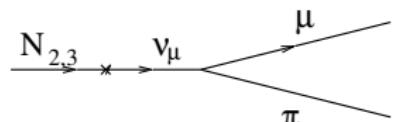
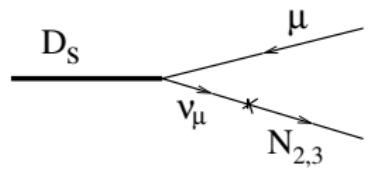
easily produced by inflaton  $XNN$

M.Shaposhnikov, I.Tkachev (2006),

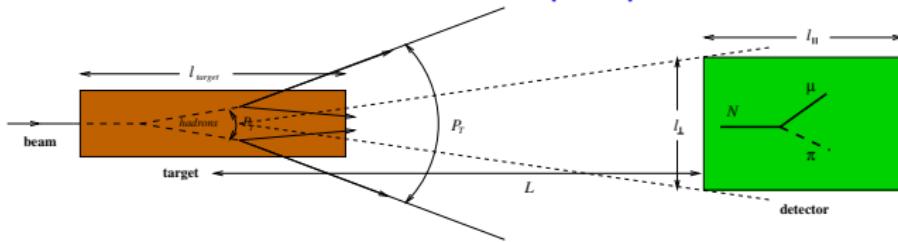
F.Bezrukov, D.G. (2009)

# Direct searches for sterile neutrinos: 2 approaches

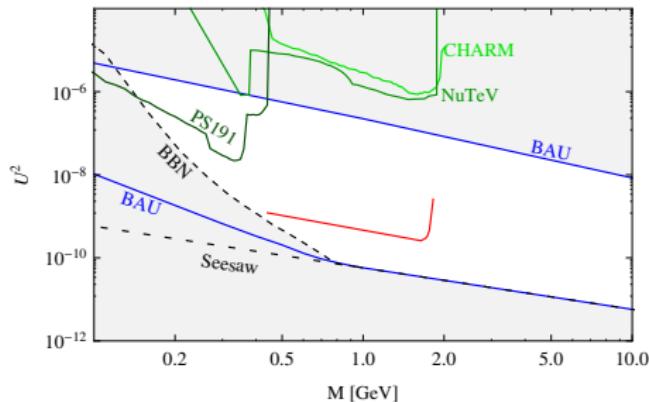
## Weak decays due to mixing



## Production in beam-dump experiments



# $\nu$ MSM parameter space



D.G, M.Shaposhnikov (2007)

$$\text{Br}(D \rightarrow l N) \lesssim 2 \cdot 10^{-8}$$

$$\text{Br}(D_s \rightarrow l N) \lesssim 3 \cdot 10^{-7}$$

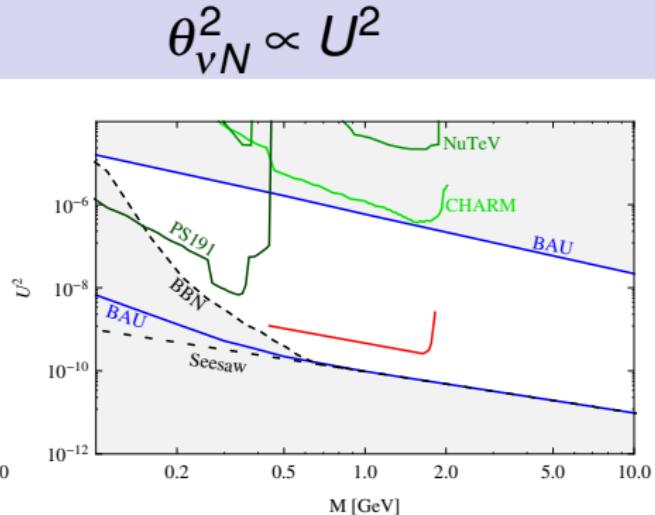
$$\text{Br}(D \rightarrow K l N) \lesssim 2 \cdot 10^{-7}$$

$$\text{Br}(D_s \rightarrow \eta l N) \lesssim 5 \cdot 10^{-8}$$

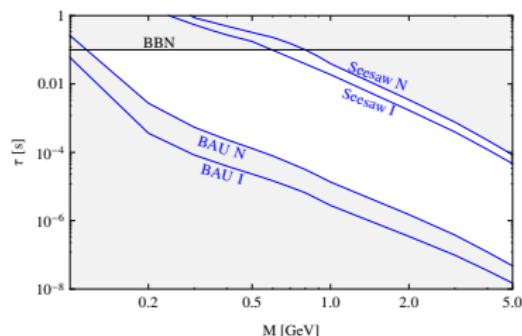
$$\text{Br}(D \rightarrow K^* l N) \lesssim 7 \cdot 10^{-8}$$

$$\text{Br}(B \rightarrow D^* l N) \lesssim 4 \cdot 10^{-7}$$

$$\text{Br}(B_s \rightarrow D_s^* l N) \lesssim 3 \cdot 10^{-7}$$



L.Canetti, M.Drewes, M.Shaposhnikov (2012)



