Scattering amplitude and pomeron loops in the perturbative QCD with large N_c

M.A. Braun, A.N. Tarasov
St. Petersburg State University

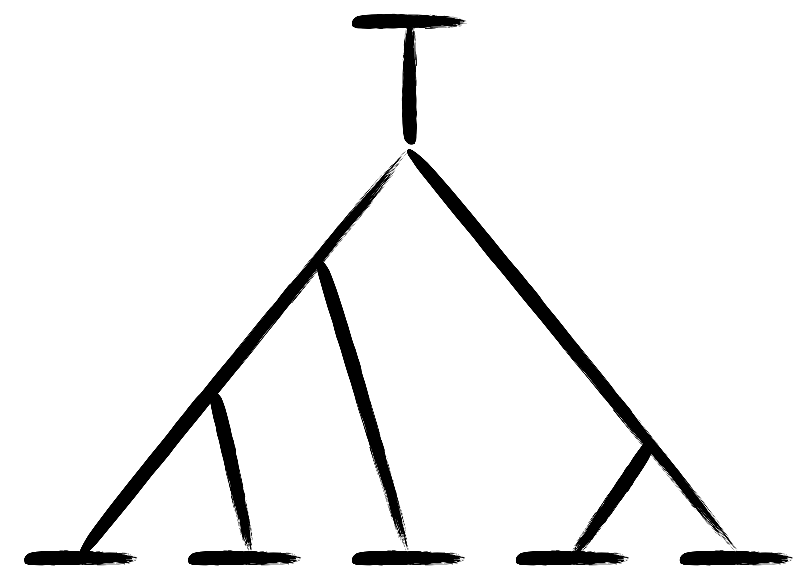
Outline

- QCD at high energies (small-x physics)
- Deep inelastic scattering
- Beyond leading logarithm approximation
- Single pomeron loop calculation
- Numerical estimations
- Conclusions

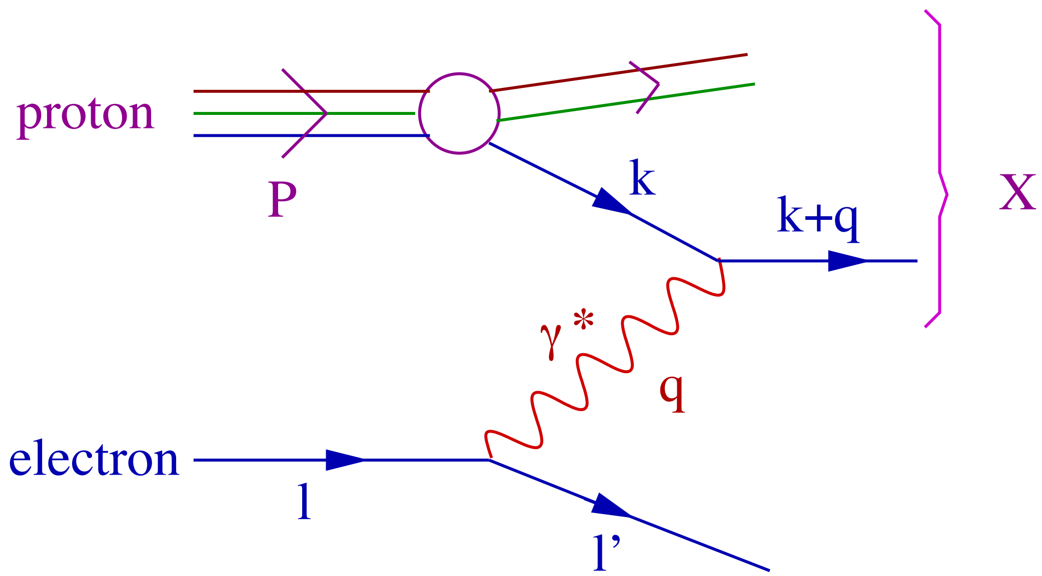
QCD at high energies (small-x physics)

In this presentation we are going to consider corrections to the BFKL dynamics

- In the framework of QCD with a large number of colours, strong interactions are mediated by the exchange of interacting BFKL pomerons
- The pomerons split and fuse by triple pomeron vertices



Deep inelastic scattering



Deep inelastic regime: $Q^2 = -q^2 \gg M^2$

Bjorken variable: $x = \dfrac{Q^2}{s + Q^2 - M^2}$

This kinematic variables are fixed by initial condition (l, P) and l'

They determine transverse area $1/Q$ and longitudinal momentum of the parton xP that is involved in the scattering

