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The construction of the electromagnetic current operator for the rho-meson decay process in the instant form of the Poincare-invariant quantum mechanics

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Poincare-invariant quantum mechanics[1]

- Construction of the electromagnetic current operator with conditions of current conservation and Lorentz – covariance.
- Good description of the meson electroweak properties of meson and deuteron.
- Asymptotic of the electromagnetic form factors at $Q^2 \to \infty$ coinciding with predictions QCD and quark counting rules.

[1] A.F. Krutov, V.E. Troitsky. Physics of Particles and Nuclei. Vol. 40, 2009, 136

Description of the processes with spin-diagonal matrix element

$$F_\pi(Q^2) = \int\int d\sqrt{s}\,d\sqrt{s'}\,g_0(s, Q^2, s')\varphi(s)\varphi(s') \quad (1)$$

modified impulse approximation

$$\langle \vec{P}, \sqrt{s}, J, l, S, m_J | j^{(0)}_\mu | \vec{P}', \sqrt{s'}, J, l', S', m'_J \rangle =$$

$$= \sum_{m''_J} \langle m_J | D^J(P,\, P') | m''_J \rangle \langle m''_J | \sum_{i=1}^{3} \left\{ F_i^{ll'SS'}\, A^i_\mu(s, Q^2, s') \right\}_+ | m'_J \rangle$$

$$F_i^{ll'SS'} = \sum_{n=0}^{2J} f_{in}^{ll'SS'}(s\,,\, Q^2\,,\, s')(iP_\mu \Gamma^\mu(P'))^n$$

Free electromagnetic form factor with (J=J'= L=L'= S=S' =0)

$$g_0(s,Q^2,s') = \frac{(s+s'+Q^2)^2 Q^2[\vartheta(s'-s_1)-\vartheta(s'-s_2)]}{2\sqrt{s-4M^2}\sqrt{s'-4M^2}[\lambda(s,Q^2,s')]^{3/2}} \tag{2}$$

$$\left[\cos(\omega_1+\omega_2)f_{10}(Q^2) - 2M\xi(s,s',Q^2)\sin(\omega_1+\omega_2)f_{30}(Q^2)\right)\right]$$

Invariant parameterization of the e.m. current for the two particles system (non-diagonal case)

Breit system

$$K'_\mu = (\sqrt{-K'^2}, 0, 0, 0), \qquad P_\mu = (\vec{q}, P_0), \qquad P'_\mu = (-\vec{q}, P'_0)$$

$$q^2 = [s^2 + s'^2 + Q^2 - 2(ss' - 2sQ^2 - 2s'Q^2)]/4[2(s+s') + Q^2] \quad (3)$$

$$K'_\mu = P_\mu + P'_\mu$$

$$\langle \vec{P}, \sqrt{s}, m, J | \tilde{J}_0(0) | \vec{P}', \sqrt{s'}, m', J' \rangle = \sum_{L', M', \tilde{m}, \tilde{m}'} D^J_{m\tilde{m}}(R(\vec{w}_1)) \cdot D^{*J'}_{m'\tilde{m}'}(R(\vec{w}_2)) \cdot$$

$$\cdot \langle J', \tilde{m}' L' M' \lambda | J \tilde{m} \rangle \cdot Y_{LM}(\theta, \varphi) \cdot G^{0,L'}_{J,J'}(s, s', Q^2) \qquad (4)$$

Invariant parameterization of the e.m. current for the two particles system (non-diagonal case)

the transition to the canonical basis

$$J_k = a_{k\mu} \cdot \tilde{J}_\mu$$

$$k = 1,2,3$$

$$a_{k\mu} = \sqrt{\frac{2\pi}{3}} \begin{vmatrix} -1 & 0 & 1 \\ i & 0 & i \\ 0 & \sqrt{2} & 0 \end{vmatrix} \qquad (5)$$

$$\langle \vec{P}, \sqrt{s}, m, J | \tilde{J}_\mu(0) | \vec{P}', \sqrt{s'}, m', J' \rangle = \sum_{j,\lambda,\tilde{m},\tilde{m}',L,M} D^{J}_{m\tilde{m}}(R(\vec{w}_1)) \cdot D^{*J'}_{m'\tilde{m}'}(R(\vec{w}_2)) \cdot$$

$$\cdot \langle J', \tilde{m}' j \lambda | J \tilde{m} \rangle \cdot \langle 1 \mu L M | j \lambda \rangle \cdot Y_{LM}(\theta, \varphi) \cdot G^{1,L,j}_{J,J'}(s, s', Q^2) \qquad (6)$$

The relevance of the radiative transition

Decay $\rho \to \pi + \gamma$

Experimental information

Theoretical information

Extraction of the pion-in-flight contribution from the elecroproduction cross section of vector mesons of the nuclei

Analysis of values of the squared four-momentum transfer for description of exclusive processes

