Heavy-ion collisions in modified AdS spaces

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Quark Gluon Plasma is the thermally equilibrated state of matter in wich quarks and gluons are deconfined from hadrons

Quark Gluon Plasma (QGP) has been discovered in Au+Au collision at energy 100 GeV for nucleon in 2005 @ RHIC

In holographic approch clasical gravity in AdS_5 discribes strong coupling field theory in 4D Minkowski space

There is hypothesis that QGP formation in 4D space corresponds to Black Holes creation in dual 5D space.

Gubser, Klebanov, Polyakov, 9802109

Witten, 9802150

The gravitational shock wave in AdS_5 space is dual to ultrarelativistic heavy-ion in 4D space-time.

Thus,

- heavy-ion collisions can be represented such as gravitational shock waves collisions in AdS_5

- QGP formation is equivalent to BH creation in AdS_5

Gubser et al.; 0805.1551, 0902.4062

Problem:

- How to get dependence of experimental multiplicity on energy from holographic model.
- Simplest holographic model is related with $\mathcal{N}=4$ SYM

  [But QCD is not SYM]

- Our goal: to study more complicate models to fit experimental data.

Main conjecture: multiplicity is proportional to entropy

$$S \sim N$$

Gubser et al.; 0805.1551

On experiments can be measured only N_{ch} : $N \sim N_{ch}$

B. B. Back et. al., 0210015[nucl-ex].

Accordingly experiment the charged-particles pseudorapidity density depends on colliding energy

$$dN_{ch}/d\eta \propto s^{0.15}$$

for Pb-Pb and Au-Au - collisions

$$dN_{ch}/d\eta \propto s^{0.11}$$

pp collision

$E=(1/2)\sqrt{s_{NN}}$ - colliding energy for nucleon

K. Aamodt et al.[ALICE Collaboration], 1011.3916 [nucl-ex].

DISCREPANCY

The simple holographic model gives

$$dN_{ch}/d\eta \propto s^{2/3}$$

The mininal black hole entropy can be estimated by trapped surface area

$$S \geq S_{trapped} = A_{trapped} / 4\mathrm{G}_N$$

The trapped surface is surface whose null normals all propagate inward.

S. W. Hawking and D. Page,Thermodynamics Of Black Holes In Anti-de Sitter Space,Commun.Math.Phys.87(1983)577

C. S. Pe¸ca, J. P. S. Lemos, 9805004 [gr-qc]

- $\mathcal{N}=4$ SYM is not QCD
- For holographic description of QCD a modified AdS_5 is used to study dependence of entropy on energy

Gursoya, Kiritsis et al.,0707.1324, 0707.1349

- Early the modification of AdS_5 space-time by introduction of wrapping factor was applied for shock waves with the specialy distributed colliding masses.

Kiritsis, 1111.1931

- We consider collisions of walls with averaged mass in modified 5D space-time.

The shock wave with mass spread over transversal surface (shock wall wave) is a simplification of a point source shock wave.

We describe heavy-ion collisions by the wall-wall shock wave collisions in AdS_5 (or in its modification)

S. Lin, E. Shuryak, 1011.1918[hep-th]

I. Y. Aref'eva, A. A. Bagrov and E. O. Pozdeeva, Holographic phase diagram of quark-gluon plasma formed in heavy-ions collisions," JHEP 1205, 117 (2012)

The Einstein equation for particle in dilaton field has the form:

$$\left(R_{\mu\nu} - \frac{g_{\mu\nu}}{2}R\right) - \frac{g_{\mu\nu}}{2}\left(-\frac{4}{3}(\partial\Phi_s)^2 + V(\Phi_s)\right) - \frac{4}{3}\partial_\mu\Phi_s\,\partial_\nu\Phi_s - g_{\mu\nu}\frac{d(d-1)}{2L^2} = 8\pi G_5 J_{\mu\nu},$$

where $$J_{++} = \frac{E}{b^3(z)}\,\delta(x^1)\delta(x^2)\delta(z-z_*)\delta(x^+).$$

We consider the shock wave metric

$$ds^2 = b^2(z)\left(dz^2 + dx^i\,dx^i - dx^+\,dx^- + \phi(z,x^1,x^2)\delta(x^+)(dx^+)^2\right)$$

Aref'eva I.Ya. 0912.5482[hep-th]

