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**The construction of the electromagnetic  
current operator for the rho-meson decay  
process in the instant form of the Poincare-  
invariant quantum mechanics**

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# Poincare-invariant quantum mechanics<sup>1</sup>

- Construction of the electromagnetic current operator with conditions of current conservation and Lorentz – covariance.
- Good description of the meson electroweak properties of meson and deuteron.
- Asymptotic of the electromagnetic form factors at  $Q^2 \rightarrow \infty$  coinciding with predictions QCD and quark counting rules.

<sup>1</sup> A.F. Krutov, V.E. Troitsky. Physics of Particles and Nuclei. Vol. 40, 2009, 136

## Description of the processes with spin-diagonal matrix element

$$F_\pi(Q^2) = \int \int d\sqrt{s}d\sqrt{s'}g_0(s, Q^2, s')\varphi(s)\varphi(s') \quad (1)$$

→ modified impulse approximation

$$\begin{aligned} & \langle \vec{P}, \sqrt{s}, J, l, S, m_J | j_\mu^{(0)} | \vec{P}', \sqrt{s'}, J, l', S', m'_J \rangle = \\ & = \sum_{m''_J} \langle m_J | D^J(P, P') | m''_J \rangle \langle m''_J | \sum_{i=1}^3 \left\{ F_i^{ll'SS'} A_\mu^i(s, Q^2, s') \right\}_+ | m'_J \rangle \end{aligned}$$

$$F_i^{ll'SS'} = \sum_{n=0}^{2J} f_{in}^{ll'SS'}(s, Q^2, s') (iP_\mu \Gamma^\mu(P'))^n$$

**Free electromagnetic form factor with  
( $\mathbf{J}=\mathbf{J}'=\mathbf{L}=\mathbf{L}'=\mathbf{S}=\mathbf{S}'=0$ )**

$$g_0(s, Q^2, s') = \frac{(s + s' + Q^2)^2 Q^2 [\vartheta(s' - s_1) - \vartheta(s' - s_2)]}{2\sqrt{s - 4M^2}\sqrt{s' - 4M^2}[\lambda(s, Q^2, s')]^{3/2}} \quad (2)$$
$$[\cos(\omega_1 + \omega_2)f_{10}(Q^2) - 2M\xi(s, s', Q^2)\sin(\omega_1 + \omega_2)f_{30}(Q^2)]$$

# Invariant parameterization of the e.m. current for the two particles system (non-diagonal case)

## Breit system

$$K'_\mu = (\sqrt{-K'^2}, 0, 0, 0), \quad P_\mu = (\vec{q}, P_0), \quad P'_\mu = (-\vec{q}, P'_0)$$

$$q^2 = [s^2 + s'^2 + Q^2 - 2(ss' - 2sQ^2 - 2s'Q^2)]/4[2(s + s') + Q^2] \quad (3)$$

$$K'_\mu = P_\mu + P'_\mu$$

$$\langle \vec{P}, \sqrt{s}, m, J | \tilde{J}_0(0) | \vec{P}', \sqrt{s'}, m', J' \rangle = \sum_{L', M', \tilde{m}, \tilde{m}'} D_{m\tilde{m}}^J(R(\vec{w}_1)) \cdot D_{m'\tilde{m}'}^{*J'}(R(\vec{w}_2)) \cdot \langle J', \tilde{m}' L' M' \lambda | J \tilde{m} \rangle \cdot Y_{LM}(\theta, \varphi) \cdot G_{J, J'}^{0, L'}(s, s', Q^2) \quad (4)$$