Deep inelastic scattering

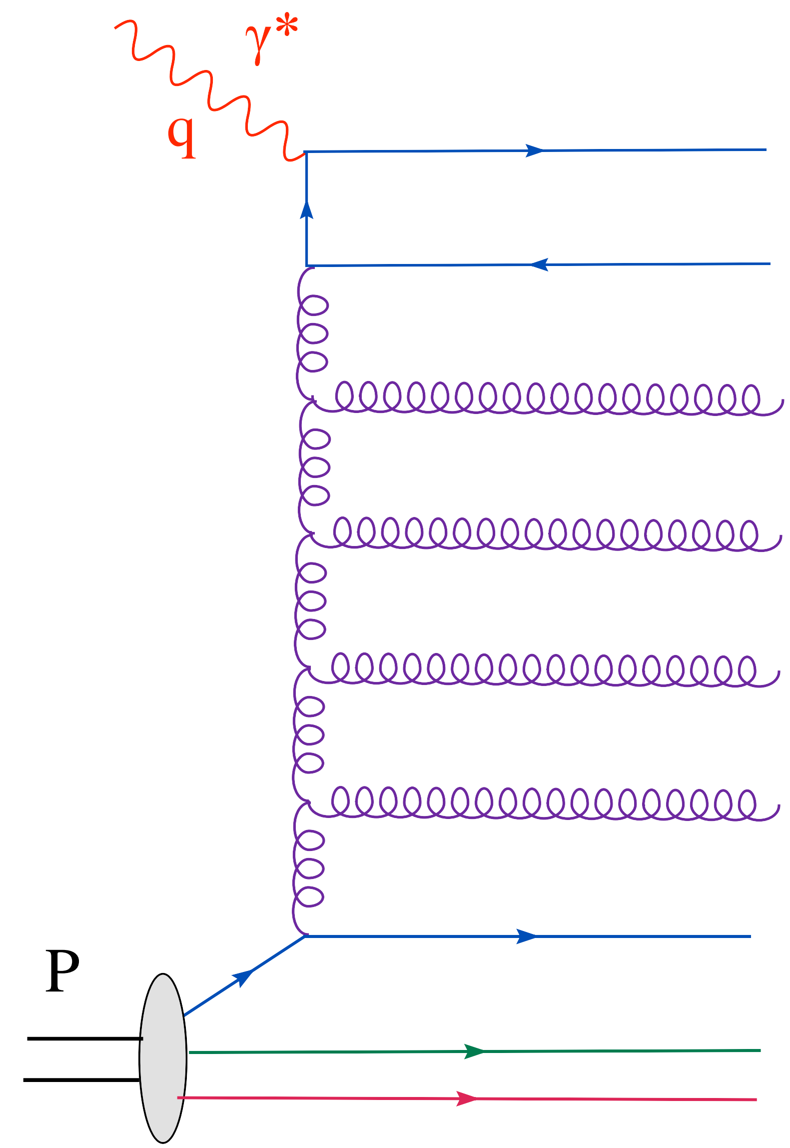
Emission of intermediate gluon is enhanced by large factor $ln(1/x)$

$$\alpha_s \ll 1, \quad \alpha_s ln(1/x) \sim 1$$

The BFKL equation sums all ladder diagrams

$$\frac{\partial N(y,r_1,r_2)}{\partial y} = \frac{\bar{\alpha}}{2\pi}\int d^2 r_3 \frac{r_{12}^2}{r_{13}^2 r_{23}^2}\Big(N(y,r_1,r_3) + N(y,r_2,r_3) - N(y,r_1,r_2)\Big)$$

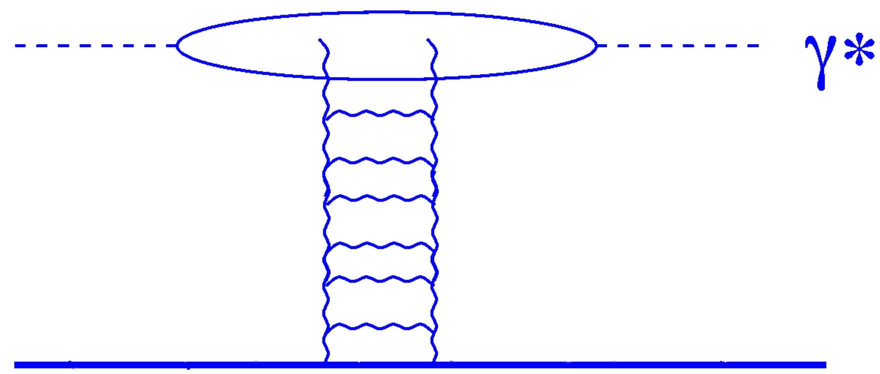
The equation sums all contribution of the order $(\alpha_s \ln s)^n$



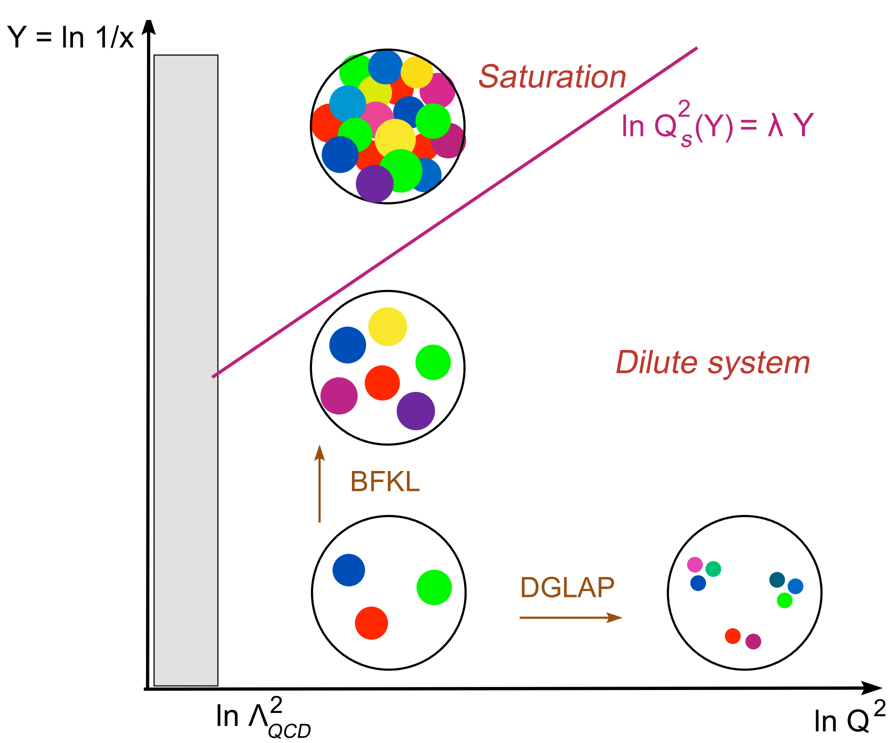
Deep inelastic scattering

$$\frac{\partial N(y,r_1,r_2)}{\partial y} = \frac{\bar{\alpha}}{2\pi}\int d^2r_3 \frac{r_{12}^2}{r_{13}^2 r_{23}^2}\Big(N(y,r_1,r_3)+N(y,r_2,r_3)-N(y,r_1,r_2)\Big)$$