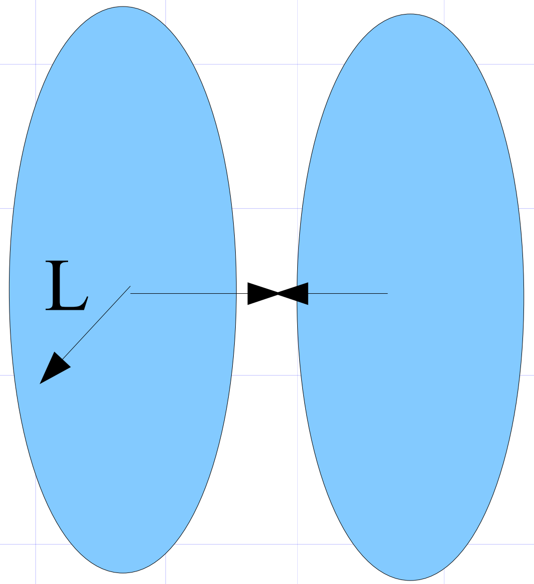
M. Hotta, M. Tanaka, Shock-wave geometry with nonvanishing cosmological constant, Class. Quantum Grav. **10,** 307, 1993

Using shock wave ansatz we reduce the Einstein equation to the differential equation for shock wave profile and two equations defining the connection of field and field potential with b-factor:

$$\left(\partial^2_{x^1}+\partial^2_{x^2}+\partial^2_z+\frac{3b'}{b}\partial_z\right)\phi(z,x_\perp)=-16\pi G_5\frac{E}{b^3}\,\delta(x^1)\delta(x^2)\delta(z_*-z)$$

$$V(\Phi_s)=\frac{3}{b^2}\left(\frac{b''}{b}+\frac{2(b')^2}{b^2}-\frac{4b^2}{L^2}\right) \qquad \Phi_s'=\pm\frac{3}{2}\sqrt{\left(\frac{2(b')^2}{b^2}-\frac{b''}{b}\right)}$$

The most interesting case is the collision of flat objects with mass uniformly distributed



The equation corresponding to the domain profile with mass distributed over the domain wall

$$\left(\partial_z^2 + 3\frac{b'}{b}\partial_z \right) \phi^W(z) = -16\pi G_5 \frac{E}{b^3}\delta(z_* - z)$$

and equation corresponding to the domain profile with mass distributed over the finite region with radius L

$$\left(\partial_z^2 + \frac{3b'}{b}\partial_z \right) \phi^w(z) = -16\pi G_5 \frac{E^*}{b^3}\delta(z_* - z), \quad E^* = \frac{E}{L^2}$$

coincide up to a constant factor L^2

The solutions to equations with mass uniformly distributed over finite and infinite surfaces coincide up to constant L^2

$$\phi^w(z) = \frac{\phi^W(z)}{L^2}$$

We identify the black hole creation with trapped surface formation. The formation conditions are applied to schock wall wave profile at the trapped surface boundary points