# Invariant parameterization of the e.m. current for the two particles system (non-diagonal case)

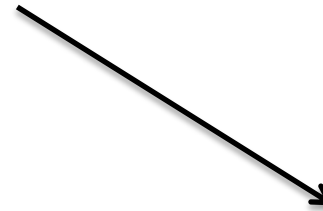
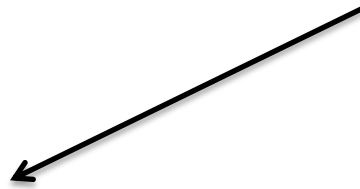
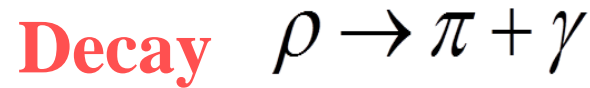
*the transition to the canonical basis*

$$J_k = a_{k\mu} \cdot \tilde{J}_\mu \quad \longrightarrow \quad a_{k\mu} = \sqrt{\frac{2\pi}{3}} \begin{vmatrix} -1 & 0 & 1 \\ i & 0 & i \\ 0 & \sqrt{2} & 0 \end{vmatrix} \quad (5)$$

$k = 1, 2, 3$

$$\langle \vec{P}, \sqrt{s}, m, J | \tilde{J}_\mu(0) | \vec{P}', \sqrt{s'}, m', J' \rangle = \sum_{j, \lambda, \tilde{m}, \tilde{m}', L, M} D_{m\tilde{m}}^J(R(\vec{w}_1)) \cdot D_{m'\tilde{m}'}^{*J'}(R(\vec{w}_2)) \cdot \langle J', \tilde{m}' j \lambda | J \tilde{m} \rangle \cdot \langle 1\mu LM | j \lambda \rangle \cdot Y_{LM}(\theta, \varphi) \cdot G_{J, J'}^{1, L, j}(s, s', Q^2) \quad (6)$$

# The relevance of the radiative transition



*Experimental information*

*Theoretical information*



**Extraction of the pion-in-flight contribution from the electroproduction cross section of vector mesons of the nuclei**

**Analysis of values of the squared four-momentum transfer for description of exclusive processes**

## Free non-diagonal form factors

$$\begin{aligned}\langle \vec{P}, \sqrt{s}, 0, 0 | J_0(0) | \vec{P}', \sqrt{s'}, m', 1 \rangle &= -\frac{1}{2\sqrt{\pi}} \cdot G_{0,1}^{0,1}(s, s', Q^2) \\ \langle \vec{P}, \sqrt{s}, 0, 0 | J_1(0) | \vec{P}', \sqrt{s'}, m', 1 \rangle &= -\frac{1}{3} \cdot G_{0,1}^{1,1,1}(s, s', Q^2)\end{aligned}\tag{7}$$

$$\langle \vec{P}, \sqrt{s}, 0, 0 | J_2(0) | \vec{P}', \sqrt{s'}, m', 1 \rangle = \frac{i}{3} \cdot (\sqrt{2} \cdot G_{0,1}^{1,0,1}(s, s', Q^2) + G_{0,1}^{1,2,1}(s, s', Q^2))$$

$$\langle \vec{P}, \sqrt{s}, 0, 0 | J_3(0) | \vec{P}', \sqrt{s'}, m', 1 \rangle = \frac{1}{3} \cdot (\sqrt{2} \cdot (G_{0,1}^{1,2,1}(s, s', Q^2) - G_{0,1}^{1,0,1}(s, s', Q^2)))$$



# Free non-diagonal form factors

*In one-particle basis:*

$$\begin{aligned}
 \langle \vec{P}, \sqrt{s}, 0, 0, 0, 0 | J_\mu(0) | \vec{P}', \sqrt{s'}, 1, 1, m' \rangle &= \int \frac{d^3 \vec{p}_1}{2p_{10}} \int \frac{d^3 \vec{p}_2}{2p_{20}} \int \frac{d^3 \vec{p}'_1}{2p'_{10}} \int \frac{d^3 \vec{p}'_2}{2p'_{20}} \cdot \\
 \cdot \langle \vec{P}, \sqrt{s}, 0, 0, 0, 0 | \vec{p}_1, m_1; \vec{p}_2, m_2 \rangle \cdot \langle \vec{p}_1, m_1; \vec{p}_2, m_2 | j_\mu(0) | \vec{p}'_1, m'_1; \vec{p}'_2, m'_2 \rangle \cdot \\
 \cdot \langle \vec{p}'_1, m'_1; \vec{p}'_2, m'_2 | \vec{P}', \sqrt{s'}, 1, 1, m' \rangle
 \end{aligned} \tag{8}$$