$$\alpha_s \ll 1, \quad \alpha_s ln(1/x) \sim 1$$



The BFKL equation is only valid at moderately high energies



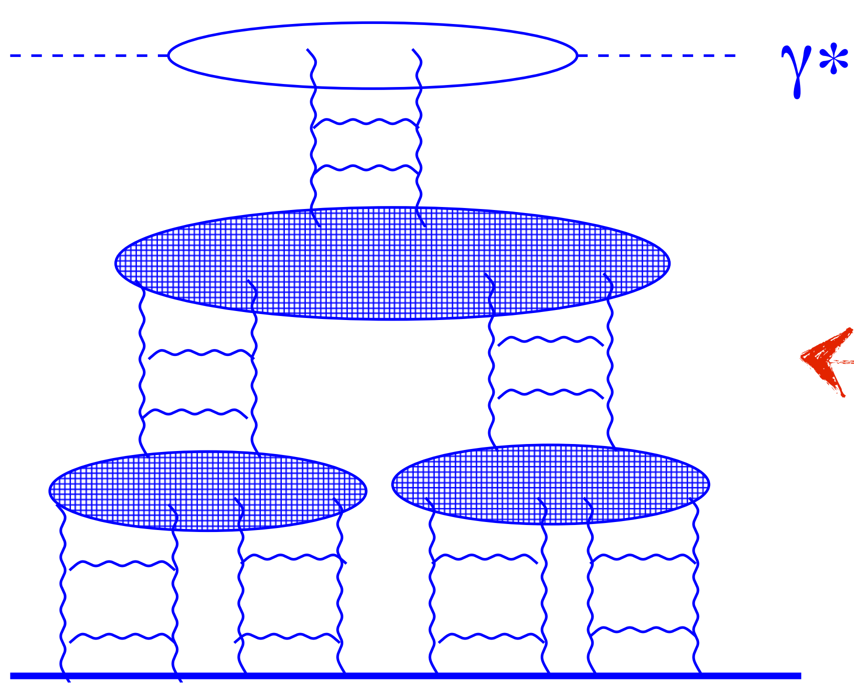
At large densities saturation is important.
One should take into account non-linear effects

There are two ways to develop the theory at extremely high energies

Beyond leading logarithm approximation:
Balitsky-Kovchegov equation

One can consider scattering on heavy nucleus target and take into account triple-pomeron vertex

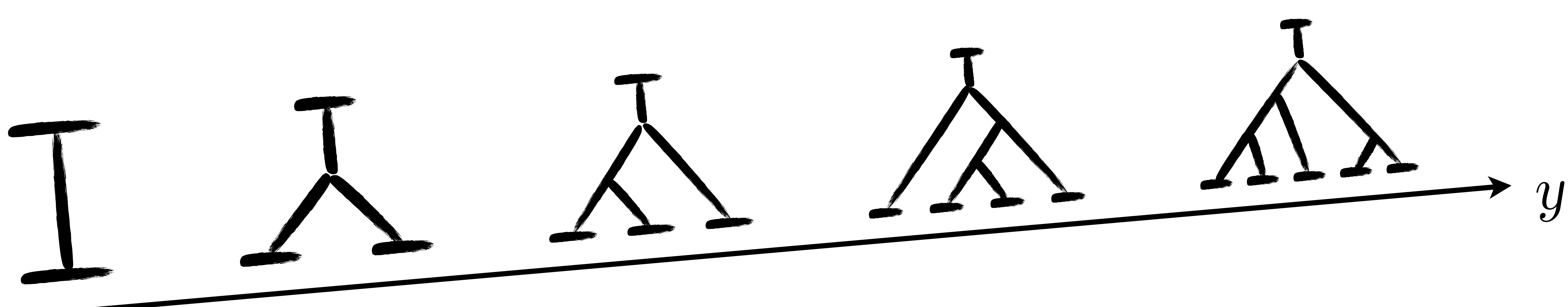
$$\frac{\partial N(y,r_1,r_2)}{\partial y} = \frac{\bar{\alpha}}{2\pi}\int d^2 r_3 \frac{r_{12}^2}{r_{13}^2 r_{23}^2}\Big(N(y,r_1,r_3) + N(y,r_2,r_3) - N(y,r_1,r_2) - \underline{N(y,r_1,r_3)N(y,r_3,r_2)}\Big)$$



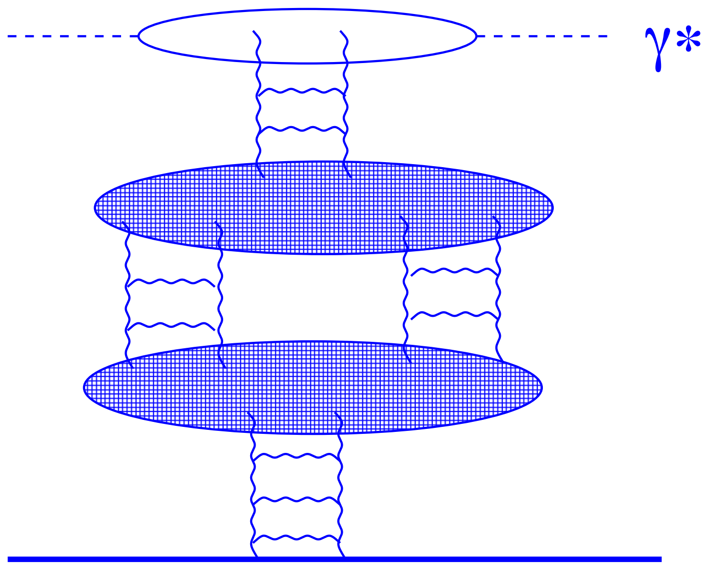
The BK equation is valid for the following kinematic conditions

$$\underbrace{\alpha_s \ll 1, \quad \alpha_s ln(1/x) \sim 1}_{\text{BFKL}}, \quad \underbrace{\alpha_s^2 A^{1/3} e^{\omega(0)y} \sim 1}_{\text{BK}}$$