$$(\partial_z \phi^w)^2 \,|_{TS} = 4$$

The trapped surface area is calculated as follows

$$S_{trap} = \frac{1}{2G_5}\int_C \sqrt{det|g_{AdS_3}|}dzd^2x_\perp$$

where $\det|g_{\mathrm{AdS}_3}|$ is the metric determinant of AdS_3

The relative area *s* dened with the formula

$$s = \frac{S_{trap}}{\int d^2x_\perp}$$

We modify AdS space-time using 4 wrapping factors types

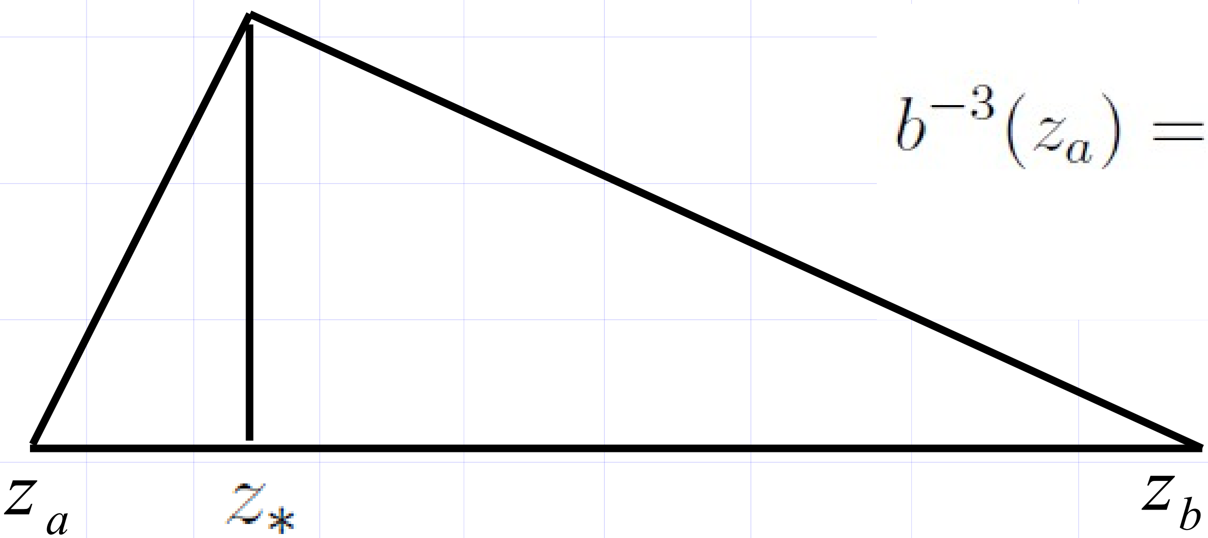
$$b = \left(\frac{L}{z - z_0}\right)^a \qquad b = \frac{L}{z}\exp\left(-\frac{z^2}{R^2}\right)$$

$$b = \exp\left(-\frac{z}{R}\right) \qquad b = \left(\frac{L}{z}\right)^a \exp\left(-\frac{z^2}{R^2}\right)$$

where a>0, R=1 fm, $L \approx 4.4$ fm

Using the solution to general form of domain equation (for any wrapping factor) and the trapped surface conditions we obtain the relations

$$F(z_*) = \frac{b^{-3}(z_b)F(z_a) + b^{-3}(z_a)F(z_b)}{b^{-3}(z_a) + b^{-3}(z_b)} \quad ; \quad \partial_z F(z) = b^{-3}(z)$$



$$b^{-3}(z_a) = \frac{b^{-3}(z_b)}{\frac{8\pi G_5 E}{L^2} b^{-3}(z_b) - 1}$$

between trapped surface boundary ($z_a < z_b$) points and collision point z_*

For the wrapping factor $b = \exp\left(-\frac{z}{R}\right)$ we have obtained following relations between boundary points and collision point

$$Z_A = \frac{L^2}{16\pi G_5 E} \cdot \frac{Z_B}{Z_B - \frac{L^2}{16\pi G_5 E}}, \qquad Z_0 = \frac{L^2}{8\pi G_5 E}$$

$$Z_0 = \exp\left(\frac{3z_*}{R}\right), \quad Z_A = \exp\left(\frac{3z_a}{R}\right), \quad Z_B = \exp\left(\frac{3z_b}{R}\right)$$

For the considered case the collision point is fixed by energy.

The relative area of trapped surface defined by

$$s = \frac{3}{2RG_5}\left(\frac{1}{\exp\left(\frac{3z_a}{R}\right)} - \frac{1}{\exp\left(\frac{3z_b}{R}\right)}\right) = \frac{3}{2RG_5}\left(\frac{1}{Z_A} - \frac{1}{Z_B}\right)$$

The maximum entropy value is obtained for $Z_b \gg 1$, in this approximation

$$Z_a \sim \frac{L^2}{16\pi G_5 E}, \qquad s \sim \frac{24\pi E}{RL^2}$$

The dependence of entropy on entropy is linear for the exponential wrapping factor

Power b-factor $b=(L/z)^a$ gives following relative area of trapped surface

$$s=\frac{1}{2G_5(3a-1)}\left(z_A\left(\frac{L}{z_A}\right)^{3a}-z_B\left(\frac{L}{z_B}\right)^{3a}\right),$$

$$z_A=\left(\frac{z_B{}^{3\,a}}{-1+z_B{}^{3\,a}C^2}\right)^{\frac{1}{3a}}\quad z_*=\left(\frac{z_A{}^{3\,a}z_B{}^{3\,a}\,(z_B+z_A)}{z_A{}^{3\,a}+z_B{}^{3\,a}}\right)^{\frac{1}{3\,a+1}}\quad C^2=\frac{8\pi G_5E}{L^{3a+2}}$$