# Neutrino production in $pp$ -collisions: meson decays

Experiment	$E$ , GeV	$N_{POT}$ , $10^{19}$	$\sigma_c/\sigma_{tot}$	$\sigma_b/\sigma_{tot}$
CNGS	400	4.5	$0.45 \times 10^{-3}$	$3 \times 10^{-8}$
NuMi	120	5	$1 \times 10^{-4}$	$10^{-10}$
T2K	30	100	$0.5 \times 10^{-5}$	$10^{-12}$
NuTeV	800	1	$1 \times 10^{-3}$	$2 \times 10^{-7}$

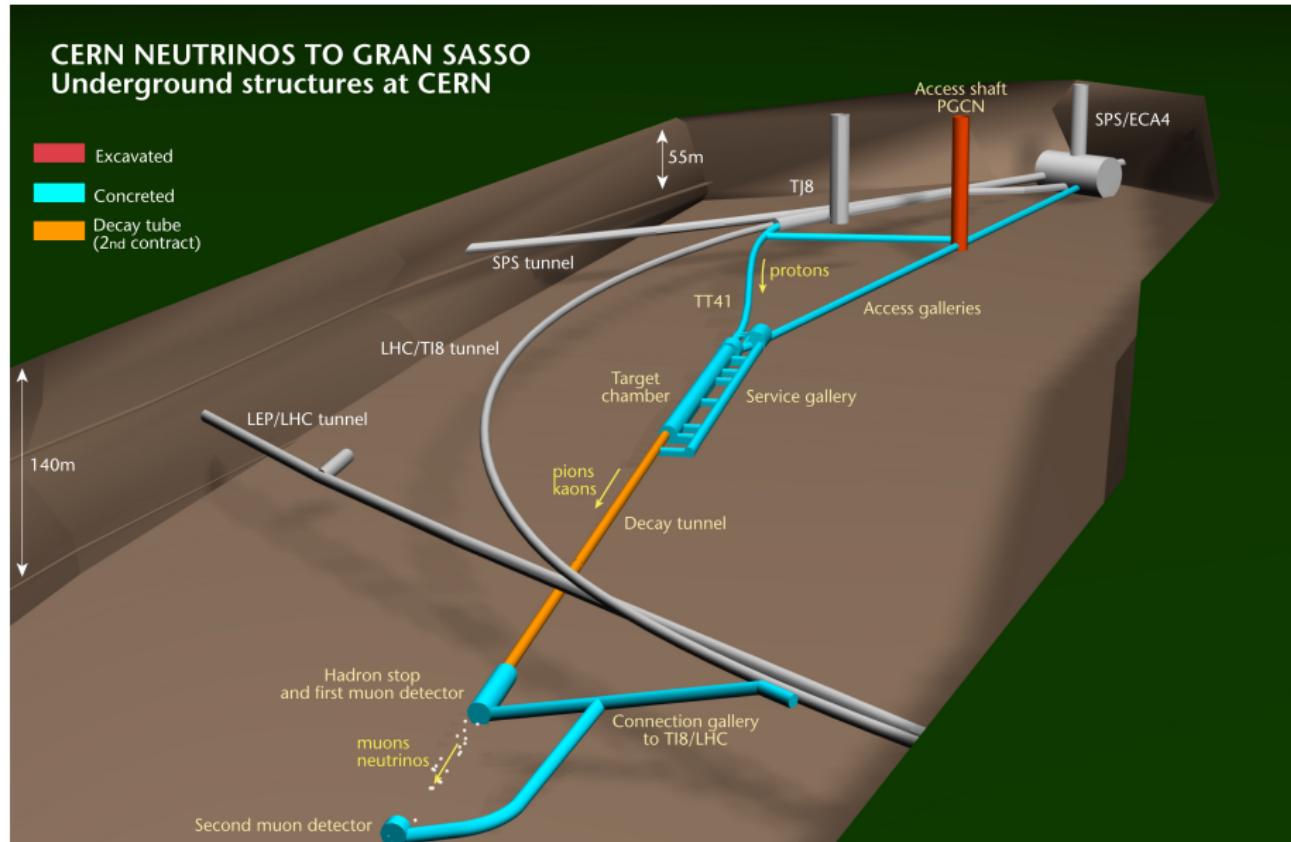
Pure geometrical factor as compared to CHARM