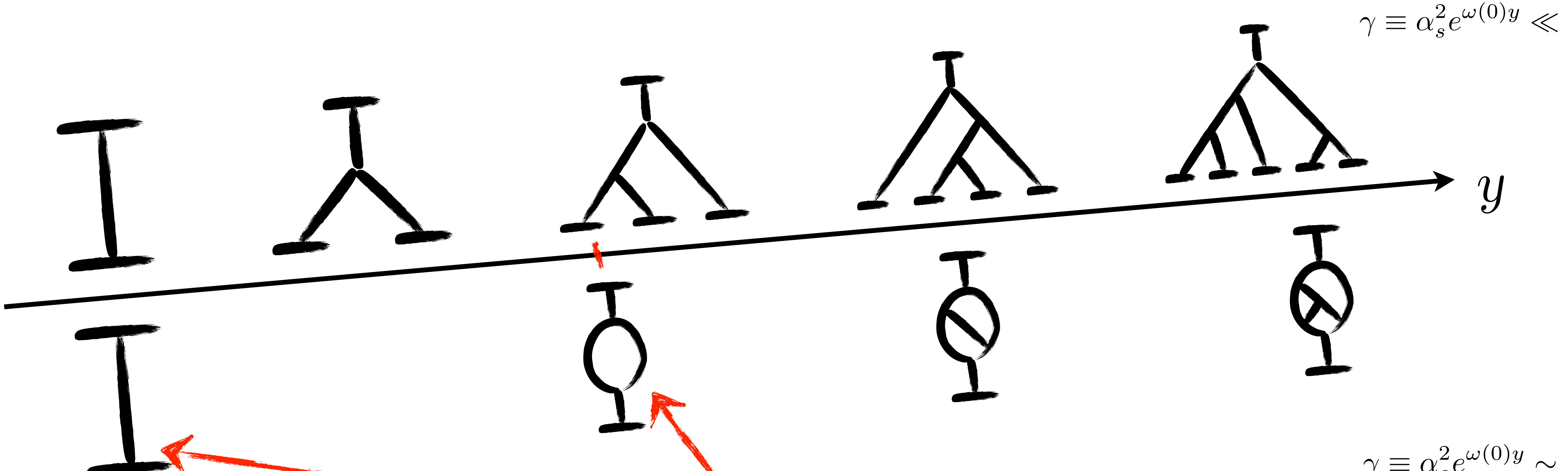
$$\gamma \equiv \alpha_s^2 e^{\omega(0)y} \ll 1$$



Beyond leading logarithm approximation: scattering on single hadrons



Non-BK pomeron loops



We are going to compare this two amplitudes

Loop contribution

- It is important to estimate the role of pomeron loops at present energies
- This will help to understand validity of the quasi-classical methods

- This seems to be straightforward. We have all instruments for this goal: the BFKL propagator and the triple pomeron vertex
- The only obstacle is the most complicated form of the letter
- A realistic calculation is a formidable task

Loop contribution

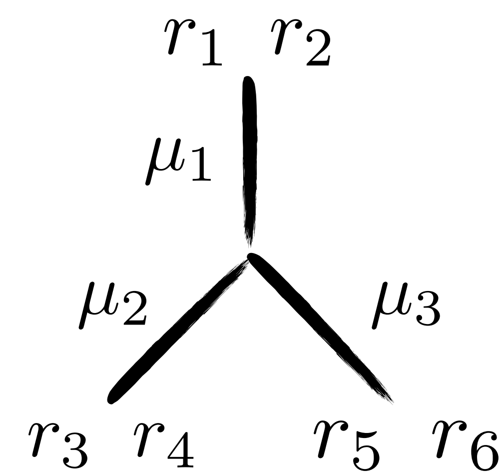
The triple pomeron vertex was introduced in:

- J. Bartels, Z. Phys. C 60, 471 (1993)
- J. Bartels, M. Wuesthoff, Z. Phys. C 66, 157 (1995)
- A.H. Mueller, B. Patel, Nucl. Phys. B 425, 471 (1994)
- M.A. Braun, G.P. Vacca, Eur. Phys. J. C 6, 147 (1999)

Interaction part of the Lagrangian of effective non-local field theory (M. Braun, Phys. Lett. B 632 (2006) 297–304):

$$L_I = \frac{2\alpha_s^2 N_c}{\pi} \int \frac{d^2 r_1 d^2 r_2 d^2 r_3}{r_{12}^2 r_{23}^2 r_{13}^2} \Phi(y, r_2, r_3) \Phi(y, r_3, r_1) L_{12} \Phi^\dagger(y, r_1, r_2) + h.c.,$$

$$L_{12} = r_{12}^4 \nabla_1^2 \nabla_2^2$$



We consider the loop by the conformal invariant technique. We present all constituents in terms of conformal basis formed by functions:

$$E_\mu(r_1, r_2) = \left(\frac{r_{12}}{r_{10} r_{02}}\right)^h \left(\frac{r_{12}^*}{r_{10}^* r_{02}^*}\right)^{\bar{h}} \qquad \mu = \{n, \nu, r_0\}$$

$$h = \frac{1+n}{2} + i\nu; \quad \bar{h} = 1 - h^*.$$

Conformal representation of the triple-pomeron vertex:

$$\Gamma(r_1, r_2 | r_3, r_4; r_5, r_6) = \sum_{\mu_1, \mu_2, \mu_3 > 0} \Gamma_{\mu_1 | \mu_2, \mu_3} E_{\mu_1}(r_1, r_2) E_{\mu_2}^*(r_3, r_4) E_{\mu_3}^*(r_5, r_6)$$

$$\sum_{\mu > 0} = \sum_{n=-\infty}^{\infty} \int_0^\infty d\nu \frac{1}{a_h} \int d^2 r_0, \quad a_h = \frac{\pi^4}{2} \frac{1}{\nu^2 + n^2/4}$$