$$\begin{aligned}
 \langle \vec{p}_1, m_1; \vec{p}_2, m_2 | j_\mu(0) | \vec{p}'_1, m'_1; \vec{p}'_2, m'_2 \rangle &= \langle \vec{p}_1, m_1 | j_{\mu 1}(0) | \vec{p}'_1, m'_1 \rangle \cdot \\
 \cdot \delta(\vec{p}_2 - \vec{p}'_2) \cdot \delta_{m_2 m'_2} + \langle \vec{p}_2, m_2 | j_{\mu 2}(0) | \vec{p}'_2, m'_2 \rangle \cdot \delta(\vec{p}_1 - \vec{p}'_1) \cdot \delta_{m_1 m'_1}
 \end{aligned} \tag{9}$$

$$\langle \vec{p}_1, m_1 | j_{\mu 1}(0) | \vec{p}'_1, m'_1 \rangle = \sum_{m''_1} D_{m_1 m''_1}^{1/2}(p_1 p'_1) \langle m_1'' | [f_{10}(Q^2) K'_\mu + i f_{30}(Q^2) R_\mu] | m'_1 \rangle \tag{10}$$

$$K'_\mu = (p + p')_\mu, \quad R_\mu = \epsilon_{\mu\nu\lambda\rho} p^\nu p'^\lambda \Gamma^\rho(p')$$

## Form factor of the free system

$$G_{0,1}^{1,1,1}(s, s', Q^2) = \frac{\sqrt{2}(s + s' + Q^2)}{2\sqrt{s - 4M^2}\sqrt{s' - 4M^2}} \cdot \left[ \cos \frac{(\omega_1 + \omega_2)}{2} f_{30} \frac{M(s - s' + Q^2)ss'}{\lambda(s, s', -Q^2)(s + s' + Q^2)} \right. \\ \left. + \sin \frac{(\omega_1 + \omega_2)}{2} \xi(s, s', Q^2) \left[ f_{30} \frac{M}{2} + \frac{8s' f_{10}}{\sqrt{\lambda(s, s', -Q^2)(s + s' + Q^2)}} \right] \right] \quad (11)$$

## Form factor of the composite system

$$G_{0,1}^{1,1,1}(Q^2) = \int \int d\sqrt{s} d\sqrt{s'} G_{0,1}^{1,1,1}(s, s', Q^2) \varphi_\pi(s) \varphi_\rho(s') \quad (12)$$

## Transition form factors measured in experiment

$$\langle P_\pi, 0, 0 | J_\mu(0) | P_\rho, \lambda, 1 \rangle = F_{\pi\rho}(Q^2) \varepsilon_{\mu\nu\alpha\beta} e^\nu(\lambda) P_\pi^\alpha P_\rho^\beta \quad (13)$$

$$F_{\pi\rho}(Q^2) = \sqrt{\frac{2}{3}} \frac{G_{0,1}^{1,1,1}(Q^2)}{q(P_\pi^0 + P_\rho^0)} \quad (14)$$

$$q^2 = [M_\pi^4 + M_\rho^4 + Q^2 - 2(M_\pi^2 M_\rho^2 - 2M_\pi^2 Q^2 - 2M_\rho^2 Q^2)] / 4[2(M_\pi^2 + M_\rho^2) + Q^2]$$

# Conclusions

- ✓ *Matrix element of the electromagnetic current was constructed for the non-diagonal case in the instant form of PIQM in the impulse approximation.*
- ✓ *The electromagnetic current satisfies the conservation law and Lorentz-covariance conditions*
- ✓ *Analytic expression was obtained for the transition form factors in the radiative decay rho meson.*