LBNE	PoT	detector length	distance to target	beam energy	detector area	charm production
HiResM <sub>v</sub>	$1.0 \times 10^{22}$	35 m	500 m	120 GeV	$4 \times 4 \text{ m}^2$	$1.0 \times 10^{-4}$
CHARM	$2.5 \times 10^{18}$	34 m	480 m	400 GeV	$3 \times 3 \text{ m}^2$	$4.5 \times 10^{-4}$

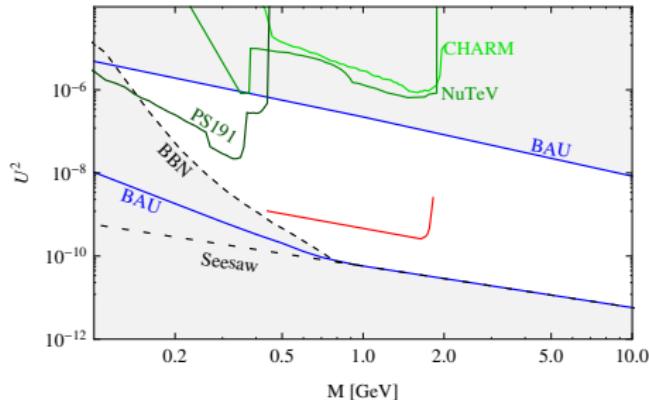
$$\frac{N_{signal}^{HiResM_v}}{N_{signal}^{CHARM}} = \frac{1.0 \times 10^{22}}{2.5 \times 10^{18}} \times \frac{35}{34} \times \left(\frac{480}{500}\right)^2 \times \left(\frac{120}{400}\right)^2 \times \frac{4 \times 4}{3 \times 3} \times \frac{1.0 \times 10^{-4}}{4.5 \times 10^{-4}} \approx 130$$

$$N_{signal} \propto \theta_{vN_2}^2 \times \theta_{vN_2}^2 \quad \text{order of magnitude improvement in } \theta_{vN_2}^2$$

# CNGS site is free after OPERA

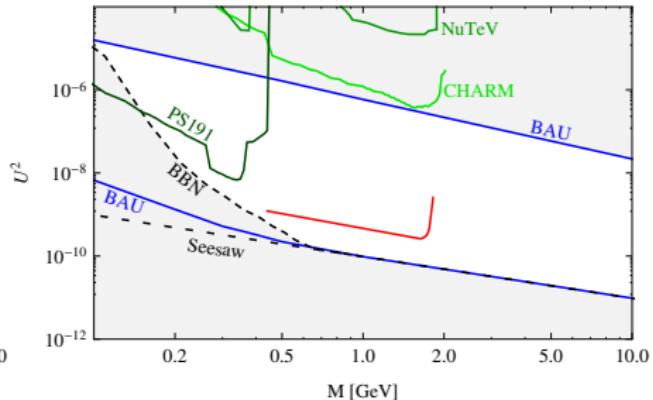


# $\nu$ MSM parameter space for $M_N < 2 \text{ GeV}$



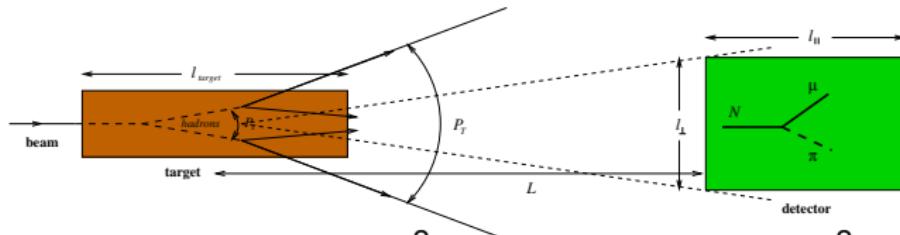
D.G, M.Shaposhnikov (2012)

S.Gninenko, D.G, M.Shaposhnikov (2012)



L.Canetti, M.Drewes, M.Shaposhnikov (2012)