Integration over $r_i, \bar{r}_j$

$\Downarrow$

Integration over n, ν, r_0

Triple-pomeron vertex

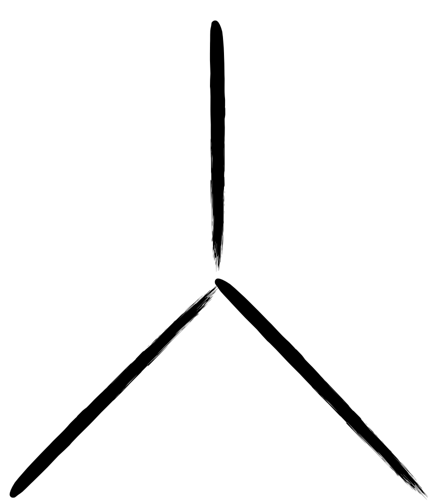
The structure of the vertex is fixed by conformal invariance:

$$\Gamma^{(0)}_{\mu_1|\mu_2,\mu_3} = R_{12}^{\alpha_{12}} R_{23}^{\alpha_{23}} R_{13}^{\alpha_{13}} \times (c.c.) \times \Omega(\bar{h}_1, h_2, h_3)$$

The explicit form of the vertex was found by Korchemsky:

$$\Omega(h_1, h_2, h_3) = \pi^3 \left[\Gamma^2(h_1)\Gamma^2(h_2)\Gamma(1-h_1)\Gamma(1-h_2)\Gamma(1-h_3)\right]^{-1} \times \sum_{a=1}^{3} J_a(h_1, h_2, h_3)\bar{J}_a(\bar{h}_1, \bar{h}_2, \bar{h}_3)$$

$\mu_1 = \{n_1, \nu_1, R_1\}$



$\mu_2 = \{n_2, \nu_2, R_2\}$ $\mu_3 = \{n_3, \nu_3, R_3\}$

J functions are convolutions of hypergeometric functions

Structure of the triple-pomeron vertex is very complex!!!

Triple-pomeron vertex

$$J_1(h_1,h_2,h_3) = \Gamma(h_1+h_2-h_3)\Gamma(1-h_1)\Gamma(h_1)\Gamma(1-h_2)\Gamma(h_2)$$

$$\times \int_0^1 dx\ (1-x)^{-h_3}{}_2F_1(h_1,1-h_1;1;x){}_2F_1(h_2,1-h_2;1;x);$$

$$J_2(h_1,h_2,h_3) = \frac{\Gamma(h_1+h_2-h_3)\Gamma(1-h_1)\Gamma(h_1)\Gamma(1-h_2)\Gamma(h_2)\Gamma^2(1-h_3)}{\Gamma(1+h_1-h_3)\Gamma(2-h_1-h_3)}$$

$$\times {}_4F_3\left(\begin{matrix} h_2,\ 1-h_2,\ 1-h_3,\ 1-h_3 \\ 1,\ 2-h_1-h_3,\ 1+h_1-h_3 \end{matrix}\Big|1\right);$$

$$J_3(h_1,h_2,h_3) = J_2(h_2,h_1,h_3);$$

$$\bar{J}_1(\bar{h}_1,\bar{h}_2,\bar{h}_3) = (-1)^{n_1+n_2}\frac{\Gamma(-\bar{h}_1+\bar{h}_2+\bar{h}_3)\Gamma(1-\bar{h}_2)\Gamma(1-\bar{h}_1)\Gamma(\bar{h}_1)}{\Gamma(-\bar{h}_1+\bar{h}_3+1)}$$

$$\times \int_0^1 dx\ x^{-\bar{h}_1}(1-x)^{\bar{h}_2-1}{}_2F_1(\bar{h}_2,-\bar{h}_1+\bar{h}_2+\bar{h}_3;-\bar{h}_1+\bar{h}_3+1;x)$$

$$\times {}_2F_1(1-\bar{h}_3,1-\bar{h}_1;1;x);$$

$$\bar{J}_2(\bar{h}_1,\bar{h}_2,\bar{h}_3) = (-1)^{n_1}\frac{\Gamma(1-\bar{h}_1)\Gamma(\bar{h}_3)\Gamma(-\bar{h}_1+\bar{h}_2+\bar{h}_3)\Gamma(1-\bar{h}_2)}{\Gamma^2(-\bar{h}_1+\bar{h}_3+1)}$$

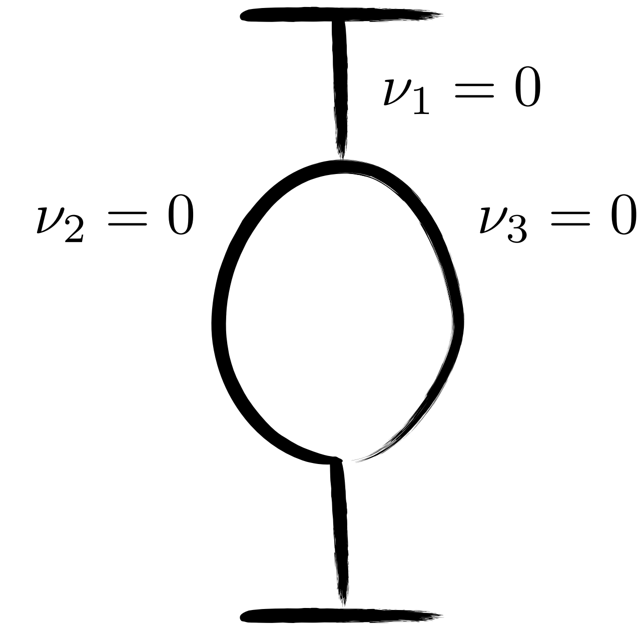
$$\times \int_0^1 dx\ x^{\bar{h}_3-\bar{h}_1}(1-x)^{\bar{h}_2-1}{}_2F_1(1-\bar{h}_1,1-\bar{h}_1;-\bar{h}_1+\bar{h}_3+1;x)$$

$$\times {}_2F_1(\bar{h}_2,-\bar{h}_1+\bar{h}_2+\bar{h}_3;-\bar{h}_1+\bar{h}_3+1;x);$$

$$\bar{J}_3(\bar{h}_1,\bar{h}_2,\bar{h}_3) = \bar{J}_3(\bar{h}_2,\bar{h}_1,\bar{h}_3).$$