The maximal entropy value will at $Z_b\gg 1$ in assumption $3a>1$

$$s\,|_{z_b\to\infty}=\frac{L^{3a}}{2G_5(3a-1)}z_A^{1-3a}=\frac{L}{2G_5}\left(\frac{8\pi G_5}{L^2}\right)^{\frac{3a-1}{3a}}E^{\frac{3a-1}{3a}}$$

For the power wrapping factor the entripy increase as $E^{1-1/3a}$

The multiplicity of particles produced in heavy ions (PbPb-and AuAu-collisions) collisions dependents on energy as $s_{NN}^{0,15}$ in the range $10-10^3$ GeV.

This model can coinside with experimental data at $a \approx 0.47$

K. Aamodt et al. [ALICE Collaboration], arXiv:1011.3916 [nucl-ex].

Mixed factor of the form $b = \frac{L}{z}\exp\left(-\frac{z^2}{R^2}\right)$ gives the another relative area of quasi-trapped surface energy dependence

$$s = \frac{L^3}{2G_5}\left(-\frac{1}{2\exp\left(\frac{3z_b^2}{R^2}\right)z_b^2} + \frac{1}{2\exp\left(\frac{3z_a^2}{R^2}\right)z_a^2} + \frac{3\,\mathrm{Ei}\left(1,\frac{3z_b^2}{R^2}\right)}{2R^2} - \frac{3\,\mathrm{Ei}\left(1,\frac{3z_a^2}{R^2}\right)}{2R^2}\right)$$

which has the maximal value at $z_b \to \infty$

$$s\,|_{z_b\to\infty} = \frac{3}{4}\frac{L^3}{G_5R^2}\left(-\mathrm{Ei}\left(1,\frac{3z_a^2}{R^2}\right) + \frac{1}{3}\frac{R^2}{\exp\left(\frac{3z_a^2}{R^2}\right)z_a^2}\right)$$

and roughly is $E^{\frac{2}{3}}(1 + 0.007\ln E) - 3$ at $10\mathrm{GeV} \le E < 1\,TeV$

The wrapping factor $b = \left(\frac{L}{z}\right)^a \exp\left(-\frac{z^2}{R^2}\right)$

gives the most complicate relative area of trapped surface energy dependence

$$s = \frac{F(z_B) - F(z_A)}{2G_5}$$

$$F(z) = \frac{\left(\frac{L}{z}\right)^{3a} z \exp\left(-\frac{3z^2}{2R^2}\right)\left(2\left(\frac{3z^2}{R^2}\right)^{\frac{3a-1}{4}} \mathrm{M}\left(\frac{-3a+1}{4}, \frac{3(-a+1)}{4}, \frac{3z^2}{R^2}\right) + 3(1-a)\exp\left(-\frac{3z^2}{2R^2}\right)\right)}{3\,(-1+3\,a)\,(-1+a)}$$

$$\mathrm{M}(\mu, \nu, z) = \exp\left(-\frac{z}{2}\right) z^{\frac{1}{2}+\nu}\, {}_1F_1\left(\frac{1}{2} + \nu - \mu, 1 + 2\nu, z\right)$$

Wich has maximal value at $z_B \to \infty$: $S \longrightarrow \frac{-F(z_A)}{2G_5}$

The entropy can be roughly estimate at a=1/2 such as

$$S \sim E^{0.3}(1 + C_1(\ln(E + 100))) - C_2$$

$C_1 = -0.738$, $C_2 = 0.393$ at $10 < E < 100$ GeV

$C_1 = -0.073$, $C_2 = 0.827$ at $100 < E < 1000$ GeV

Conclusions

The black holes formation in the domain wall-wall collisions is investigated in the deformed AdS_5 with b-factors.

The several b-factor types: power, exponential and mixed are considered.

The dependence of the entropy on the energy for different b-factors is analyzed.

These results (with the account of AdS/CFT-duality) allow to simulate the dependence of multiplicity on the energy of the colliding heavy-ions

$$b = (L/z)^a, \quad a \approx 0.47, \quad S \sim E^{0.3}$$

(in agreement with experimental data $s_{NN}^{0,15}$).

The additional logarithms appear when considering the mixed factor.