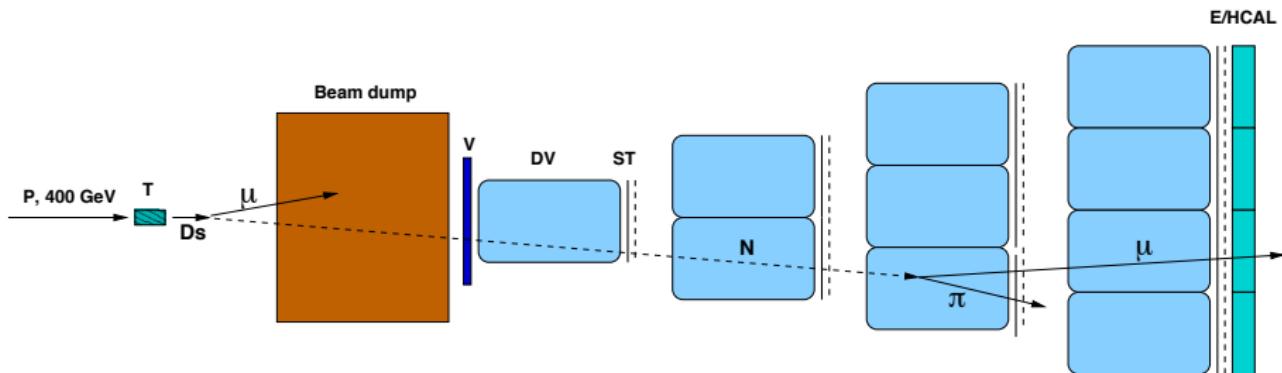
$$L=100 \text{ m} \quad I_{\perp} \sim 5 \text{ m} \quad I_{\parallel} \sim 100 \text{ m}$$



$$\text{Production rate} \propto U^2,$$

$$\text{Decay rate} \propto U^2$$

# To fully explore the region $M_N < 2 \text{ GeV}$



$$N_{\text{PoT}} \simeq 10^{20}, \quad l_{\text{total}} \simeq 3 \text{ km}$$

multisectional detector (presumably on surface)

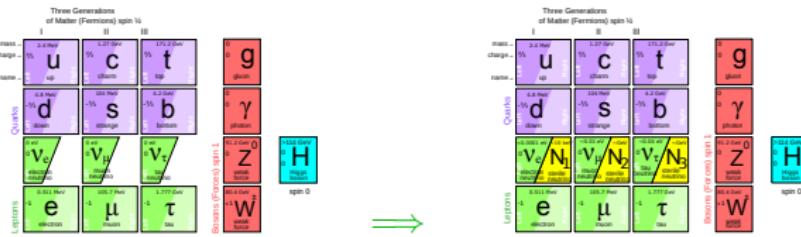
# Sterile neutrinos: dedicated experiment is needed

- Most economic explanation of neutrino oscillations within renormalizable approach:  
 $N = 2$  Majorana neutrinos
- Capable of explaining baryon asymmetry of the Universe even with  $\delta_{CP} = 0$
- One more neutrino can serve as (naturally Warm) dark matter

$\nu_{\text{MSM}}$

direct searches are feasible for  $M_N < 2 \text{ GeV}$  ( $5 \text{ GeV}$ )

- 100-m length detector at SNGS site operated on upgraded SPS beam allows to cover major part of parameter space
- 3-km scale detector is needed to fully explore the model





# Backup slides

# Heavy sterile neutrinos: $M_N \simeq 1 \text{ keV}-5 \text{ GeV}$ $\nu\text{MSM}$

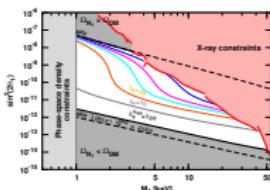
T.Asaka, S.Blanchet, M.Shaposhnikov (2005)

- Good fact: small finite quantum corrections  $\delta m_H^2 \propto f^2 M_N^2 \ll m_H^2$   
True low-energy scale modification of the SM
- Good fact: At  $T > 100 \text{ GeV}$  active-sterile neutrino oscillations produce lepton asymmetry in the early Universe, if  $\Delta M_{N_{2,3}} \ll M_{N_2}, M_{N_3}$  E.Akhmedov, V.Rubakov, A.Smirnov (1998)
- To make phenomenologically complete: Dark Matter?
  - NOT a seesaw neutrino!  $m_\nu \ll m_{\text{atm,sol}}$  general statement

$$\tau_{N \rightarrow 3\nu} \sim 1 / (G_F^2 M_N^5 \theta_{\alpha N}^2) \sim 1 / (G_F^2 M_N^4 m_\nu) \sim 10^{11} \text{ yr} (10 \text{ keV}/M_N)^4$$

either decay or equilibrate and then contribute to hot dark matter

- production in primordial plasma due to mixing with active neutrinos is ruled out from searches at X-ray telescopes



$$\Gamma_{N \rightarrow \nu\gamma} \simeq 5.5 \times 10^{-22} \theta_1^2 \left( \frac{M_1}{1 \text{ keV}} \right)^5 \text{ s}^{-1} \quad \text{a narrow line} \quad (\delta E_\gamma/E_\gamma \sim v \sim 10^{-3})$$