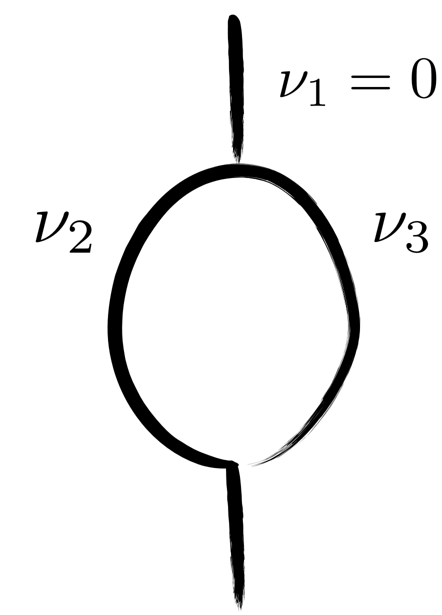
Previous attempts

J. Bartels, M. Ryskin, G.P.Vacca, Eur. Phys. J. C 27, 101 (2003)



- The pomeron loop was calculated with an <u>approximate</u> form of the triple-pomeron vertex (was taken with fixed conformal parameters)
- The single-loop contribution to <u>scattering amplitude</u> was found
- It was found that the loop gives no significant contribution up to extraordinary high energies (rapidities of the order of 40)

M.A. Braun, Eur. Phys. J. C 63, 287 (2009)

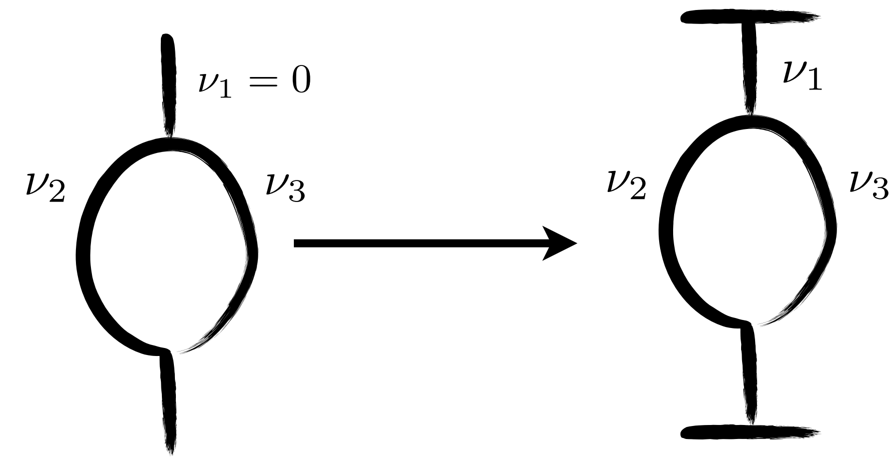


- The pomeron loop was calculated with an <u>exact</u> form of the triple-pomeron vertex (dependence on internal parameters was considered)
- The single loop contribution to the BFKL pomeron <u>propagator</u> was considered (an external parameter was fixed)
- The loop begins to dominate already at rapidities of the order of 10−15

Previous attempts

M.A. Braun, Eur. Phys. J. C 63, 287 (2009)

- The pomeron loop was calculated with an exact form of the triple-pomeron vertex
- The single loop contribution to the BFKL pomeron propagator was considered
- The loop begins to dominate already at rapidities of the order of 10−15



1. This result did not solve the real physical problem, the contribution of the loop to the scattering amplitude, which is obtained after integration with external particles
2. The full dependence on conformal parameters should be used
3. This can change estimation of the loop magnitude

Scattering amplitude with bare pomeron exchange

The amplitude for scattering of two hadrons in terms of complex angular momentum:

$$A(s,t) = is \int_{\sigma - i\infty}^{\sigma + i\infty} \frac{d\omega}{2\pi i} s^{\omega} f_{\omega}(q^2)$$

The amplitude in the lowest order is a convolution of the bare pomeron propagator with two impact factors:

$$f_{\omega}(q^2) = \int d^2 r d^2 r' \;\; \Phi_1(r,q) g_{\omega}^{q}(r,r') \Phi_2^{*}(r',q)$$

It is sufficient to consider forward scattering amplitude. The conformal representation of the propagator at $q^2 \to 0$ was found in L.N. Lipatov, Zh. Eksp. Teor. Fiz. 90 1536 (1986)

$$g_{\omega}^{0}(r,r') = \frac{1}{\pi^2} |rr'| \sum_{n} \int_{0}^{\infty} d\nu \left| \frac{r}{r'} \right|^{2i\nu} \left(\frac{r^* r'}{r r'^*} \right)^{n/2} g_{\omega,h}$$

We consider the leading contribution at $n = 0$

$$g_{\omega}^{0}(r,r') = \frac{1}{\pi^2} \int_{0}^{\infty} d\nu \;\; |r|^{1+2i\nu} |r'|^{1-2i\nu} g_{\omega,\nu}$$

Impact factors

The forward scattering amplitude is

$$f_\omega(0) = \frac{1}{\pi^2}\int_0^\infty d\nu\ g_{\omega,\nu}\int d^2r\Phi(r,0)|r|^{1+2i\nu}\int d^2r'\Phi^*(r',0)|r'|^{1-2i\nu}$$