Free non-diagonal form factors

$$\langle \vec{P}, \sqrt{s}, 0, 0 | J_0(0) | \vec{P'}, \sqrt{s'}, m', 1 \rangle = -\frac{1}{2\sqrt{\pi}} \cdot G^{0,1}_{0,1}(s, s', Q^2)$$
$$\langle \vec{P}, \sqrt{s}, 0, 0 | J_1(0) | \vec{P'}, \sqrt{s'}, m', 1 \rangle = -\frac{1}{3} \cdot G^{1,1,1}_{0,1}(s, s', Q^2) \quad (7)$$

$$\langle \vec{P}, \sqrt{s}, 0, 0 | J_2(0) | \vec{P'}, \sqrt{s'}, m', 1 \rangle = \frac{i}{3} \cdot (\sqrt{2} \cdot G^{1,0,1}_{0,1}(s, s', Q^2) + G^{1,2,1}_{0,1}(s, s', Q^2))$$

$$\langle \vec{P}, \sqrt{s}, 0, 0 | J_3(0) | \vec{P'}, \sqrt{s'}, m', 1 \rangle = \frac{1}{3} \cdot (\sqrt{2} \cdot (G^{1,2,1}_{0,1}(s, s', Q^2) - G^{1,0,1}_{0,1}(s, s', Q^2))$$

Free non-diagonal form factors

In one-particle basis:

$$\langle\vec{P},\sqrt{s},0,0,0,0|J_\mu(0)|\vec{P'},\sqrt{s'},1,1,m'\rangle = \int\frac{d^3\vec{p_1}}{2p_{10}}\int\frac{d^3\vec{p_2}}{2p_{20}}\int\frac{d^3\vec{p_1'}}{2p_{10}'}\int\frac{d^3\vec{p_2'}}{2p_{20}'}\cdot$$
$$\cdot\langle\vec{P},\sqrt{s},0,0,0,0|\vec{p_1},m_1;\vec{p_2},m_2\rangle\cdot\langle\vec{p_1},m_1;\vec{p_2},m_2|j_\mu(0)|\vec{p_1'},m_1';\vec{p_2'},m_2'\rangle\cdot$$
$$\cdot\langle\vec{p_1'},m_1';\vec{p_2'},m_2'|\vec{P'},\sqrt{s'},1,1,m'\rangle \qquad (8)$$

$$\langle\vec{p_1},m_1;\vec{p_2},m_2|j_\mu(0)|\vec{p_1'},m_1';\vec{p_2'},m_2'\rangle = \langle\vec{p_1},m_1|j_{\mu1}(0)|\vec{p_1'},m_1'\rangle\cdot$$
$$\cdot\delta(\vec{p_2}-\vec{p_2'})\cdot\delta_{m_2m_2'} + \langle\vec{p_2},m_2|j_{\mu2}(0)|\vec{p_2'},m_2'\rangle\cdot\delta(\vec{p_1}-\vec{p_1'})\cdot\delta_{m_1m_1'} \qquad (9)$$

$$\langle\vec{p_1},m_1|j_{\mu1}(0)|\vec{p_1'},m_1'\rangle = \sum_{m_1''}D^{1/2}_{m_1m_1''}(p_1p_1')\langle m_1''|[f_{10}(Q^2)K_\mu' + if_{30}(Q^2)R_\mu]|m_1'\rangle \qquad (10)$$

$$K_\mu' = (p+p')_\mu\,,\qquad R_\mu = \epsilon_{\mu\nu\lambda\rho}\,p^\nu\,p'^{\lambda}\,\Gamma^\rho(p')$$

Form factor of the free system

$$G_{0,1}^{1,1,1}(s,s',Q^2) = \frac{\sqrt{2}(s+s'+Q^2)}{2\sqrt{s-4M^2}\sqrt{s'-4M^2}} \cdot \left[\cos\frac{(\omega_1+\omega_2)}{2} f_{30} \frac{M(s-s'+Q^2)ss'}{\lambda(s,s',-Q^2)(s+s'+Q^2)}\right.$$

$$\left. + \sin\frac{(\omega_1+\omega_2)}{2}\xi(s,s',Q^2)\left[f_{30}\frac{M}{2} + \frac{8s' f_{10}}{\sqrt{\lambda(s,s',-Q^2}(s+s'+Q^2)}\right]\right] \quad (11)$$

Form factor of the composite system

$$G_{0,1}^{1,1,1}(Q^2) = \int\int d\sqrt{s}\, d\sqrt{s'}\, G_{0,1}^{1,1,1}(s,s',Q^2)\varphi_\pi(s)\varphi_\rho(s') \quad (12)$$

Transition form factors measured in experiment

$$\langle P_\pi, 0, 0 | J_\mu(0) | P_\rho, \lambda, 1\rangle = F_{\pi\rho}(Q^2)\varepsilon_{\mu\nu\alpha\beta}e^\nu(\lambda)P_\pi^\alpha P_\rho^\beta \quad (13)$$

$$F_{\pi\rho}(Q^2) = \sqrt{\frac{2}{3}}\frac{G_{0,1}^{1,1,1}(Q^2)}{q(P_\pi^0 + P_\rho^0)} \quad (14)$$

$$q^2 = [M_\pi^4 + M_\rho^4 + Q^2 - 2(M_\pi^2 M_\rho^2 - 2M_\pi^2 Q^2 - 2M_\rho^2 Q^2)]/4[2(M_\pi^2 + M_\rho^2) + Q^2]$$

Conclusions

- ***Matrix element of the electromagnetic current was constructed for the non-diagonal case in the instant form of PIQM in the impulse approximation.***

- ***The electromagnetic current satisfies the conservation law and Lorentz-covariance conditions***

- ***Analytic expression was obtained for the transition form factors in the radiative decay rho meson.***