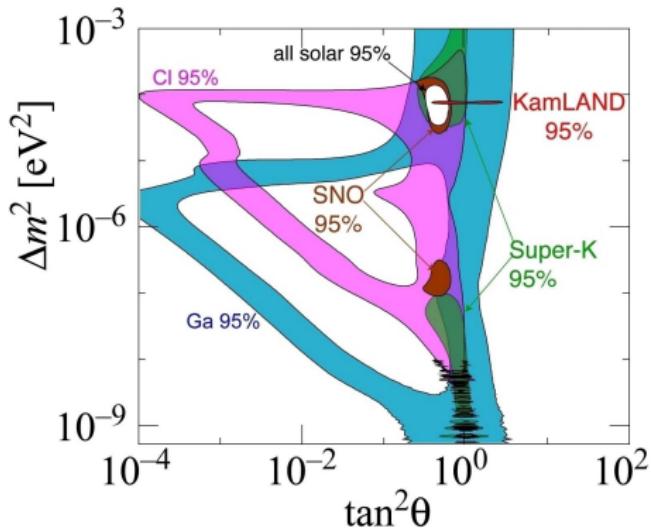
$$\text{at } E_\gamma = M_N/2$$

- Possible for 1-50 keV (WDM-CDM range) either with further unbelievable fine-tuning in  $M_{N_1}$  ( $\Delta M_N \sim 10^{-7} \text{ eV}$ ) to get  $L \gg B$  and use the resonant production or with ANOTHER source of production, e.g. inflaton decays.. then untestable

M.Shaposhnikov, I.Tkachev (2006), F.Berukova, D.G. (2009)

# Neutrino oscillations: masses and mixing angles

## Solar $2 \times 2$ “subsector”

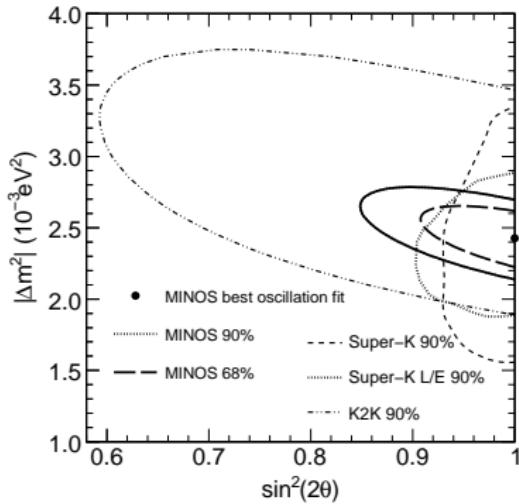


<http://hitoshi.berkeley.edu/neutrino/>

$$m_1 > 0.008 \text{ eV}$$

DAYA-BAY, RENO:  $\sin^2 2\theta_{13} \approx 0.1$

## Atmospheric $2 \times 2$ “subsector”



arXiv:0806.2237

$$m_2 > 0.05 \text{ eV}$$

also T2K (talk by M.Khabibullin)

# Cosmological limits on active neutrino masses

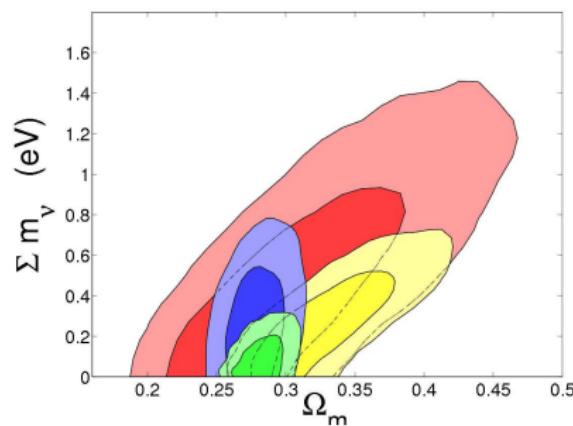
## Neutrino contributions:

Start of structure formation

Gravity potentials at recombination

Late-time structure formation

Universe expansion

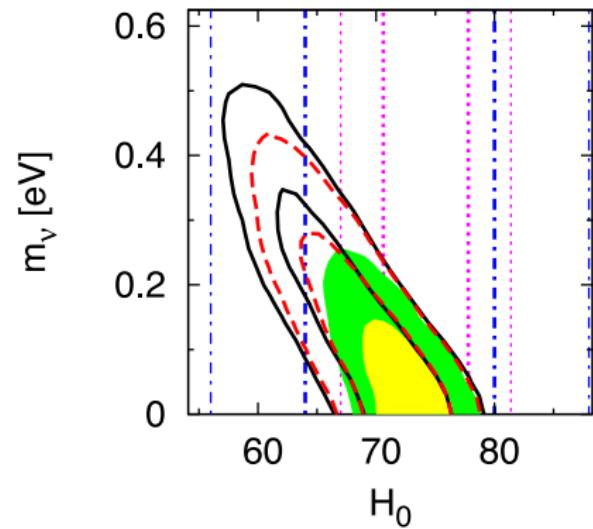


LRG+BAO+WMAP5+SNe

$\sum m_\nu < 0.28 \text{ eV}$  (95% CL)