It is natural to choose the impact factor in a gaussian form:

$$\Phi(r,0) = \frac{\lambda b}{\pi}e^{-br^2}$$

As a result integration with impact factors can be trivially done:

$$\int d^2r\Phi(r,0)|r|^{1+2i\nu} = \frac{\lambda b}{\pi}\pi b^{-3/2-i\nu}\Gamma(3/2+i\nu)$$

We find:

$$f_\omega(0) = \frac{1}{\pi^2}\int_0^\infty d\nu\ g_{\omega,\nu}\times\frac{\lambda^2}{b}\Big(\nu^2+\frac{1}{4}\Big)\frac{\pi}{\cosh(\pi\nu)}$$

Loop contribution

To take into account the loop contribution one has to substitute the bare propagator by the full Green function. With the single loop insertion:

$$G_{\omega,\nu} = \frac{1}{1/g_{\omega,\nu} + l_{0\nu}^2 \Sigma_{\omega,\nu}} = \frac{1}{l_{0\nu}} \frac{1}{\omega - \omega_\nu} - \frac{\Sigma_{\omega,\nu}}{(\omega - \omega_\nu)^2}$$

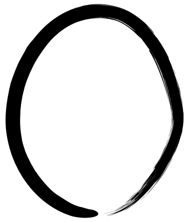
The first term in comes from exchange of the bare pomeron. It is not difficult to show that

$$A_y^{(1)}(0) = \frac{1}{16\pi^2} \frac{\lambda^2}{b} \int_0^\infty d\nu \; \frac{1}{\left(\nu^2 + \frac{1}{4}\right)} \frac{\pi}{\cosh(\pi\nu)} e^{\omega_\nu y}$$

The second term corresponds to the lowest order loop contribution.

The explicit form of the pomeron self-mass was found in
M.A. Braun, Eur. Phys. J. C 63, 287 (2009)

$$\Sigma_{\omega,\nu} = \frac{\alpha_s^4 N_c^2}{8\pi^{10}} \int_0^\infty d\nu_1 d\nu_2 \frac{\nu_1^2}{\left(\nu_1^2 + \frac{1}{4}\right)^2} \frac{\nu_2^2}{\left(\nu_2^2 + \frac{1}{4}\right)^2} \frac{\Omega^2(1/2 + i\nu, 1/2 + i\nu_1, 1/2 + i\nu_2)}{\omega - \omega(0,\nu_1) - \omega(0,\nu_2)}$$



Scattering amplitudes

The total forward scattering amplitude with the lowest order loop correction is a sum

$$A_y(0) = A_y^{(1)}(0) + A_y^{(2)}(0)$$

Bare pomeron exchange:

$$A_y^{(1)}(0) = \frac{1}{16\pi^2}\frac{\lambda^2}{b}\int_0^\infty d\nu\ \frac{1}{\left(\nu^2+\frac{1}{4}\right)}\frac{\pi}{\cosh(\pi\nu)}e^{\omega_\nu y}$$

Single-loop contribution:

$$A_y^{(2)}(0) = -\frac{1}{16\pi^2}\frac{\lambda^2}{b}\int_0^\infty d\nu\ 16\left(\nu^2+\frac{1}{4}\right)\frac{\pi}{\cosh(\pi\nu)}$$

$$\times\left(\frac{e^{\omega_\nu y}y}{\omega_\nu-\omega_{\nu_1}-\omega_{\nu_2}} - \frac{e^{\omega_{\nu_1}y}}{\left(\omega_\nu-\omega_{\nu_1}-\omega_{\nu_2}\right)^2} + \frac{e^{(\omega_{\nu_1}+\omega_{\nu_2})y}}{\left(\omega_\nu-\omega_{\nu_1}-\omega_{\nu_2}\right)^2}\right)$$

$$\times\frac{\alpha_s^4 N_c^2}{8\pi^{10}}\int_0^\infty d\nu_1 d\nu_2 \frac{\nu_1^2}{\left(\nu_1^2+\frac{1}{4}\right)^2}\frac{\nu_2^2}{\left(\nu_2^2+\frac{1}{4}\right)^2}\Omega^2(1/2+i\nu, 1/2+i\nu_1, 1/2+i\nu_2)$$

Details of numerical studies

We have set up a program which calculates the bare pomeron exchange amplitude and single-loop contribution

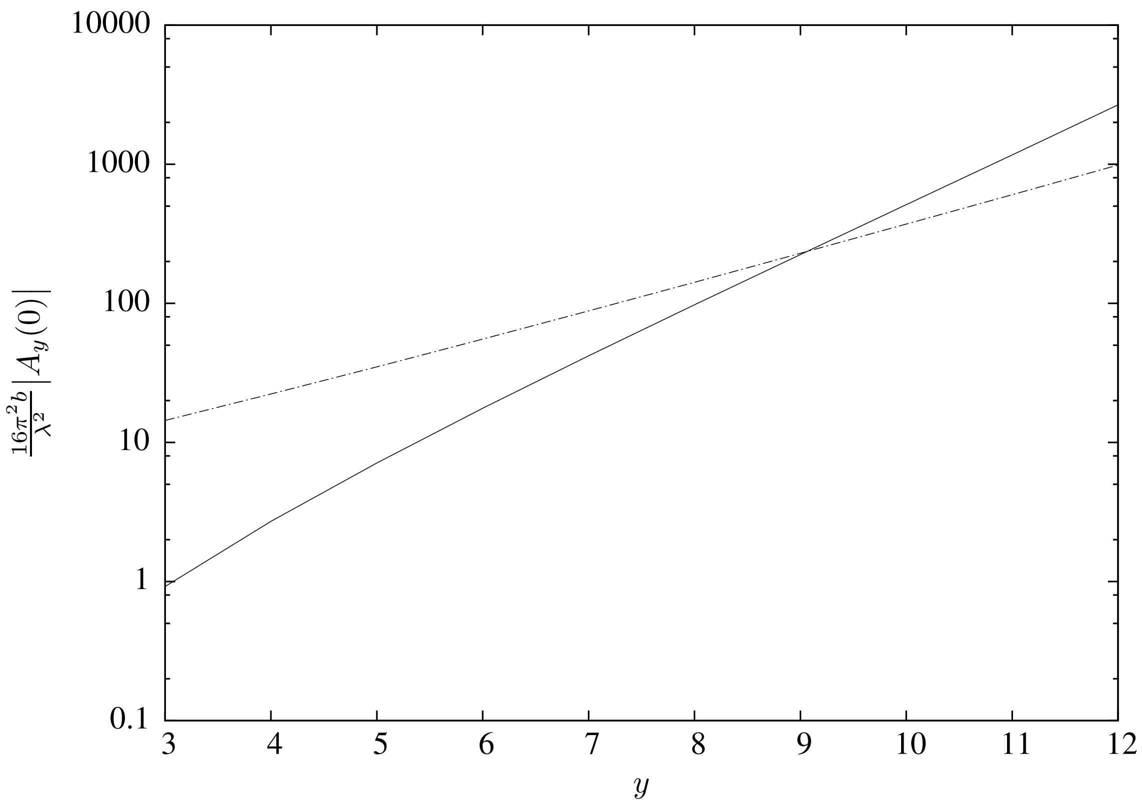
- The most difficult part is the computation of the triple-pomeron vertex Ω
- We have restricted conformal variables to lie in the interval $0 < \nu < 3.0$ and introduced a grid dividing this interval into N=20 points and found the vertex on this grid
- The value of the vertex in between the grid points was found by interpolation