0911.5291, see also 1112.4940

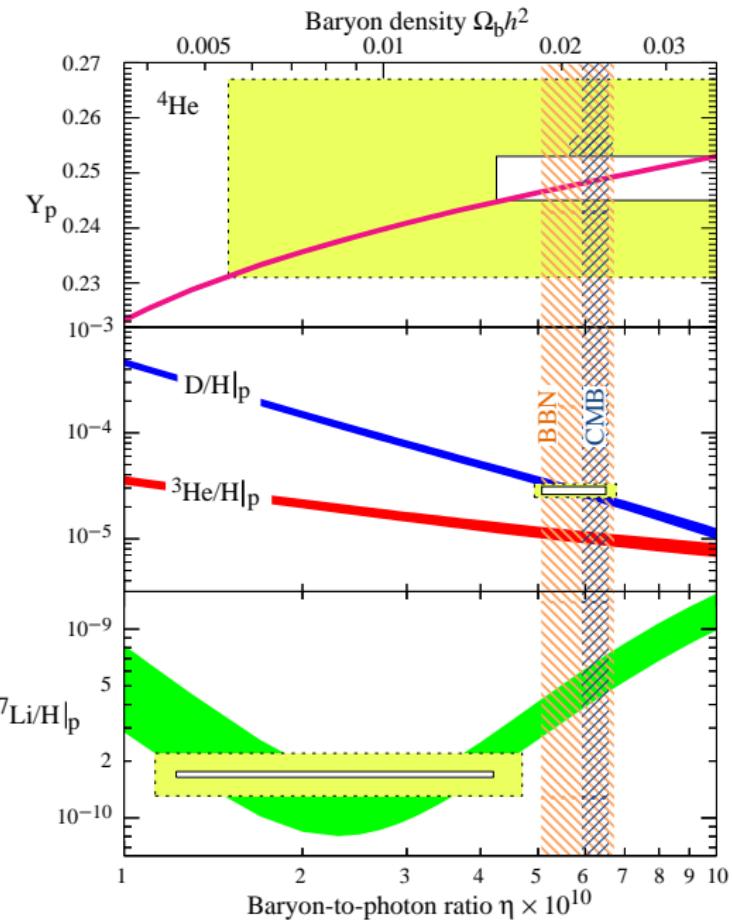
For  $N_\nu = 3$  light neutrinos



CMB+Hubble measurements

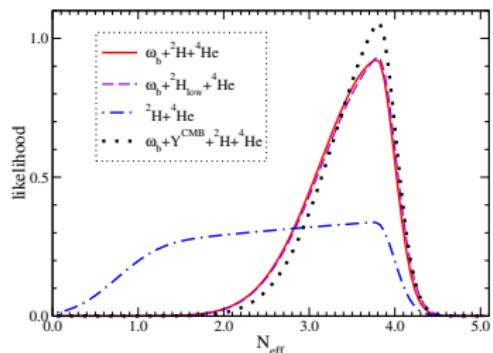
$\sum m_\nu < 0.20 \text{ eV}$  (95% CL)

0911.0976, see also 1202.2889



$$Y_p = 0.2581 \pm 0.025, \\ D/H|_p = (2.87 \pm 0.21) \times 10^{-5}$$

1103.1261



similar results from other recent studies including structure formation

1001.4440, 1001.5218, 1202.2889

$N_\nu < 4.2 @ 95\% \text{CL}$

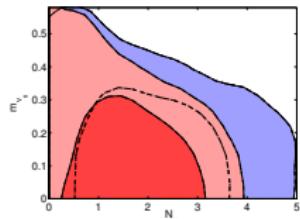
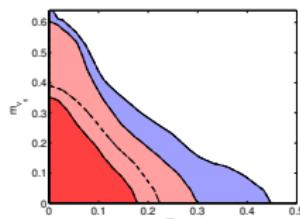
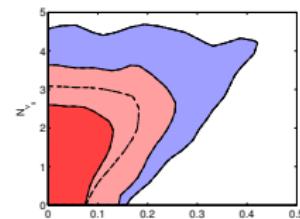
$N_\nu < 3.6 \text{ from D/H,}$

1205.3785



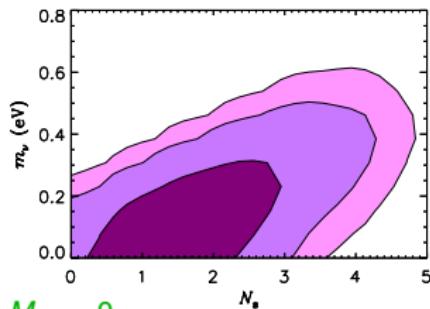
# Combined analysis for sterile and active neutrinos

WMAP7+LRG+HST

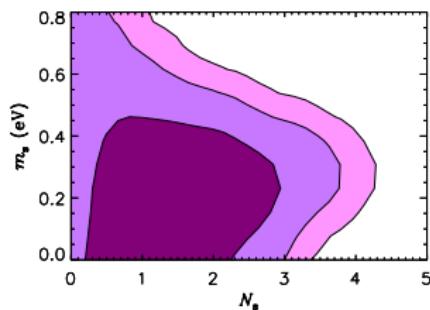


flat  $\Lambda$ CDM      1102.4774

CMB+SDSS+HST



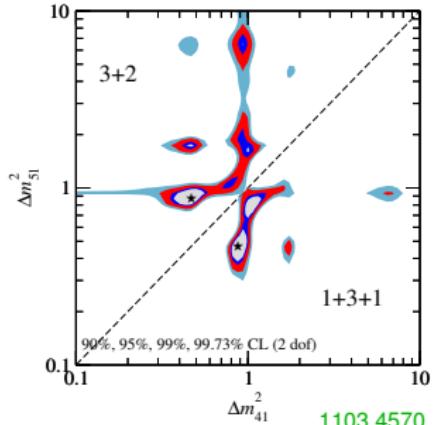
$M_{\nu_s} = 0$



$m_\nu = 0$   
flat  $\Lambda$ CDM

1006.5276

LSND+MiniBooNE



1103.4570

"3+1" :

$$\Delta m_{41}^2 = 1.76 \text{ eV}^2, |U_{e4}| = 0.151$$

"3+2" :

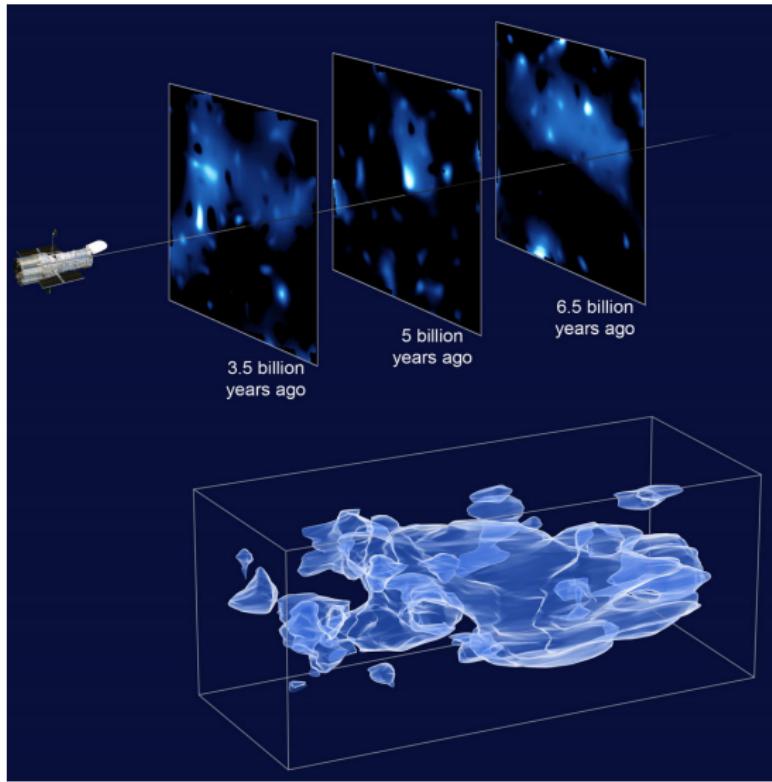
$$\Delta m_{41}^2 = 0.46 \text{ eV}^2, |U_{e4}| = 0.108$$

$$\Delta m_{51}^2 = 0.89 \text{ eV}^2, |U_{e5}| = 0.124$$

BBN rules out "3+2"

For "3+1" to allow  $M_N \gtrsim 1 \text{ eV}$  for CMB and LSS we need a new ingredient