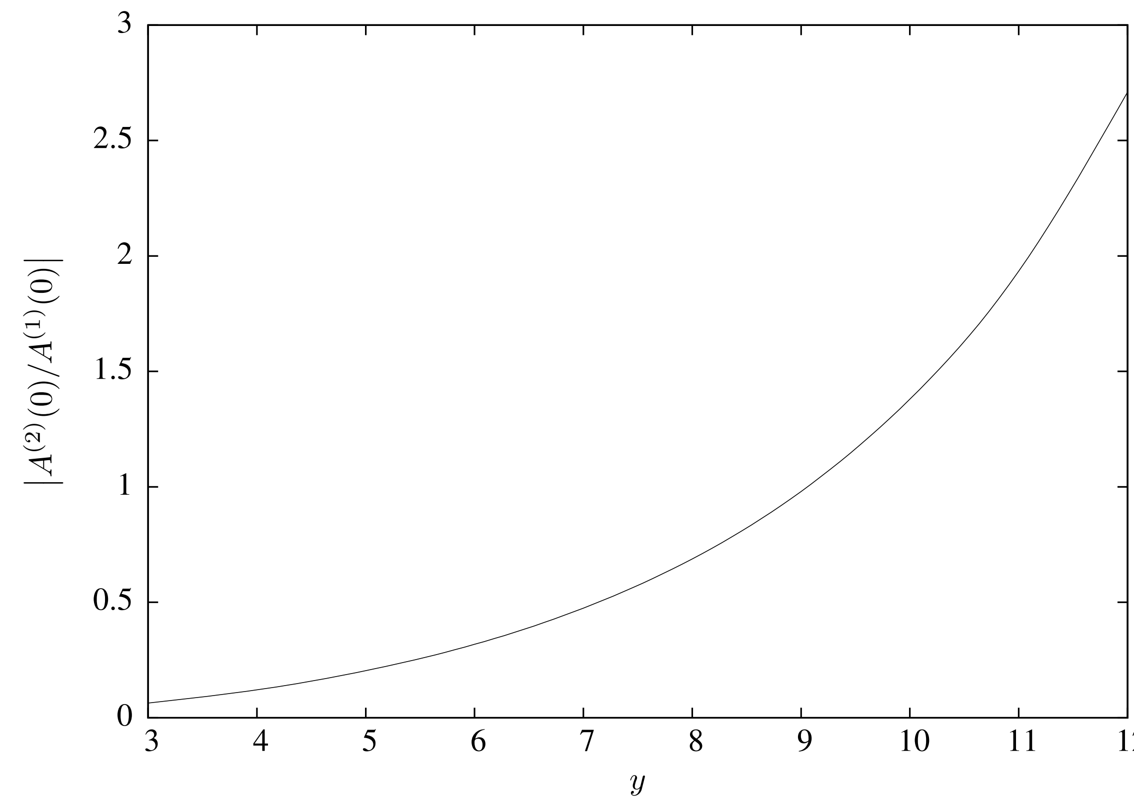
The scattering amplitudes were calculated by the Newton-Cotes integration formulas

- The limits were taken as $0 < \nu < 3.0$
- The number of sample points was chosen to provide relative error 10^{-3}

We have performed calculations for the standard value of the QCD coupling constant $\alpha_s = 0.2$ and $N_c = 3$

Numerical results





- The behavior of the bare amplitude is determined by the initial pole of the conformal BFKL propagator
- The curve grows with rapidity roughly as $e^{\Delta y}$, where $\Delta \approx 0.48$
- The single-loop contribution with very good accuracy grows twice faster as $\sim e^{2\Delta y}$
- For small rapidities the loop term is suppressed by the smallness of the QCD coupling constant
- However, its faster growth with rapidity compensates this very early
- The loop contribution becomes visible already at rapidities 3-8 and starts to dominate at 8-10

Conclusions

- We have studied a single loop contribution to the scattering amplitude of two colliding hadrons
- We have found expression for the amplitude in a framework of conformal invariant technique with more or less general form of the impact factors
- The triple-pomeron vertex with full dependence on the intermediate conformal weights was calculated and used
- Numerical analysis shows that smallness of the QCD coupling constant is compensated by rapid growth of the single-loop amplitude with rapidity
- We found that loop contribution manifests itself at relatively small rapidities 3-8 and dominates the bare pomeron exchange amplitude already at 8-10