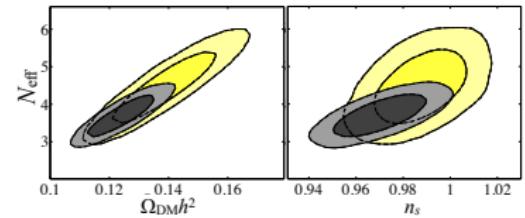
# LSS: SZ-clusters, Weak lensing of CMB



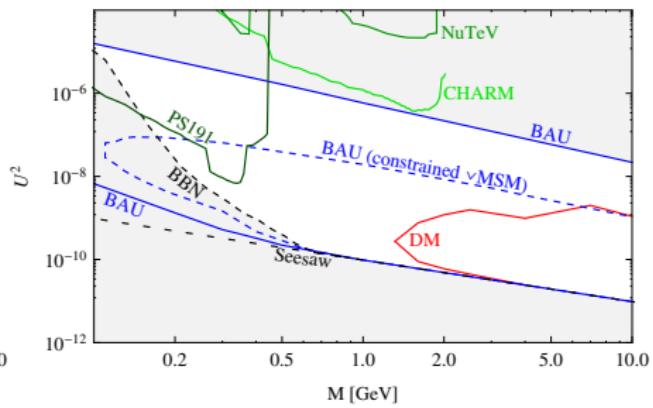
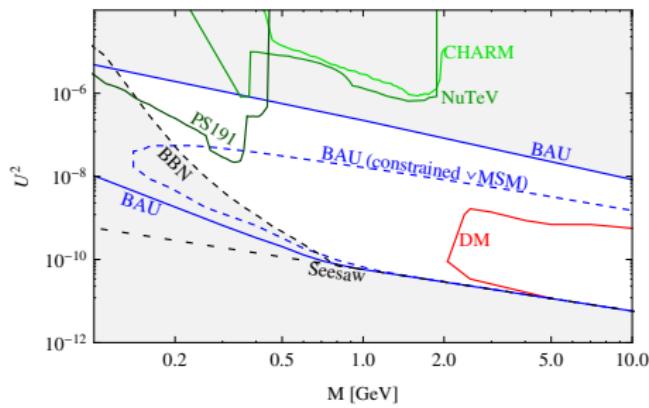
- $\Delta N_\nu$  amplifies shear power: cancel with quintessence contribution and flattening of spectrum,  $n_s \rightarrow 1$
- $M_N$  reduces power

$$N_{\text{eff}} = 3 \rightarrow M_\nu < 0.46 \text{ eV}$$
$$M_\nu = 0 \rightarrow N_{\text{eff}} = 3.8 \pm 0.4$$
$$M_\nu < 0.62 \text{ eV} \rightarrow N_{\text{eff}} = 3.9 \pm 0.4$$

1212.3608



# $\nu$ MSM parameter space with resonant DM



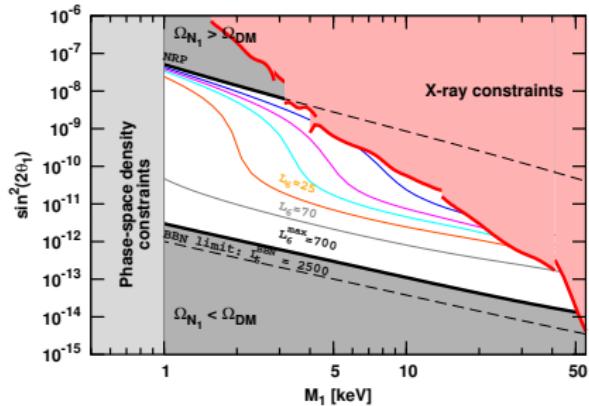
L.Canetti, M.Drewes, M.Shaposhnikov 1204.3902

# Lightest sterile neutrino $N_1$ as Dark Matter

Non-resonant production  
(active-sterile mixing) is ruled out

Resonant production (lepton asymmetry) requires  
 $\Delta M_{2,3} \lesssim 10^{-16}$  GeV

arXiv:0804.4542, 0901.0011, 1006.4008



Dark Matter production  
from inflaton decays in plasma at  $T \sim m_\chi$

Not seesaw neutrino!

M.Shaposhnikov, I.Tkachev (2006)

$$M_{N_I} \bar{N}_I^c N_I \leftrightarrow f_I X \bar{N}_I N_I$$

Can be “naturally” Warm ( $250 \text{ MeV} < m_\chi < 1.8 \text{ GeV}$ )

F.Bezrukov, D.G. (2009)

$$M_1 \lesssim 15 \times \left( \frac{m_\chi}{300 \text{ MeV}} \right) \text{ keV}$$